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Research Article

Synthesis of Multicriteria Controller by Means of Fuzzy Logic Approach

Andrew Lozynskyy and Lyubomyr Demkiv

Lviv Polytechnic National University, 12 Bandery Street, Lviv 79026, Ukraine

Correspondence should be addressed to Lyubomyr Demkiv; demkivl@gmail.com

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A two-mass fuzzy control system is considered. For fuzzification process, classical both linear and nonlinear membership functions are used. To find optimal values of membership function's parameters, genetic algorithm is used. To take into account values of both output and intermediate parameters of the system, a penalty function is considered. Research is conducted for the case of speed control system and displacement control system. Obtained results are compared with the case of the system with classical, crisp controller.

1. Introduction

In the optimal controls synthesis, various approaches are used. Among them are: the method of analytical design of controllers [1], the Pontryagin maximum principle, and Bellman dynamic programming [2, 3], and also root finding methods. The disadvantages of these approaches are that they do not take into account changing conditions of the system, changes of the subject, and so forth.

Nonlinear control theory, including feedback linearization [4], is not widely used due to complexity of determining the aggregate variables in technical systems. As well as method of geometric control theory [5] are rather not widespread.

Application of fuzzy logic at synthesis of optimal control is frequently used. In particular, in [6], for each of the subsystems LQR, optimal control is synthesized. Switching between subsystems is done by means of the fuzzy sets theory. In the paper [7], fuzzy control in conjunction with genetic algorithms is used for the synthesis of optimal control for continuous stirred tank reactor. In [8], an optimal control algorithm for T-S fuzzy descriptor systems with time domain hard constraints including control input constraints and state constraints was proposed.

Many studies are devoted to selecting the type of membership function (e.g., [9, 10]), that is, mapping between the set of operation error admissible values and the interval $[0, 1]$. However, in industrial problems standard types of membership functions, trapezoid, triangular sigmoid, and so forth (see, e.g., [11]), are often used. When one uses these functions there is a problem of choosing the parameters of membership function which would provide the desired transients in the system.

One approach to solving this problem is the synthesis of control actions that ensure a minimum level of quality desired integral performance index. By means of genetic algorithms, it is possible to determine the unknown parameters of the membership functions.

2. Materials and Methods

2.1. Problem Statement. Research was conducted for the case of two-mass dynamic system. In Subsection 3.1 for a study of the speed control system, (1) was done, and, in Subsection 3.2 similar studies were conducted for the case of positional two-mass system (1)-(2). Consider

$$T_{M_1} p\omega_1(p) = M_1 - M_{12}(p),$$
$$T_C p M_{12}(p) = \omega_1(p) - \omega_2(p), \quad (1)$$
$$T_{M_2} p\omega_2(p) = M_{12}(p),$$
$$p\varphi(p) = \omega_2(p). \quad (2)$$

Variables that are described in (1)-(2) are normalized physical quantities whose values depend on the type of two-mass system: with elastic deformations twisting, stretching, or bending. In particular, in the case of two-mass system with twisting elastic deformations, T_{M1}, T_{M2} are mechanical time constants of the motor and load machine, respectively, T_C is the stiffness time constant, $\omega_1^*(p)$ and $\omega_2^*(p)$ are motor and load speeds, M_{12}^* is shaft torque, M_1^* is electromagnetic torque, and $\varphi(p)$ is the angle of shaft rotation. When one applies the fuzzy logic at controller synthesis, one of the most problematic places is choice of the terms number for each fuzzy variable.

In the work [12], fuzzification of only one variable is proposed. This significantly simplifies the rule base. And, in the work [13], number of terms was limited to two. It is shown that this does not significantly affect the output signal of the controller. In this case, the base of the rules will have the following form:

$$\begin{aligned} &\text{IF } (e \text{ in } B) \text{ THEN } u(t) = f_B(\overline{x}), \\ &\text{IF } (e \text{ in } S) \text{ THEN } u(t) = f_S(\overline{x}), \end{aligned} \tag{3}$$

where $e(t)$ is the error between the reference value and output value, $f_B(\overline{x})$ and $f_S(\overline{x})$ are corresponding functions of $\overline{x}$—the state vector of the system. Center of gravity defuzzification was used here.

At synthesis of these regulators, we arrive to a structure that can be represented as

$$u(t) = \sum_i \lambda_i(\overline{x}) u_i, \tag{4}$$

where $u_i = u(\overline{x}(t), \overline{k})$ is vector function and $\overline{k}$ is vector of coefficients.

