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Research Article

Z Number Based Fuzzy Inference System for Dynamic Plant Control

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Frequently the reliabilities of the linguistic values of the variables in the rule base are becoming important in the modeling of fuzzy systems. Taking into consideration the reliability degree of the fuzzy values of variables of the rules the design of inference mechanism acquires importance. For this purpose, Z number based fuzzy rules that include constraint and reliability degrees of information are constructed. Fuzzy rule interpolation is presented for designing of an inference engine of fuzzy rule-based system. The mathematical background of the fuzzy inference system based on interpolative mechanism is developed. Based on interpolative inference process Z number based fuzzy controller for control of dynamic plant has been designed. The transient response characteristic of designed controller is compared with the transient response characteristic of the conventional fuzzy controller. The obtained comparative results demonstrate the suitability of designed system in control of dynamic plants.

1. Introduction

In industry some dynamic plants are characterized by uncertainty of environment and fuzziness of information. Deterministic models used to control these dynamic plants usually prove to be insufficient to adequately describe the processes. One of the effective ways for modeling and controlling of these plants is the use of fuzzy systems based on If-Then rules. The rules of the fuzzy systems are basically constructed using the knowledge of experts or experienced specialists. Sometimes the people have made a rational decision using uncertain, incomplete information. Fuzzy logic allows handling uncertain and imprecise knowledge and provides a powerful framework for reasoning. The design of the rules and also the reliabilities of the linguistic values of the variables in these rules are an important issue in the modeling of the systems. Taking into consideration the reliability degree of fuzzy values used in the fuzzy If-Then rules the design of decision-making module acquires importance. The description of this decision-making mechanism is very hard and its simulation is difficult. Zadeh suggested Z number to deal with uncertain information with the degree of its reliability. Z number provides fuzzy restriction and reliability information [1].

Recently fuzzy set theory is efficiently applied for solution different real world problems. These research works are basically based on type-1 or type-2 fuzzy systems. In these researches the reliability of information is taken into consideration. Z number provided in [1] allows representing fuzzy constraint and reliability of the fuzzy system. Z number based systems have been discussed and designed in many research works [1–3]. Comparing with the classical fuzzy set, Z number based fuzzy set more adequately describes the knowledge of human. The theory can describe both constraint and reliability of information [1]. The concept of Z number is investigated and also applied to solve different problems. In [3, 4] Z numbers are used to solve the multicriteria decision-making problem. However the approach proposed in these researches are based on converting Z number to fuzzy numbers on the base of the approach presented in [5]. Reference [6] uses Z number for multicriteria decision-making, where Z number is converted to the interval-valued

fuzzy set with a footprint of uncertainty and then by computing the centroid it is converted to the crisp numbers for decision-making. These researchers present the advantages of the presented approach based on their low computational complexity. However the converting Z numbers in real numbers may lead to the significant loss of information and, therefore, this may affect the performance of designed fuzzy system.

The paper [7] suggests the use of Z number in the perception of the uncertainty of the information conveyed by a natural language statement and to merge human-affective perspectives with Computing with Words (CWW). Reference [8] considers an application of Z number based approach to the AHP. The presented work is also based on the approach proposed in [5]. In [9] a fair price approach for a participation in a decision-making under interval, set-valued, fuzzy, and Z number uncertainty is considered.

As shown, using Z numbers, a set of research works on arithmetic operations and decision-making have been done. The use of Z numbers in decision-making, control, and modeling needs to use efficient inference mechanism for the designed system. Reference [2] suggests using Z interpolation in inference process. In [10] Kóczy and Hirota purposed interpolative reasoning in the sparse fuzzy rule base. This method maintains the logical interpretation of modus ponens. The method is distance based approximate fuzzy reasoning [10, 11]. There are many different distance definitions for fuzzy sets [12]. These are absolute distance, Euclidian distance, disconsistency measure, Hausdorff measure, and Kaufman and Gupta measure. Reference [12] points that the main deficiency of these measures is that the information of the shape of the membership function of the fuzzy sets is mostly lost. Using distance measure of two fuzzy sets A and B and given fuzzy set A', it is impossible to construct the second fuzzy set B'. Reference [12] mentions that Kóczy measure can be used to solve this problem. Kóczy distance is based on α-cuts of two fuzzy sets. In [10, 11] using Kóczy measure a fuzzy interpolative reasoning was proposed. The interpolative approximate reasoning is applied to the solution of different problems [13, 14]. In this paper, the derivation of the interpolative approximate reasoning for Z number based fuzzy rules is considered. The designed method is then applied for solving of dynamic plants control.

