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Research Article

Common Fixed Points of Intuitionistic Fuzzy Maps for Meir-Keeler Type Contractions

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The main purpose of this paper is to establish and prove some new common fixed point theorems for intuitionistic fuzzy maps in the context of (α, β)-cut sets of intuitionistic fuzzy sets on a complete metric space in association with the Hausdorff metric. Furthermore, the technique of Meir-Keeler (shortly, M-K) contraction is applied to obtain common fixed point of intuitionistic fuzzy compatible maps and fixed points of Kannan type intuitionistic fuzzy set-valued contractive mappings. Our results generalize M-K type fixed point theorem along with its various generalizations. Some nontrivial examples have been furnished in the support of the main results.

1. Introduction

The theory of fixed points has been revealed as a very powerful and important tool in the study of nonlinear phenomena. In particular, fixed point techniques have been applied in such diverse fields as biology, chemistry, economics, engineering, game theory, and physics. The Banach fixed point theorem [1] (also known as a contraction mapping principle) is an important tool in nonlinear analysis. It guarantees the existence and uniqueness of fixed points of self-mappings on complete metric spaces and provides a constructive method to find fixed points. Many extensions of this principle have been done up to now. In 1976, Jungck [2] studied coincidence and common fixed points of commuting mappings and improved the Banach contraction principle. In 1986, Jungck [3] introduced the notion of compatible maps for a pair of self-mappings and existence of common fixed points. In 1969, Meir and Keeler [4] obtained a valuable fixed point theorem for single valued mappings $\Phi : X \longrightarrow X$ that satisfies the following condition:

$$\begin{gathered}\text{Given } \epsilon > 0, \text{ there exists a } \delta > 0 \text{ such that} \\ \epsilon \leq d(x, y) < \epsilon + \delta \text{ implies } d(\Phi x, \Phi y) < \epsilon.\end{gathered} \tag{1}$$

In 1981, Park and Bae [5] extended it to a pair of commuting single valued mappings. A variety of extensions, generalizations, and applications of this followed; e.g., see [6, 7]. In 1993, Beg and Shahzad [8] derived and proved random fixed point of two random multivalued operators satisfying the M-K [4] condition in Polish spaces. In 2001, Lim [9] wrote on characterization of M-K-contractive maps. In 2012, Abdeljawad et al.'s paper [10] contains a study of M-K type coupled fixed point on ordered partial metric spaces and Chen et al. [11] established and proved common fixed point theorems for the stronger M-K cone-type function in cone ball-metric spaces. In 2013, Karapinar et al. [12] studied the existence and uniqueness of a fixed point of the multidimensional operators in partially ordered metric space which satisfied M-K type contraction condition and improved the results mentioned above and the recent results on these topics in the literature. In the same year, Abdeljawad [13] established and proved M-K-contractive fixed point and common fixed point theorems. Patel et al. [14] formulated and proved a more generalized version of [13]. In 2014, Singh et al. [15] derived a new common fixed point theorem for Suzuki-M-K contractions. In 2015, Redjel et al. [16] proved fixed point theorems for (α, ψ)-Meir-Keeler-Khan mappings.

Abtahi [17, 18] established and proved fixed point theorems in 2016 and common fixed point theorems in 2017 for M-K type contractions in metric spaces. Popa and Patriciu [19] derived and proved a general theorem of M-K type for mappings satisfying an implicit relation in partial metric spaces in 2017.

