Research Article

Two-Stage Stratified Randomized Response Model with Fuzzy Numbers

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We consider an allocation problem in two-stage stratified Warner’s randomized response model and minimize the variance subject to cost constraint. The costs (measurement costs and total budget of the survey) in the cost constraint are assumed as fuzzy numbers, in particular triangular and trapezoidal fuzzy numbers due to the ease of use. The problem formulated is solved by using Lagrange multipliers technique and the optimum allocation obtained in the form of fuzzy numbers is converted into crisp form using α-cut method at a prescribed value of α. An illustrative numerical example is presented to demonstrate the proposed problem.

1. Introduction

Sample survey is a method of drawing an inference about the characteristic of a population or universe by observing only a part of the population. In modern complex surveys it is not possible to obtain true measurements on all the characteristics of interest on all the units in the sample because they are affected by two types of errors, that is, sampling errors and nonsampling errors. Nonsampling error is further classified into two types, response and nonresponse errors. Reduction in the reliability of measurements results in response error which can be minimized over repeated measurements. Whereas nonresponse errors are due to the nonavailability of information about some selected units for one or the other reason.

A major source of nonresponse errors in sample surveys is the difficulty to obtain true responses when respondents are asked questions of highly personal or controversial nature, for example, questions on accumulated savings, intentional tax evasion, consumption of illegal drugs, and extramarital affairs. To avoid providing the requisite information or to avoid embarrassment, some respondents may refuse to answer or may intentionally give wrong answers. Thus the estimates obtained from a direct survey on such topics would be subject to high bias and any inference drawn from these would be erroneous. In order to solve this problem Warner [1] introduced a randomized response technique (RRT) which was developed subsequently by different authors.

Some other authors who introduce other randomized response techniques are Mangat and Singh [2], Chua and Tsui [3], Padmawar and Vijayan [4], Chang and Huang [5], Chaudhuri [6], and so forth. Kim and Warde [7] suggested a stratified randomized response using optimum allocation. Mangat and Singh [2] proposed a two-stage randomized response model.

Traditional decision making problems are handled either by the deterministic approach or by probabilistic approach. Deterministic approach completely avoiding the uncertainty provides an approximate solution while probabilistic approach on an assumption represents any uncertainty as a probability distribution. Both of these approaches only partially capture reality. Uncertainty also is involved in decision problems due to vagueness or impreciseness associated with linguistic information; then in this case optimization using fuzzy mathematical theories becomes more relevant. The idea of fuzzy decision making problems was proposed by Bellmann and Zadeh [8] and this idea was used in problems of mathematical programming by Tanaka and Asai [9]. Many authors use fuzzy data/fuzzy numbers in decision making problems such as Mahapatra and Roy [10], Pramanik and
Roy [11], Abbasbandy and Hajjari [12], Kaur and Kumar [13], Ebrahimnejad [14], Sen et al. [15], and Gupta and Bari [16].

In this paper, the deterministic problem formulated by Ghufran et al. [17] is extended by considering it into an uncertain environment. Here we consider the measurement cost and total budget for the survey as fuzzy numbers and formulate a fuzzy nonlinear programming problem. Then, the fuzzy nonlinear problem is solved by Lagrange multipliers technique after converting it into crisp problem using α-cut. For demonstrating the proposed problem an illustrative example is presented.

2. Preliminary

Before formulating the problem of interest, we should know the basic definitions of fuzzy sets, fuzzy numbers, and so forth, which are reproduced here, from Bector and Chandra [18], Mahapatra and Roy [10], Hassanzadeh et al. [19], and Aggarwal and Sharma [20] as follows.

Fuzzy Set. A fuzzy set \( \tilde{A} \) in a universe of discourse \( X \) is defined as the following set of pairs \( \tilde{A} = \{(x, \mu(A)(x)) : x \in X \} \). Here \( \mu(A) : X \rightarrow [0, 1] \) is a mapping called the membership function of the fuzzy set \( \tilde{A} \) and \( \mu(A) \) is called the membership value or degree of membership of \( x \in X \) in the fuzzy set \( \tilde{A} \). The larger the value of \( \mu(A) \), the stronger the grade of membership in \( \tilde{A} \).

