Research Article

Morphism of m -Polar Fuzzy Graph

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The main purpose of this paper is to introduce the notion of m -polar h -morphism on m -polar fuzzy graphs. The action of m -polar h -morphism on m -polar fuzzy graphs is studied. Some elegant theorems on weak and coweak isomorphism are obtained. Also, some properties of highly irregular, edge regular, and totally edge regular m -polar fuzzy graphs are studied.

1. Introduction

Akram [1] introduced the notion of bipolar fuzzy graphs describing various methods of their construction as well as investigating some of their important properties. Bhutani [2] discussed automorphism of fuzzy graphs. Chen et al. [3] generalized the concept of bipolar fuzzy set to obtain the notion of m -polar fuzzy set. The notion of m -polar fuzzy set is more advanced than fuzzy set and eliminates doubtfulness more absolutely. Ghorai and Pal [4–6] studied some operations and properties of m -polar fuzzy graphs. Rashmanlou et al. [7] discussed some properties of bipolar fuzzy graphs and some of its results are investigated. Ramprasad et al. [8] studied product m -polar fuzzy graph, product m -polar fuzzy intersection graph, and product m -polar fuzzy line graph. Values between 0 and 1 are used to develop a set theory based on fuzziness by Zadeh [9–11].

In the present work the authors introduce the concepts of m -polar h -morphism, edge regular m -polar fuzzy graph, totally edge regular m -polar fuzzy graph, and highly irregular m -polar fuzzy graph in order to strengthen the decision-making in critical situations.

2. Preliminaries

Definition 1. Throughout the paper, $[0, 1]^m$ (m copies of the closed interval $[0, 1]$) is considered to be a poset with pointwise order $\leq$, where m is a natural number, $\leq$ is given

by $w \leq f \Leftrightarrow$ for all $i = 1, 2, \dots, m$, $p_i(w) \leq p_i(f)$, where $w, f \in [0, 1]^m$ and $p_i : [0, 1]^m \rightarrow [0, 1]$ is the i th projection mapping. An m -polar fuzzy set (or a $[0, 1]^m$ -set) on X is a mapping $W : X \rightarrow [0, 1]^m$.

Definition 2. Let W be an m -polar fuzzy set on X . An m -polar fuzzy relation on W is an m -polar fuzzy set F of $X \times X$ such that $F(uv) \leq \min\{W(u), W(v)\}$ for all $u, v \in V$, that is, for each $i = 1, 2, \dots, m$, for all $u, v \in X$, $p_i \circ F(uv) \leq \min\{p_i \circ W(u), p_i \circ W(v)\}$.

Definition 3. A generalized m -polar fuzzy graph of a graph $G^* = (V, E)$ is a pair $G = (V, W, F)$, where $W : V \rightarrow [0, 1]^m$ is an m -polar fuzzy set in V and $F : V \times V \rightarrow [0, 1]^m$ is an m -polar fuzzy set in $V \times V$ such that $F(uv) \leq \min\{W(u), W(v)\}$ for all $uv \in V \times V$ and $F(uv) = 0$ for all $uv \in ((\bar{V}^2) - E)$ ($0 = 0, 0 \dots 0$) is the smallest element in $[0, 1]^m$. W is called the m -polar fuzzy set of G and F is called m -polar fuzzy edge set of G .

Definition 4. An m -polar fuzzy graph $G = (V, W, F)$ of the graph $G^* = (V, E)$ is said to be strong if $p_i \circ F(uv) = \min\{p_i \circ W(u), p_i \circ W(v)\}$ for all $uv \in E$, $i = 1, 2, \dots, m$.

3. A New Theory of Regularity in m -Polar Fuzzy Graphs

Using the existing graph theories a new m -polar fuzzy graph theory is introduced in this section.

Definition 5. Let $G = (V, W, F)$ be an m -polar fuzzy graph. Then the degree of a vertex u is defined for $i = 1, 2, \dots, m$ as

$$d_G(u) = \sum_{uv \in E, u \neq v} p_i \circ F(uv) = \left\langle \sum_{uv \in E, u \neq v} p_1 \circ F(uv), \sum_{uv \in E, u \neq v} p_2 \circ F(uv), \sum_{uv \in E, u \neq v} p_3 \circ F(uv), \dots, \sum_{uv \in E, u \neq v} p_m \circ F(uv) \right\rangle. \quad (1)$$

Definition 6. The degree of an edge $xy \in E$ in an m -polar fuzzy graph $G = (V, W, F)$ is defined as

$$d_G(xy) = d_G(x) + d_G(y) - 2 \langle p_1 \circ F(xy), p_2 \circ F(xy), p_3 \circ F(xy), \dots, p_m \circ F(xy) \rangle. \quad (2)$$

Definition 7. The total degree of an edge $xy \in E$ in an m -polar fuzzy graph $G = (V, W, F)$ is defined as

$$td_G(xy) = d_G(x) + d_G(y) - \langle p_1 \circ F(xy), p_2 \circ F(xy), p_3 \circ F(xy), \dots, p_m \circ F(xy) \rangle. \quad (3)$$

Definition 8. The degree of an edge $w_j w_k \in E$ in a crisp graph G^* is $d_{G^*}(w_j w_k) = d_{G^*}(w_j) + d_{G^*}(w_k) - 2$.

Example 9. Consider an m -polar fuzzy graph $G = (V, W, F)$ of $G^* = (V, E)$, where

$$\begin{aligned} V &= \{K, L, M, N\}, \\ E &= \{KL, LM, MN, NK\}, \\ W &= \left\{ \frac{\langle 0.2, 0.5, 0.6 \rangle}{K}, \frac{\langle 0.3, 0.5, 0.7 \rangle}{L}, \frac{\langle 0.3, 0.6, 0.7 \rangle}{M}, \frac{\langle 0.4, 0.7, 0.8 \rangle}{N} \right\}, \\ F &= \left\{ \frac{\langle 0.2, 0.4, 0.5 \rangle}{KL}, \frac{\langle 0.3, 0.4, 0.6 \rangle}{LM}, \frac{\langle 0.3, 0.5, 0.6 \rangle}{MN}, \frac{\langle 0.2, 0.3, 0.6 \rangle}{NK} \right\}, \end{aligned} \quad (4)$$

as in Figure 1.