Taking into account that technical systems which can operate at different points in state space, which are characterized by different constraints and different requirements to quality control, are imposed, traditionally one uses a compromise setting and shape control action based on the following criterion:

$$J = \int_0^\infty \left(\sum_i \lambda_i F_i^*(\overline{x}(t)) + u^2(t) \right) dt, \tag{5}$$

where λ_i are constant coefficients, determined on the basis of peer review, and $F_i^*(\overline{x}(t))$ are arbitrary quality functionals which provide desired system behavior.

Optimal control $u(t) = u^*(t)$ synthesized for each subsystem provides minimization of a single functional F_i. This functional is formed for the ith point of the state space. In classical control theory, at finding the optimal control for the entire system, the generalized functionality would have the following form:

$$I = \sum_i \lambda_i F_i, \tag{6}$$

where i is index of individual subsystem's model and $\lambda_i =$ const is based on expert assessments or theory of Pareto optimal solutions.

At the application of fuzzy sets theory, we do not form some sort of trajectory that is optimal for all subsystems of the family but implement an optimal transition from one trajectory to another, specified for a particular subsystem. This approach makes it possible to improve the quality characteristics of the system.

That is, the criterion will be formed as follows:

$$\begin{gathered} J = \int_0^\infty \left(\sum_i \lambda_i(\overline{x}(t)) F_i^*(\overline{x}(t)) + u^2(t) \right) dt, \\ \sum_i \lambda_i(\overline{x}(t)) = 1. \end{gathered} \tag{7}$$

Each subsystem can generate different types of transitions at different speeds. It is possible to form different transition paths to a given point in output signal's space by switching between corresponding control actions.

Obviously, at the beginning of system operation (when the error is large), controller which ensures faster transients should be applied to the system, when approaching the area of the set-point switching to a subsystem that provides a smooth behavior of the system must be done.

As an example, a study for the functional described in [14] was conducted. Based on the results obtained there, it is easy to show that the optimal control has the following forms:

$$\begin{aligned} u_1^{\text{opt}}(t) = {} & (\omega_0 + 1) T_{M_1} \ddot{x}(t) \\ & + \left(T_C T_{M_1} T_{M_2} \omega_0^3 - (\omega_0 + 1) T_{M_1} \right) x(t) \\ & + \left(\left(\omega + \omega^2 - \frac{1}{T_C T_{M_2}} \right) T_C T_{M_1} - 1 \right) \dot{x}(t) \end{aligned} \tag{8}$$

in the case of standard linear Butterworth form and

$$\begin{aligned} u_2^{\text{opt}}(t) = {} & 3\omega_0 T_{M_1} \ddot{x}(t) \\ & + \left(\left(3\omega_0^2 - \frac{1}{T_C T_{M_2}} \right) T_C T_{M_1} - 1 \right) \dot{x}(t) \\ & + \left(T_C T_{M_2} T_{M_1} \omega_0^3 - 3 T_{M_1} \omega_0 \right) x(t) \end{aligned} \tag{9}$$

in the case of standard linear binomial form. Here, ω_0 is frequency of the system.

To investigate the influence of the values of the membership functions parameters on the behavior of the system, classic performance indexes values (see, e.g., [15]) were determined:

$$I_1 = \gamma_1 \int_0^T t\,|e(t)|\,dt, \qquad I_2 = \gamma_2 \int_0^T |e(t)|\,dt. \tag{10}$$

At synthesis of the controller in real electrical facilities one should comply with the physical feasibility of the processes occurring in them. To take into account constraints

