Research Article

Final State Interaction Effects on the $B^+ \rightarrow J/\psi \rho^+$ Decay

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1. Introduction

The importance of FSI in weak nonleptonic $B$ meson decays is investigated by using a relativistic chiral unitary approach based on coupled channels [1–3]. The chiral Lagrangian approach is proved to be reliable for evaluating hadronic processes, but there are too many free parameters which are determined by fitting data, so that its applications are much constrained. Therefore, we have tried to look for some simplified models which can give rise to reasonable estimation of FSI [4, 5]. The FSI can be considered as a rescattering process of some intermediate two-body states with one particle exchange in the $t$-channel and computed via the absorptive part of the hadronic loop level (HLL) diagrams. The calculation with the single-meson-exchange scenario is obviously much simpler and straightforward. Moreover, some theoretical uncertainties are included in an off-shell form factor which
modifies the effective vertices. Since the particle exchanged in the t-channel is off shell and since final state particles are hard, form factors or cutoffs must be introduced to the strong vertices to render the calculation meaningful in perturbation theory. If the intermediate two body mesons are hard enough, so that the perturbative calculation can make sense and work perfectly well, but the FSI can be modelled as the soft rescattering of the intermediate mesons. When one or two intermediate meson can reach a low-energy region where they are not sufficiently hard, one can be convinced that at this region the perturbative QCD approach fails or cannot result in reasonable values. If the intermediate mesons are soft, one can conjecture that at this region the nonperturbative QCD would dominate, and it could be attributed into the FSI effects. Because all FSI processes are concerning nonperturbative QCD [6], we have to rely on phenomenological models to analyze the FSI effects in certain reactions. In fact, after weak decays of heavy mesons, the particles produced can rescatter into other particle states through nonperturbative strong interaction. In the FSI the decay $B^+ \rightarrow J/\psi \rho^+$ is a very interesting mode [7]. We calculated the $B^+ \rightarrow J/\psi \rho^+$ decay according to QCDF method and selected the leading order Wilson coefficients at the scale $m_b$ and obtained the $\text{BR}(B^+ \rightarrow J/\psi \rho^+) = (1.42 \pm 0.36) \times 10^{-5}$. The FSI can give sizable corrections, and we can utilize it [8]. Rescattering amplitude can be derived by calculating the absorptive part of triangle diagrams. In this case, intermediate states are $D^+D_0^0$, $D^{*+}D_0^0$, $D^{**}D_0^0$, and $D^{*+}D_0^{*0}$. Then, we calculated the $B^+ \rightarrow J/\psi \rho^+$ decay according to HLL method. Taken FSI corrections into account the branching ratio of $B^+ \rightarrow J/\psi \rho^+$ becomes $(4.2 - 5.8) \times 10^{-5}$ and the experimental result of this decay is $(5 \pm 0.8) \times 10^{-5}$ [9].

This paper is organized as follows. We present the calculation of QCDF for $B^+ \rightarrow J/\psi \rho^+$ decay in Section 2. In Section 3, we calculate the amplitudes of the intermediate states of $B^+ \rightarrow D^{(*)}D_0^{(*)}$ decays. Then, we present the calculation of HLL for $B^+ \rightarrow J/\psi \rho^+$ decay in Section 4. In Section 5, we give the numerical results, and in the last section, we have a short conclusion.

## 2. $B^+ \rightarrow J/\psi \rho^+$ Decay in QCD Factorization

A detailed discussion of the QCDF approach can be found in [10, 11]. Factorization is a property of the heavy-quark limit, in which we assume that the $b$ quark mass is parametrically large. The QCDF formalism allows us to compute systematically the matrix elements of the effective weak Hamiltonian in the heavy-quark limit for certain two-body final states $J/\psi \rho^+$. In this section, we obtain the amplitude of $B^+ \rightarrow J/\psi \rho^+$ decay using QCDF method. In factorization approach, there are color-suppressed tree and allowed penguin diagrams to $B^+ \rightarrow J/\psi \rho^+$ decay. We adopt leading order Wilson coefficients at the scale $m_b$ for QCDF
Figure 2: $B^+ \to D^+ \bar{D}^0$ decay diagrams.