Then, we have

$$\begin{aligned} d_G(K) &= \langle 0.4, 0.7, 1.1 \rangle, \\ d_G(L) &= \langle 0.5, 0.8, 1.1 \rangle, \\ d_G(M) &= \langle 0.6, 0.9, 1.2 \rangle, \\ d_G(N) &= \langle 0.5, 0.8, 1.2 \rangle, \\ d_G(KL) &= \langle 0.5, 0.7, 1.2 \rangle, \\ d_G(LM) &= \langle 0.5, 0.9, 1.1 \rangle, \\ d_G(MN) &= \langle 0.5, 0.7, 1.2 \rangle, \\ d_G(NK) &= \langle 0.5, 0.9, 1.1 \rangle, \end{aligned}$$

FIGURE 1: 3-polar fuzzy graph G .

$$\begin{aligned} td_G(KL) &= \langle 0.7, 1.1, 1.7 \rangle, \\ td_G(LM) &= \langle 0.8, 1.3, 1.7 \rangle, \\ td_G(MN) &= \langle 0.8, 1.2, 1.8 \rangle, \\ td_G(NK) &= \langle 0.7, 1.2, 1.7 \rangle. \end{aligned} \quad (5)$$

Definition 10. If every vertex in an m -polar fuzzy graph $G = (V, W, F)$ has the same degree $\langle l_1, l_2, \dots, l_m \rangle$, then $G = (V, W, F)$ is called regular m -polar fuzzy graph or m -polar fuzzy graph of degree $\langle l_1, l_2, \dots, l_m \rangle$.

Definition 11. If every edge in an m -polar fuzzy graph $G = (V, W, F)$ has the same degree $\langle l_1, l_2, \dots, l_m \rangle$, then $G = (V, W, F)$ is called an edge regular m -polar fuzzy graph.

Definition 12. If every edge in an m -polar fuzzy graph $G = (V, W, F)$ has the same total degree $\langle l_1, l_2, \dots, l_m \rangle$, then $G = (V, W, F)$ is called totally edge regular m -polar fuzzy graph.

Example 13. Consider an m -polar fuzzy graph $G = (V, W, F)$ of $G^* = (V, E)$, where

$$\begin{aligned} V &= \{K, L, M, N\}, \\ E &= \{KL, LM, MN, NK\}, \\ W &= \left\{ \frac{\langle 0.2, 0.6, 0.1 \rangle}{K}, \frac{\langle 0.3, 0.5, 0.1 \rangle}{L}, \frac{\langle 0.3, 0.5, 0.2 \rangle}{M}, \frac{\langle 0.4, 0.6, 0.3 \rangle}{N} \right\}, \\ F &= \left\{ \frac{\langle 0.2, 0.2, 0.1 \rangle}{KL}, \frac{\langle 0.2, 0.2, 0.1 \rangle}{LM}, \frac{\langle 0.2, 0.5, 0.1 \rangle}{MN}, \frac{\langle 0.2, 0.5, 0.1 \rangle}{NK} \right\}, \end{aligned} \quad (6)$$

as in Figure 2.

FIGURE 2: An edge regular m -polar fuzzy graph G .

Then, we have $d_G(KL) = d_G(LM) = d_G(MN) = d_G(NK) = (0.4, 0.7, 0.2)$.

Theorem 14. Let $G = (V, W, F)$ be an m -polar fuzzy graph on a cycle $G^* = (V, E)$. Then

$$\sum_{w_j \in V} d_G(w_j) = \sum_{w_j w_k \in E, j \neq k} d_G(w_j w_k). \quad (7)$$

Proof. Suppose that $G = (V, W, F)$ is an m -polar fuzzy graph and G^* is a cycle $w_1 w_2 w_3 \cdots w_n w_1$.

Now, we get

$$\begin{aligned} \sum_{j=1}^n d_G(w_j w_{j+1}) &= d_G(w_1 w_2) + d_G(w_2 w_3) + \cdots \\ &\quad + d_G(w_n w_1), \quad \text{where } w_{n+1} = w_1, \\ &= d_G(w_1) + d_G(w_2) - 2 \langle p_1 \circ F(w_1 w_2), p_2 \\ &\quad \circ F(w_1 w_2), \dots, p_m \circ F(w_1 w_2) \rangle + d_G(w_2) \\ &\quad + d_G(w_3) - 2 \langle p_1 \circ F(w_2 w_3), p_2 \circ F(w_2 w_3), \dots, \\ &\quad p_m \circ F(w_2 w_3) \rangle + \cdots + d_G(w_n) + d_G(w_1) - 2 \langle p_1 \\ &\quad \circ F(w_n w_1), p_2 \circ F(w_n w_1), \dots, p_m \circ F(w_n w_1) \rangle \\ &= 2 \sum_{w_j \in V} d_G(w_j) - 2 \sum_{j=1}^n \langle p_1 \circ F(w_j w_{j+1}), p_2 \\ &\quad \circ F(w_j w_{j+1}), \dots, p_m \circ F(w_j w_{j+1}) \rangle \\ &= \sum_{w_j \in V} d_G(w_j) + \sum_{w_j \in V} d_G(w_j) - 2 \sum_{j=1}^n \langle p_1 \\ &\quad \circ F(w_j w_{j+1}), p_2 \circ F(w_j w_{j+1}), \dots, p_m \end{aligned}$$

$$\begin{aligned} &\circ F(w_j w_{j+1}) \rangle = \sum_{w_j \in V} d_G(w_j) + 2 \sum_{j=1}^n \langle p_1 \\ &\circ F(w_j w_{j+1}), p_2 \circ F(w_j w_{j+1}), \dots, p_m \\ &\circ F(w_j w_{j+1}) \rangle - 2 \sum_{j=1}^n \langle p_1 \circ F(w_j w_{j+1}), p_2 \\ &\circ F(w_j w_{j+1}), \dots, p_m \circ F(w_j w_{j+1}) \rangle \\ &= \sum_{w_j \in V} d_G(w_j). \end{aligned} \quad (8)$$

Hence,

$$\sum_{w_j \in V} d_G(w_j) = \sum_{w_j w_k \in E, j \neq k} d_G(w_j w_k). \quad (9)$$

□

Remark 15. Let $G = (V, W, F)$ be an m -polar fuzzy graph on a crisp graph G^* . Then

$$\begin{aligned} \sum_{w_j w_k \in E} d_G(w_j w_k) &= \sum_{w_j w_k \in E} d_{G^*}(w_j w_k) \langle p_1 \\ &\circ F(w_j w_k), p_2 \circ F(w_j w_k), \dots, p_m \circ F(w_j w_k) \rangle, \end{aligned} \quad (10)$$

where $d_{G^*}(w_j w_k) = d_{G^*}(w_j) + d_{G^*}(w_k) - 2$, for all $w_j w_k \in E$.

