Research Article

Cosmological New Massive Gravity and Galilean Conformal Algebra in 2 Dimensions

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1. Introduction

Recently, there has been some interest in extending the AdS/CFT correspondence to non-relativistic field theories [1, 2]. The Kaluza-Klein-type framework for nonrelativistic symmetries, used in [1, 2], is basically identical to the one introduced in [3, 4] (see also [5, 6]). The study of a different nonrelativistic limit was initiated in [7, 8], where the nonrelativistic conformal symmetry was obtained by a parametric contraction of the relativistic conformal group. Galilean conformal algebra (GCA) arises as contraction relativistic conformal algebras [7–10], where in 3+1 space-time dimensions the Galilean conformal group is a fifteen-parameter group which contains the ten parameter Galilean subgroup. Infinite-dimensional Galilean conformal group has been reported in [9, 10], the generators of this group are $L^n = -(n+1)t^n x_i \partial_i - t^{n+1} \partial_t$, $M_i^n = t^{n+1} \partial_i$, and $J_{ij}^n = -t^n (x_j \partial_i - x_i \partial_j)$ for an arbitrary integer $n$, where $i$ and $j$ are specified by the spatial directions. There is a finite-dimensional subgroup of the infinite-dimensional Galilean conformal group which generated by $(J_{ij}^0, L^2, L^0, M_i^1, M_0^0)$. These generators are obtained by contraction ($t \to t, x_i \to \epsilon x_i, \epsilon \to 0, v_i \sim \epsilon$) of
the relativistic conformal generators. Recently the authors of [11] (see also [12, 13]) have shown that the GCA\textsubscript{2} is the asymptotic symmetry of cosmological topologically massive gravity (CTMG) in the nonrelativistic limit. They have obtained the central charges of GCA\textsubscript{2}, and also a nonrelativistic generalization of Cardy formula. In the present paper we want to investigate similar problem for cosmological new massive gravity (CNMG).

Recently, a new theory of massive gravity in three dimensions was proposed by Bergshoeff et al. [14, 15]. This theory, referred to as new massive gravity (NMG), seems to offer a possibility for formulating a consistent theory of quantum gravity in three dimensions. NMG is defined by adding higher derivative term to the EH term in action, with coupling $1/m^2$. If we add to it the negative cosmological constant we can refer to this three-dimensional gravity as cosmological new massive gravity (CNMG). In this theory, the linearized excitations about the anti de Sitter vacuum describe a propagating massive graviton. One can show that CNMG admits the BTZ black holes as solutions [14, 15], moreover in NMG regular warped black holes have been obtained by Clément [16] (see also [17, 18]).

In this paper we propose the contracted BTZ and warped AdS\textsubscript{3} black hole solution of CNMG as a gravity dual of 2d GCA in the context of the nonrelativistic AdS\textsubscript{3}/CFT\textsubscript{2} correspondence. The rest of the paper is organized as follows: in Section 2 we give a brief review of 2d CFT and its contraction, the GCA parameters were realized in term of CFT parameters. In Sections 3 and 4 we study GCA realization of BTZ, and warped AdS\textsubscript{3} black hole solutions of NMG, respectively, in these sections GCA parameters were constructed in terms of gravity parameters, and finally we obtained finite entropy in nonrelativistic limit. The last section is devoted to the conclusion.

2. Galilean Conformal Algebra in 2 Dimensions

Galilean conformal algebra in 2d can be obtained from contracting 2d conformal symmetry [19]. 2d conformal algebra at the quantum level is described by two copy of Virasoro algebra. In two-dimensions space-time $(z = t + x, \bar{z} = t - x)$, the CFT generators

$$\mathcal{L}_n = -z^{n+1} \partial_z, \quad \mathcal{L}_{\bar{n}} = -\bar{z}^{n+1} \partial_{\bar{z}}$$  \hspace{1cm} (2.1)$$

obey the centrally extended Virasoro algebra

$$[\mathcal{L}_m, \mathcal{L}_n] = (m-n)\mathcal{L}_{m+n} + \frac{c_R}{12} m(m^2 - 1) \delta_{m+n,0},$$

$$[\mathcal{L}_m, \mathcal{L}_{\bar{n}}] = (m-n)\mathcal{L}_{m+n} + \frac{c_L}{12} m(m^2 - 1) \delta_{m+n,0}. \hspace{1cm} (2.2)$$

