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Research Article

Optimization of Risk and Return Using Fuzzy Multiobjective Linear Programming

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Stock selection poses a challenge for both the investor and the finance researcher. In this paper, a hybrid approach is proposed for asset allocation, offering a combination of several methodologies for portfolio selection, such as investor topology, cluster analysis, and the analytical hierarchy process (AHP) to facilitate ranking the assets and fuzzy multiobjective linear programming (FMOLP). This paper considers some important factors of stock, like relative strength index (RSI), coefficient of variation (CV), earnings yield (EY), and price to earnings growth ratio (PEG ratio), apart from the risk and return and stocks which are included within these same factors. Employing fuzzy multiobjective linear programming, optimization is performed using seven objective functions viz., return, risk, relative strength index (RSI), coefficient of variation (CV), earnings yield (EY), price to earnings growth ratio (PEG ratio), and AHP weighted score. The FMOLP transforms the multiobjective problem to a single objective problem using the "weighted adaptive approach" in which the weights are calculated by AHP or choices by the investors. The FMOLP model permits choices in solution.

1. Introduction

Due to the uncertainty of return it is not easy to select the stocks. The main aim of portfolio selection is to obtain an accurate ratio of the assets to ensure that the investor gets the maximum return with minimum risk.

Professor Markowitz initially presented the problem of portfolio selection [1]. He proposed the Markowitz model or mean-variance model (MV) for portfolio selection reiterating the fact that investing in more than one stock is less risky than investing in a single stock. Konno and Yamazaki [2] introduced an improved version of the Markowitz model in which the risk is calculated by the mean absolute deviation (MAD). Speranza [3] advanced a linear programming model, in which the risk is calculated by the semiabsolute deviation method. Gupta et al. [4] projected the hybrid approach for portfolio selection using a combination of multiple methodologies like investor's behavioral survey, cluster analysis, analytical hierarchy process, and fuzzy mathematical programming. Ganasekaran and Ramaswami [5] proffered a portfolio optimization model applying the neurofuzzy framework. Gupta et al. [6] obtained ethical stock performance using the AHP technique and portfolio selection done by the FMCDM technique. Mehlawat [7] presented a detailed computation procedure of the AHP and applied FMCDM technique. Sanokolaei [8] proposed the fuzzy method for portfolio optimization based on the mean absolute deviation risk function. Sadati and Doniavi [9] advocated their portfolio selection model based on the possibility model with the fuzzy random variable parameter and applied the harmony search algorithm. Konak and Bagei [10] applied fuzzy linear programming for portfolio optimization. Wang et al. [11] introduced a new risk index variable called equilibrium risk value (ERV) of the random fuzzy expected value (EV) and the EV-ERV model was used for portfolio selection.

A literature survey revealed several drawbacks in the K-means algorithm used for clustering and improper scaling because it involves identification of the number of clusters. In AHP, the stocks are ranked based the criteria of return, risk, liquidity, dividend, alpha, beta and stock prices, etc.

This study presents a hybrid approach for portfolio selection with multiple methodology. First, the X-means algorithm needs to be performed for cluster analysis, which is an extended version of the K-means clustering. The drawbacks

have been improved in X-means. In X-means, the number of clusters does not need to be specified. Then by applying the AHP, the stocks for all three clusters must be ranked. In this paper, some new features for stock selection have been included, such as relative strength index (RSI), coefficient of variation (CV), earnings yield (EY), and price to earnings growth ratio (PEG ratio), which have not been used earlier in the AHP. Optimization is done using fuzzy multiobjective linear programming with seven objective functions viz., return, risk, relative strength index (RSI), coefficient of variation (CV), earnings yield (EY), price to earnings growth ratio (PEG ratio), and AHP weighted score. The daily closing price, number of shares, turnover rate, earning per share, price to earnings ratio, price to earnings growth ratio, and market cap for all the 15 stocks selected are taken from the BSE, Bombay Stock Exchange, Mumbai, India (https://www.bseindia.com), from February '15 to January '16.