Theorem 16. Let $G = (V, W, F)$ be an m -polar fuzzy graph on a c -regular crisp graph G^* . Then

$$\sum_{w_j w_k \in E} d_G(w_j w_k) = (c - 1) \sum_{w_j \in V} d_G(w_j). \quad (11)$$

Proof. From Remark 15, we have

$$\begin{aligned} \sum_{w_j w_k \in E} d_G(w_j w_k) &= \sum_{w_j w_k \in E} d_{G^*}(w_j w_k) \langle p_1 \\ &\circ F(w_j w_k), p_2 \circ F(w_j w_k), \dots, p_m \circ F(w_j w_k) \rangle \\ &= \sum_{w_j w_k \in E} (d_{G^*}(w_j) + d_{G^*}(w_k) - 2) \langle p_1 \\ &\circ F(w_j w_k), p_2 \circ F(w_j w_k), \dots, p_m \circ F(w_j w_k) \rangle. \end{aligned} \quad (12)$$

Since G^* is a regular crisp graph, we have the degree of every vertex in G^* as c .

That is, $d_{G^*}(w_j) = c$, so

$$\begin{aligned} \sum_{w_j w_k \in E} d_G(w_j w_k) &= (c + c - 2) \sum_{w_j w_k \in E} \langle p_1 \\ &\circ F(w_j w_k), p_2 \circ F(w_j w_k), \dots, p_m \circ F(w_j w_k) \rangle, \\ \sum_{w_j w_k \in E} d_G(w_j w_k) &= 2(c - 1) \sum_{w_j w_k \in E} \langle p_1 \circ F(w_j w_k), p_2 \end{aligned}$$

$$\begin{aligned} & \circ F(w_j w_k), \dots, p_m \circ F(w_j w_k) \rangle, \\ \sum_{w_j w_k \in E} d_G(w_j w_k) &= (c-1) \sum_{w_j w_k \in E} (d_G(w_j)) \end{aligned} \quad (13)$$

□

Theorem 17. Let $G = (V, W, F)$ be an m -polar fuzzy graph on a crisp graph G^* . Then

$$\begin{aligned} \sum_{w_j w_k \in E} td_G(w_j w_k) &= \sum_{w_j w_k \in E} d_{G^*}(w_j w_k) \langle p_1 \\ & \circ F(w_j w_k), p_2 \circ F(w_j w_k), \dots, p_m \circ F(w_j w_k) \rangle \\ &+ \sum_{w_j w_k \in E} \langle p_1 \circ F(w_j w_k), p_2 \circ F(w_j w_k), \dots, p_m \\ & \circ F(w_j w_k) \rangle. \end{aligned} \quad (14)$$

Proof. From the definition of total edge degree of G , we get

$$\begin{aligned} \sum_{w_j w_k \in E} td_G(w_j w_k) &= \sum_{w_j w_k \in E} (d_G(w_j w_k) + \langle p_1 \\ & \circ F(w_j w_k), p_2 \circ F(w_j w_k), \dots, p_m \circ F(w_j w_k) \rangle) \\ &= \left(\sum_{w_j w_k \in E} d_G(w_j w_k) + \sum_{w_j w_k \in E} \langle p_1 \circ F(w_j w_k), p_2 \right. \\ & \left. \circ F(w_j w_k), \dots, p_m \circ F(w_j w_k) \rangle \right). \end{aligned} \quad (15)$$

From Remark 15, we have

$$\begin{aligned} \sum_{w_j w_k \in E} td_G(w_j w_k) &= \sum_{w_j w_k \in E} d_{G^*}(w_j w_k) \langle p_1 \\ & \circ F(w_j w_k), p_2 \circ F(w_j w_k), \dots, p_m \circ F(w_j w_k) \rangle \\ &+ \sum_{w_j w_k \in E} \langle p_1 \circ F(w_j w_k), p_2 \circ F(w_j w_k), \dots, p_m \\ & \circ F(w_j w_k) \rangle. \end{aligned} \quad (16)$$

□

Theorem 18. Let $G = (V, W, F)$ be an m -polar fuzzy graph. Then the function $\langle p_1 \circ F, p_2 \circ F, \dots, p_m \circ F \rangle$ is a constant function if and only if the following conditions are equivalent.

- (i) G is an edge regular m -polar fuzzy graph.
- (ii) G is a totally edge regular m -polar fuzzy graph.

Proof. Suppose that $\langle p_1 \circ F, p_2 \circ F, \dots, p_m \circ F \rangle$ is a constant function. Then

$$\begin{aligned} & \langle p_1 \circ F(xy), p_2 \circ F(xy), \dots, p_m \circ F(xy) \rangle \\ &= \langle k_1, k_2, \dots, k_m \rangle \quad \forall xy \in E, \end{aligned} \quad (17)$$

where $k_1, k_2, \dots, k_m$ are constants and $k_1, k_2, \dots, k_m \in [0, 1]$. Let G be an edge regular m -polar fuzzy graph. Then, for all $w_j w_k \in E$, $d_G(w_j w_k) = \langle r_1, r_2, \dots, r_m \rangle$.

Now we have to show that G is a totally edge regular m -polar fuzzy graph.

Now

$$\begin{aligned} td_G(w_j w_k) &= d_G(w_j w_k) + \langle p_1 \circ F(w_j w_k), p_2 \\ & \circ F(w_j w_k), \dots, p_m \circ F(w_j w_k) \rangle = \langle r_1, r_2, \dots, r_m \rangle \\ &+ \langle k_1, k_2, \dots, k_m \rangle = \langle r_1 + k_1, r_2 + k_2, \dots, r_m \\ &+ k_m \rangle \quad \forall w_j w_k \in E. \end{aligned} \quad (18)$$

Thus G is a totally edge regular graph.