The paper is organized as follows. Section 2 presents the interpolative approximate reasoning method. Section 3 considers the use of the interpolative approximate reasoning method for Z numbers. Section 4 presents the application of the designed interpolative approximate reasoning for the development of control system of the dynamic plant.

2. Fuzzy Rule Interpolation

The inference technique of fuzzy logic resembles human reasoning capabilities. In the literature various fuzzy reasoning methods are purposed to process uncertain information and increase the performance of the designed systems. These fuzzy reasoning methods are mainly based on the compositional inference rule, analogy and similarity, interpolation, and the concept of distance. The speed, processing capabilities, and complexity of these reasoning methods are important issues.

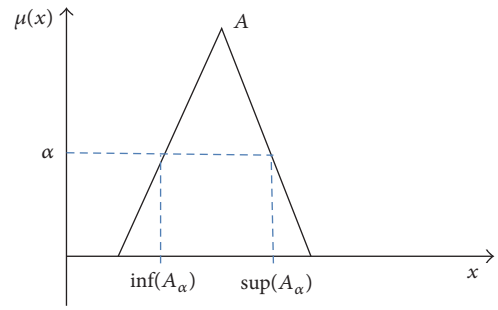

Figure 1: Membership function, infimum, and supremum for α-cut.

In the paper inference based on fuzzy interpolation of rules is used. Fuzzy rule interpolation is proposed by Kóczy and Hirota [10, 11] and is designated for use for sparse rule base. The inference engine requires the satisfaction of the following conditions: the used fuzzy sets should be continuous, convex, and normal, with bounded support.

The proposed inference method is based on distance measure. Here the distance measure of two fuzzy sets is described using a fuzzy set defined in the interval [0, 1]. The used distance measure is based on α-cut. Let us consider some concepts used for two fuzzy sets A_1 and A_2. α-cut of fuzzy sets A_1 and A_2 will be denoted as $A_{1\alpha}$ and $A_{2\alpha}$. We say that fuzzy set A_1 is less than A_2; that is, $A_1 < A_2$, iff

$$\begin{aligned} \inf\{A_{1\alpha}\} &< \inf\{A_{2\alpha}\}, \\ \sup\{A_{1\alpha}\} &< \sup\{A_{2\alpha}\}, \end{aligned} \tag{1}$$

where $\inf\{A_{1\alpha}\}$ and $\inf\{A_{2\alpha}\}$ are infimum of A_1 and A_2; $\sup\{A_{1\alpha}\}$ and $\sup\{A_{2\alpha}\}$ are supremum of A_1 and A_2 (Figure 1).

Let us consider the following case. Assume that we have fuzzy rule base for the controller and in the result of observation we detect that input variable X is A^*. Let us determine the output Y of the rule-based system. Assume that A^* is located between fuzzy sets A_1 and A_2. Let us determine the output of fuzzy system using the rules which includes A_1 and A_2 fuzzy sets.

$$\begin{array}{rl} & \text{Given that } X \text{ is } A^* \\ \text{Rules:} & \text{If } X \text{ is } A_1 \text{ Then } Y \text{ is } B_1 \\ & \text{If } X \text{ is } A_2 \text{ Then } Y \text{ is } B_2 \\ \hline & \text{Find Conclusion } Y = B^*? \end{array} \tag{2}$$

If $A_1 < A^* < A_2$ and $B_1 < B_2$ then using linear interpolation [10] shows that

$$\frac{d(A^*, A_1)}{d(A^*, A_2)} = \frac{d(B^*, B_1)}{d(B^*, B_2)}. \tag{3}$$

Here $d(^*)$ is the distance between two fuzzy sets. Various formulas are applied to calculate $d(^*)$ distance between two sets. These are the following: Hamming distance, Euclidian distance, Hausdorff distance, and Kaufman-Gupta distance.