This paper is organized in four sections as follows: Section 2 includes an account of the research methodology, the FMOLP algorithm, and its working process with reference to each of the seven objectives, viz., return, risk, relative strength index, coefficient of variation, earning yield, price to earnings growth ratio, and AHP weighted score. Section 3 presents the numerical illustrations, while Section 4 contains the concluding remarks.

2. Methodology

To solve the multiobjective linear programming problem, the following step-by-step strategy is used.

2.1. Investor Behavior Pattern. Investor behavior plays an important role in the selection of stocks as each individual stock-holder will have a specific decision-making style. Three main categories of investors can be identified, viz., money makers, liquidity lovers, and risk averse investors, according to their investment topology [12]. The survey done above is based only on the characteristics of return, risk, and liquidity. Return, risk, and liquidity are the basic factors used in stock selection; however, some more important features, as listed below, need to be considered prior to selecting the stocks:

(i) J. Welles Wilder introduced the relative strength index in 1978. This evaluates the current and historical performance of a stock based on today's closing prices. RSI normally falls within the 30-70 range.

(ii) Coefficient of variation enables the evaluation of the value of instability relative to the return rate.

(iii) Earning yield is the percentage of each amount invested in the stock which the company has received.

(iv) A comparative calculation or relation between the stock price, EPS, and the growth of the companies is defined by the price to earnings growth ratio.

(v) Market cap is used to classify the company size, which is of greater importance than the stock price.

2.2. Cluster. For every investor, the approaches employed in stock selection are different. Generally, however, the investors predominantly observe all the three aspects of return, risk, and liquidity. Therefore, based on these three points, stocks can be better categorized under three groups, with qualities like high return, minimum risk, and liquid stocks. Cluster analysis is a technique used to divide data into groups by which similar objects are placed within the same cluster which is different from the other cluster objects. To formulate the clusters, the X-means [13] clustering algorithm is used. It is an extended version of the K-means which attempts to automatically determine the number of clusters. It starts with just one centroid and then iteratively increases the centroid, as required. If a cluster is divided into two subclusters, then the data distribution is done using the Bayesian Information Criteria (BIC) which is a statistical model.

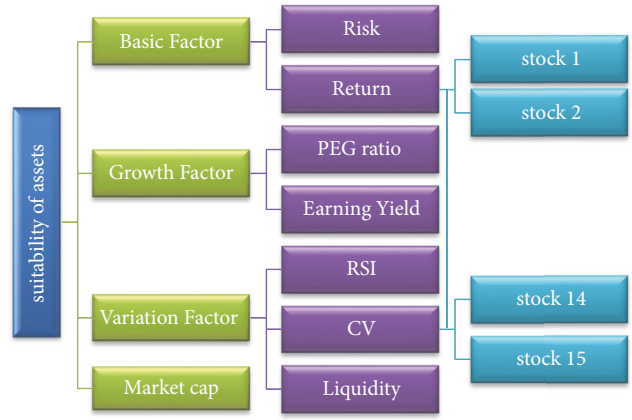

Figure 1: Hierarchy structure.

The proposed research includes investor topology, clustering, the AHP, and optimization technique for portfolio selection. Different investors employ different approaches for investing in the stock market. Based on the preferences, the investors are divided into three different clusters:

(a) Investors who are willing to take only higher returns

(b) Investors who are not interested in taking more risks, even if the returns are less

(c) Investors who are neither in favor of greater risk nor favor low returns and who only desire secure investment (liquidity lovers)

Therefore, based on these three points, stocks are divided into three groups, with qualities like high return, minimum risk, and liquid stocks.

2.3. AHP. AHP technique developed by Thomas L. Saaty [14] is a multicriteria decision-making (MCDM) tool. It has a particular application in group decision-making. Hierarchy structure design, weight analysis, and consistency proof are the three main steps of AHP for ranking the object. Figure 1 shows the 4-level hierarchy structure of AHP. Firstly, form a pair wise comparison matrix for each criterion with respect to its parent criteria.