Now, let G be a $\langle h_1, h_2, \dots, h_m \rangle$ -totally edge regular m -polar fuzzy graph. Then

$$td_G(w_j w_k) = \langle h_1, h_2, \dots, h_m \rangle \quad \forall w_j w_k \in E. \quad (19)$$

So, we have

$$\begin{aligned} td_G(w_j w_k) &= d_G(w_j w_k) + \langle p_1 \circ F(w_j w_k), p_2 \\ & \circ F(w_j w_k), \dots, p_m \circ F(w_j w_k) \rangle \\ &= \langle h_1, h_2, \dots, h_m \rangle. \end{aligned} \quad (20)$$

Hence

$$\begin{aligned} d_G(w_j w_k) &= \langle h_1, h_2, \dots, h_m \rangle - \langle p_1 \circ F(w_j w_k), p_2 \\ & \circ F(w_j w_k), \dots, p_m \circ F(w_j w_k) \rangle = \langle h_1 - p_1 \\ & \circ F(w_j w_k), h_2 - p_2 \circ F(w_j w_k), \dots, h_m - p_m \\ & \circ F(w_j w_k) \rangle = \langle h_1 - k_1, h_2 - k_2, \dots, h_m - k_m \rangle. \end{aligned} \quad (21)$$

Then G is an $\langle h_1 - k_1, h_2 - k_2, \dots, h_m - k_m \rangle$ -edge regular m -polar fuzzy graph.

Conversely, suppose that G is an edge regular m -polar fuzzy graph and G is a totally edge regular m -polar fuzzy graph which are equivalent. We have to prove that $\langle p_1 \circ F, p_2 \circ F, \dots, p_m \circ F \rangle$ is a constant function. In a contrary way, we suppose that $\langle p_1 \circ F, p_2 \circ F, \dots, p_m \circ F \rangle$ is not a constant function. Then

$$\begin{aligned} & \langle p_1 \circ F(w_j w_k), p_2 \circ F(w_j w_k), \dots, p_m \circ F(w_j w_k) \rangle \\ & \neq \langle p_1 \circ F(w_r w_s), p_2 \circ F(w_r w_s), \dots, p_m \\ & \circ F(w_r w_s) \rangle \end{aligned} \quad (22)$$

for at least one pair of edges $w_j w_k, w_r w_s \in E$. Let G be an $\langle r_1, r_2, \dots, r_m \rangle$ -edge regular m -polar fuzzy graph. Then

$d_G(w_j w_k) = d_G(w_r w_s) = \langle r_1, r_2, \dots, r_m \rangle$. Hence, for every $w_j w_k \in E$ and for every $w_r w_s \in E$,

$$\begin{aligned} td_G(w_j w_k) &= d_G(w_j w_k) + \langle p_1 \circ F(w_j w_k), p_2 \\ &\quad \circ F(w_j w_k), \dots, p_m \circ F(w_j w_k) \rangle \\ &= \langle r_1, r_2, \dots, r_m \rangle + \langle p_1 \circ F(w_j w_k), p_2 \\ &\quad \circ F(w_j w_k), \dots, p_m \circ F(w_j w_k) \rangle = \langle r_1 + p_1 \\ &\quad \circ F(w_j w_k), r_2 + p_2 \circ F(w_j w_k), \dots, r_m + p_m \\ &\quad \circ F(w_j w_k) \rangle, \end{aligned} \quad (23)$$

$$\begin{aligned} td_G(w_r w_s) &= d_G(w_r w_s) + \langle p_1 \circ F(w_r w_s), p_2 \\ &\quad \circ F(w_r w_s), \dots, p_m \circ F(w_r w_s) \rangle = \langle r_1, r_2, \dots, r_m \rangle \\ &\quad + \langle p_1 \circ F(w_r w_s), p_2 \circ F(w_r w_s), \dots, p_m \\ &\quad \circ F(w_r w_s) \rangle = \langle r_1 + p_1 \circ F(w_r w_s), r_2 + p_2 \\ &\quad \circ F(w_r w_s), \dots, r_m + p_m \circ F(w_r w_s) \rangle. \end{aligned}$$

Since

$$\begin{aligned} &\langle p_1 \circ F(w_j w_k), p_2 \circ F(w_j w_k), \dots, p_m \circ F(w_j w_k) \rangle \\ &\neq \langle p_1 \circ F(w_r w_s), p_2 \circ F(w_r w_s), \dots, p_m \\ &\quad \circ F(w_r w_s) \rangle, \end{aligned} \quad (24)$$

we have $td_G(w_j w_k) \neq td_G(w_r w_s)$. Hence, G is not a totally edge regular m -polar fuzzy graph. This is a contradiction to our assumption. Hence, $\langle p_1 \circ F, p_2 \circ F, \dots, p_m \circ F \rangle$ is a constant function. In the same way, we can prove that $\langle p_1 \circ F, p_2 \circ F, \dots, p_m \circ F \rangle$ is a constant function, when G is a totally edge regular m -polar fuzzy graph. $\square$

Theorem 19. Let G^* be a h -regular crisp graph and $G = (V, W, F)$ be an m -polar fuzzy graph on G^* . Then, $\langle p_1 \circ F, p_2 \circ F, \dots, p_m \circ F \rangle$ is a constant function if and only if G is both regular m -polar fuzzy graph and totally edge regular m -polar fuzzy graph.

Proof. Let $G = (V, W, F)$ be an m -polar fuzzy graph on G^* and let G^* be a h -regular crisp graph. Assume that $\langle p_1 \circ F, p_2 \circ F, \dots, p_m \circ F \rangle$ is a constant function. Then

$$\begin{aligned} &\langle p_1 \circ F(xy), p_2 \circ F(xy), \dots, p_m \circ F(xy) \rangle \\ &= \langle k_1, k_2, \dots, k_m \rangle \quad \forall xy \in E, \end{aligned} \quad (25)$$

where $k_1, k_2, \dots, k_m$ are constants and $k_1, k_2, \dots, k_m \in [0, 1]$. From the definition of degree of a vertex, we get

$$\begin{aligned} d_G(w_j) &= \sum_{w_j w_k \in E} \langle p_1 \circ F(w_j w_k), p_2 \\ &\quad \circ F(w_j w_k), \dots, p_m \circ F(w_j w_k) \rangle \end{aligned}$$

$$= \sum_{w_j w_k \in E} \langle k_1, k_2, \dots, k_m \rangle = \langle hk_1, hk_2, \dots, hk_m \rangle$$

for every $w_j \in V$.