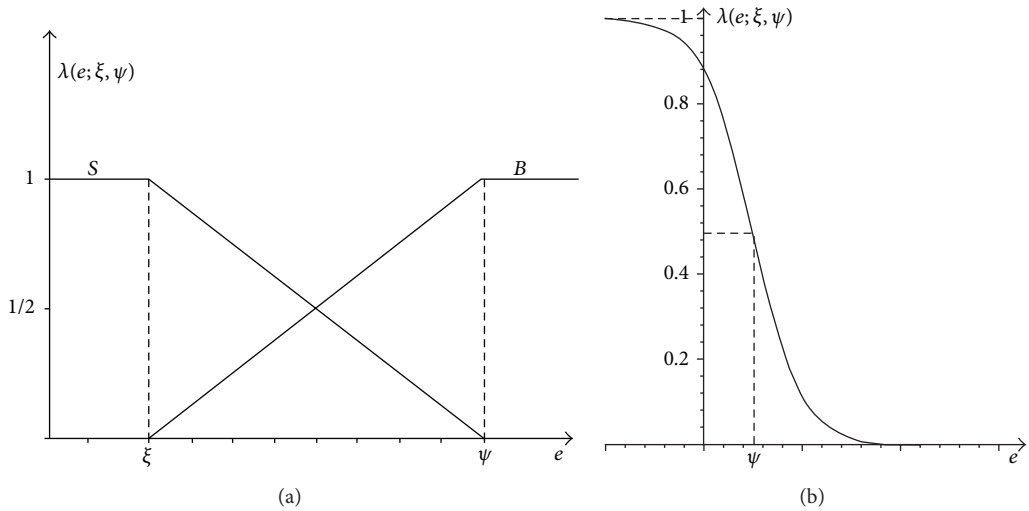

Figure 1: Membership functions. (a) S and Z type (b) sigmoid.

on intermediate coordinates value of the following penalty function is calculated

$$F_{\text{penalty}} = \gamma_3 \left(\frac{x_1(t)}{x_{1,\max}}\right)^2 H\left(\frac{x_1(t)}{x_{1,\max}}\right) + \gamma_4 \left(\frac{x_3(t)}{x_{3,\max}}\right)^2 H\left(\frac{x_3(t)}{x_{3,\max}}\right) + \gamma_5 \left(x^t_{1,\max} - x^t_{1,\min}\right)^2, \tag{11}$$

where $H(\cdot)$ is Heaviside function, $x_{1,\max}$, $x_{3,\max}$ are defined maximum allowable overshoots and, in this case, they are 10% and 5%, respectively, and $x^t_{1,\max}$ $x^t_{1,\min}$ are maximum and minimum values of $x_1(t)$ after first achieving of the set-point and coefficients γ_i, $i = \overline{1\ldots5}$ are chosen for reasons of commensurability of studied variables among each other.

Hence, fitness function has the following form:

$$I^*_i = I_i + F_{\text{penalty}}, \quad i = \overline{1\ldots2}, \tag{12}$$

where coefficients γ_i, $i = \overline{1,\ldots,5}$ are chosen for reasons studied variables commensurability among each other.

The case of a compromise setting of the system which consists of two subsystems ($\lambda_1 = \lambda_2 = 0.5$) had also been studied. In this case, the optimal control has the form

$$\begin{aligned} u^{\text{opt}}(t) &= \left(1 + \sqrt{1+3\lambda_1}\right)\omega_0 T_{M_1}\ddot{x}(t) \\ &+ \left(\left(\left(1+\sqrt{1+3\lambda_1}\right)\omega_0^2 - \frac{1}{T_C T_{M_2}}\right)T_C T_{M_1} - 1\right) \\ &\times \dot{x}(t) \\ &+ \left(T_C T_{M_1} T_{M_2}\omega_0^3 - \left(1+\sqrt{1+3\lambda_1}\right)\omega_0 T_{M_1}\right)x(t). \end{aligned} \tag{13}$$

Note that, with this approach, the values of the coefficients do not depend on the state of the system at any current moment of time.

When using fuzzy logic, both linear and nonlinear membership functions have been applied. In this case, the system transient will depend on the parameters of these functions ξ and ψ.

2.2. Membership Functions. There are a number of membership functions. However, in the case of two terms, it is possible and appropriate to use limited number of such functions. Conducted studies suggest the possibility of applying the proposed approach to both linear and nonlinear membership functions.

2.2.1. Linear Membership Function. In the case of a linear membership function, study was carried out on the example of the functions of S and Z types (14), as shown in Figure 1(a):

$$\mu(e;\xi,\psi) = \begin{cases} 1, & x \le \xi \\ \dfrac{(x-\psi)}{(\xi-\psi)}, & \xi < x \le \psi \\ 0, & x > \psi. \end{cases} \tag{14}$$

In the case of such function, parameters ξ and ψ define the degree of each subsystem's influence on the overall trajectory of the system and on smooth switching between subsystems.

2.2.2. Nonlinear Membership Function. We believe that, among nonlinear functions, the most common is sigmoid function (15). Consider

$$\mu(e;\xi,\psi) = \frac{e^{-\xi(x-\psi)}}{1+e^{-\xi(x-\psi)}}, \tag{15}$$

where parameter ψ specifies the value of the error at which the value of membership function is equal to 0.5 (Figure 1(b)) and the parameter ξ specifies the degree of the membership function's slope at $\mu(e;\xi,\psi) = 0.5$.