Each entry of the judgmental matrix A is formed by the following rules:

$$a_{ij} > 0$$
$$a_{ij} = \frac{1}{a_{ji}} \tag{1}$$
$$a_{ij} = 1,$$
$$\text{when } i = j, \text{ for all } i, j$$

To compare two things, we have a well-defined 1-9 scale which is given by Satty.

For the matrix A of order "n" the normalized Eigenvector is called Priority vector.

$$Aw = \lambda_{max} w, \tag{2}$$

where "w" is known as weight of the objects. λ_{max} is the highest Eigen value of the matrix A. The consistency index (CI) for each nth order matrix is calculated as

$$CI = \frac{(\lambda_{max} - n)}{(n-1)} \tag{3}$$

The consistency ratio (CR) is calculated as

$$CR = \frac{CI}{RI} \tag{4}$$

where RI the random index is determined by on the order of the matrix.

The matrix is consistent if CR $\leq$ 0.10. However, if CR> 0.10, inconsistencies exist and pairwise comparisons need revision.

2.4. Portfolio Selection Model. The fuzzy multiobjective linear programming (FMOLP) [15] technique is commonly used for optimization. The MOLP can be changed to a single objective utilizing the membership functions.

2.4.1. Portfolio Selection Problem. The multiobjective portfolio selection problem with seven objective functions such as return, risk, relative strength index, coefficient of variation, earning yield, price to earnings growth ratio, and AHP weight and some notations are introduced as follows:

r_i: return of the ith stock,

x_i: the proportion of the total fund invested in the ith stock,

b_i: the binary variable indicating whether the ith stock is contained in the portfolio or not, i.e., $b_i = \{1, if\ i^{th} stock\ contained\ in\ portfolio, 0, if\ not\ containing\ in\ portfolio\}$

k_i: risk of the ith stock,

R_i: relative strength index of the ith stock,

C_i: coefficient of variation of the ith stock,

w_{AHPi}: the AHP weight of the ith stock,

E_i: earning yield of the ith stock,

P_i: p/e growth ratio of the ith stock,

u_i: the maximum fraction of the ith stock,

l_i: the minimum fraction of the ith stock,

n: total number of stocks in each cluster (15),

a: number of stock in a selected portfolio (5).

2.4.2. Parameters Used

(i) Return. The return of the portfolio is written as

$$f_1(x) = \sum_{i=1}^{n} r_i x_i \tag{5}$$

Where $r_i = (1/12)\sum_{i=1}^{12} r_{it}$.

(ii) Risk. The semiabsolute deviation of return of the portfolio below the expected return over the past period t, t = 1, 2, ... T, can be written as

$$\begin{aligned} k_t(x) &= \left| \min\left\{0, \sum_{i=1}^{n}(r_{it} - r_i)\right\} x_i \right| \\ &= \frac{\left|\sum_{i=1}^{n}(r_{it} - r_i)x_i\right| + \sum_{i=1}^{n}(r_i - r_{it})x_i}{2} \end{aligned} \tag{6}$$

Consequently, the expected semiabsolute deviation of return of the portfolio $x = (x_1, x_2, x_3, \ldots x_n)$ below the expected return becomes

$$\begin{aligned} f_2(x) = k(x) &= \frac{1}{T}\sum_{t=1}^{T} k_t(x) \\ &= \sum_{t=1}^{T} \frac{\left|\sum_{i=1}^{n}(r_{it} - r_i)x_i\right| + \sum_{i=1}^{n}(r_i - r_{it})x_i}{2T}, \end{aligned} \tag{7}$$

where k(x) represents portfolio risk.