\( \alpha \)-Cut. The \( \alpha \)-cut for a fuzzy set \( \tilde{A} \) is shown by \( \tilde{A}_\alpha \) and for \( \alpha \in [0, 1] \) is defined to be

\[
\tilde{A}_\alpha = \{ x | \mu(A)(x) \geq \alpha, x \in X \},
\]

where \( X \) is the universal set.

Upper and lower bounds for any \( \alpha \)-cut \( \tilde{A}_\alpha \) are given by \( \tilde{A}_\alpha^U \) and \( \tilde{A}_\alpha^L \), respectively.

Fuzzy Number. A fuzzy set \( A \) in \( \mathbb{R} \) is called a fuzzy number if it satisfies the following conditions:

(i) \( A \) is convex and normal.
(ii) \( A_\alpha \) is a closed interval for every \( \alpha \in (0, 1] \).
(iii) The support of \( A \) is bounded.

Triangular Fuzzy Number (TFN). A fuzzy number \( \tilde{A} = (p, q, r) \) is said to be a triangular fuzzy number if its membership function is given by

\[
\mu(A)(x) =
\begin{cases}
\frac{x - p}{q - p}, & \text{if } p \leq x \leq q, \\
\frac{q - x}{r - q}, & \text{if } q \leq x \leq r, \\
0, & \text{otherwise.}
\end{cases}
\]

Trapezoidal Fuzzy Number (TrFN). A fuzzy set \( \tilde{A} = (p, q, r, s) \) on real numbers \( \mathbb{R} \) is called a trapezoidal fuzzy number with membership function as follows:

\[
\mu(A)(x) =
\begin{cases}
0, & x \leq p, \\
\frac{x - p}{q - p}, & p \leq x \leq q, \\
1, & q \leq x \leq r, \\
\frac{s - x}{s - r}, & r \leq x \leq s, \\
0, & s \leq x.
\end{cases}
\]

3. Statement of the Problem of Two-Stage Randomized Response Model

Consider a stratified population of size \( N \) partitioned into \( L \) disjoint strata of size \( N_h, h = 1, 2, \ldots, L \), and \( N = \sum_{h=1}^{L} N_h \). Let \( W_h = N_h/N \) denote stratum weights, \( n_h \) denotes sample size, and \( n = \sum_{h=1}^{L} n_h \) is the total sample size for the stratum \( h \).

In the first stage an individual respondent in the sample is instructed to use the randomization device \( R_{1h} \) which consists of the following two statements.

(i) “I belong to the sensitive group” and (ii) “I do not belong to the sensitive group” with known probabilities \( M_h \) and \( 1 - M_h \), respectively.

In the second stage the respondents are instructed to use the randomization device \( R_{2h} \) which consists of the following two statements.

(i) “I belong to the sensitive group” and (ii) “I do not belong to the sensitive group” with known probabilities \( P_h \) and \( 1 - P_h \), respectively.

Assuming that the “Yes” or “No” reports are made truthfully for different outcomes and \( M_h \) and \( P_h \) are set by the interviewer, then the probability of a “Yes” answer in stratum \( h \) is given by

\[
Y_h = M_h \pi_{sh} + (1 - M_h) \left[ P_h \pi_{sh} + (1 - P_h) (1 - \pi_{sh}) \right];
\]

and \( \pi_{sh} \) is the proportion of respondents belonging to the sensitive group from stratum \( h \). The maximum likelihood estimate of \( \pi_{sh} \) is

\[
\tilde{\pi}_{sh} = \frac{\bar{Y}_h - (1 - M_h) (1 - P_h)}{2P_h - 1 + 2M_h (1 - P_h)}; \quad h = 1, 2, \ldots, L,
\]

where \( \bar{Y}_h \) is the estimated proportion of “Yes” answers which follows a binomial distribution \( B(n_h, Y_{sh}) \). It can be seen that the estimator \( \tilde{\pi}_{sh} \) is unbiased for \( \pi_{sh} \) with variance

\[
V(\tilde{\pi}_{sh}) = \frac{\pi_{sh} (1 - \pi_{sh})}{n_h} + \frac{(1 - M_h) (1 - P_h) \left[ 1 - (1 - M_h) (1 - P_h) \right]}{n_h \left[ 2P_h - 1 + 2M_h (1 - P_h) \right]^2}.
\]
If the suffix “\(h\)” is removed then the expressions 1, 2, and 3 will be reduced in Mangat and Singh’s expressions.