$$\begin{aligned} d(X,Y) &= |X - Y|; \\ d(X,Y) &= \sqrt{(X-Y)^2} \\ d(X,Y) &= \max\left\{\sup_{x\in X}\inf_{y\in Y} d(x,y), \sup_{y\in Y}\inf_{x\in X} d(x,y),\right\} \\ d(X,Y) &= \frac{(|x_1 - y_1| + |x_2 - y_2|)}{2(\beta_2 - \beta_1)}. \end{aligned} \tag{4}$$

Here $[a_1, a_2]$ and $[b_1, b_2]$ are supports of A and B, respectively. $[\beta_1, \beta_2]$ is the support of both A and B. Reference [12] notices that these distance measures do not give information about the shape of the membership functions of the fuzzy sets. Kóczy and Hirota [10, 11] introduced distance based on α-cuts of the two fuzzy sets. Using this distance the final fuzzy set can easily be constructed. Kóczy and Hirota [10, 11] based on α-cut calculated lower d_L and upper d_U distances between $A_{1\alpha}$ and $A_{2\alpha}$:

$$\begin{aligned} d_L(A_{1\alpha}, A_{2\alpha}) &= d(\inf\{A_{1\alpha}\}, \inf\{A_{2\alpha}\}); \\ d_U(A_{1\alpha}, A_{2\alpha}) &= d(\sup\{A_{1\alpha}\}, \sup\{A_{2\alpha}\}). \end{aligned} \tag{5}$$

Here

$$\begin{aligned} d_L(A^*_\alpha, A_{1\alpha}) &= d(\inf\{A^*_\alpha\}, \inf\{A_{1\alpha}\}) \\ &= \inf\{A^*_\alpha\} - \inf\{A_{1\alpha}\} \\ d_U(A^*_\alpha, A_{2\alpha}) &= d(\sup\{A^*_\alpha\}, \sup\{A_{2\alpha}\}) \\ &= \sup\{A^*_\alpha\} - \sup\{A_{2\alpha}\} \\ d_L(B^*_\alpha, B_{1\alpha}) &= d(\inf\{B^*_\alpha\}, \inf\{B_{1\alpha}\}) \\ &= \inf\{B^*_\alpha\} - \inf\{B_{1\alpha}\} \\ d_U(B^*_\alpha, B_{2\alpha}) &= d(\sup\{B^*_\varepsilon\}, \sup\{B_{2\alpha}\}) \\ &= \sup\{B^*_\alpha\} - \sup\{B_{2\alpha}\}. \end{aligned} \tag{6}$$

It is needed to mention that Hamming or Euclidian formula can be used in (5) to measure the distances. Taking (5) into (3) we can obtain the following:

$$\begin{aligned} \inf\{B^*_\alpha\} &= \frac{(1/d_{\alpha L}(A_{1\alpha}, A^*_\alpha))\inf\{B_{1\alpha}\} + (1/d_{\alpha L}(A_{2\alpha}, A^*_\alpha))\inf\{B_{2\alpha}\}}{1/d_{\alpha L}(A_{1\alpha}, A^*_\alpha) + 1/d_{\alpha L}(A_{2\alpha}, A^*_\alpha)} \\ \sup\{B^*_\alpha\} &= \frac{(1/d_{\alpha U}(A_{1\alpha}, A^*_\alpha))\sup\{B_{1\alpha}\} + (1/d_{\alpha U}(A_{2\alpha}, A^*_\alpha))\sup\{B_{2\alpha}\}}{1/d_{\alpha U}(A_{1\alpha}, A^*_\alpha) + 1/d_{\alpha U}(A_{2\alpha}, A^*_\alpha)}. \end{aligned} \tag{7}$$

Then the output α-level set in the consequent part for every α will be

$$B^*_\alpha = [\inf\{B^*_\alpha\}, \sup\{B^*_\alpha\}]. \tag{8}$$

Reference [10] extended the formula for $2k$ rules.

$$\begin{aligned} \inf\{B^*_\alpha\} &= \frac{\sum_{i=1}^{2k}(1/d_{\alpha L}(A_{i\alpha}, A^*_\alpha))\inf\{B_{i\alpha}\}}{\sum_{i=1}^{2k}(1/d_{\alpha L}(A_{i\alpha}, A^*_\alpha))}; \\ \sup\{B^*_\alpha\} &= \frac{\sum_{i=1}^{2k}(1/d_{\alpha U}(A_{i\alpha}, A^*_\alpha))\sup\{B_{i\alpha}\}}{\sum_{i=1}^{2k}(1/d_{\alpha U}(A_{i\alpha}, A^*_\alpha))}. \end{aligned} \tag{9}$$

In the next section we are applying the above formulas to obtain output of fuzzy system based on Z numbers.