Above risk function converted into linear function as optimization technique is for linear problem

$$\begin{aligned} f_2(x) = k(x) &= \frac{1}{T}\sum_{t=1}^{T} k_t(x) \\ &= \sum_{t=1}^{T} \frac{\left|\sum_{i=1}^{n}(r_{it} - r_i)x_i\right| + \sum_{i=1}^{n}(r_i - r_{it})x_i}{2} \\ f_2(v) = k(v) &= \frac{1}{T}\sum_{t=1}^{T} v_t, \\ &\text{where } v_t + \sum_{t=1}^{T}(r_{it} - r_i)x_i \geq 0. \end{aligned} \tag{8}$$

(iii) Relative Strength Index (RSI). The RSI of the portfolio is written as

$$f_3(x) = \sum_{i=1}^{n} R_i x_i, \tag{9}$$

where $R_i = 100 - 100/(1 + avg_i)$ and $avg_i = avg\ gain/avg\ loss$.

(iv) Coefficient of Variation (CV). The CV of the portfolio is written as

$$f_4(x) = \sum_{i=1}^{n} C_i x_i, \tag{10}$$

where $c_i = SD_i/return_i$ of the ith stock.

(v) Earning Yield (EY). The EY of the portfolio is written as

$$f_5(x) = \sum_{i=1}^{n} E_i x_i, \tag{11}$$

where $E_i = 1/(p/e)ratio$ of the ith stock.

(vi) Price to Earnings Growth Ratio (PEG Ratio). The PEG ratio of the portfolio is written as

$$f_6(x) = \sum_{i=1}^{n} P_i x_i, \tag{12}$$

where $P_i = (p/e)ratio/growth\ ratio$ of the ith stock.

(vii) AHP Weight. The AHP weight of the portfolio is written as

$$f_7(x) = \sum_{i=1}^{n} w_{AHPi} x_i \tag{13}$$

where w_{AHPi} is weight of ith stock.

2.4.3. Constraints. Investment economical restriction on the stocks:

(i) Sum of proportion of stocks should be 1

$$\sum_{i=1}^{n} x_i = 1 \tag{14}$$

(ii) Number of stocks held in a portfolio is

$$\sum_{i=1}^{n} b_i = a \tag{15}$$

(iii) The maximum percentage of the investment which can be invested in a stock:

$$x_i \le u_i b_i, \quad i = 1, 2, \ldots, n, \tag{16}$$

(iv) The minimum percentage of the investment which can be invested in a stock is

$$x_i \ge l_i b_i, \quad i = 1, 2, \ldots, n, \tag{17}$$

The upper and lower bounds have been taken to avoid too many large investments and in the same manner too many small investments.

2.4.4. The Decision Problem

$$\max \quad f_1(x) = \sum_{i=1}^{n} r_i x_i \tag{18}$$

$$\min \quad f_2(v) = \sum_{i=1}^{n} k_i x_i \tag{19}$$

$$\max \quad f_3(x) = \sum_{i=1}^{n} R_i x_i \tag{20}$$

$$\min \quad f_4(x) = \sum_{i=1}^{n} C_i x_i \tag{21}$$

$$\max \quad f_5(x) = \sum_{i=1}^{n} E_i x_i \tag{22}$$

$$\max \quad f_6(x) = \sum_{i=1}^{n} P_i x_i \tag{23}$$

$$\max \quad f_7(x) = \sum_{i=1}^{n} w_{AHPi} x_i \tag{24}$$

$$\text{subject to} \quad v_t + \sum_{t=1}^{T} (r_{it} - r_i) x_i \ge 0, \tag{25}$$

$$\sum_{i=1}^{n} x_i = 1, \tag{26}$$

$$\sum_{i=1}^{n} b_i = a, \tag{27}$$

$$x_i \le u_i b_i, \quad i = 1, 2, \ldots, n, \tag{28}$$

$$x_i \ge l_i b_i, \quad i = 1, 2, \ldots, n, \tag{29}$$

$$x_i \ge 0, \quad i = 1, 2, \ldots, n, \tag{30}$$

$$b_i \epsilon \{0, 1\}, \quad i = 1, 2, \ldots, n. \tag{31}$$

Assuming that, after solving (18) with the constraints ((25)–(31)), the solution is X_1, then the other objective functions are also similarly calculated at X_1.When this process is repeated for (19) through to (24) you will get seven solutions with respect to each objective.