By taking the nonrelativistic limit $(t \rightarrow t, x \rightarrow \epsilon x$ with $\epsilon \rightarrow 0$), the GCA generators $L_n$ and $M_n$ are constructed from Virasoro generators by

$$L_n = \lim_{\epsilon \rightarrow 0} \left( \mathcal{L}_n + \mathcal{L}_{\bar{n}} \right), \quad M_n = \lim_{\epsilon \rightarrow 0} \epsilon \left( \mathcal{L}_n - \mathcal{L}_{\bar{n}} \right). \hspace{1cm} (2.3)$$
From (2.2) and (2.3), one obtains centrally extended 2d GCA

\[
[L_m, L_n] = (m - n)L_{m+n} + C_1m^2(m^2 - 1)\delta_{m+n,0},
\]

\[
[L_m, M_n] = (m - n)M_{m+n} + C_2m^2(m^2 - 1)\delta_{m+n,0},
\]

(2.4)

\[
[M_n, M_m] = 0.
\]

Note that \([M_n, M_m]\) cannot have any central extension. The GCA central charges \((C_1, C_2)\) are related to CFT central charges \((c_L, c_R)\) as

\[
C_1 = \lim_{\epsilon \to 0} \frac{c_L + c_R}{12}, \quad C_2 = \lim_{\epsilon \to 0} \left( \epsilon \frac{c_L - c_R}{12} \right).
\]

(2.5)

Similarly, rapidity \(\xi\) and scaling dimensions \(\Delta\), which are the eigenvalues of \(M_0\) and \(L_0\) respectively, are given by

\[
\Delta = \lim_{\epsilon \to 0} (h + \bar{h}), \quad \xi = \lim_{\epsilon \to 0} (h - \bar{h}),
\]

(2.6)

where \(h\) and \(\bar{h}\) are eigenvalues of \(L_0\) and \(\bar{L}_0\), respectively. So the 2d GCA was obtained by the nonrelativistic limit of the Virasoro CFT2.

### 3. GCA Realization of BTZ Black Hole Solution of NMG

In this section we would like to propose that the contracted BTZ black hole solution of three-dimensional new massive gravity (NMG) is gravity dual of 2d GCA in the context of AdS/CFT correspondence. It is notable that the NMG (as a gravity dual) has to yield finite parameters \((\Delta, \xi, C_1, C_2, \text{ and entropy } S_{\text{GCA}})\) for GCA. The action of the cosmological new massive gravity in three dimensions is [14, 15]

\[
S[g_{\mu\nu}] = \frac{1}{16\pi G} \int \sqrt{-g} \left( R - 2\lambda + \frac{1}{m^2} \mathcal{L}_{\text{NMG}} \right),
\]

(3.1)

where the NMG term is

\[
\mathcal{L}_{\text{NMG}} = R_{\mu\nu}R^{\mu\nu} - \frac{3}{8} R^2.
\]

(3.2)

The Einstein equation of motion of this action is

\[
G_{\mu\nu} + \lambda m^2 g_{\mu\nu} - \frac{1}{2m^2} K_{\mu\nu} = 0,
\]

(3.3)
where

\[ K_{\mu \nu} = -\frac{1}{2} \nabla^2 R g_{\mu \nu} - \frac{1}{2} \nabla_\mu \nabla_\nu R + 2 \nabla^2 R_{\mu \nu} + 4 R_{\mu \alpha \nu \beta} R^{\alpha \beta} \]

\[ - \frac{3}{2} R R_{\mu \nu} - R_{\alpha \beta} R^{\alpha \beta} g_{\mu \nu} + \frac{3}{2} R^2 g_{\mu \nu}, \]  

(3.4)

where \( g^{\mu \nu} K_{\mu \nu} = \mathcal{L}_{NMG} \). The parameter \( \lambda \) is dimensionless and characterizes the cosmological constant term while \( m \) has the dimension of mass and provides the coupling to the NMG term. The solution of BTZ black hole is given by

\[ ds^2 = \left( -f(r) + \frac{16G^2J^2}{r^2} \right) dt^2 + \frac{dr^2}{f(r)} + r^2 d\varphi^2 + 8GJ dt d\varphi, \]

(3.5)

where

\[ f(r) = \left( \frac{r^2}{l^2} - 8GM + \frac{16G^2J^2}{r^2} \right), \quad l^2 = 2m^2 \left( -1 \pm \sqrt{1 - \frac{\lambda}{m^2}} \right). \]

(3.6)

The parameters \( M \) and \( J \) correspond to the mass and angular momentum in the case without the new massive gravity term, but their definitions in the case with NMG term are [16]

\[ M_1 = \left( 1 - \frac{1}{2m^2l^2} \right) M, \quad J_1 = \left( 1 - \frac{1}{2m^2l^2} \right) J. \]

(3.7)