Table 1: Comparison of the characteristics of dynamic system (1) with the fuzzy controller and membership function (14).

	I_1^*	I_2^*	F_{penalty}	$t_{5\%}$	t_1	t_{fin}	$\max(x_3(t))$
But	0.1198	0.1571	0.0451	0.78	0.4718	2.185	1.081
Bin	0.09375	0.125	0.0394	0.7518	1.404	1.742	1
Equ	0.06621	0.1093	0.0246	0.5698	0.7215	1.654	1.008
Fuz	0.05904	0.1026	0.0232	0.4805	0.5805	1.766	1.019

Table 2: Comparison of the characteristics of dynamic system (1) with the fuzzy controller and membership function (15).

	I_1^*	I_2^*	F_{penalty}	$t_{5\%}$	t_1	t_{fin}	$\max(x_3(t))$
But	0.1198	0.1571	0.0451	0.78	0.4718	2.185	1.081
Bin	0.09375	0.125	0.0394	0.7518	1.404	1.742	1
Equ	0.06621	0.1093	0.0246	0.5698	0.7215	1.654	1.008
Fuz	0.0561	0.09807	0.0215	0.4748	0.5308	1.484	1.038

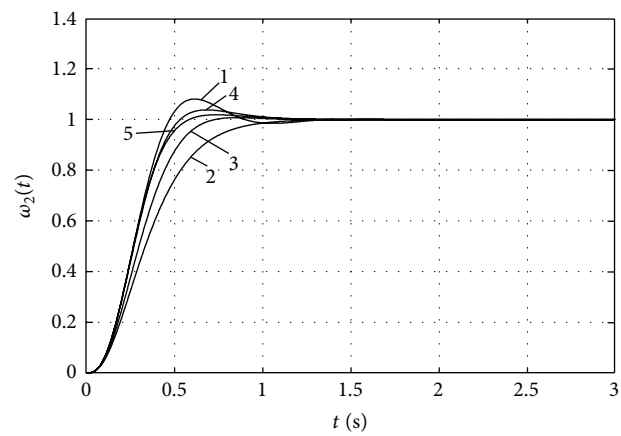

Figure 2: Output signal of the system (1) with the following controllers: 1, (8); 2, (9); 3, (13); 4, fuzzy with (14); 5, fuzzy with (15).

3. Results and Discussion

Research was conducted for the case of the systems (1) and (1)-(2). For solution of the optimization problems genetic algorithm was used (see [16]).

For the comparison of the obtained in this paper results with a classic case a system with crisp controller which is tuned only for one linear standard form (8)-(9) (see [14, 17]) was also considered. Note that all cases are studied with the same value of ω_0.

Hereinafter, $\max(x_1(t))$ is maximum overshoot of $x_1(t)$; $\max(x_3(t))$ is maximum overshoot of $x_3(t)$; $t_{5\%}$ is the time of entering the 5% zone; t_1 is the time of first set-point achievement; t_{fin} is the time of transition to steady state operation.

3.1. The Case of System with Speed Control. Research is carried out for the case of linear (8) and nonlinear (9) membership functions. The main results of the research, for the third order system, are given in Tables 1 and 2.

In the given tables the following notations are used: But corresponds to the system with controller (8), Bin corresponds to the system with controller (9), Equ corresponds to the system with controller (13), and Fuz corresponds to the system with fuzzy controller.

The corresponding transients are shown in Figures 2 and 3.

3.2. The Case of System with Displacement Control. Similar studies were conducted for the case of the fourth order system (1)-(2). The results of calculations are presented in Tables 3 and 4.

It should be noted that the proposed approach can be generalized to the case of arbitrary linear or nonlinear functions.

The corresponding transients are shown in Figures 4 and 5.

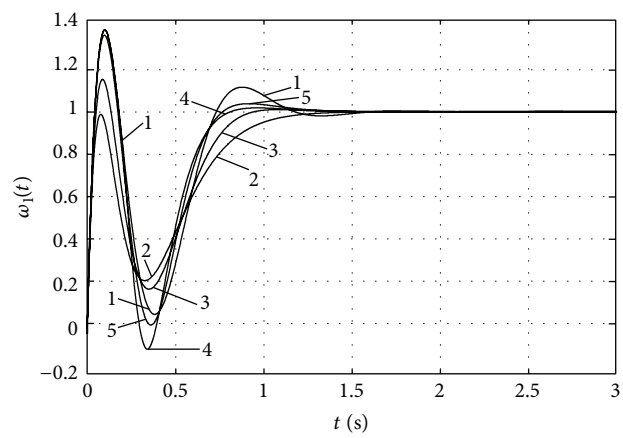

Figure 3: Time dependence of $\omega_1(t)$ in the case of the system (1) with the following controllers: 1, (8); 2, (9); 3, (13); 4, fuzzy with (14); 5, fuzzy with (15).