approach. The diagrams describing this decay are shown in Figure 1. According to the QCDF, the amplitude of $B^+ \to J/\psi \rho^+$ decay is given by

$$A_{\text{QCDF}}(B^+ \to J/\psi \rho^+) = -i \frac{G_F}{\sqrt{2}} f_{J/\psi} m_{J/\psi} \left\{ \left( \epsilon_{J/\psi} \cdot \epsilon_{\rho} \right) (m_B + m_\rho) A_1^{B \rho} \left( m_{J/\psi}^2 \right) - \left( \epsilon_{J/\psi} \cdot p_B \right) \left( \epsilon_{\rho} \cdot p_B \right) \frac{2 A_2^{B \rho} \left( m_{J/\psi}^2 \right)}{m_B + m_\rho} \right\}$$

$$-i \epsilon_{\mu \nu \alpha \beta} \epsilon_{\mu}^{J/\psi} \epsilon_{\nu}^{\rho} \epsilon_{\rho}^{\bar{p}} p_{B 1}^{\bar{p}} \times \frac{2 V_{cb} V_{c d}^* a_2 - V_{tb} V_{t d}^* \left[ a_3 + r_X^{J/\psi} (a_5 + a_7 + a_9) \right] \}}{m_B + m_\rho} \right\},$$

where

$$r_X^{J/\psi} = \frac{2 m_{J/\psi} f_{J/\psi}}{m_b} \frac{f_{J/\psi}}{m_b} \frac{2 m_{J/\psi} f_{J/\psi}}{m_b} \frac{f_{J/\psi}}{m_b}$$

The effective coefficients $a_i$, which are specific to the factorization approach, and defined as

$$a_i = c_i^{\text{eff}} + \frac{1}{N_c} c_{i+1}^{\text{eff}} \quad (i = \text{odd}),$$

$$a_i = c_i^{\text{eff}} + \frac{1}{N_c} c_{i-1}^{\text{eff}} \quad (i = \text{even}),$$

(2.3)
where the quantities of $c_{i}^{\text{eff}}$ are effective Wilson coefficients at the renormalization scale $\mu$ for the $b \to \bar{d}$ transition. In the above amplitude the determination of $a_{2}$ in the $b \to c$ current-current transitions has received a lot of attention, the quantities of $a_{3}, a_{5}$ and $a_{7}, a_{9}$ are the QCD-penguin and electroweak-penguin coefficients, respectively. Numerical values of $a_{i}$ $(i = 1, \ldots, 10)$ for representative value of the phenomenological parameter $N_{c}$ are displayed in Section 5.

3. Amplitudes of Intermediate State

To estimate the $B^{+} \to D^{(*)} \bar{D}^{0(*)}$ decays amplitudes of intermediate states [12], the QCDF method is again used. For $B^{+} \to D^{+} \bar{D}^{0}$ decay, Feynman diagrams are shown in Figure 2, and the amplitude comes

$$A(B^{+} \to D^{+} \bar{D}^{0}) = i \frac{G_{F}}{\sqrt{2}} f_{D} \left\{ F_{0}^{BD} \left( m_{B}^{2} - m_{D}^{2} \right) \right.$$ \n
\hspace{1cm} $\times \left\{ (a_{1} + a_{2})V_{cb}V_{cd}^{*} + \left[ a_{4} + a_{10} + r_{X}^{D}(a_{6} + a_{8}) \right] \lambda_{p} \right\}$ \hspace{1cm} (3.1) \n
\hspace{1cm} $+ f_{B}f_{D} \left( b_{3} + \frac{1}{2} b_{3,EW} \right) \lambda_{p} \right\},$

where

$$\lambda_{p} = \sum_{p=uc} V_{pb}V_{pd}^{*},$$

$$b_{3} = \frac{C_{F}}{N_{c}^{2}} \left[ c_{3} A_{3}^{i} + c_{5} \left( A_{3}^{i} + A_{3}^{f} \right) + N_{c} c_{6} A_{3}^{f} \right],$$