Since \(n_h\) are drawn independently from each stratum, the estimators for individual strata can be added to obtain the estimator for the whole population. Thus an unbiased estimate of \(\pi_{sh}\) is given by

\[
\hat{\pi}_s = \sum_{h=1}^{L} W_h \hat{\pi}_{sh},
\]

Using (5)

\[
\hat{\pi}_s = \sum_{h=1}^{L} W_h \left\{ \frac{\bar{Y}_h - (1 - M_h) (1 - P_h)}{2P_h - 1 + 2M_h (1 - P_h)} \right\}
\]

with a sampling variance

\[
V(\hat{\pi}_s) = \sum_{h=1}^{L} W_h^2 V(\hat{\pi}_{sh})
\]

or

\[
V(\hat{\pi}_s) = \sum_{h=1}^{L} W_h^2 \pi_{sh} (1 - \pi_{sh})
\]

To find the optimum allocation we either maximize the precision for fixed budget or minimize the cost for fixed precision.

A linear cost function which is an adequate approximation of the actual cost incurred will be

\[
C = c_0 + \sum_{h=1}^{L} c_h n_h,
\]

where \(c_h\) is per unit cost of measurement in the \(h\)th stratum and \(c_0\) is overhead cost.

In view of (4) to (11) the problem of finding optimum allocation is formulated as nonlinear programming problem (NLPP) as follows:

Minimize \(V(\hat{\pi}_s)\),

Subject to \(\sum_{h=1}^{L} c_h n_h \leq C_0\), \(1 \leq n_h \leq N_h; h = 1, 2, \ldots, L\).

4. Fuzzy Formulation of Two-Stage Randomized Response Problem

Generally, real-world situations involve a lot of parameters such as cost and time, whose values are assigned by the decision makers and in the conventional approach, they are required to fix an exact value to the aforementioned parameters. However decision-makers frequently do not precisely know the value of those parameters. Therefore, in such cases it is better to consider those parameters or coefficients in the decision-making problems as fuzzy numbers. The mathematical modeling of fuzzy concepts was presented by Zadeh in [21]. Therefore, the fuzzy formulation of problem (12) with fuzzy cost constraint is given by considering two cases of fuzzy numbers, that is, triangular fuzzy number (TFN) and trapezoidal fuzzy number (TrFN).

4.1. Case 1: Nonlinear Problem with TFN. Consider

Minimize \(\sum_{h=1}^{L} \frac{W_h^2}{n_h} [\pi_{sh} (1 - \pi_{sh}) + A_h]\),

Subject to \(\sum_{h=1}^{L} (c_h^{(1)}, c_h^{(2)}, c_h^{(3)}) n_h \leq (C_0^{(1)}, C_0^{(2)}, C_0^{(3)})\), \(1 \leq n_h \leq N_h, h = 1, 2, \ldots, L\),

where

\[
A_h = \frac{(1 - M_h) (1 - P_h) [1 - (1 - M_h) (1 - P_h)]}{[2P_h - 1 + 2M_h (1 - P_h)]^2}
\]

and \(\bar{c}_h = (c_h^{(1)}, c_h^{(2)}, c_h^{(3)})\) is triangular fuzzy numbers with membership function

\[
\mu_{\bar{c}_h}(x) = \begin{cases} 
\frac{x - c_h^{(1)}}{c_h^{(2)} - c_h^{(1)}}, & \text{if } c_h^{(1)} \leq x \leq c_h^{(2)}, \\
\frac{c_h^{(3)} - x}{c_h^{(3)} - c_h^{(2)}}, & \text{if } c_h^{(2)} \leq x \leq c_h^{(3)}, \\
0, & \text{otherwise.}
\end{cases}
\]