3. Interpolative Reasoning Using Z Number Based Fuzzy Rules

Let us consider Z If-Then rules and its inference mechanism. Assume that multiinput-single output fuzzy Z rules are given as follows.

Z Number Based If-Then Rule Base is as follows:

$$\begin{aligned} &\text{If } x_1 \text{ is } (A_{11}, R_{11}) \text{ and} \ldots \text{and } x_m \text{ is } (A_{1m}, R_{1m}) \\ &\quad \text{Then } y \text{ is } (B_1, R_1) \\ &\text{If } x_1 \text{ is } (A_{21}, R_{21}) \text{ and} \ldots \text{and } x_m \text{ is } (A_{2m}, R_{2m}) \\ &\quad \text{Then } y \text{ is } (B_2, R_2) \\ &\qquad \vdots \\ &\text{If } x_1 \text{ is } (A_{n1}, R_{n1}) \text{ and} \ldots \text{and } x_m \text{ is } (A_{nm}, R_{nm}) \\ &\quad \text{Then } y \text{ is } (B_n, R_n). \end{aligned} \tag{10}$$

Let us consider the application of interpolation approach given in Section 2 to the inference process of the fuzzy system based on Z numbers. At first iteration using α-cut the differences between coming input signals and fuzzy values of variables in antecedent parts are calculated. For this purpose, α-cuts can be used with Hamming, Euclidian, or one of the distances given in (4). The distances will be calculated for constraint and reliability variables in antecedent part separately.

Here formulas (5) and (6) are used to find lower and upper distances of membership function. After finding the distances using formula (9) in consequent part the lower and upper parts of the fuzzy output signal are calculated. These operations can be expressed using the following formulas also. We can obtain lower and upper parts through α-cuts. For this purpose, at first iteration the distances are calculated as

$$d_L\left(A^{\alpha}_{X_{i,j}}, X^{\alpha}_j\right) = \left|A^{\alpha}_{X_{i,j}} - X^{\alpha}_j\right|;$$
$$d^{\alpha}_i = \sum_j^m d_L\left(A^{\alpha}_{X_{i,j}}, X^{\alpha}_j\right), \tag{11}$$

where $j = 1, \ldots, m$ and m is a number of input signals, $i = 1, \ldots, n$, and n is a number of rules. For special case $\alpha = \{0, 1\}$. $d_L(A^{\alpha}_{X_i}, X^{\alpha}_i)$ is a distance between two fuzzy sets. Formula (11) is used to find distance dc^{α}_i for constraint parameter A and also the same formula can be adapted to find distance dr^{α}_i for reliability parameter. The total distance will be sum of two distances computed for constraint and reliability

$$d^{\alpha}_i = dc^{\alpha}_i + dr^{\alpha}_i. \tag{12}$$

Here dr^{α}_i is the distance computed for the reliability parameter. The fuzzy set in the output of the rule is calculated by using the following equation.

$$\left(Y^{\alpha}, R^{\alpha}_Y\right) = \frac{\sum_{i=1}^n \left(1/d^{\propto}_i\right)\left(B_{Y,i}, R_{Y,i}\right)^{\propto}}{1/\sum_{i=1}^n \left(1/d^{\propto}_i\right)}. \tag{13}$$

Formula (13) is applied for finding inf, sup, and center values in output fuzzy sets. If we combine (11) and (13) then we can get the following:

$$\left(Y^{\alpha}, R^{\alpha}_Y\right) = \frac{\sum_{i=1}^n \left(1/\sum_j^m d_L\left(A^{\alpha}_{X_{i,j}}, X^{\alpha}_j\right)\right)\left(B_{Y,i}, R_{Y,i}\right)^{\propto}}{1/\sum_{i=1}^n \left(1/\sum_j^m d_L\left(A^{\alpha}_{X_{i,j}}, X^{\alpha}_j\right)\right)}. \tag{14}$$