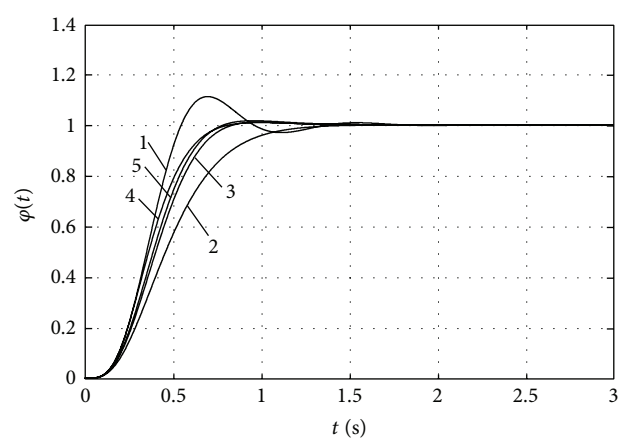

Figure 4: Output signal of the system (1)-(2) with the following controllers: 1, (8); 2, (9); 3, (13); 4, fuzzy with (14); 5, fuzzy with (15).

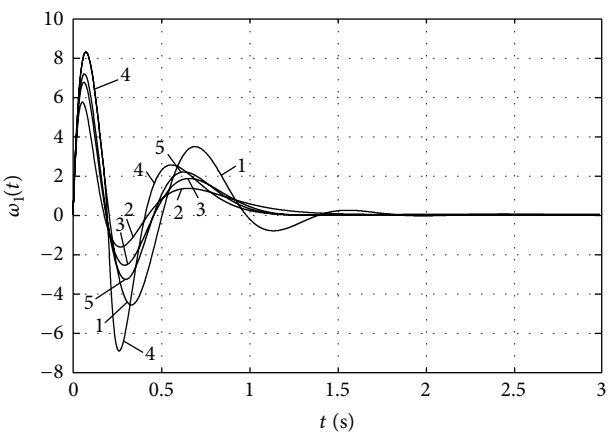

Figure 5: Time dependence of $\omega_1(t)$ in the case of the system (1)-(2) with the following controllers: 1, (8); 2, (9); 3, (13); 4, fuzzy with (14); 5, fuzzy with (15).

Table 3: Comparison of the characteristics of dynamic system (1)-(2) with the fuzzy controller and membership function (14).

	I_1^*	I_2^*	F_{penalty}	$t_{5\%}$	t_1	t_{fin}	$\max(x_3(t))$
But	0.2672	0.2949	0.0114	0.8808	0.547	2.853	1.111
Bin	0.183	0.1934	0.0712	0.9305	1.633	1.99	1
Equ	0.1469	0.1862	0.0557	0.696	0.84	1.826	1.011
Fuz	0.1455	0.1813	0.0549	0.6458	0.815	1.96	1.009

Table 4: Comparison of the characteristics of dynamic system (1)-(2) with the fuzzy controller and membership function (15).

	I_1^*	I_2^*	F_{penalty}	$t_{5\%}$	t_1	t_{fin}	$\max(x_3(t))$
But	0.2672	0.2949	0.0114	0.8808	0.547	2.853	1.111
Bin	0.183	0.1934	0.0712	0.9305	1.633	1.99	1
Equ	0.1469	0.1862	0.0557	0.696	0.84	1.826	1.011
Fuz	0.1445	0.1802	0.0547	0.7348	0.547	1.906	1.016

4. Conclusions

The synthesis system based on functional with variable parameters is proposed in this paper. These parameters are implemented by using fuzzy control and on the basis of genetic algorithms.

This approach can be used as a variant of the optimal control synthesis for both linear and nonlinear systems, which are described as the set of dynamic subsystems. This approach provides a gain compared with systems synthesized based on Pareto optimal solutions.

The proposed approach can be applied to any system, any integral quality indexes, and any membership functions without considerable modifications.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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