Next, identify the best upper bound (ub) and worst lower bound (lb) for all the objectives.

The membership function for $f(x)_1$, $f(x)_3$, $f(x)_5$, $f(x)_6$, and $f(x)_7$ is defined by

$$\mu_{f_k(x)} = \begin{cases} 1, & if\ f_k(x) \ge ub \\ \dfrac{f_k(x) - lb}{ub - lb}, & lb \le f_k(x) \le ub \\ 0, & if\ f_k(x) \le lb, \end{cases} \quad k = 1, 3, 5, 6, 7 \tag{32}$$

$$\mu_{f_h(x)} = \begin{cases} 0, & if\ f_h(x) \ge ub \\ \dfrac{ub - f_h(x)}{ub - lb}, & lb \le f_h(x) \le ub \\ 1, & if\ f_h(x) \le lb, \end{cases} \quad h = 2, 4 \tag{33}$$

where $\mu_{f(x)}$ is the satisfaction degree of the objective function for a given solution X.

Convert the multiobjective problem into a single objective using "weighted adaptive approach" based on AHP-criteria weight in respect of each objective.

Table 1: Cluster result.

Parameters	Cluster 1 (46 stocks)	Cluster 2 (78 stocks)	Cluster 3 (23stocks)
average return	0.0441	0.0154	0.0731
average risk	0.0547	0.0344	0.0744
turnover rate	0.0010	0.0005	0.0010
Category	Liquid	less risky	high return

Table 2: Stocks for each cluster.

Symbol	Cluster 1	Cluster 2	Cluster 3
S1	Whbrady	Blue Star	Kinetic Eng.
S2	Nelco Ltd.	Great Estate	Tokyo Plast
S3	Nocil Ltd	Swaraj Engine	Force Motor
S4	Ceat Limited	Bajfinance	Kg Denim
S5	Nucleus S/w Exports Ltd.	Finolex Ind.	Zenith Fiber
S6	Sauras.Cem.	Bharat Pet.	Jenson Nicolson
S7	Fedder.Llyod	Lakshmi Mill	NIIT Ltd.
S8	Dcw Ltd.	Jsw steel	Tata Elxsi
S9	Eveready Ind. India Ltd.	Pel	Century Ext
S10	Himachal Fertilizer	Swan Eng	Jasch Indust
S11	Timex Group	Pfizer Ltd.	Medi-caps
S12	Camph.& All	Sri Adhikari Brothers Tel. Net. Ltd.	Pas.Acrylon
S13	Andhra Petro	Kajaria Cer.	Modi Rubber
S14	Sha Eng Pla	Asian Paints	Mafatlal Ind
S15	Majestic Aut	Lic Housing Finance	Panyam Cement

Model I

$$\begin{aligned}
\max \quad & 0.1961a_1 + 0.1569a_2 + 0.0642a_3 \\
& + 0.0642a_4 + 0.1176a_5 + 0.1176a_6 \\
& + 0.0712a_7 \\
\text{Subject to} \quad & f_k(x) \\
& - (\textit{upper bound} - \textit{lower bound})\, a_k \\
& \geq \textit{lower bond}, \quad k = 1, 3, 5, 6, 7 \\
& f_h(x) \\
& + (\textit{upper bound} - \textit{lower bound})\, a_h \\
& \leq \textit{upper bond}, \quad h = 2, 4 \\
& 0 \leq a_j \leq 1, \quad j = 1, 2, \ldots, 7 \\
& \sum_{i=1}^{n} x_i = 1 \\
& \sum_{i=1}^{n} b_i = a \\
& x_i - u_i b_i \geq 0, \quad i = 1, 2, \ldots n, \\
& x_i - l_i b_i \leq 0, \quad i = 1, 2, \ldots n, \\
& x_i \geq 0, \quad i = 1, 2, \ldots n.
\end{aligned} \tag{34}$$

The solution obtained on solving Model I is the first iteration. The old lower bound will be replaced by the first iteration, only when improvement is required. This process must be repeated until the investors are satisfied with the solution.