Due to NMG term, the Bekenstein-Hawking entropy is renormalized by the same factor \( 1 - 1/2m^2l^2 \)

\[ S_{BH} = \frac{\pi r_h}{2G} \left( 1 - \frac{1}{2m^2l^2} \right), \]

(3.8)

here \( r_h = \sqrt{2G(lM + J)} + \sqrt{2G(lM - J)} \). In the paper [20] Liu and Sun have analyzed the Brown-Henneaux boundary condition for the NMG. They have calculated the conserved charges corresponding to the generators of the asymptotical symmetry under the Brown-Henneaux boundary condition (see Appendix). Then they have obtained the central charges of the Virasoro algebra as [20–22]

\[ c_L = c_R = \frac{3l}{2G} \left( 1 - \frac{1}{2m^2l^2} \right). \]

(3.9)

For the BTZ black hole solution in NMG, \( h \) and \( \tilde{h} \) are calculated as

\[ h = \frac{1}{2} (lM_1 + J_1) + \frac{c_L}{24}, \quad \tilde{h} = \frac{1}{2} (lM_1 - J_1) + \frac{c_R}{24}. \]

(3.10)
then the microscopic entropy is expressed by Cardy formula

\[ S_{\text{CFT}} = 2\pi \left( \sqrt{\frac{c_L \hbar}{6}} + \sqrt{\frac{c_R \hbar}{6}} \right) = \frac{\pi \hbar}{2G} \left( 1 - \frac{1}{2m^2l^2} \right), \]  

(3.11)

which agrees with renormalized Bekenstein-Hawking entropy formula (3.8). Now let us consider nonrelativistic limit in three-dimensional gravity:

\[ t \rightarrow t, \quad r \rightarrow r, \quad \varphi \rightarrow \epsilon \varphi. \]  

(3.12)

Accordingly, the parameters \( M \) and \( J \) in the BTZ solution must scale like

\[ M \rightarrow M, \quad J \rightarrow \epsilon J. \]  

(3.13)

As have been discussed in [11], the black hole metric (3.5) degenerates and looks singular in the Galilean limits (3.12), (3.13). This is similar to the usual Newtonian approximation \( c \rightarrow \infty \). This situation in the bulk gravity is describe by the Newton-Cartan-like geometry for the geometry with the AdS\(_2\) base [9, 11].

Since the Virasoro generators corresponds to \( \mathcal{L}_r^a = i\xi_n^{R, L} \), from the gravity side using (A.3), (3.12), and (2.3) one can define the generators \( L_n \) and \( M_n \) as

\[ L_n = e^{in\tau} \left[ \left( 1 - 2e^{-2\rho n^2} \right) \partial_{\tau} + i\phi \left( 1 + 2e^{-2\rho n^2} \right) \partial_{\phi} - i\rho \partial_{\rho} \right], \]

\[ M_n = e^{in\tau} \left( 1 + 2e^{-2\rho n^2} \right) \partial_{\phi}, \]  

(3.14)

these generators satisfy the centerless version of GCA algebra (2.4). From (2.5) and (3.9) the GCA central charges \( C_1 \) and \( C_2 \) in this case are

\[ C_1 = \frac{l}{4G} \left( 1 - \frac{1}{2m^2l^2} \right), \quad C_2 = 0. \]  

(3.15)

So, one of the GCA central charges vanishes because of the parity invariance of the NMG.

Similarly, from (2.6) and (3.10), scaling dimensions \( \Delta \) and rapidity \( \xi \), which are the eigenvalue of \( L_0 \) and \( M_0 \) are given by

\[ \Delta = \lim_{\epsilon \rightarrow 0} \left( lM_1 + \frac{c_L + c_R}{24} \right) = \left( 1 - \frac{1}{2m^2l^2} \right) lM + \frac{C_1}{2}, \]

\[ \xi = \lim_{\epsilon \rightarrow 0} \left( J + \frac{c_L - c_R}{24} \right) = 0. \]  

(3.16)

When \( M \) is large enough, the last term \( C_1/2 \) can be neglected.
Now we want to obtain the entropy of the GCA. The scaling limit \((M \to M, J \to \epsilon J)\) requires that the event horizon of the BTZ black hole should scale \(r_h \to 2l\sqrt{2G\epsilon}\), so the black hole entropy (3.8) is given by

\[
\lim_{\epsilon \to 0} S_{BH} = \lim_{\epsilon \to 0} S_{CFT} = \pi \left(1 - \frac{1}{2m^2l^2}\right) \sqrt{\frac{2l^2M}{G}}.
\] (3.17)

From the expressions of the central charge \(C_1\) (3.15) and scaling dimension \(\Delta\) (3.16), we can rewrite the entropy as

\[
S_{GCA} = \pi \sqrt{8\Delta C_1}.
\] (3.18)

This expression is the entropy for the GCA in two dimensions.