Fuzzy sets were introduced by Zadeh [20] in 1965 to represent/manipulate data and information possessing nonstatistical uncertainties. In 1986, the concept of an intuitionistic fuzzy set (IFS) was put forward by Atanassov [21], which can be viewed as an extension of fuzzy set. Intuitionistic fuzzy sets not only define the degree of membership of an element, but also characterize the degree of nonmembership. IFS has much attention due to its significance to remove the vagueness or uncertainty in decision-making. IFS is a tool in modeling real life problems such as psychological investigation and career determination. Abbasizadeh and Davvaz [22] introduced intuitionistic fuzzy topological polygroups. In 2017, Azam et al. [23] proved coincidence and fixed point theorems of intuitionistic fuzzy mappings with applications. In the same year, Azam and Tabassum [24] established and proved fixed point theorems of intuitionistic fuzzy mappings in quasi-pseudometric spaces. Recently, Kumam et al. [25] and Shoaib et al. [26] derived and proved some fuzzy fixed point results for fuzzy mappings in complete b-metric spaces. Humaira et al. [27] established and proved fuzzy fixed point results for $\Phi-$contractive mapping and gave some applications. Ertrk and Karakaya [28] stated and proved n-tuplet coincidence point theorems in intuitionistic fuzzy normed spaces. Xia Li et al. [29] worked on the intuitionistic fuzzy metric spaces and the intuitionistic fuzzy normed spaces (see also [30–32]).

In this paper, the main focus is to establish the existence of fixed point and common fixed point theorems of M-K type contraction for intuitionistic fuzzy set-valued maps in complete metric spaces. Some nontrivial examples have been furnished in the support of the main results.

2. Preliminaries

We start this section by recalling some pertinent concepts.

Definition 1 (see [33]). Let (X, d) be a metric space. The set of all nonempty closed and bounded subsets of X is denoted by $CB(X)$. The function H defined on $CB(X) \times CB(X)$ by

$$H(A, B) = \max\left(\sup_{a\in A} d(a, B), \sup_{b\in B} d(A, b)\right) \tag{2}$$

for all $A, B \in CB(X)$ is a metric on $CB(X)$ called the Hausdorff metric of d,

where

$$d(x, A) = \inf_{y\in A} d(x, y). \tag{3}$$

Definition 2 (see [20]). Let X be an arbitrary nonempty set. A fuzzy set in X is a function with domain X and values in $[0, 1]$. If A is a fuzzy set and $x \in X$, then the function-value $A(x)$ is called the grade of membership of x in A. I^X stands for the collection of all fuzzy sets in X unless and until stated otherwise.

Definition 3 (see [21]). Let X be a nonempty set. An intuitionistic fuzzy set is defined as

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}, \tag{4}$$

where $\mu_A : X \longrightarrow [0, 1]$ and $\nu_A : X \longrightarrow [0, 1]$ denote the degree of membership and degree of nonmembership of each element x to the set A, respectively, such that

$$0 \le \mu_A(x) + \nu_A(x) \le 1, \quad \forall x, y \in X. \tag{5}$$

The collection of all intuitionistic fuzzy sets in X is denoted by $(IFS)^X$.

Definition 4 (see [34]). Let A be an intuitionistic fuzzy set and $x \in X$; then $\alpha-$level set of an intuitionistic fuzzy set A is denoted by $[A]_\alpha$ and is defined as

$$[A]_\alpha = \{x \in X : \mu_A(x) \ge \alpha,\ \nu_A(x) \le 1 - \alpha\}, \quad \text{if } \alpha \in (0, 1]. \tag{6}$$

A generalized version of $\alpha-$level set of an intuitionistic fuzzy set A was investigated in [35, 36].

Definition 5 (see [35, 36]). Let $L = \{(\alpha, \beta) : \alpha+\beta \le 1,\ (\alpha, \beta) \in (0, 1] \times [0, 1)\}$ and let A be an IFS on X; then $(\alpha, \beta)-$cut set of A is defined as

$$A_{(\alpha,\beta)} = \{x \in X : \mu_A(x) \ge \alpha,\ \nu_A(x) \le \beta\}. \tag{7}$$

Definition 6 (see [23]). Let X be an arbitrary set and let Y be a metric space. A mapping $S : X \longrightarrow (IFS)^Y$ is called an intuitionistic fuzzy mapping.