Similarly, the membership function for available budget can be expressed as

\[
\mu_{\bar{C}_0}(x) = \begin{cases} 
\frac{x - C_0^{(1)}}{C_0^{(2)} - C_0^{(1)}}, & \text{if } C_0^{(1)} \leq x \leq C_0^{(2)}, \\
\frac{C_0^{(3)} - x}{C_0^{(3)} - C_0^{(2)}}, & \text{if } C_0^{(2)} \leq x \leq C_0^{(3)}, \\
0, & \text{otherwise.}
\end{cases}
\]

4.2. Case 2: Nonlinear Problem with TrFN. Consider

Minimize \(\sum_{h=1}^{L} \frac{W_h^2}{n_h} [\pi_{sh} (1 - \pi_{sh}) + A_h]\),

Subject to \(\sum_{h=1}^{L} (c_h^{(1)}, c_h^{(2)}, c_h^{(3)}, c_h^{(4)}) n_h \leq (C_0^{(1)}, C_0^{(2)}, C_0^{(3)}, C_0^{(4)})\), \(1 \leq n_h \leq N_h, h = 1, 2, \ldots, L\).
\[ \begin{align*}
\phi(n_h, \lambda) &= \sum_{h=1}^{L} \frac{W_h^2}{n_h} \{\pi_{sh} (1 - \pi_{sh}) + A_h\} \\
&+ \lambda \left\{ \sum_{h=1}^{L} (\tilde{c}_{h}^{(1)}, \tilde{c}_{h}^{(2)}, \tilde{c}_{h}^{(3)}, \tilde{c}_{h}^{(4)}) n_h - (C_0^{(1)}, C_0^{(2)}, C_0^{(3)}) \right\}.
\end{align*} \]

Differentiating (21) with respect to \( n_h \) and \( \lambda \) and equating to zero, we get the following sets of equations:

\[ \frac{\partial \phi}{\partial n_h} = - \frac{W_h^2}{n_h^2} \{\pi_{sh} (1 - \pi_{sh}) + A_h\} + \lambda (\tilde{c}_{h}^{(1)}, \tilde{c}_{h}^{(2)}, \tilde{c}_{h}^{(3)}) = 0 \]  

(22)

or

\[ n_h = \frac{1}{\sqrt{\lambda}} \sqrt{\pi_{sh} (1 - \pi_{sh}) + A_h} \left( \tilde{c}_{h}^{(1)}, \tilde{c}_{h}^{(2)}, \tilde{c}_{h}^{(3)} \right). \]  

(23)

Also,

\[ \frac{\partial \phi}{\partial \lambda} = \sum_{h=1}^{L} \left( \tilde{c}_{h}^{(1)}, \tilde{c}_{h}^{(2)}, \tilde{c}_{h}^{(3)} \right) n_h - \left( C_0^{(1)}, C_0^{(2)}, C_0^{(3)} \right) = 0 \]  

(24)

which gives

\[ \sum_{h=1}^{L} \left( \tilde{c}_{h}^{(1)}, \tilde{c}_{h}^{(2)}, \tilde{c}_{h}^{(3)} \right) W_h \sqrt{\frac{\pi_{sh} (1 - \pi_{sh}) + A_h}{\lambda (\tilde{c}_{h}^{(1)}, \tilde{c}_{h}^{(2)}, \tilde{c}_{h}^{(3)})}} - \left( C_0^{(1)}, C_0^{(2)}, C_0^{(3)} \right) = 0 \]  

(25)

or

\[ \frac{1}{\sqrt{\lambda}} \sum_{h=1}^{L} \left( \tilde{c}_{h}^{(1)}, \tilde{c}_{h}^{(2)}, \tilde{c}_{h}^{(3)} \right) W_h \sqrt{\frac{\pi_{sh} (1 - \pi_{sh}) + A_h}{\lambda (\tilde{c}_{h}^{(1)}, \tilde{c}_{h}^{(2)}, \tilde{c}_{h}^{(3)})}} = \left( C_0^{(1)}, C_0^{(2)}, C_0^{(3)} \right). \]  