3. Numerical Illustration

The results of an experimental study built on a data set of 147 assets registered in the BSE, Mumbai, India (from February-'15 to January-'16), are as follows.

3.1. Cluster Analysis. For performing cluster analysis, X-means tool of the Rapid Miner version 5.2 software is used. And the initial distribution of first centroid is performed by K-means clustering. The result of the X-means algorithm is shown in Table 1.

As per the topology of investors discussed in Section 2.1,

(i) **Cluster 1** includes liquid stocks as the liquidity is highest when compared with the other clusters, and risk is medium. This cluster is suitable for those investors who are interested in liquid stocks and medium risk.

(ii) **Cluster 2** includes high return stocks, as the average value of return is higher in comparison to the other clusters. This cluster is meant for those investors who are focused only on maximum returns.

(iii) **Cluster 3** contains less risky stocks, as the average risk value is low when compared with the other clusters. This cluster is good for those investors who are risk averse.

Symbolic representations of stocks from each cluster are shown in Table 2.

Table 3: Weight of criteria and subcriteria.

Criteria	weight	sub-criteria	Weight
Basic factor	0.3529	Risk	0.1569
		Return	0.1961
Growth factor	0.2353	PEG Ratio	0.1176
		Earning Yield	0.1176
Variation factor	0.2353	Relative Strength Index	0.0642
		Coefficient of Variation	0.0642
Market cap	0.1765	Liquidity	0.1070

Table 4: Input data for Cluster 1.

Stocks	Return	Risk	RSI	CV	EY	PEG	AHP weight
S1	0.0628	0.0402	56.897	1.5707	5.61	3.24	0.1126
S2	0.0296	0.0638	50.689	5.2433	5.29	0.51	0.0466
S3	0.0369	0.0471	52.540	3.0668	6.88	0.75	0.0592
S4	0.0314	0.0544	50.626	4.8066	4.757	1.72	0.1101
S5	0.0326	0.0534	51.306	3.7950	6.04	2.5	0.0553
S6	0.0512	0.0620	51.430	3.8625	5.9	-0.97	0.0501
S7	0.0323	0.0661	50.823	4.9407	13.41	-1.23	0.0534
S8	0.0417	0.0366	52.184	2.2204	2.73	0	0.0486
S9	0.0371	0.0534	54.159	3.4756	3.86	1.3	0.0655
S10	0.0301	0.0522	50.700	3.8591	5.05	1.68	0.0713
S11	0.0673	0.0483	55.167	1.9584	0.94	39.64	0.1287
S12	0.0653	0.0486	53.923	1.7890	6.86	0.51	0.0532
S13	0.0308	0.0592	49.333	4.6172	6.6	3.12	0.0475
S14	0.0628	0.0402	56.897	1.5707	3.99	1.43	0.0552
S15	0.0556	0.0669	49.732	3.1074	5.04	0	0.0428

Table 5: Input data for Cluster 2.