$$b_{3,EW} = \frac{C_{F}}{N_{c}^{2}} \left[ c_{9} A_{3}^{i} + c_{7} \left( A_{3}^{i} + A_{3}^{f} \right) + N_{c} c_{8} A_{3}^{f} \right],$$

$$A_{3}^{i} \approx 2\pi a_{s} \left[ 9 \left( X_{A} - 4 + \frac{\pi^{2}}{3} \right) + r_{X}^{D} r_{X}^{D'} \left( 2X_{A}^{2} - X_{A} \right) \right],$$

$$A_{3}^{f} = 0,$$

$$A_{5}^{i} \approx 6\pi a_{s} \left( r_{X}^{D'} + r_{X}^{D} \right) \left( 2X_{A}^{2} - X_{A} \right),$$

$$X_{A} = \frac{1 + \rho e^{i\phi}}{\Lambda_{\text{QCD}}},$$

where

There are also some similar diagrams such as $B^{+} \to D^{(*)} \bar{D}^{0(*)}$ and $B^{+} \to D^{(*)} \bar{D}^{0}(D^{(*)} \bar{D}^{0})$ decays, which have polarization vectors. In their amplitude, just contribution of $a_{1}$ is
considered, which has the dominant contribution. So the amplitude of $B^+ \to D^{*+} \bar{D}^{*0}$ decay, read as

$$A \left( B^+ \to D^{*+} \bar{D}^{*0} \right) = -i \frac{G_F}{\sqrt{2}} f_{D^* \bar{D}^*} V_{cb} V_{cd}^* \left\{ (\epsilon_1 \cdot \epsilon_2^*) (m_{B^+} + m_{D^{*+}}) A_1^{BD^*} \left( m_{D^{*+}}^2 \right) - (\epsilon_1 \cdot p_B) (\epsilon_2^* \cdot p_B) \frac{2 A_2^{BD^*} (m_{D^{*+}}^2)}{m_{B^+} + m_{D^{*+}}} \right\}. \tag{3.3}$$

where $A_1^{BD^*}$ and $A_2^{BD^*}$ are form factors for $B \to D^*$ [13], and the amplitude of $B^+ \to D^{*+} \bar{D}^{*0}$ is given by

$$A \left( B^+ \to D^{*+} \bar{D}^{*0} \right) = \sqrt{2} G_F f_{D^* \bar{D}^*} V_{cb} V_{cd}^* (\epsilon_{D^*} \cdot p_{D^*}) \left[ f_{D^* \bar{D}^*} F_1^{BD^*} \left( m_{D^{*+}}^2 \right) + f_{D^* \bar{D}^*} A_2^{BD^*} \left( m_{D^{*+}}^2 \right) \right]. \tag{3.4}$$

### 4. Final State Interaction of $B^+ \to J/\psi \rho^+$ Decay

In our approach, we consider only leading contributions in charmless FSI, when the FSI for decay is calculated, two-body intermediate states, such as, $D^+ \bar{D}^0, D^{*+} \bar{D}^{*0}, D^{*+} \bar{D}^{0},$ and $D^+ \bar{D}^{*0}$ via the exchange of $D^-$ and $D^{*-}$ are contributed to the $B^+ \to J/\psi \rho^+$ decay. The quark model for $B^+ \to D^{*+} \bar{D}^{*0} \to J/\psi \rho^+$ diagram is shown in Figure 3. This diagram just shows tree level ($a_1$ contribution), while there are other contributions that were calculated in the last section. The diagrams which determine the HLL effects on the rate of $B^+ \to J/\psi \rho^+$ decay are shown in Figure 4. At the HLL, the charge exchange reaction between the $J/\psi$ and $\rho$ can proceed either by exchange of a $D$ or $D^*$ meson. In order to calculate the various Feynman diagrams for the reactions $D + \bar{D} \to J/\psi + \rho$. And so forth (were calculated in the nonrelativistic quark model [14]) we need the construct the effective three mesons vertices, such as, $\phi DD$ and $\rho DD$. We begin with an SU(4) symmetry so as to include charm, and we introduce pseudoscalar ($P = \phi_a \lambda_a$) and vector ($V^\mu = v^\mu_a \lambda_a$) meson matrices, where $\phi_a$ and
Figure 4: HLL diagrams for long-distance $t$-channel contribution to $B^+ \rightarrow J/\psi \rho^+$. 