### 4. GCA Realization on Warped AdS\(_3\) Black Hole Solution of NMG

In this section we consider the warped black holes in NMG. The metric of warped NMG can be written as [16–18]

\[
ds^2 = dt^2 + \frac{dr^2}{\zeta^2 \eta^2 (r-r_+)(r-r_-)} + |\zeta|(r + \eta \sqrt{r_+ r_-}) dt d\varphi
\] (4.1)

\[
+ \frac{\zeta^2}{4} r \left( (1 - \eta^2) r + \eta^2 (r_+ + r_-) + 2 \eta \sqrt{r_+ r_-} \right) d\varphi^2.
\]

The ranges of the coordinates are \(t \in (-\infty, +\infty), r \in [0, +\infty), \) and \(\varphi \in [0, 2\pi]\). There are two horizons which are located at \(r_+\) and \(r_-\). The parameters \(\eta, \zeta\) are given by

\[
\eta^2 = \frac{21}{4} + \frac{2m^2l^2}{\zeta^2}, \quad \zeta^2 = \frac{4m^2l^2}{21} \left( -6 \pm \sqrt{3(5 - 7\lambda)} \right).
\] (4.2)

The Bekenstein-Hawking entropy in this case is

\[
S_{BH} = \frac{\pi l \zeta^3}{2m^2G} (r_+ + \eta \sqrt{r_+ r_-}).
\] (4.3)

The microscopic entropy of the dual CFT can be computed by the Cardy formula which matches with the black hole Bekenstein-Hawking entropy [17, 18]

\[
S_{CFT} = \frac{\pi^2 l}{3} (c_L T_L + c_R T_R).
\] (4.4)

The left and right moving temperatures introduced in [17, 18]:

\[
T_L = \frac{\eta^2 \zeta^2}{8\pi l} (r_+ + r_- + 2 \eta \sqrt{r_+ r_-}), \quad T_R = \frac{\eta^2 \zeta^2}{8\pi l} (r_+ - r_-).
\] (4.5)
By the calculation such as we review briefly in the Appendix, one can obtain the central charges of warped AdS$_3$ black hole solution of NMG as (see also [17, 18])

$$c_L = c_R = \frac{6l^2 \zeta}{G\eta^2 m^2}. \quad (4.6)$$

From the above central charges and temperatures (4.5) we have

$$S_{\text{CFT}} = \frac{\pi l^3}{2m^2 G} (r_+ + \eta \sqrt{r_+ r_-}), \quad (4.7)$$

which agrees precisely with the gravity result presented in [17, 18]. Now we consider nonrelativistic limit in three-dimensional new massive gravity:

$$t \to t, \quad r \to r, \quad \varphi \to e \varphi. \quad (4.8)$$

GCA central charges $C_1$ and $C_2$ are defined in terms of CFT central charges (2.5)

$$C_1 = \frac{l_\zeta}{G\eta^2 m^2}, \quad C_2 = 0. \quad (4.9)$$

CFT entropy (4.4) with the limit $(\epsilon \to 0)$ is converted to Galilean conformal entropy:

$$S_{\text{GCA}} = \lim_{\epsilon \to 0} \left( \frac{\pi^2}{3} \left[ 6C_1(T_L + T_R) + 6C_2 \left( \frac{T_L - T_R}{\epsilon} \right) \right] \right). \quad (4.10)$$

We define Galilean conformal temperatures as follows:

$$T_1 = \lim_{\epsilon \to 0} 6(T_L + T_R), \quad T_2 = \lim_{\epsilon \to 0} \frac{T_L - T_R}{\epsilon}, \quad (4.11)$$

the GCA entropy is

$$S_{\text{GCA}} = \frac{\pi^2}{3} (C_1 T_1 + C_2 T_2). \quad (4.12)$$

To make the GCA entropy finite, $T_L + T_R \sim O(1)$ and $T_L - T_R \sim O(\epsilon)$

$$T_L + T_R = \frac{\eta^2 l^2}{4\pi l} (r_+ + \eta \sqrt{r_+ r_-}), \quad T_L - T_R = \frac{\eta^2 l^2}{4\pi l} (r_- + \eta \sqrt{r_+ r_-}). \quad (4.13)$$

The parameters $r_+$ and $r_-$ in the nonrelativistic warped NMG must scale like

$$r_+ \to r_+, \quad r_- \to e^2 r_. \quad (4.14)$$
Then, $T_L + T_R = (\eta^2 \zeta^2 / 4 \pi l) r_s \sim O(1)$ and $T_L - T_R = (\eta^2 \zeta^2 / 4 \pi l) \epsilon \sim O(\epsilon)$, and the GCA entropy is finite

$$S_{GCA} = \frac{\pi l \zeta^3}{2m^2 G} r_s,$$  \hspace{1cm} (4.15)

which agrees with Bekenstein-Hawking entropy of warped CNMG in nonrelativistic limit (4.14).