(26)

Now using (23) and (26), we obtain

\[ n_h^* = \frac{\left( \tilde{c}_{h}^{(1)}, \tilde{c}_{h}^{(2)}, \tilde{c}_{h}^{(3)} \right) W_h \sqrt{\frac{\pi_{sh} (1 - \pi_{sh}) + A_h}{\lambda (\tilde{c}_{h}^{(1)}, \tilde{c}_{h}^{(2)}, \tilde{c}_{h}^{(3)})}}}{\sum_{h=1}^{L} \left( \tilde{c}_{h}^{(1)}, \tilde{c}_{h}^{(2)}, \tilde{c}_{h}^{(3)} \right) W_h \sqrt{\frac{\pi_{sh} (1 - \pi_{sh}) + A_h}{\lambda (\tilde{c}_{h}^{(1)}, \tilde{c}_{h}^{(2)}, \tilde{c}_{h}^{(3)})}}}. \]  

(27)

In similar manner, the optimum allocation of NLPP (17) with trapezoidal fuzzy numbers can be obtain as

\[ n_h^* = \frac{\left( C_0^{(1)}, C_0^{(2)}, C_0^{(3)} \right) W_h \sqrt{\frac{\pi_{sh} (1 - \pi_{sh}) + A_h}{\lambda (\tilde{c}_{h}^{(1)}, \tilde{c}_{h}^{(2)}, \tilde{c}_{h}^{(3)}, \tilde{c}_{h}^{(4)})}}}{\sum_{h=1}^{L} \left( C_0^{(1)}, C_0^{(2)}, C_0^{(3)} \right) W_h \sqrt{\frac{\pi_{sh} (1 - \pi_{sh}) + A_h}{\lambda (\tilde{c}_{h}^{(1)}, \tilde{c}_{h}^{(2)}, \tilde{c}_{h}^{(3)}, \tilde{c}_{h}^{(4)})}}}. \]  

(28)
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The allocations obtained in (27) and (28) are fuzzy in nature, so we have to convert fuzzy allocations into a crisp allocation by $\alpha$-cut method at a prescribed value of $\alpha$.

6. Procedure for the Conversion of Fuzzy Numbers

To convert the fuzzy allocation into crisp allocation $\alpha$-cut method is used as follows.

Let $\tilde{A} = (p, q, r)$ be a TFN. An $\alpha$-cut for $\tilde{A}$, $\tilde{A}_{\alpha}$ is computed as

$$\alpha = \frac{r - x}{r - q} \implies \tilde{A}_{L, \alpha} = x = (q - p)\alpha + p,$$

and

$$\alpha = \frac{x - p}{q - p} \implies \tilde{A}_{U, \alpha} = x = (q - p)\alpha + p.$$

Similarly, if $\tilde{A} = (p, q, r, s)$ is a TrFN, then the $\alpha$-cut for $\tilde{A}$, $\tilde{A}_{\alpha}$ is computed as

$$\alpha = \frac{x - p}{q - p} \implies \tilde{A}_{L, \alpha} = x = (q - p)\alpha + p,$$

$$\alpha = \frac{s - x}{s - r} \implies \tilde{A}_{U, \alpha} = x = (s - r)\alpha.$$

The allocations obtained by (30) and (32) provide the solution to NLPP (13) and (17) if it satisfies the restriction $1 \leq n_h \leq N_h$; $h = 1, 2, \ldots, L$. The allocations obtained in (30) and (32) may not be integer allocations, so to get integer allocations, round off the allocations to the nearest integer values. After rounding off we have to be careful in rechecking that the round-off values satisfy the cost constraint.