Stocks	Return	Risk	RSI	CV	EY	PEG	AHP weight
S1	0.0110	0.0151	51.256	3.701	2.8	1.53	0.0443
S2	0.0008	0.0161	50.669	55.590	10.91	-4.49	0.0617
S3	0.0111	0.0172	51.076	3.810	4.99	4.94	0.0609
S4	0.0342	0.0174	44.017	1.320	5.27	1.3	0.0880
S5	0.0048	0.0188	49.703	10.388	5.76	0.86	0.0416
S6	0.0192	0.0190	52.799	2.506	7.91	0.88	0.0904
S7	0.0017	0.0197	49.607	31.856	5.33	7.06	0.0712
S8	0.0058	0.0203	50.001	9.464	8.16	1.49	0.0739
S9	0.0102	0.0213	51.640	5.128	5.87	0.51	0.0489
S10	0.0186	0.0215	52.378	2.956	0.47	-20.5	0.0739
S11	0.0128	0.0218	50.822	4.196	6.37	-3.28	0.0447
S12	0.0349	0.0220	56.296	1.718	5.08	0	0.0702
S13	0.0207	0.0229	53.777	2.986	3.47	2.36	0.0590
S14	0.0073	0.0232	51.026	7.824	2.69	4.74	0.1038
S15	0.0052	0.0235	50.436	12.596	9.08	1.03	0.0674

3.2. Numerical Calculation of AHP Weights. In this segment under the criteria and subcriteria in AHP, stocks are ranked according to the investor preference. The weights are given in Table 3.

Tables 4–6 represent the input data for all three clusters.

3.3. FMOLP Calculation. Upper and lower bound for each cluster are given by Table 7.

Table 6: Input data for Cluster 3.

Stocks	Return	Risk	RSI	CV	EY	PEG	AHP weight
S1	0.0924	0.0668	53.803	1.796	-5.03	0.00	0.0379
S2	0.0915	0.0721	53.639	1.891	6.19	0.84	0.0717
S3	0.0907	0.0873	55.482	2.482	4.68	-0.99	0.1235
S4	0.0892	0.0650	53.335	1.778	14.13	-6.02	0.0592
S5	0.0883	0.0478	59.405	1.269	15.15	-13.41	0.0569
S6	0.0860	0.0669	51.985	2.230	-8.41	0.00	0.0385
S7	0.0807	0.0797	54.253	2.446	1.27	0.00	0.0578
S8	0.0805	0.0647	56.497	1.887	5.34	0.56	0.1658
S9	0.0779	0.1038	49.735	3.147	12.92	-2.13	0.0446
S10	0.1027	0.0758	54.414	1.989	10.03	1.70	0.1056
S11	0.0740	0.0602	52.276	2.414	3.79	-3.25	0.0417
S12	0.0734	0.0689	50.528	4.414	17.19	0.41	0.0703
S13	0.0704	0.0790	52.559	2.946	1.16	-1.34	0.0328
S14	0.0701	0.0537	55.462	1.839	-1.57	0.00	0.0397
S15	0.0698	0.0794	52.630	3.182	15.67	-217.43	0.0540

Table 7: Upper bound and lower bound.

	cluster 1		cluster 2		cluster 3	
Objective	Ub	Lb	Ub	Lb	Ub	Lb
Return	0.0661	0.0311	0.0333	0.0014	0.0980	0.0725
Risk	0.0643	0.0385	0.0222	0.0156	0.0743	0.0511
RSI	56.7298	50.7244	55.0748	50.2265	58.0193	51.4598
CV	5.0742	1.6272	43.6201	1.7508	3.8445	1.5350
EY	10.4818	2.9446	9.9886	4.4508	16.4061	4.2379
PEG ratio	23.4067	-0.2626	5.9784	-2.1260	1.2829	-82.2285
AHP weight	0.1195	0.0507	0.0971	0.0522	0.1442	0.0526

Table 8: Iterations for Cluster 1.

Cluster 1	f1	f2	f3	f4	f5	f6	f7
iteration 1	0.0661	0.0643	56.7298	5.0742	10.4818	23.4067	0.1195
iteration 2	0.0311	0.0385	50.7244	1.6272	2.9446	-0.2626	0.0507
iteration 3	0.0647	0.0455	55.7323	1.8668	3.1830	23.2575	0.1176
iteration 4	0.0643	0.0451	55.2060	1.7007	6.1264	2.4610	0.0774
iteration 5	0.0643	0.0451	55.2060	1.7007	6.1264	2.4610	0.0774

The iterations for each cluster are given by Tables 8–10.