5. Conclusion

In the present paper we have considered the BTZ and warped AdS$_3$ black hole solution of CNMG in the nonrelativistic limit. The BTZ solution of CNMG has been obtained from a BTZ solution of pure Einstein gravity only by redefinition of mass and angular momentum parameter of metric, as (3.7), [16]. We have shown that the contracted BTZ solution has a dual description in term of a 2-dimensional GCA. We have obtained the central charges $C_1, C_2$ and also the scaling dimension $\Delta$ and rapidity $\xi$ as (3.15) and (3.16), respectively. Using these results and scaling limits ($M \rightarrow M, J \rightarrow \epsilon J$), we have obtained the entropy of GCA$_2$ by (3.18), this expression exactly agrees with the nonrelativistic limit of Bekenstein-Hawking entropy of the black hole. After that we studied the warped AdS$_3$ black hole solution in nonrelativistic limit. Using the central charges $C_1, C_2$ for GCA$_2$, we obtained the entropy of nonrelativistic CFT$_2$, as (4.10). Then we have defined Galilean conformal temperatures by (4.11). Using these results we have obtained final form of GCA entropy by (4.15), which is exactly the nonrelativistic limit of entropy formula (4.3).

Appendix

Here we give a brief review of Brown-Henneaux boundary condition for the NMG (to see more about it refer to [20]). The NMG have an AdS$_3$ solution

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = L^2 \left( -\cosh^2 \rho \, d\tau^2 + \sinh^2 \rho \, d\phi^2 + d\rho^2 \right),$$ \hspace{1cm} (A.1)

by expanding $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$ around AdS$_3$, one could obtain the equation of motion for the linearized excitations $h_{\mu\nu}$. The Brown-Henneaux boundary condition for the linearized gravitational excitations in asymptotical AdS$_3$ spacetime in the global coordinate system can be written as

$$\begin{pmatrix}
  h_{++} = O(1) & h_{+-} = O(1) & h_{+\rho} = O(e^{-2\rho}) \\
  h_{-+} = h_{++} & h_{-\rho} = O(1) & h_{-\rho} = O(e^{-2\rho}) \\
  h_{\rho+} = h_{+\rho} & h_{\rho-} = h_{-\rho} & h_{\rho\rho} = O(e^{-2\rho})
\end{pmatrix},$$ \hspace{1cm} (A.2)
The corresponding asymptotic Killing vectors are

\[
\xi = \xi^+ \partial_+ + \xi^- \partial_- + \xi^\rho \partial_\rho \\
= \left[ e^\phi (\tau^+) + 2 e^{-2 \phi} \partial_\tau e^{-\psi} (\tau^-) + \mathcal{O}(e^{-4 \phi}) \right] \partial_+ + \left[ e^{-\phi} (\tau^-) + 2 e^{-2 \phi} \partial_\tau e^\psi (\tau^+) + \mathcal{O}(e^{-4 \phi}) \right] \partial_- \\
- \frac{1}{2} \left[ \partial_\tau e^\phi (\tau^+) + \partial_\tau e^{-\psi} (\tau^-) + \mathcal{O}(e^{-2 \phi}) \right] \partial_\rho,
\]

where \( \tau^\pm = \tau \pm \phi, \partial_\pm = 1/2(\partial_+ \pm \partial_-) \). Since \( \phi \) is periodic, one could choose the basis \( \xi^+_{\pm n} = e^{i n \tau^+} \) and \( \xi^-_{\pm n} = e^{i n \tau^-} \) and denote the corresponding Killing vectors as \( \xi^L_{\pm n} \) and \( \xi^R_{\pm n} \). The algebra structure of these vectors is

\[
i [\xi^L_{\pm m}, \xi^L_{\pm n}] = (m - n) \xi^L_{\pm m+n}, \quad i [\xi^R_{\pm m}, \xi^R_{\pm n}] = (m - n) \xi^R_{\pm m+n}, \\
i [\xi^L_{\pm m}, \xi^R_{\pm n}] = 0.
\]

So these asymptotic Killing vectors give two copies of Virasoro algebra. Then by calculating the conserved charges, the authors of [20] have shown that the left moving and right moving conserved charges fulfill two copies of Virasoro algebra with central charges (3.9).

References


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