Definition 7. Mappings $\Phi : X \longrightarrow (IFS)^X$ and $\psi : X \longrightarrow X$ are said to be (α, β) compatible if whenever there is a sequence $\{x_n\} \subseteq X$ satisfying $\lim_{n\longrightarrow\infty}\psi x_n \in \lim_{n\longrightarrow\infty}[\Phi x_n]_{(\alpha,\beta)}$ (provided $\lim_{n\longrightarrow\infty}\psi x_n$ and $\lim_{n\longrightarrow\infty}[\Phi x_n]_{(\alpha,\beta)}$ exist and $\psi[\Phi x_n]_{(\alpha,\beta)} \in CB(X)$), then $\lim_{n\longrightarrow\infty}H(\psi[\Phi x_n]_{(\alpha,\beta)}, [\Phi\psi x_n]_{(\alpha,\beta)}) = 0$.

Lemma 8 (see [37]). *Let $\{\Upsilon_n\}$ be a sequence in $CB(X)$ and $H(\Upsilon_n, \Upsilon) \longrightarrow 0$ for $\Upsilon \in CB(X)$. If $x_n \in \Upsilon_n$ and $\lim_{n\longrightarrow\infty}d(x_n, x) = 0$, then $x \in \Upsilon$.*

3. Main Results

Theorem 9. *Let X be a complete metric space and let $\Phi : X \longrightarrow (IFS)^X$, $\psi : X \longrightarrow X$ be (α, β) compatible mappings. Suppose for each $x \in X$ there exists $(\alpha, \beta) \in (0, 1] \times [0, 1)$*

such that $[\Phi x]_{(\alpha,\beta)} \in CB(X)$ *and* $\cup_{x\in X}[\Phi x]_{(\alpha,\beta)} \subseteq \psi X$ *and the following condition is satisfied:*

$$\text{for } \epsilon > 0 \text{ there exists a } \delta > 0 \text{ such that} \tag{8}$$

$$\epsilon \le d(\psi x, \psi y) < \epsilon + \delta \text{ implies } d(u, v) < \epsilon, \tag{9}$$

$$\begin{aligned} &u \in [\Phi x]_{(\alpha,\beta)}, \\ &v \in [\Phi y]_{(\alpha,\beta)}, \\ &[\Phi x]_{(\alpha,\beta)} = [\Phi y]_{(\alpha,\beta)} \\ &\qquad\qquad \text{when } \psi x = \psi y. \end{aligned} \tag{10}$$

If ψ *is continuous, then* Φ *and* ψ *have a common fixed point.*

Proof. Let $x_0 \in X$, and consider the following sequences x_n and y_n in X and $\Upsilon_n \in CB(X)$, $y_n = \psi x_n \in [\Phi x_{n-1}]_{(\alpha,\beta)}$, $n \ge 0$ (which is possible due to the hypothesis $\cup_{x\in X}[\Phi x]_{(\alpha,\beta)} \subseteq \psi X$). Then for each $\epsilon > 0$ there exists $\delta > 0$ such that $\epsilon \le (\psi x_m, \psi x_n) < \epsilon + \delta$ implies $d(\psi x_{m+1}, \psi x_{n+1}) < \epsilon$. It follows that $d(y_n, y_{n+1}) < d(y_{n-1}, y_n)$. Thus, the sequence $\{d(y_n, y_{n+1})\}$ is nonincreasing and converges to the greatest lower bound of its range, which we denote by l.

Now $l \ge 0$; in fact, $l = 0$. Otherwise, if $l > 0$, take N so that $n \ge N$ implies $l \le d(y_n, y_{n+1}) < l + \delta$. It implies that $d(y_{n+1}, y_{n+2}) < l$ which contradicts the fact that $l = \inf_n d(y_{n+1}, y_{n+2})$. Hence $d(\psi x_n, [\Phi x_n]_{(\alpha,\beta)}) \le d(\psi x_n, \psi x_{n+1}) \longrightarrow 0$. Now it is to prove that $\{y_n\}$ is a Cauchy sequence. Suppose that $d(y_n, y_{n+1}) = 0$ for some $n > 0$. Then $d(y_m, y_{m+1}) = 0$ for all $m > n$; otherwise $d(y_n, y_{n+1}) = 0 < d(y_{n+1}, y_{n+2})$, a contradiction. Hence, $\{y_n\}$ is a Cauchy sequence.