4. Assets Allocation

The numerical results for each cluster are shown in Table 11.

Thus, from the results it is clear that Cluster 1 contains the high liquidity stocks, Custer 2 includes the low risk stocks, and Cluster 3 has the high return stocks, although the main objective of minimization of risk and maximization of return is to be preserved.

4.1. Comparison. Risk/return ratio (CV) is very helpful to choosing the stocks. Investors are risk averse, as they want to consider stocks with a low risk and a high degree return.

Proposed approach gives better results as compared to the approach presented in Gupta et al. [4] as the CV (risk/return) is 0.81, 0.70, and 0.61 for Cluster 1, Cluster 2, and Cluster 3, respectively, in this model while Dr. Gupta's [4] methodology results are 0.77, 1.12, and 0.65 for the same clusters. Based on the above results, the investor would like to invest with lower

Table 9: Iterations for Cluster 2.

Cluster 2	f1	f2	f3	f4	f5	f6	f7
iteration 1	0.0333	0.0222	55.0748	43.6201	9.9886	5.9784	0.0971
iteration 2	0.0014	0.0156	50.2265	1.7508	4.4508	-2.1260	0.0522
iteration 3	0.0332	0.0202	51.3810	2.2922	5.1847	0.7219	0.0772
iteration 4	0.0329	0.0202	51.3191	2.4010	5.1672	0.7754	0.0782
iteration 5	0.0329	0.0202	51.3191	2.4010	5.1672	0.7754	0.0782

Table 10: Iterations for Cluster 3.

Cluster 3	f1	f2	f3	f4	f5	f6	f7
iteration 1	0.0980	0.0743	58.0193	3.8445	16.4061	1.2829	0.1442
iteration 2	0.0725	0.0511	51.4598	1.5350	4.2379	-82.2285	0.0526
iteration 3	0.0888	0.0562	56.8504	1.5181	14.1940	-9.6902	0.0628
iteration 4	0.0849	0.0737	55.8689	2.1708	5.4825	0.0042	0.1442
iteration 5	0.0901	0.0734	54.3120	2.0392	10.3194	-3.9619	0.0847

Table 11: Results for each cluster.

Stock	cluster 1	cluster 2	cluster 3
S1	0.0225	0	0
S2	0	0	0.3435
S3	0	0	0.0560
S4	0	0.0225	0.0225
S5	0	0	0.0225
S6	0	0.5555	0
S7	0	0.0225	0
S8	0	0	0
S9	0	0	0
S10	0	0	0.5555
S11	0.5555	0	0
S12	0.3770	0.3770	0
S13	0	0	0
S14	0.0225	0.0225	0
S15	0.0225	0	0

CV, since the lower value of risk/return ratio indicate a better risk-return trade-off.

5. Conclusion

This paper presented a hybrid approach that was adopted while investigating the problem of portfolio selection. The hybrid approach involved important components such as Behavior Survey, Cluster Analysis, AHP, and FMOLP. Cluster analysis is done using the X-means algorithm, which gives a better fit to the data in the clusters as the number of clusters was decided by itself. In this paper, a few new and important criteria like RSI, CV, EY, and the PEG ratio have been considered, which are very helpful for beginners and a good start for stock selection. The FMOLP transforms a multiobjective problem to a single objective one using the "weighted adaptive approach" in which the weights are calculated by the AHP or chosen by the investors. The FMOLP model permits choices in solution. The main advantage of the model proposed is—if the investor is not satisfied with the portfolio he/she can change the weights of objective functions or recalculate the AHP model—based on the preferences of the decision-maker and thus achieves improved results. This approach gives better results as risk/return ratio is lower which indicates better risk-return trade-off.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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