Using (14) we can derive the output Z fuzzy signal of the system. It is needed to note that for finding output signal we use B_{Yi}; for finding the reliability we use R_{Yi} variables in right side of (13) (or (14)). After getting output signals, the converting of Z number to the crisp number is performed. Formula $Y = ((Y_l + 4 * Y_m + Y_r)/6) * ((R_{Yl} + 4 * R_{Ym} + R_{Yr})/6)$ given in [4, 5] is used to drive the crisp value of output signal. Here Y is fuzzy output value and R_Y is the reliability. It is needed to mention that in the antecedent and consequent parts of the rule base we are using triangular type fuzzy sets for the input and output parameters. If we are considering $\alpha = 0$ and $\alpha = 1$ levels then we can get left Y_l, middle Y_m, and right Y_r values of the output signal. Left (Y_l, R_{Yl}) and right (Y_r, R_{Yr}) values are corresponding to $\alpha = 0$ level; middle (Y_m, R_{Ym}) value is corresponding to $\alpha = 1$ level.

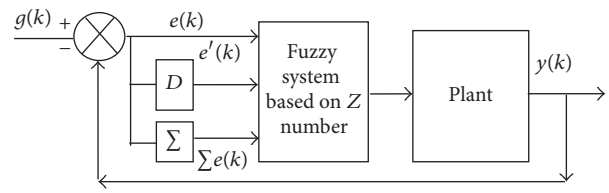

Figure 2: Structure of type-2 FNS based control system.

4. Simulation Studies: Control of Dynamic Plant

In industry, many dynamic plants are operating in an uncertain environment and characterized by the fuzziness of information. These dynamic plants are characterized with uncertainties related to the structure and the parameters. The deterministic models cannot adequately describe these dynamic plants and using these models it was difficult to obtain the required control performance. The use of fuzzy set theory for constructing control systems can be a valuable alternative to solve the problem [15–19]. In this paper, the fuzzy Z number based system described in the above section is used for the control of dynamic plants.

The proposed fuzzy Z number based control system is used for control of dynamic plants. The structure of the control system is given in Figure 2. In this structure the difference $e(k)$ between the plant's output signal $y(k)$ and the set-point signal $g(k)$ is determined. Using $e(k)$ the change of error $e'(k)$ and the sum of error $\sum e(k)$ are determined. In the figure, D indicates differentiation operation and $\sum$ indicates integration operation. Using these input signals the fuzzy Z number based system is used for closed-loop control system.

The kernel of fuzzy controller is the knowledge base that has If-Then form. The accuracy of a fuzzy control system depends on input variables as well as the expert rules and the membership functions; therefore it is important for them to be chosen carefully. The antecedent part of If-Then rules include input and the consequent part of the rules is the control signal to be applied to the plant. The analyses have shown that the control of dynamic plants is basically implemented using input variables error and change-in-error. At first iteration, using these parameters the knowledge base that has If-Then form is developed. In knowledge base, the If-Then rules demonstrate the association between input parameters and output control signal. On the base analysis of the dynamic plants, the KB describing input-output association is designed. Using error and change of error the corresponding value of control signal is determined.

During designing KB the input variables are described by linguistic values. The forms of these linguistic values are taken in triangle form. Each of them is represented by triangle membership functions. During designing these

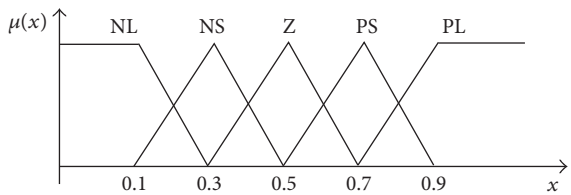

Figure 3: Membership functions of input variables.

Table 1: Knowledge of the expert.

Control u		Change-in-error e'				
		NL, U	NS, U	Z, U	PS, U	PL, U
Error e	NL, U	PL, U	PL, U	PL, U	PS, U	Z, U
	NS, U	PL, U	PL, U	PS, U	Z, U	NS, U
	Z, U	PL, U	PS, U	Z, U	NS, U	NL, U
	PS, U	PS, U	Z, U	NS, U	NL, U	NL, U
	PL, U	Z, U	NS, U	NL, U	NL, U	NL, U

triangle membership functions for the linguistic values it is necessary to determine the universe of discourse and number of linguistic values for each input parameter. In the paper, the number of membership functions is taken to be equal to 5. The linguistic values for the input and output parameters are denoted as NL (negative large), NS (negative small), Z (zero), PS (positive small), and PL (positive large). For simplicity, we scale the range of variables between 0 and 1. The membership functions used to describe input and output variables are shown in Figure 3. After defining membership functions for each parameter, the construction of fuzzy If-Then rules is performed. Each rule has two-input and one-output variables. Table 1 describes the relationships between input parameters, error and change-in-error, and output parameter, control signal of the controller. Using the table fuzzy production rules can be obtained. In (15) using Table 1 the fragment of rule base is given.