(26)

So $d_G(w_j) = \langle hk_1, hk_2, \dots, hk_m \rangle$. Therefore, G is regular m -polar fuzzy graph.

Now, for $i = 1, 2, \dots, m$,

$$\begin{aligned} td_G(w_j w_k) &= \sum (p_i \circ F(w_j w_k)) \\ &\quad + \sum (p_i \circ F(w_k w_l)) \\ &\quad + (p_i \circ F(w_j w_l)) \\ &= \sum_{\substack{w_j w_k \in E \\ j \neq k}} \langle k_1, k_2, \dots, k_m \rangle \\ &\quad + \sum_{\substack{w_k w_l \in E \\ k \neq l}} \langle k_1, k_2, \dots, k_m \rangle \\ &\quad + \langle k_1, k_2, \dots, k_m \rangle \\ &= (h-1) \langle k_1, k_2, \dots, k_m \rangle \\ &\quad + (h-1) \langle k_1, k_2, \dots, k_m \rangle \\ &\quad + \langle k_1, k_2, \dots, k_m \rangle \\ &= (2h-1) \langle k_1, k_2, \dots, k_m \rangle \quad \forall w_j w_k \in E. \end{aligned} \quad (27)$$

Hence, G is also a totally edge regular m -polar fuzzy graph.

Conversely, assume that G is both regular and totally edge regular m -polar fuzzy graph. Now we have to prove that $\langle p_1 \circ F, p_2 \circ F, \dots, p_m \circ F \rangle$ is a constant function. Since G is regular, $d_G(w_j) = \langle l_1, l_2, \dots, l_m \rangle$ for all $w_j \in V$. Also G is totally edge regular. Hence, $td_G(w_j w_k) = \langle h_1, h_2, \dots, h_m \rangle$ for all $w_j w_k \in E$. From the definition of total edge degree, we get

$$\begin{aligned} td_G(w_j w_k) &= d_G(w_j) + d_G(w_k) - \langle p_1 \\ &\quad \circ F(w_j w_k), p_2 \circ F(w_j w_k), \dots, p_m \circ F(w_j w_k) \rangle \\ &\quad \forall w_j w_k \in E, \end{aligned} \quad (28)$$

$$\begin{aligned} \langle h_1, h_2, \dots, h_m \rangle &= \langle l_1, l_2, \dots, l_m \rangle + \langle l_1, l_2, \dots, l_m \rangle \\ &\quad - \langle p_1 \circ F(w_j w_k), p_2 \circ F(w_j w_k), \dots, p_m \\ &\quad \circ F(w_j w_k) \rangle, \end{aligned}$$

so

$$\begin{aligned} &\langle p_1 \circ F(w_j w_k), p_2 \circ F(w_j w_k), \dots, p_m \circ F(w_j w_k) \rangle \\ &= 2 \langle l_1, l_2, \dots, l_m \rangle - \langle h_1, h_2, \dots, h_m \rangle \\ &= \langle 2l_1 - h_1, 2l_2 - h_2, \dots, 2l_m - h_m \rangle \quad \forall w_j w_k \in E. \end{aligned} \quad (29)$$

Hence $\langle p_1 \circ F, p_2 \circ F, \dots, p_m \circ F \rangle$ is a constant function. $\square$

FIGURE 3: h -morphism of m -polar fuzzy graphs G_1 and G_2 .

4. h -Morphism on m -Polar Fuzzy Graphs

Definition 20. Let $G_1 = (V_1, W_1, F_1)$ and $G_2 = (V_2, W_2, F_2)$ be two m -polar fuzzy graphs of the graphs $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$, respectively.

A homomorphism g from G_1 to G_2 is a mapping $g : V_1 \rightarrow V_2$ such that, for each $i = 1, 2, \dots, m$,

$$\begin{aligned} p_i \circ W_1(u) &\leq p_i \circ W_2(g(u)), \quad \forall u \in V_1, \\ p_i \circ F_1(uv) &\leq p_i \circ F_2(g(u)g(v)), \quad \forall uv \in E_1. \end{aligned} \quad (30)$$

An isomorphism g from G_1 to G_2 is a bijective mapping $g : V_1 \rightarrow V_2$ which satisfies the following conditions:

$$\begin{aligned} p_i \circ W_1(u) &= p_i \circ W_2(g(u)), \quad \forall u \in V_1, \\ p_i \circ F_1(uv) &= p_i \circ F_2(g(u)g(v)), \quad \forall uv \in E_1. \end{aligned} \quad (31)$$

A weak isomorphism g from G_1 to G_2 is a bijective mapping $g : V_1 \rightarrow V_2$ which satisfies the following conditions:

$$g \text{ is homomorphism and } p_i \circ W_1(u) = p_i \circ W_2(g(u)), \quad \forall u \in V_1.$$

A coveak isomorphism g from G_1 to G_2 is a bijective mapping $g : V_1 \rightarrow V_2$ which satisfies the following conditions:

$$g \text{ is homomorphism and } p_i \circ F_1(uv) = p_i \circ F_2(g(u)g(v)), \quad \forall uv \in E_1.$$

Definition 21. The order of an m -polar fuzzy graph G is defined as

$$\begin{aligned} O(G) &= \sum_{u \in V} p_i \circ W(u) = \left\langle \sum_{u \in V} p_1 \circ W(u), \right. \\ &\quad \left. \sum_{u \in V} p_2 \circ W(u), \dots, \sum_{u \in V} p_m \circ W(u) \right\rangle. \end{aligned} \quad (32)$$

Definition 22. The size of an m -polar fuzzy graph G is defined as

$$S(G) = \sum_{\substack{u \neq v \\ uv \in E}} p_i \circ F(uv) = \left\langle \sum_{\substack{u \neq v \\ uv \in E}} p_1 \circ F(uv), \right. \\ \left. \sum_{\substack{u \neq v \\ uv \in E}} p_2 \circ F(uv), \dots, \sum_{\substack{u \neq v \\ uv \in E}} p_m \circ F(uv) \right\rangle. \quad (33)$$

Definition 23. Let G_1 and G_2 be two m -polar graphs on (V_1, E_1) and (V_2, E_2) , respectively.