$v_a^\mu$ are pseudoscalar and vector multiplets and the $\lambda_s$ are SU(4) generators. The free meson lagrangian is read [15]

$$\mathcal{L}_0 = \text{Tr} \left( \partial^{\mu} P^{\dagger} \partial_{\mu} P \right) - \frac{1}{2} \text{Tr} \left( \partial^{\mu} V^{\dagger,\nu} - \partial^{\nu} V^{\dagger,\mu} \right) (\partial_{\mu} V_{\nu} - \partial_{\nu} V_{\mu}) \quad (4.1)$$

Interaction Lagrangians are generated by replacing the space-time derivative with a gauge covariant one $\hat{\partial}_\mu f \rightarrow D_\mu f = \partial_\mu f + [A_\mu, f]$, where $A_\mu = -ig/2V_\mu$. The vector mesons are
recognizable as playing roles of gauge bosons. As is usual in effective field theory strategies, to keep gauge consistency, we must collect terms of order $g^2$. The effective lagrangian is read

$$\mathcal{L}_{\text{int}} = ig\text{Tr}(PV\mu\partial_\mu P - \partial_\mu PV\mu P) + \frac{1}{2}g^2\text{Tr}(PV\mu P - PV^\mu PV_\mu)$$

$$+ ig\text{Tr}(\partial^\nu V^\mu [V_\mu, V_\nu] + [V^\mu, V^\nu]\partial_\mu V_\nu) + g^2\text{Tr}(V^\mu V^\nu [V_\mu, V_\nu]).$$

The order $g$ terms deliver three-point vertices and the order $g^2$ terms are responsible for so-called contact terms or four-point couplings which are necessary for a gauge invariant theory. The interactions of interest for this study are the following [5]:

$$\mathcal{L} = igt_{\mu\nu\rho}(D^\mu\partial_\nu D_\rho - \partial_\nu D^\mu D_\rho),$$

$$\mathcal{L} = -igt_{\mu\nu\rho}(D\partial_\mu \rho_\nu D_\rho + \partial_\mu D^\nu \partial_\nu D^\mu D_\rho),$$

$$\mathcal{L} = igt_{\mu\nu}(D^\mu \partial_\nu D_\rho - \partial_\nu D^\mu D_\rho),$$

$$\mathcal{L} = igt_{\mu\nu\rho}(\partial_\mu D^\nu D_\rho - D^\nu \partial_\mu D_\rho),$$

$$\mathcal{L} = igt_{\mu\nu\rho}(D^\mu \partial_\nu D_\rho - \partial_\nu D^\mu D_\rho),$$

$$\mathcal{L} = igt_{\mu\nu\rho}(\partial_\mu D^\nu D_\rho - D^\nu \partial_\mu D_\rho),$$

Here $\epsilon_{0123} = +1$ and we define the charm meson iso-doublets as

$$\overline{D}^T = (D^0, D^-), \quad D = (D^0, D^+),$$

$$\overline{D}^{\ast T} = (D^{0*}, D^{--}), \quad D^{\ast} = (D^{0*}, D^{++}).$$

In this approach there are several coupling constants in (4.3). Methods for constraining them will be discussed below. The $O(g^2)$ terms carry one power of coupling constant for each three-point vertices from which the contact term collapses. The coupling constants $g_{\psi DD}, g_{\rho DD},$
$g_{\rho D^*D^*}$ and $g_{\psi D^*D^*}$ can be derived in the standard framework of the vector meson dominance model (VDM) [16–18]. By using VDM to the coupling of $\gamma DD$ we have

$$
\gamma_\rho g_{\rho DD} = \sqrt{\frac{4}{3} \alpha_{em}}, \quad \gamma_{J/\psi} g_{\psi DD} = \frac{4}{3} \sqrt{\frac{4 \alpha_{em}}{3}}
$$