Fragment of Fuzzy Rule Base is as follows:

$$\begin{aligned}
&\textbf{IF}\ \text{error is } (\mathrm{NL}, U)\ \textbf{and}\ \text{change-in-error is } (\mathrm{NL}, U)\\
&\quad \textbf{Then}\ \text{Control signal is } (\mathrm{PL}, U)\\
&\textbf{IF}\ \text{error is } (\mathrm{Z}, U)\ \textbf{and}\ \text{change-in-error is } (\mathrm{PS}, U)\\
&\quad \textbf{Then}\ \text{Control signal is } (\mathrm{NS}, U)\\
&\qquad\qquad \vdots\\
&\textbf{IF}\ \text{error is } (\mathrm{PS}, U)\ \textbf{and}\ \text{change-in-error is } (\mathrm{NL}, U)\\
&\quad \textbf{Then}\ \text{Control signal is } (\mathrm{PS}, U)
\end{aligned} \tag{15}$$

Example 1. The presented fuzzy system based on Z numbers using rule base is applied for control of dynamic plants. The designed system is validated using plant models in order to check if the system performance is acceptable. In the first example, the designed fuzzy controller based on Z number is applied for the control of dynamic plant given by the following equation.

$$y(k) = \frac{y(k-1)\,y(k-2)\left(y(k-1)+2.5\right)}{\left(1+y(k-1)^2+y(k-2)^2\right)} + u(k), \tag{16}$$

where $y(k-1)$, $y(k-2)$ are one- and two-step delayed output of the dynamic plant, and $u(k)$ is input control signal.

The control of the plant is performed using different values of the reference signal. The period of the reference signal is 200 samples, and the mathematical expression for the reference signal is given as follows:

$$\text{Reference}(k) = \begin{cases} 10, & 0 \le k < 50 \\ 15, & 50 \le k < 100 \\ 10, & 100 \le k < 150 \\ 15, & 150 \le k < 200 \end{cases}. \tag{17}$$

In the paper the root-mean-square error (RMSE) is used as a performance criterion.

$$\mathrm{RMSE} = \sqrt{\frac{\sum_{i=1}^{K}\left(y_i^d - y_i\right)^2}{K}}, \tag{18}$$

where K is the total number of the samples.

The simulation of the fuzzy Z number based control system has been performed for the plant (16). The performance of Z number based fuzzy control system is compared with the performance of conventional fuzzy control system using the same initial conditions. Figure 4 depicts the time response characteristics of the conventional fuzzy and Z number based control system. In the figure, the solid line is the response characteristic of the control system with Z number based fuzzy controller; dashed line is the response characteristic of the conventional fuzzy control system. As shown from the figure the values of static errors for both controllers are zero; transient overshoots are the same (near 19%). Sums of

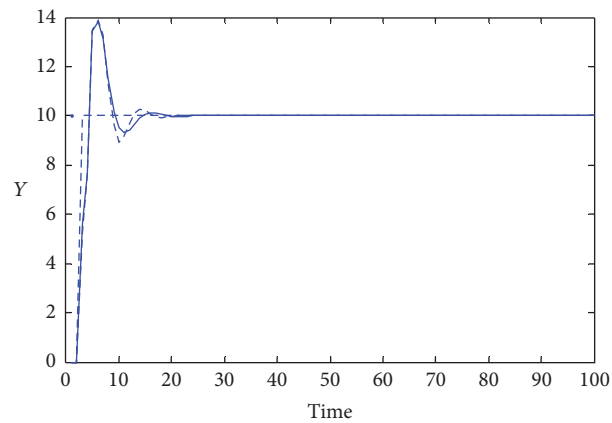

Figure 4: Transient response characteristic of Z number based (solid curve) and conventional fuzzy (dashed line) controller.