A bijective function $h : V_1 \rightarrow V_2$ is called an m -polar morphism or m -polar h -morphism if there exists two numbers $l_1 > 0$ and $l_2 > 0$ such that $p_i \circ W_2(h(u)) = l_1 p_i \circ W_1(u)$, $\forall u \in V_1$, $p_i \circ F_2(h(u)h(v)) = l_2 p_i \circ F_1(uv)$, $\forall uv \in E_1$, $i = 1, 2, \dots, m$. In such a case, h will be called an (l_1, l_2) m -polar h -morphism from G_1 to G_2 . If $l_1 = l_2 = l$, we call h , an m -polar l -morphism.

Example 24. Consider two m -polar fuzzy graphs $G_1 = (V_1, W_1, F_1)$ and $G_2 = (V_2, W_2, F_2)$ as shown in Figure 3.

An m -polar fuzzy graph $G_1 = (V_1, W_1, F_1)$ is shown in Figure 3(a) where

$$\begin{aligned} V_1 &= \{K, L, M\}, \\ E_1 &= \{KL, LM, MK\}, \\ W_1 &= \left\{ \frac{\langle 0.3, 0.45, 0.2 \rangle}{K}, \frac{\langle 0.3, 0.5, 0.3 \rangle}{L}, \frac{\langle 0.4, 0.46, 0.3 \rangle}{M} \right\}, \\ F_1 &= \left\{ \frac{\langle 0.2, 0.3, 0.1 \rangle}{KL}, \frac{\langle 0.1, 0.2, 0.2 \rangle}{LM}, \frac{\langle 0.2, 0.3, 0.1 \rangle}{MK} \right\}. \end{aligned} \quad (34)$$

Another m -polar fuzzy graph $G_2 = (V_2, W_2, F_2)$ is shown in Figure 3(b) where

$$\begin{aligned} V_2 &= \{K', L', M'\}, \\ E_2 &= \{K'L', L'M', M'K'\}, \end{aligned}$$

$$\begin{aligned}
W_2 &= \left\{ \frac{\langle 0.6, 0.9, 0.4 \rangle}{K'}, \frac{\langle 0.6, 1.0, 0.6 \rangle}{L'}, \frac{\langle 0.8, 0.92, 0.6 \rangle}{M'} \right\}, \\
F_2 &= \left\{ \frac{\langle 0.6, 0.9, 0.3 \rangle}{K'L'}, \frac{\langle 0.3, 0.6, 0.6 \rangle}{L'M'}, \frac{\langle 0.6, 0.9, 0.3 \rangle}{M'K'} \right\}.
\end{aligned} \tag{35}$$

Here, there is an m -polar h -morphism such that $h(K) = K'$, $h(L) = L'$, $h(M) = M'$, $l_1 = 2$, and $l_2 = 3$.

Theorem 25. *The relation h -morphism is an equivalence relation in the collection of m -polar fuzzy graphs.*

Proof. Consider the collection of m -polar fuzzy graphs. Define the relation $G_1 \approx G_2$ if there exists a (l_1, l_2) h -morphism from G_1 to G_2 where both $l_1 \neq 0$ and $l_2 \neq 0$. Consider the identity morphism G_1 to G_1 . It is a $(1, 1)$ -morphism from G_1 to G_1 and hence $\approx$ is reflexive. Let $G_1 \approx G_2$. Then there exists a (l_1, l_2) morphism from G_1 to G_2 for some $l_1 \neq 0$ and $l_2 \neq 0$. Therefore,

$$\begin{aligned}
p_i \circ W_2(h(u)) &= l_1 p_i \circ W_1(u), \quad \forall u \in V_1, \\
p_i \circ F_2(h(u)h(v)) &= l_2 p_i \circ F_1(uv), \quad \forall uv \in E_1.
\end{aligned} \tag{36}$$

Consider $h^{-1} : G_2 \rightarrow G_1$. Let $m, n \in V_2$. Since h^{-1} is bijective, $m = h(u)$, $n = h(v)$, for some $u, v \in V_2$. Now,

$$\begin{aligned}
p_i \circ W_1(h^{-1}(m)) &= p_i \circ W_1(h^{-1}(h(u))) \\
&= p_i \circ W_1(u) = \frac{1}{l_1} p_i \circ W_2(h(u)) = \frac{1}{l_1} p_i \circ W_2(m), \\
p_i \circ F_1(h^{-1}(m)h^{-1}(n)) &= p_i \circ F_1(h^{-1}(h(u)h(v))) = p_i \circ F_1(uv) \\
&= \frac{1}{l_2} p_i \circ F_2(h(u)h(v)) = \frac{1}{l_2} p_i \circ F_2(mn).
\end{aligned} \tag{37}$$

Thus there exists $(1/l_1, 1/l_2)$ morphism from G_2 to G_1 . Therefore, $G_2 \approx G_1$ and hence $\approx$ is symmetric.

Let $G_1 \approx G_2$ and $G_2 \approx G_3$. Then there exists a (l_1, l_2) morphism from G_1 to G_2 , say h for some $l_1 \neq 0$ and $l_2 \neq 0$, and there exists (l_3, l_4) morphism from G_2 to G_3 , say q for some $l_3 \neq 0$ and $l_4 \neq 0$. So, for $i = 1, 2, \dots, m$,

$$\begin{aligned}
p_i \circ W_3(q(x)) &= l_3 p_i \circ W_2(h(x)), \quad x \in V_2, \\
p_i \circ F_3(q(x)q(y)) &= l_4 p_i \circ F_2(h(x)h(y)), \quad \forall xy \in E_2.
\end{aligned} \tag{38}$$

Let $r : q \circ p : G_1 \rightarrow G_3$.