(4.5)

where

$$
\gamma_\rho = \sqrt{\frac{3 \Gamma_\rho \rightarrow e^+e^-}{m_\rho \alpha_{em}}}, \quad \gamma_{J/\psi} = \sqrt{\frac{3 \Gamma_{J/\psi} \rightarrow e^+e^-}{m_{J/\psi} \alpha_{em}}}.
$$

(4.6)

The same method can be used for the $\gamma D^*D^*$ coupling. Vector dominance gives

$$
g_{\rho D^*D^*} = g_{\rho DD}, \quad g_{\psi D^*D^*} = g_{\psi DD}.
$$

(4.7)

For the $\rho D^*D$ and $\psi D^*D$ couplings the VDM can be applied to the radiative decays of $D^*$ into $D$, that is, $D^* \rightarrow D\gamma$. By using the same technique we have

$$
\gamma_\rho g_{\rho D^*D} = \frac{1}{3} \sqrt{\frac{4 \alpha_{em}}{3}} (g_{\overline{V}} + 2 g_{V}^0), \quad \gamma_{J/\psi} g_{\psi DD} = \frac{4}{3} \sqrt{\frac{4 \alpha_{em}}{3}} g_{\overline{V}} = \frac{4}{3} \sqrt{\frac{4 \alpha_{em}}{3}} g_{V}^0
$$

(4.8)

where $g_{\overline{V}V}^0$ are calculated in the model prediction based on relativistic potential model that we follow [19]. With the above preparation we can write out the decay amplitude involving HLL contributions with the following formula:

$$
A(B(p_B) \rightarrow M(p_1) M(p_2) \rightarrow M(p_3) M(p_4)) = \frac{1}{2} \int \frac{d^3\vec{p}_1}{2E_1(2\pi)^3} \frac{d^3\vec{p}_2}{2E_2(2\pi)^3}
$$

$$
\times (2\pi)^4 \delta^4(p_B - p_1 - p_2) M(B \rightarrow M_1 M_2)
$$

$$
\times G(M_1 M_2 \rightarrow M_3 M_4),
$$

(4.9)

for which both intermediate mesons ($M_1, M_2$) are pseudoscalar. And

$$
A(B(p_B) \rightarrow M(p_1) M(p_2) \rightarrow M(p_3) M(p_4)) = -i G_F \frac{1}{2\sqrt{2}} \int \frac{d^3\vec{p}_1}{2E_1(2\pi)^3} \frac{d^3\vec{p}_2}{2E_2(2\pi)^3}
$$

$$
\times (2\pi)^4 \delta^4(p_B - p_1 - p_2) f_D m_D V_{cb} V_{cd}^*.
$$
\[
\times \left[ (e_1^+ \cdot e_2^+) (m_B + m_1) A_1^{BM} \left( m_2^2 \right) \\
- (e_1^+ \cdot p_B) (e_2^+ \cdot p_B) \frac{2A_2^{BM} \left( m_2^2 \right)}{m_B + m_1} \right] \\
\times G(M_1 M_2 \rightarrow M_3 M_4),
\]
(4.10)
in which both meson are vector. Also \(G(M_1 M_2 \rightarrow M_3 M_4)\) involves hadronic vertices factor, which is defined as

\[
\langle D(p_3) \rho(e_2, p_2) | i\mathcal{E}| D(p_1) \rangle = -i g_{DD\rho} (p_1 + p_3), \\
\langle D^*(e_3, p_3) \rho(e_2, p_2) | i\mathcal{E}| D(p_1) \rangle = -i v(\rho_{DD\rho} e_2^{\mu} e_3^{\nu} p_1^\mu p_2^\nu) \\
\langle D^*(e_3, p_3) \psi(e_2, p_2) | i\mathcal{E}| D(p_1) \rangle = -i g_{DD\psi} (p_1 + p_3), \\
\langle D^*(e_3, p_3) \psi(e_2, p_2) | i\mathcal{E}| D^*(e_1, p_1) \rangle = -ie_1^+ e_2^+ e_3^+ \left[ 2p_{1v} g_{\alpha\beta} - (p_1 + p_2)_\alpha g_{\alpha\beta} + 2p_2 g_{\alpha\beta} \right],
\]
(4.11)
The dispersive part of the rescattering amplitude can be obtained from the absorptive part via the dispersion relation [6, 20]:

\[
\text{DisM} \left( m_B^2 \right) = \frac{1}{\pi} \int_s^{\infty} \frac{\text{Abs M}(s')}{{s'}^2 - m_B^2} ds',
\]
(4.12)
where \(s'\) is the square of the momentum carried by the exchanged particle and \(s\) is the threshold of intermediate states, in this case \(s > m_B^2\). Unlike the absorptive part, the dispersive contribution suffers from the large uncertainties arising from the complicated integrations. So the amplitudes of the mode \(B^+ \rightarrow D^*(p_1) \overline{D}^0 (p_2) \rightarrow J/\psi(p_3, \epsilon_3) \rho^+(p_4, \epsilon_4)\), where \(D\) and \(D^*\) mesons are exchanged, respectively, are given by

\[
\text{Abs}(4a) = \int_{-1}^{1} \frac{|\bar{p}_1| d(\cos \theta)}{16\pi m_B} \mathcal{A}(B^+ \rightarrow D^* \overline{D}^0) (-i) g_{\rho DD} (2\epsilon_3 \cdot p_1) \\
\times (-i) g_{DD\rho} (2\epsilon_4 \cdot p_2) \frac{F^2(q^2, m_D^2)}{q^2 - m_D^2} \\
= - \int_{-1}^{1} \frac{|\bar{p}_1| d(\cos \theta)}{4\pi m_B} g_{\rho DD} g_{DD\rho} \mathcal{A}(B^+ \rightarrow D^* \overline{D}^0) \frac{F^2(q^2, m_D^2)}{q^2 - m_D^2} H_1,
\]
(4.13)
where $\theta$ is the angle between $\vec{p}_1$ and $\vec{p}_3$, $q$ is the momentum of the exchange $D$ meson, and

\begin{equation}
H_1 = \epsilon_3 p_i^\mu \epsilon_4 v_2^\nu \left( p_i^0 - |\vec{p}_1| \right) \left( p_2^0 - |\vec{p}_2| \right),
\end{equation}

\begin{equation}
q^2 = m_1^2 + m_2^2 - 2E_1E_3|\vec{p}_1| |\vec{p}_2| \cos \theta,
\end{equation}

and $F(q^2, m^2_D)$ is the form factor defined to take care of the off-shell of the exchange particles, which introduced as [6, 21]

\begin{equation}
F(q^2, m^2_D) = \left( \frac{\Lambda^2 - m^2_D}{\Lambda^2 - q^2} \right)^n.
\end{equation}

The form factor (i.e., $n = 1$) normalized to unity at $q^2 = m^2_D$. $m_D$ and $q$ are the physical parameters of the exchange particle and $\Lambda$ is a phenomenological parameter. It is obvious that for $q^2 \to 0$, $F(q^2, m^2_D)$ becomes a number. If $\Lambda \gg m_i$ then $F(q^2, m^2_D)$ turns to be unity, whereas, as $q^2 \to \infty$ the form factor approaches to zero and the distance becomes small and the hadron interaction is no longer valid. Since $\Lambda$ should not be far from the $m_D$ and $q$, we choose

\begin{equation}
\Lambda = m_D + \eta \Lambda_{QCD},
\end{equation}

where $\eta$ is the phenomenological parameter that its value in the form factor is expected to be of the order of unity and can be determined from the measured rates, and