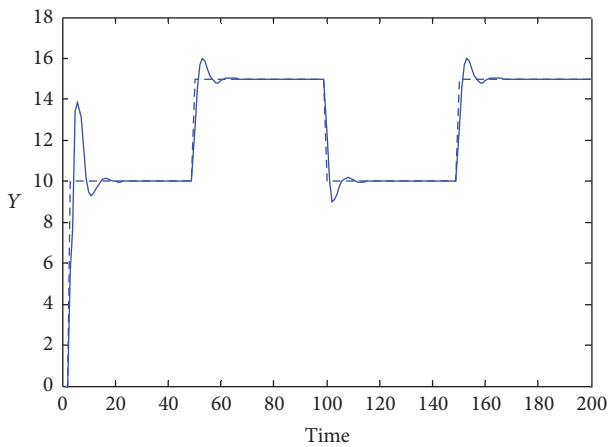

Figure 5: Time response characteristic of control systems with Z number based controller for different values of set-point input signals. Dashed line is set-point signal; solid line is plant output.

Table 2: RMSE values.

Controller	RMSE values	
	100 iterations (Figure 4)	200 iterations (Figure 5)
Fuzzy	70.2697	2626.501988
Z number based controller	64.49811	2614.209756

RMSE values of control systems for 100 iterations are given in Table 2. RMSE value of Z number based control system is less than conventional fuzzy control system.

In next simulation, the performance fuzzy Z number based control system has been tested using different set-point signals given by formula (14). Figure 5 depicts the time response characteristics of Z number based control system using different values of set-point signals. The RMSE values for both simulations are given in Table 2. As can be seen, the RMSE value for Z number based fuzzy control system is less than that of the conventional fuzzy controller.

Example 2. In the second experiment, the simulation of the control system of the temperature of rectifier column K-2 in oil refinery plant is performed. The process is described by the following differential equations:

$$a_0 y(2)(t) + a_1 y(1)(t) + a_2 y(t) = b_0 u(t), \qquad (19)$$

where $a_0 = 0.072\,\text{min}^2$, $a_1 = 0.056\,\text{min}$, and $a_2 = 1$, $b_0 = 60^\circ\text{C}/(\text{kgf/cm}^2)$; here $y(t)$ is regulation parameter of object; $u(t)$ is output of fuzzy controller.

The rule base given in Table 2 is used for the design of Z number based controller for the plant (18). Analogously to Example 1, the designed control system is tested using input signals given in Example 1, for 100 and 200 iterations. Simulation results of designed fuzzy control system based on Z number are compared with the simulation results of control systems based on the conventional fuzzy controller. In Table 3 the results of comparative estimation of RMSE values of the response characteristic of control systems are given.

Table 3: RMSE values.

Controller	RMSE values	
	100 iterations (Figure 4)	200 iterations (Figure 5)
Fuzzy	215.65172	2419.03285
Z number based controller	97.693416	2219.600649

The simulation results demonstrate the efficiency of using Z number based controller in control of dynamic plants.

5. Conclusions

The paper presents the design of Z number based fuzzy inference system. The interpolative inference mechanism of the fuzzy system has been presented and the same inference mechanism is developed for Z number based fuzzy system. Z fuzzy rule base is designed and the reliability degrees of the fuzzy values of the variables are estimated. Interpolative fuzzy inference engine process using Z number based system has been designed. The designed algorithm is used for control of dynamic plants. The If-Then rule base using the knowledge of experts is designed. The reliability degrees of the fuzzy values of the variables are estimated. Using rule base and fuzzy interpolative reasoning the control of dynamic plants is performed. The obtained results demonstrate the suitability of designed system in control of dynamic plants.

Competing Interests

The author has no competing interests.

References

[1] L. A. Zadeh, "A note on Z-numbers," *Information Sciences*, vol. 181, no. 14, pp. 2923–2932, 2011.

[2] R. A. Aliev, O. H. Huseynov, R. R. Aliyev, and A. A. Alizadeh, *The arithmetic of Z-numbers*, World Scientific Publishing, 2015.

[3] R. A. Aliev and L. M. Zeinalova, "Decision making under Z-information," in *Human-Centric Decision-Making Models for Social Sciences (Studies in Computational Intelligence)*, P. Guo and W. Pedrycz, Eds., pp. 233–252, Springer, Berlin, Germany, 2014.