Now,

$$\begin{aligned}
p_i \circ W_3(r(u)) &= p_i \circ W_3((q \circ h)(u)) \\
&= p_i \circ W_3(q(h(u))) \\
&= l_3 p_i \circ W_2(h(u)) \\
&= l_3 l_1 p_i \circ W_1(u), \\
p_i \circ F_3(r(u)r(v)) &= p_i \circ F_3((q \circ h)(u)(q \circ h)(v)) \\
&= p_i \circ F_3(q(h(u))q(h(v))) \\
&= l_4 p_i \circ F_2(h(u)h(v)) \\
&= l_4 l_2 p_i \circ F_1(uv).
\end{aligned} \tag{39}$$

Thus there exists $(l_3 l_1, l_4 l_2)$ morphism r from G_1 to G_3 . Therefore, $G_1 \approx G_3$ and hence $\approx$ is transitive. So, the relation h -morphism is an equivalence relation in the collection of m -polar fuzzy graphs. $\square$

Theorem 26. *Let G_1 and G_2 be two m -polar fuzzy graphs such that G_1 is (l_1, l_2) m -polar morphism to G_2 for some $l_1 \neq 0$ and $l_2 \neq 0$. The image of a strong edge in G_1 is also a strong edge in G_2 if and only if $l_1 = l_2$.*

Proof. Let xy be a strong edge in G_1 such that $h(x)h(y)$ is also a strong edge in G_2 .

Now as $G_1 \approx G_2$ for $i = 1, 2, \dots, m$, we have

$$\begin{aligned}
l_2 p_i \circ F_1(xy) &= p_i \circ F_2(h(x)h(y)) \\
&= p_i \circ W_2(h(x) \wedge h(y)) \\
&= l_1 p_i \circ W_1(x) \wedge l_1 p_i \circ W_1(y) \\
&= l_1 (p_i \circ W_1(x) \wedge p_i \circ W_1(y)) \\
&= l_1 p_i \circ F_1(xy) \quad \forall xy \in E_1.
\end{aligned} \tag{40}$$

Hence,

$$l_2 p_i \circ F_1(xy) = l_1 p_i \circ F_1(xy), \quad \forall xy \in E_1. \tag{41}$$

The equation holds if and only if $l_1 = l_2$. $\square$

Theorem 27. *If an m -polar fuzzy graph G_1 is coweak isomorphic to G_2 and if G_1 is regular then G_2 is also regular.*

Proof. As an m -polar fuzzy graph G_1 is coweak isomorphic to G_2 , there exists a coweak isomorphism $h : G_1 \rightarrow G_2$ which is bijective for $i = 1, 2, \dots, m$ that satisfies

$$\begin{aligned}
p_i \circ W_1(u) &\leq p_i \circ W_2(h(u)), \quad \forall u \in V_1, \\
p_i \circ F_1(uv) &= p_i \circ F_2(h(u)h(v)), \quad \forall uv \in E_1.
\end{aligned} \tag{42}$$

As G_1 is regular, for $u \in V$, $\sum_{u \neq v, v \in V_1} p_i \circ F_1(uv) = \text{constant}$. Now $\sum_{h(u) \neq h(v)} p_i \circ F_2(h(u)h(v)) = \sum_{u \neq v, v \in V_1} p_i \circ F_1(uv) = \text{constant}$. Therefore, G_2 is regular. $\square$

Theorem 28. *Let G_1 and G_2 be two m -polar fuzzy graphs. If G_1 is weak isomorphic to G_2 and if G_1 is strong then G_2 is also strong.*

Proof. As G_1 is an m -polar fuzzy graph which is weak isomorphic with G_2 , then there exists a weak isomorphism $h : G_1 \rightarrow G_2$ which is bijective for $i = 1, 2, \dots, m$ that satisfies

$$\begin{aligned} p_i \circ W_1(u) &= p_i \circ W_2(h(u)), \quad \forall u \in V_1, \\ p_i \circ F_1(uv) &\leq p_i \circ F_2(h(u)h(v)), \quad \forall uv \in E_1. \end{aligned} \quad (43)$$

As G_1 is strong, $p_i \circ F_1(uv) = \min(p_i \circ W_1(u), p_i \circ W_1(v))$. Now, we get

$$\begin{aligned} p_i \circ F_2(h(u)h(v)) &\geq p_i \circ F_1(uv) \\ &= \min(p_i \circ W_1(u), p_i \circ W_1(v)) \\ &= \min(p_i \circ W_2(h(u)), p_i \circ W_2(h(v))). \end{aligned} \quad (44)$$

By the definition, $p_i \circ F_2(h(u)h(v)) \leq \min(p_i \circ W_2(h(u)), p_i \circ W_2(h(v)))$. Therefore, $p_i \circ F_2(h(u)h(v)) = \min(p_i \circ W_2(h(u)), p_i \circ W_2(h(v)))$. So G_2 is strong. $\square$

Theorem 29. If an m -polar fuzzy graph G_1 is coweak isomorphic with a strong regular m -polar fuzzy graph G_2 , then G_1 is strong regular m -polar fuzzy graph.

Proof. As an m -polar fuzzy graph G_1 is coweak isomorphic to G_2 . Then there exists a coweak isomorphism $h : G_1 \rightarrow G_2$ which is bijective for $i = 1, 2, \dots, m$ that satisfies

$$\begin{aligned} p_i \circ W_1(u) &\leq p_i \circ W_2(h(u)), \quad \forall u \in V_1, \\ p_i \circ F_1(uv) &= p_i \circ F_2(h(u)h(v)), \quad \forall uv \in E_1. \end{aligned} \quad (45)$$

Now, we get

$$\begin{aligned} p_i \circ F_1(uv) &= p_i \circ F_2(h(u)h(v)) \\ &= \min(p_i \circ W_2(h(u)), p_i \circ W_2(h(v))) \\ &\geq \min(p_i \circ W_1(u), p_i \circ W_1(v)). \end{aligned} \quad (46)$$

But, by the definition, we have

$$p_i \circ F_1(uv) \leq \min(p_i \circ W_1(u), p_i \circ W_1(v)). \quad (47)$$

So, $p_i \circ F_1(uv) = \min(p_i \circ W_1(u), p_i \circ W_1(v))$.