\begin{equation}
\text{Abs}(4b) = \int_{-1}^{1} \frac{|\vec{p}_1|}{16\pi m_B} \frac{d(cos \theta)}{A(B^+ \rightarrow D^+ \overline{D}^0)} (-i) \chi g_{D^* D^0} e^{\rho\lambda\nu\sigma} e_3 p_1^\rho p_2^\lambda p_3^\mu \left( g_{\sigma\nu} - \frac{q_\sigma q_\nu}{m^2_D} \right) \frac{F^2(q^2, m^2_D)}{q^2 - m^2_D} - \int_{-1}^{1} \frac{|\vec{p}_1|}{16\pi m_B} \frac{d(cos \theta)}{A(B^+ \rightarrow D^+ \overline{D}^0)} \frac{F^2(q^2, m^2_D)}{q^2 - m^2_D} H_2,
\end{equation}

where

\begin{equation}
H_2 = -2 \left[ (p_1 \cdot p_4)(p_2 \cdot p_3) - (p_1 \cdot p_2)(p_3 \cdot p_4) \right] + p_3^0 \left[ p_1^0 (p_1 \cdot p_2) - (p_2^0 - |\vec{p}_2|)(p_1 \cdot p_4) \right] + (p_1^0 - |\vec{p}_1|) \left[ (p_2^0 - |\vec{p}_2|)(p_3 \cdot p_4) - p_4^0 (p_2 \cdot p_3) \right].
\end{equation}
For diagrams (4c) and (4d) in Figure 4 the absorptive part of the amplitude corresponding to the process of $B^+ \to D^{**}(p_1, e_1)D^0(p_2, e_2) \to J/ψ(p_3, e_3)ρ^+(p_4, e_4)$, where $D^-$ and $D^{**}$ mesons are exchanged, respectively, is read as

$$
\text{Abs}(4c) = -\frac{iG_F}{2\sqrt{2}} \int \frac{d^3p_1}{2E_1(2\pi)^3} \frac{d^3p_2}{2E_2(2\pi)^3} (2\pi)^4 \delta^4(p_B - p_1 - p_2) 
\times (-i)g_{ρD^0D}e_3^μe_3^νe_1^σe_1^τp_1^μp_1^ν p_2^σp_2^τ \frac{F^2(q^2, m_B^2)}{q^2 - m_D^2} 
\times f_{D^-}\bar{m}_{D^-}\lambda_{Cb}V_{cd} \left\{ (e_1^αe_2^ρ) (m_B + m_{D^0}) A_{1}^{BD} \left(m_{D^0}^2, H_3 - \frac{2A_{2}^{BD} \left(m_{D^0}^2, H_3\right)}{m_B + m_{D^0}} \right) \right\},
$$

where

$$
H_3 = e_μ e_ν e_ρ e_τ e_3^μ e_3^ν e_1^ρ e_1^τ p_1^μ p_1^ν p_2^ρ p_2^τ \left(-g_ρ^α + \frac{p_1^ρ p_1^τ}{m_{D^0}^2}\right) \left(-g_τ^α + \frac{p_2^ρ p_2^τ}{m_{D^0}^2}\right)
$$

$$
= -2[(p_1 \cdot p_2)(p_3 \cdot p_4) - (p_1 \cdot p_4)(p_2 \cdot p_3)] + p_4^0 \left[(p_1^0 - |p_1|^2)(p_2 \cdot p_3) - p_3^0(p_1 \cdot p_2)\right]
+ (p_2^0 - |p_2|^2) \left[p_3^0(p_1 \cdot p_4) - (p_1^0 - |p_1|^2)(p_3 \cdot p_4)\right],
$$

$$
H_3' = e_μ e_ν e_ρ e_τ e_3^μ e_3^ν e_1^ρ e_1^τ p_1^μ p_1^ν p_2^ρ p_2^τ \left(-g_ρ^α + \frac{(p_1 \cdot p_2)p_1^ρ}{m_{D^0}^2}\right) \left(-g_τ^α + \frac{(p_2 \cdot p_3)p_2^τ}{m_{D^0}^2}\right)
$$