Now assume that $d(y_n, y_{n+1}) \ne 0$ for each n. Define $\zeta = 2\epsilon$ and choose (without loss of generality) δ, $0 < \delta < \epsilon$, such that (9) is satisfied. Since $d(y_n, y_{n+1}) \longrightarrow 0$, there exists an integer N such that $(y_i, y_{i+1}) < \delta/6$ for $i \ge N$. We now let $q > p > N$ and show that $d(y_p, y_q) \le \zeta$, to prove that $\{y_n\}$ is indeed Cauchy. Suppose that

$$d(y_p, y_q) \ge 2\epsilon = \zeta. \tag{11}$$

First, we show that there exists an integer $m > p$ such that

$$\epsilon + \frac{\delta}{3} < d(y_p, y_m) < \epsilon + \delta, \tag{12}$$

where p and m are of opposite parity. Let k be the smallest integer greater than p such that

$$d(y_p, y_k) > \epsilon + \frac{\delta}{2} \tag{13}$$

(which is possible due to (11) as $\delta < \epsilon$). Moreover,

$$d(y_p, y_k) < \epsilon + \frac{2\delta}{3}. \tag{14}$$

For otherwise,

$$\epsilon + \frac{2\delta}{3} \le d(y_p, y_{k-1}) + d(y_{k-1}, y_k). \tag{15}$$

Since $k - 1 \ge p \ge N$, therefore $d(y_{k-1}, y_k) < \delta/6$. It implies that

$$d(y_p, y_{k-1}) > \epsilon + \frac{\delta}{2}, \tag{16}$$

which contradicts the fact that k is the smallest such that (13) is satisfied. Thus,

$$\epsilon + \frac{\delta}{2} < d(y_p, y_k) < \epsilon + \frac{2\delta}{3}. \tag{17}$$

If p and k are of opposite parity, we can take $k = m$ in (17) to obtain (12). If p and k are of the same parity, p and $k + 1$ are of opposite parity. In this case,

$$\begin{aligned} d(y_p, y_{k+1}) &\le d(y_p, y_k) + d(y_k, y_{k+1}) \le \epsilon + \frac{2\delta}{3} + \frac{\delta}{6} \\ &= \epsilon + \frac{5\delta}{6}. \end{aligned} \tag{18}$$

Moreover,

$$\begin{aligned} &d(y_p, y_k) \le d(y_p, y_{k+1}) + d(y_{k+1}, y_k), \\ &d(y_p, y_k) - d(y_{k+1}, y_k) \le d(y_p, y_{k+1}), \\ &\epsilon + \frac{\delta}{2} - \frac{\delta}{6} < d(y_p, y_{k+1}), \\ &\epsilon + \frac{\delta}{3} < d(y_p, y_{k+1}). \end{aligned} \tag{19}$$

Thus,

$$\epsilon + \frac{\delta}{3} < d(y_p, y_{k+1}) < \epsilon + \frac{5\delta}{6}. \tag{20}$$

Putting $m = k + 1$, we obtain (12). Hence (12) holds. Now,

$$\begin{aligned} \epsilon + \frac{\delta}{3} &< d(y_p, y_m) \\ &\le d(y_p, y_{p+1}) + d(y_{p+1}, y_{m+1}) + d(y_{m+1}, y_m) \\ &< \frac{\delta}{6} + \epsilon + \frac{\delta}{6} = \epsilon + \frac{\delta}{3}, \end{aligned} \tag{21}$$

a contradiction.

Hence $\{y_n\} = \{\psi x_n\}$ is a Cauchy sequence. By completeness of the space, there exists an element $z \in X$ such that $d(y_n, z) \longrightarrow 0$; continuity of ψ implies that $d(\psi y_n, \psi z) \longrightarrow 0$. Hence, $H([\Phi y_n]_{(\alpha,\beta)}, [\Phi z]_{(\alpha,\beta)}) \le \sup\{d(u, v) : u \in [\Phi y_n]_{(\alpha,\beta)}, v \in [\Phi z]_{(\alpha,\beta)}\} < d(\psi y_n, \psi z) \longrightarrow 0$.