7. Some Other Allocation Techniques

7.1. Equal Allocation. In this method, the total sample size $n$ is divided equally among all the strata, that is, for the $h$th stratum

$$n_h = \frac{n}{L},$$

where $n$ can be obtained from the cost constraint equation as follows:

$$\sum_{h=1}^{L} \left[ (c_h^{(2)} - c_h^{(1)}) + c_h^{(1)} \right] n_h = C_0^{(4)} - C_0^{(3)}.$$

7.2. Proportional Allocation. This allocation was originally proposed by Bowley [22]. This procedure of allocation is very common in practice because of its simplicity. When no other information except $N_h$, the total number of units in the $h$th stratum, is available, the allocation of a given sample of size $n$ to different strata is done in proportion to their sizes, that is, in the $h$th stratum

$$n_h = n \frac{N_h}{N}.$$
and obtained an optimum allocation using LMT as

\[ n_h^* = \frac{C_0 W_h \sqrt{[\pi_{sh} (1 - \pi_{sh}) + A_h] / c_h}}{\sum_{h=1}^{L} W_h \sqrt{[\pi_{sh} (1 - \pi_{sh}) + A_h] c_h}}. \]  

(39)

8. Numerical Illustration

A hypothetical example is given to illustrate the computational details of the proposed problem. Let us suppose the population size is 1000 with total available budget of the survey as TFNs and TrFNs are \((3500, 4000, 4800)\) and \((3500, 4000, 4400, 4600)\) units, respectively. The other required relevant information is given in Table 1. By using the values of Table 1, we compute the values of \(A_h\) which is given in Table 2.

After substituting all the values from Tables 1 and 2 in (13), the required FNLP is given as

\[
\begin{align*}
\text{Minimize} & \quad 0.03098772 n_1 + 0.20718425 n_2 \\
\text{Subject to} & \quad (1, 2, 4) n_1 + (18, 20, 24) n_2 \leq (3500, 4000, 4800), \\
& \quad 1 \leq n_1 \leq 300; \quad 1 \leq n_2 \leq 700.
\end{align*}
\]  

(40)

The required optimum allocations for problem (13) obtained by substituting the values from Tables 1 and 2 in (30) at \(\alpha = 0.5\) will be

\[
\begin{align*}
n_1 &= \frac{(4800 - 800\alpha) \times 0.3 \sqrt{[0.4 (1 - 0.4) + 0.104308]} / (\alpha + 1)}{0.3 \sqrt{[0.4 (1 - 0.4) + 0.104308]} (\alpha + 1) + 0.3 \sqrt{[0.6 (1 - 0.6) + 0.182825]} (2\alpha + 18)} \\
&= 287.51 \approx 288,
\end{align*}
\]  

(41)

\[
\begin{align*}
n_2 &= \frac{(4800 - 800\alpha) \times 0.7 \sqrt{[0.6 (1 - 0.6) + 0.182825]} / (2\alpha + 18)}{0.3 \sqrt{[0.4 (1 - 0.4) + 0.104308]} (\alpha + 1) + 0.3 \sqrt{[0.6 (1 - 0.6) + 0.182825]} (2\alpha + 18)} \\
&= 208.88 \approx 209.
\end{align*}
\]

In similar manner, optimum allocations for problem (17) obtained by substituting the values from Tables 1 and 2 in (32) at \(\alpha = 0.5\) will be

\[
\begin{align*}
n_1 &= \frac{3750 \times 0.3 \sqrt{[0.4 (1 - 0.4) + 0.104308]} (\alpha + 1)}{0.3 \sqrt{[0.4 (1 - 0.4) + 0.104308]} (\alpha + 1) + 0.3 \sqrt{[0.6 (1 - 0.6) + 0.182825]} (2\alpha + 18)} \\
&= 280.97 \approx 281,
\end{align*}
\]
Table 1: Data for two strata.