[4] B. Kang, D. Wei, Y. Li, and Y. Deng, "Decision making using Z-numbers under uncertain environment," *Journal of Computational Information Systems*, vol. 8, no. 7, pp. 2807–2814, 2012.

[5] B. Kang, D. Wei, Y. Li, and Y. Deng, "A method of converting z-number to classical fuzzy number," *Journal of Computational Information System*, vol. 9, no. 3, pp. 703–709, 2012.

[6] Z.-Q. Xiao, "Application of Z-numbers in multi-criteria decision making," in *Proceedings of the International Conference on Informative and Cybernetics for Computational Social Systems (ICCSS '14)*, pp. 91–95, IEEE, Qingdao, China, October 2014.

[7] S. K. Pal, R. Banerjee, S. Dutta, and S. S. Sarma, "An insight into the Z-number approach to CWW," *Fundamenta Informaticae*, vol. 124, no. 1-2, pp. 197–229, 2013.

[8] A. Azadeh, M. Saberi, N. Z. Atashbar, E. Chang, and P. Pazhoheshfar, "Z-AHP: a Z-number extension of fuzzy analytical hierarchy process," in *Proceedings of the International Conference*

on Digital Ecosystems and Technologies (DEST '13), pp. 141–147, Menlo Park, Calif, USA, July 2013.

[9] J. Lorkowski, V. Kreinovich, and R. A. Aliev, "Towards decision making under interval, set-valued, fuzzy, and Z-number uncertainty: a fair price approach," in *Proceedings of the IEEE International Conference on Fuzzy Systems (FUZZ-IEEE '14)*, pp. 2244–2253, Beijing, China, July 2014.

[10] L. T. Kóczy and K. Hirota, "Approximate reasoning by linear rule interpolation and general approximation," *International Journal of Approximate Reasoning*, vol. 9, no. 3, pp. 197–225, 1993.

[11] L. T. Kóczy and K. Hirota, "Interpolative reasoning with insufficient evidence in sparse fuzzy rule bases," *Information Sciences*, vol. 71, no. 1-2, pp. 169–201, 1993.

[12] Z. C. Johanyák and S. Kovács, "Distance based similarity measures of fuzzy sets," in *Proceedings of the 3rd Slovakian-Hungarian Joint Symposium on Applied Machine Intelligence*, pp. 265–276, Herl'any, Slovakia, January 2005.

[13] S. Kovács and L. T. Kóczy, "Application of interpolation-based fuzzy logic reasoning in behaviour-based control structures," in *Proceedings of the IEEE International Conference on Fuzzy Systems, UZZIEEE*, vol. 6, pp. 25–29, Budapest, Hungary, July 2004.

[14] Z. Huang and Q. Shen, "A new fuzzy interpolative reasoning method based on center of gravity," in *Proceedings of the 12th IEEE International Conference on Fuzzy Systems (FUZZ '03)*, vol. 1, pp. 25–30, IEEE, May 2003.

[15] R. H. Abiyev and O. Kaynak, "Type 2 fuzzy neural structure for identification and control of time-varying plants," *IEEE Transactions on Industrial Electronics*, vol. 57, no. 12, pp. 4147–4159, 2010.

[16] R. H. Abiyev and O. Kaynak, "Fuzzy wavelet neural networks for identification and control of dynamic plants—a novel structure and a comparative study," *IEEE Transactions on Industrial Electronics*, vol. 55, no. 8, pp. 3133–3140, 2008.

[17] R. Abiyev, "Fuzzy inference system based on neural network for technological process control," *Mathematical and Computational Applications*, vol. 8, no. 2, pp. 245–252, 2003.

[18] R. H. Abiyev and O. Kaynak, "Identification and control of dynamic plants using fuzzy wavelet neural networks," in *Proceedings of the IEEE International Symposium on Intelligent Control (ISIC '08)*, pp. 1295–1301, IEEE, San Antonio, Tex, USA, September 2008.

[19] R. Abiyev, "Neuro-fuzzy system for technological processes control," in *Proceedings of the 6th World Multi-Conference on Systemics, Cybernetics and Informatics/8th International Conference on Information Systems Analysis and Synthesis (SCI-ISAS '02)*, vol. 12, pp. 249–253, Orlando, Fla, USA, July 2002.