Therefore, G_1 is strong. Also for $u \in V_1$, $\sum_{u \neq v, v \in V_1} p_i \circ F_1(uv) = \sum p_i \circ F_2(h(u)h(v)) = \text{constant}$ as G_2 is regular. Therefore, G_1 is regular. $\square$

Theorem 30. Let G_1 and G_2 be two isomorphic m -polar fuzzy graphs; then G_1 is strong regular if and only if G_2 is strong regular.

Proof. As an m -polar fuzzy graph G_1 is isomorphic with an m -polar fuzzy graph G_2 , there exists an isomorphism $h : G_1 \rightarrow G_2$ which is bijective for $i = 1, 2, \dots, m$ that satisfies

$$\begin{aligned} p_i \circ W_1(u) &= p_i \circ W_2(h(u)), \quad \forall u \in V_1, \\ p_i \circ F_1(uv) &= p_i \circ F_2(h(u)h(v)), \quad \forall uv \in E_1. \end{aligned} \quad (48)$$

Now, G_1 is strong if and only if $p_i \circ F_1(uv) = \min(p_i \circ W_1(u), p_i \circ W_1(v))$, if and only if $p_i \circ F_2(h(u)h(v)) = \min(p_i \circ W_2(h(u)), p_i \circ W_2(h(v)))$, and if and only if G_2 is strong.

FIGURE 4: A highly irregular m -polar fuzzy graph.

G_1 is regular if and only if, for $u \in V_1$, $\sum_{u \neq v, v \in V_1} p_i \circ F_1(uv) = \text{constant}$, if and only if $\sum_{p(u) \neq p(v)} p_i \circ F_2(h(u)h(v)) = \text{Constant}$, for all $h(u) \in V_2$, and if and only if G_2 is regular. $\square$

Definition 31. Let $G = (V, W, F)$ be a connected m -polar fuzzy graph. Then G is said to be a highly irregular m -polar fuzzy graph if every vertex of G is adjacent to vertices with distinct degrees.

Example 32. Consider an m -polar fuzzy graph $G = (V, W, F)$ of $G^* = (V, E)$, where

$$V = \{K, L, M, N\},$$

$$E = \{KL, LM, MN, NK\},$$

$$W = \left\{ \frac{\langle 0.3, 0.4, 0.5 \rangle}{K}, \frac{\langle 0.2, 0.5, 0.9 \rangle}{L}, \frac{\langle 0.5, 0.5, 0.5 \rangle}{M}, \frac{\langle 0.4, 0.3, 0.8 \rangle}{N} \right\}, \quad (49)$$

$$F = \left\{ \frac{\langle 0.1, 0.3, 0.4 \rangle}{KL}, \frac{\langle 0.2, 0.5, 0.5 \rangle}{LM}, \frac{\langle 0.3, 0.2, 0.5 \rangle}{MN}, \frac{\langle 0.1, 0.2, 0.4 \rangle}{NK} \right\},$$

as in Figure 4.

By usual calculations, we get

$$\begin{aligned} d_G(K) &= \langle 0.2, 0.5, 0.8 \rangle, \\ d_G(L) &= \langle 0.3, 0.8, 0.9 \rangle, \\ d_G(M) &= \langle 0.5, 0.7, 1.0 \rangle, \\ d_G(N) &= \langle 0.4, 0.4, 0.9 \rangle. \end{aligned} \quad (50)$$

We see that every vertex of G is adjacent to vertices with distinct degrees.

Theorem 33. For any two isomorphic highly irregular m -polar fuzzy graphs, their order and size are the same.

Proof. If $h : G_1 \rightarrow G_2$ is an isomorphism between the two highly irregular m -polar fuzzy graphs G_1 and G_2 with

the underlying sets V_1 and V_2 , respectively, then, for $i = 1, 2, \dots, m$,

$$\begin{aligned} p_i \circ W_1(u) &= p_i \circ W_2(h(u)), \quad \forall u \in V_1, \\ p_i \circ F_1(uv) &= p_i \circ F_2(h(u)h(v)), \quad \forall uv \in E_1. \end{aligned} \quad (51)$$

So, we get

$$\begin{aligned} O(G_1) &= \sum_{x_1 \in V_1} p_i \circ W_1(x_1) = \sum_{x_1 \in V_1} p_i \circ W_2(h(x_1)) \\ &= \sum_{x_2 \in V_2} p_i \circ W_2(x_2) = O(G_2), \\ S(G_1) &= \sum_{x_1 y_1 \in E_1} p_i \circ F_1(x_1 y_1) \\ &= \sum_{x_1 y_1 \in E_1} p_i \circ F_2(h(x_1)h(y_1)) \\ &= \sum_{x_2 y_2 \in E_2} p_i \circ F_2(x_2 y_2) = S(G_2). \end{aligned} \quad (52)$$

□

Theorem 34. *If G_1 and G_2 are isomorphic highly irregular m -polar fuzzy graphs, then, the degrees of the corresponding vertices u and $h(u)$ are preserved.*

Proof. If $h : G_1 \rightarrow G_2$ is an isomorphism between the highly irregular m -polar fuzzy graphs G_1 and G_2 with the underlying sets V_1 and V_2 , respectively, then, for $i = 1, 2, \dots, m$,

$$p_i \circ F_1(uv) = p_i \circ F_2(h(u)h(v)) \quad \forall u, v \in V_1. \quad (53)$$

Therefore,

$$\begin{aligned} d_{G_1}(u) &= \sum_{u,v \in V_1} p_i \circ F_1(uv) = \sum_{u,v \in V_1} p_i \circ F_2(h(u)h(v)) \\ &= d_{G_2}(h(u)). \end{aligned} \quad (54)$$

That is, the degrees of the corresponding vertices of G_1 and G_2 are the same. □

5. Conclusion

Any dissimilar fuzzy graph hypothesis needs large data for training to be able to help in decision-making which is crucial to utilitarian research in science and technology. The new method developed in this paper based on the pattern of unique cases helps us to make a better choice in contrast to the established fuzzy graph solutions. The concept of h -morphism, highly irregular m -polar fuzzy graphs is discussed in this paper.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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