Since $\{\psi x_n\}$ is a Cauchy sequence in X and

$$\begin{aligned} H(\Upsilon_m, \Upsilon_n) &= H([\Phi x_m]_{(\alpha,\beta)}, [\Phi x_n]_{(\alpha,\beta)}) \\ &\le \sup\{d(u, v) : u \in [\Phi x_m]_{(\alpha,\beta)}, v \in [\Phi x_n]_{(\alpha,\beta)}\} \\ &< d(\psi x_m, \psi x_n), \end{aligned} \tag{22}$$

it follows that Υ_n is a Cauchy sequence in $CB(X)$. By completeness of $CB(X)$, there exists $\Upsilon \in CB(X)$ such that

$H(\Upsilon_n, \Upsilon) \longrightarrow 0$. Since $y_{n+1} \in \Upsilon_n$ and $d(y_{n+1}, z) \longrightarrow 0$, Lemma 8 implies that $z \in \Upsilon$, that is, $\lim_{n\longrightarrow\infty}\psi x_n \in \lim_{n\longrightarrow\infty}[\Phi x_n]_{(\alpha,\beta)}$. Compatibility of ψ and Φ further implies that

$$\lim_{n\longrightarrow\infty} H\left(\psi\left[\Phi x_n\right]_{(\alpha,\beta)}, \left[\Phi\psi x_n\right]_{(\alpha,\beta)}\right) = 0. \tag{23}$$

Since $d(\psi y_{n+1}, [\Phi y_n]_{\alpha,\beta}) \leq H(\psi[\Phi x_n]_{(\alpha,\beta)}, [\Phi\psi x_n]_{(\alpha,\beta)})$, therefore $\psi z \in [\Phi z]_{\alpha,\beta}$, that is, $\lim_{n\longrightarrow\infty}\psi y_n \in \lim_{n\longrightarrow\infty}[\Phi y_n]_{(\alpha,\beta)}$ and $\lim_{n\longrightarrow\infty}H(\psi[\Phi x_n]_{(\alpha,\beta)}, [\Phi\psi x_n]_{(\alpha,\beta)}) = H(\psi[\Phi z]_{(\alpha,\beta)}, [\Phi\psi z]_{(\alpha,\beta)}) = 0$.

Let $b = \psi z$; then, by (9) we have

$$\begin{aligned} &d(b, \psi b) \\ &\quad \leq \sup\left\{d(u,v) : u \in [\Phi z]_{(\alpha,\beta)},\ v \in \psi[\Phi z]_{(\alpha,\beta)}\right\} \\ &\quad \leq \sup\left\{d(u,v) : u \in [\Phi z]_{(\alpha,\beta)},\ v \in [\Phi\psi z]_{(\alpha,\beta)}\right\} \\ &\quad < d(\psi z, \psi\psi z) = d(b, \psi b). \end{aligned} \tag{24}$$

Thus $b = \psi b$.

Now,

$$\begin{aligned} &d\left(b, [\Phi b]_{(\alpha,\beta)}\right) \leq d\left(\psi z, [\Phi\psi z]_{(\alpha,\beta)}\right) \\ &\quad \leq \sup\left\{d(u,v) : u \in [\Phi z]_{(\alpha,\beta)},\ v \in [\Phi\psi z]_{(\alpha,\beta)}\right\} \\ &\quad < d(\psi z, \psi\psi z) = d(b, \psi b) = 0. \end{aligned} \tag{25}$$