<table>
<thead>
<tr>
<th>Stratum</th>
<th>( h )</th>
<th>( W_h )</th>
<th>( \pi_h )</th>
<th>( M_h )</th>
<th>( P_h )</th>
<th>( (c^{(1)}_h, c^{(2)}_h, c^{(3)}_h) )</th>
<th>( (C^{(1)}_h, C^{(2)}_h, C^{(3)}_h) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.3</td>
<td>0.4</td>
<td>0.8</td>
<td>0.6</td>
<td>(1, 2, 4)</td>
<td>(1, 2, 4, 7)</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.7</td>
<td>0.6</td>
<td>0.6</td>
<td>0.7</td>
<td>(18, 20, 24)</td>
<td>(18, 20, 24, 26)</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Computation for \( A_h \).

\[
\begin{array}{ccccccccccc}
\text{Stratum} & M_h & P_h & (1 - M_h) & (1 - P_h) & (4)(5) & 1 - (6) & (2P_h - 1) & 2(5)(6) & [8 + 9]^2 & (6)(7) & A_h = (11)/(10)
\end{array}
\]

<table>
<thead>
<tr>
<th>Stratum</th>
<th>( M_h )</th>
<th>( P_h )</th>
<th>( 1 - M_h )</th>
<th>( 1 - P_h )</th>
<th>( (4)(5) )</th>
<th>( 1 - (6) )</th>
<th>( (2P_h - 1) )</th>
<th>( 2(5)(6) )</th>
<th>( [8 + 9]^2 )</th>
<th>( (6)(7) )</th>
<th>( A_h = (11)/(10) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.8</td>
<td>0.6</td>
<td>0.2</td>
<td>0.4</td>
<td>0.08</td>
<td>0.2</td>
<td>0.64</td>
<td>0.7056</td>
<td>0.0736</td>
<td>0.104308</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.6</td>
<td>0.7</td>
<td>0.4</td>
<td>0.3</td>
<td>0.12</td>
<td>0.4</td>
<td>0.36</td>
<td>0.5776</td>
<td>0.1056</td>
<td>0.182825</td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Optimum allocations.

<table>
<thead>
<tr>
<th>Allocations</th>
<th>( n_1 )</th>
<th>( n_2 )</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>LMT (optimum allocation)</td>
<td>Case of TFN</td>
<td>288</td>
<td>209</td>
</tr>
<tr>
<td></td>
<td>Case of TrFN</td>
<td>281</td>
<td>204</td>
</tr>
<tr>
<td>Equal allocation</td>
<td>Case of TFN</td>
<td>160</td>
<td>160</td>
</tr>
<tr>
<td></td>
<td>Case of TrFN</td>
<td>156.5</td>
<td>156.5</td>
</tr>
<tr>
<td>Proportional allocation</td>
<td></td>
<td>41</td>
<td>95</td>
</tr>
<tr>
<td>Ghufran’s allocation</td>
<td></td>
<td>86</td>
<td>157</td>
</tr>
</tbody>
</table>

\[
n_2 = \frac{(4400 - 200\alpha) \times 0.7 \sqrt{[0.6(1 - 0.6) + 0.182825] / (2\alpha + 18)}}{0.3 \sqrt{[0.4(1 - 0.4) + 0.104308] (\alpha + 1) + 0.3 \sqrt{[0.6(1 - 0.6) + 0.182825] (2\alpha + 18)}} = 204.13 = 204.
\]

By using \( \alpha \)-cut and LMT, the optimum allocation after rounding-off is obtained and summarized in Table 3 with the equal allocation, proportional allocation, and Ghufran’s allocation.

### 9. Conclusion

The optimum allocation problem in two-stage stratified warner’s randomized response model with fuzzy costs is formulated as a problem of fuzzy nonlinear programming problem. The problem is then solved by using Lagrange multipliers technique for obtaining optimum allocation. The optimum allocation obtained in the form of fuzzy numbers is converted into an equivalent crisp number by using \( \alpha \)-cut method at a prescribed value of \( \alpha \).

On comparing the result of LMT with the result of equal allocation, proportional allocation, and Ghufran’s allocation, it is seen that LMT gives the best allocation. But it is not necessary that optimum allocation obtained by Lagrange multipliers technique always gives the feasible or optimal solution (proved by [17]) and also for practical purposes we need integer sample sizes. Therefore, in future instead of rounding off the continuous solution, we can obtain integer solution by Dakin’s Method [23] or formulating the problem as fuzzy integer nonlinear programming problem and obtain the integer solution by LINGO software.

### Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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