Hence, $b \in [\Phi b]_{(\alpha,\beta)}$. □

Definition 10 (see [38]). Let (X, d) be a metric space and let $\Phi : X \longrightarrow (IFS)^X$ be an intuitionistic fuzzy map. A single valued map $\psi : X \longrightarrow X$ is said to be a selection of $\Phi : X \longrightarrow (IFS)^X$, if there exists $(\alpha, \beta) \in (0, 1] \times [0, 1)$ such that

$$\psi x \in [\Phi x]_{(\alpha,\beta)}, \quad x \in X. \tag{26}$$

Theorem 11. *Let Y be a compact subset of a complete metric space X and let $\Phi : Y \longrightarrow (IFX)^Y$ be a mapping which satisfies the following conditions:*

$$\begin{aligned} &\text{Given } \epsilon > 0, \text{ there } \textit{exists a } \delta > 0 \\ &\text{such that for all } x, y \in Y, \end{aligned} \tag{27}$$

$$\begin{aligned} &\epsilon \leq \max\left\{d(x, [\Phi x])_{(\alpha,\beta)}, d\left(y, [\Phi y]_{(\alpha,\beta)}\right)\right\} \\ &\quad < \epsilon + \delta \\ &\textit{implies } H\left([\Phi x]_{(\alpha,\beta)}, [\Phi y]_{(\alpha,\beta)}\right) < \epsilon. \end{aligned} \tag{28}$$

Then, there exists a subset W of Y such that $[\Phi w]_{(\alpha,\beta)} = W$ for each $w \in W$. Moreover, for each $w \in W$ there exists a selection of Φ having w as a unique fixed point.

Proof. Let x_0 be an arbitrary fixed element of X. Two sequences $\{x_n\}$ and $\{r_n\}$ of elements in X and R, respectively, will be constructed. $[\Phi x_0]_{(\alpha,\beta)}$ is a closed subset of Y and therefore is compact. There exists a point $x_1 \in [\Phi x_0]_{(\alpha,\beta)}$ such that $d(x_0, x_1) = d(x_0, [\Phi x_0]_{(\alpha,\beta)}) = r_0$. Similarly, there exists $x_2 \in [\Phi x_1]_{(\alpha,\beta)}$ such that $d(x_1, x_2) = d(x_1, [\Phi x_1]_{(\alpha,\beta)}) = r_1$. By induction, we prove that sequences $\{x_n\}$ and $\{r_n\}$ are such that $x_n \in [\Phi x_{n-1}]_{(\alpha,\beta)}$, $d(x_n, x_{n+1}) = d(x_n, [\Phi x_n]_{(\alpha,\beta)}) = r_n$, $n \geq 0$. From inequality (28), we have

$$\begin{aligned} d\left(x_n, [\Phi x_n]_{(\alpha,\beta)}\right) &\leq H\left([\Phi x_{n-1}]_{(\alpha,\beta)}, [\Phi x_n]_{(\alpha,\beta)}\right) \\ &< \max\left\{d\left(x_{n-1}, [\Phi x_{n-1}]_{(\alpha,\beta)}\right), d\left(x_n, [\Phi x_n]_{(\alpha,\beta)}\right)\right\}. \end{aligned} \tag{29}$$

If $d(x_n, [\Phi x_n]_{(\alpha,\beta)}) > d(x_{n-1}, [\Phi x_{n-1}]_{(\alpha,\beta)})$, then (29) implies that $d(x_n, [\Phi x_n]_{(\alpha,\beta)}) < d(x_n, [\Phi x_n]_{(\alpha,\beta)})$, a contradiction. Hence,

$$d\left(x_n, [\Phi x_n]_{(\alpha,\beta)}\right) < d\left(x_{n-1}, [\Phi x_{n-1}]_{(\alpha,\beta)}\right). \tag{30}$$

Thus, $\{r_n\}$ is a monotone nonincreasing sequence of nonnegative real numbers. Therefore, $\{r_n\}$ converges to $\inf\{r_n : n \geq 0\}$. Suppose $\inf\{r_n : n \geq 0\} = r > 0$. Take N so that $n \geq N$ implies that

$$r \leq r_n < r + \delta. \tag{31}$$

It follows that

$$r_{n+1} \leq H\left([\Phi x_n]_{(\alpha,\beta)}, [\Phi x_{n+1}]_{(\alpha,\beta)}\right) < r, \tag{32}$$

which is a contradiction to the assumption that $\inf\{r_n : n \geq 0\} = r > 0$. Hence $r_n \longrightarrow 0$.

That is,

$$d\left(x_n, [\Phi x_n]_{(\alpha,\beta)}\right) \longrightarrow 0. \tag{33}$$

It follows that $H([\Phi x_n]_{(\alpha,\beta)}, [\Phi x_m]_{(\alpha,\beta)}) \longrightarrow 0$. By completeness of $(CB(Y), H)$ (see [39]), there exists a set $W \in CB(Y)$ such that $H([\Phi x_n]_{(\alpha,\beta)}, W) \longrightarrow 0$. Let $w \in W$; then $w \in [\Phi w]_{(\alpha,\beta)}$. If not, let $d(w, [\Phi w]_{(\alpha,\beta)}) = c > 0$; then

$$\begin{aligned} c = d\left(w, [\Phi w]_{(\alpha,\beta)}\right) &\leq H\left([\Phi w]_{(\alpha,\beta)}, W\right) \\ &\leq H\left([\Phi w]_{(\alpha,\beta)}, [\Phi x_n]_{(\alpha,\beta)}\right) + H\left([\Phi x_n]_{(\alpha,\beta)}, W\right) \\ &< \max\left\{d(w, [\Phi w]_{(\alpha,\beta)}, d\left(x_n, [\Phi x_n]_{(\alpha,\beta)}\right)\right\} \\ &\quad + H\left([\Phi x_n]_{(\alpha,\beta)}, W\right). \end{aligned} \tag{34}$$

In a limiting case when $n \longrightarrow \infty$, we have $c < c$, a contradiction. Hence,

$$w \in [\Phi w]_{(\alpha,\beta)}. \tag{35}$$

Now,

$$\begin{aligned} H\left([\Phi w]_{(\alpha,\beta)}, W\right) &= \lim_{n\longrightarrow\infty} H\left([\Phi w]_{(\alpha,\beta)}, [\Phi x_n]_{(\alpha,\beta)}\right) \\ &< \lim_{n\longrightarrow\infty} \max\left\{d(w, [\Phi w])_{(\alpha,\beta)}, d\left(x_n, [\Phi x_n]_{(\alpha,\beta)}\right)\right\} \\ &= 0. \end{aligned} \tag{36}$$

Hence, $[\Phi w]_{(\alpha,\beta)} = W$ for all $w \in W$.

Next, we will prove that there exists a selection of Φ which has a unique fixed point. For each $u \in Y$, $[\Phi u]_{(\alpha,\beta)}$ is compact. Therefore, for $w \in Y$ there exists $u_w \in [\Phi u]_{(\alpha,\beta)})$ such that

$$d(w, u_w) = d\left(w, [\Phi u]_{(\alpha,\beta)}\right)). \tag{37}$$

Let $\psi : Y \longrightarrow Y$ defined as $\psi u = u_w$ be a selection of $\Phi : Y \longrightarrow (IFX)^Y$. Then, for each $u \in Y$ we have $\psi u = u_w \in [\Phi u]_{(\alpha,\beta)}$. Let $\psi w = v(= w_x)$; then $d(w, v) = d(w, [\Phi w]_{(\alpha,\beta)}) = 0$. This implies that

$$v = w = \psi w. \tag{38}$$

Now,

$$\begin{aligned} d(\psi u, \psi v) &\le d(\psi u, w) + d(w, \psi v) \\ &\le d(u_w, w) + d(w, v_w) \\ &\le d\left(w, [\Phi u]_{(\alpha,\beta)}\right) + d\left(w, [\Phi v]_{(\alpha,\beta)}\right) \\ &\le H\left([\Phi w]_{(\alpha,\beta)}, [\Phi u]_{(\alpha,\beta)}\right) \\ &\quad + H\left([\Phi w]_{(\alpha,\beta)}, [\Phi v]_{(\alpha,\beta)}\right) \\ &< d\left(u, [\Phi u]_{(\alpha,\beta)}\right) + d\left(v, [\Phi v]_{(\alpha,\beta)}\right) \\ &< d(u, \psi u) + d(v, \psi v). \end{aligned} \tag{39}$$

It follows that the fixed point of ψ is unique. □

The following examples show that our results generalize a number of previous theorems.

Example 12. Let X be the set of all nonnegative integers with the Euclidean metric. Let $\psi : X \longrightarrow X$ be defined as $\psi x = 2x^2$ and let $\Phi : X \longrightarrow (IFS)^X$ be an intuitionistic fuzzy map defined as

$$\begin{aligned} \mu_{\Phi x}(t) &= \begin{cases} \dfrac{3}{4}, & \text{if } t \in \Omega_x, \\ \dfrac{1}{4}, & \text{if } t \notin \Omega_x. \end{cases} \\ \nu_{\Phi x}(t) &= \begin{cases} \dfrac{1}{5}, & \text{if } t \in \Omega_x, \\ \dfrac{4}{5}, & \text{if } t \notin \Omega_x, \end{cases} \end{aligned} \tag{40}$$

where

$$\Omega_x = \{u \in \psi X : u \le x\}. \tag{41}$$

For $\alpha = 3/4$ and $\beta = 1/5$,

$$[\Phi x]_{(3/4,1/5)} = \{t \in \psi X : t \le x\}. \tag{42}$$

For $\epsilon > 0$, there exists $\delta(= \epsilon)$ such that all the hypotheses of Theorem 9 are valid to obtain common fixed point of ψ and Φ. Previously known results are not applicable to this example (even in the case when Φ is single valued, that is, $\Phi x = \max\{t \in \psi X : \ t \le x\}$) since $\psi\Phi x \ne \Phi\psi x$ at $x \ne 0$.

Example 13. Let $X = R$ with the Euclidean metric, $Y = [-20, 20]$, and $A =]10, 20[$.

For $x \in Y$, define

$$\Gamma_x = \left\{t : 2 - \frac{1}{x} \le t \le 4 - \frac{1}{x}\right\}. \tag{43}$$

Define intuitionistic fuzzy map $\Phi : Y \longrightarrow (IFS)^Y$ as follows:

when $x \in A$,

$$\begin{aligned} \mu_{\Phi x}(t) &= \begin{cases} \dfrac{1}{2}, & \text{if } t \in \Gamma_x, \\ \dfrac{1}{3}, & \text{if } t \notin \Gamma_x. \end{cases} \\ \nu_{\Phi x}(t) &= \begin{cases} \dfrac{2}{3}, & \text{if } t \in \Gamma_x, \\ \dfrac{3}{4}, & \text{if } t \notin \Gamma_x. \end{cases} \end{aligned} \tag{44}$$

When $x \notin A$,

$$\begin{aligned} \mu_{\Phi x}(t) &= \begin{cases} \dfrac{1}{2}, & \text{if } t \in [2, 4], \\ \dfrac{1}{3}, & \text{if } t \notin [2, 4]. \end{cases} \\ \nu_{\Phi x}(t) &= \begin{cases} \dfrac{2}{3}, & \text{if } t \in [2, 4], \\ \dfrac{3}{4}, & \text{if } t \notin [2, 4]. \end{cases} \end{aligned} \tag{45}$$

For $\alpha = 1/2$ and $\beta = 2/3$,

$$[\Phi x]_{(1/2,2/3)} = \begin{cases} \Gamma_x, & \text{if } x \in A, \\ [2, 4], & \text{if } x \notin A. \end{cases} \tag{46}$$

For $\epsilon > 0$, there exists $\delta(= 59\epsilon)$, such that Φ satisfies all the assumptions of Theorem 11. In this case, $W = [2, 4] \in CB(Y)$ such that $[\Phi w]_{(1/2,2/3)} \in W$ for all $w \in W$ and corresponding to $w \in W$ the mapping $\psi : Y \longrightarrow Y$ defined as

$$\psi x = \begin{cases} w, & \text{if } w \in [\Phi u]_{(1/2,2/3)}, \\ 4 - \dfrac{1}{u}, & \text{if } w \notin [\Phi u]_{(1/2,2/3)}, \end{cases} \tag{47}$$

is a selection of Φ.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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