$$
= m_B^2 \left[p_1^0(p_2 - |p_2|^2)(p_3 \cdot p_4) - 2[(p_1 \cdot p_2)(p_3 \cdot p_4) - (p_1 \cdot p_4)(p_2 \cdot p_3)]\right] + p_4^0 \left[(p_2^0 - |p_2|^2)(p_1 \cdot p_4) - p_3^0(p_1 \cdot p_2)\right]
+ [p_3 \cdot p_4 - p_2^0 p_4^0, p_4^0(p_2 - |p_2|^2) + p_2^0(p_1^0 - |p_1|^2) - p_1 \cdot p_2] + 3p_4^0 p_3^0(p_1 \cdot p_2) - p_1^0(p_2 \cdot p_3) + p_2^0 p_4^0(p_1^0 - |p_1|^2) + p_2^0 p_3^0(p_2^0 - |p_2|^2)] - 2p_2^0 p_4^0(p_1 \cdot p_4) - p_1^0(p_3 \cdot p_4) + p_2^0 p_4^0(p_1^0 - |p_1|^2),
$$

$$
\text{Abs}(4d) = -\frac{iG_F}{2\sqrt{2}} \int \frac{d^3p_1}{2E_1(2\pi)^3} \frac{d^3p_2}{2E_2(2\pi)^3} (2\pi)^4 \delta^4(p_B - p_1 - p_2) 
\times (-i)g_{ρD^0D}e_3^μe_3^ν[2p_1ρg_ρ^β - (p_1 + p_3)_β g_ρ^β + 2p_3μg_ρ^β] \left(g_ρ^α + \frac{q^α q^β}{m_{D^0}^2}\right)
$$
\[
\times (-i) g^{D^0 D^0}_F e_2^e_4 (2p_{21} g_{a v} - (p_2 + p_4)_{a} g_{a 1} + 2p_{4 v} g_{a 1}) \frac{F^2(q^2, m^2_{D^0})}{q^2 - m^2_{D^0}}
\]

\[
\times f_{D^0 m_{D^0}} V_{cd} V^*_{cd} \left\{ (e_1^e_2^e_4^e_2^e_4) (m_B + m_{D^0}) A^{BD}_1 \left( m^2_{D^0} \right) - (e_1^e_2^e_4^e_2^e_4) (m_B + m_{D^0}) A^{BD}_2 \left( m^2_{D^0} \right) \right\}
\]

\[
= \frac{G_F}{16 \sqrt{2} \pi m_B} f_{D^0 m_{D^0}} V_{cd} V^*_{cd} \int_{-1}^{1} |\vec{p}_1| d(\cos \theta) \frac{F^2(q^2, m^2_{D^0})}{q^2 - m^2_{D^0}}
\]

\[
\times \left\{ (m_B + m_{D^0}) A^{BD}_1 \left( m^2_{D^0} \right) H_4 - \frac{2 A^{BD}_2 \left( m^2_{D^0} \right)}{m_B + m_{D^0}} H_4 \right\},
\]

(4.20)

where

\[
H_4 = \left( -g^{a b} + \frac{q^a q^b}{m^2_{D^0}} \right) \left( -g^{\mu \nu} + \frac{p^\mu_1 p^\nu_1}{m^2_{D^0}} \right) \left( -g^{\nu \lambda} + \frac{p^\mu_2 p^\lambda_2}{m^2_{D^0}} \right) \epsilon_4^e_4 \]

\[
\times \left( 2p_{1 b} g_{\mu b} - (p_1 + p_3)_{\mu b} g_{\mu b} + 2p_{3 b} g_{\mu b} \right) (2p_{21} g_{a v} - (p_2 + p_4)_{a} g_{a 1} + 2p_{4 v} g_{a 1}),
\]

\[
H'_4 = \left( -g^{a b} + \frac{q^a q^b}{m^2_{D^0}} \right) \left( -p^\mu_B + \frac{(p_1 \cdot p_B) p^\mu_1}{m^2_{D^0}} \right) \left( -p^\nu_B + \frac{(p_2 \cdot p_B) p^\nu_2}{m^2_{D^0}} \right) \epsilon_4^e_4 \]

\[
\times \left( 2p_{1 b} g_{\mu b} - (p_1 + p_3)_{\mu b} g_{\mu b} + 2p_{3 b} g_{\mu b} \right) (2p_{21} g_{a v} - (p_2 + p_4)_{a} g_{a 1} + 2p_{4 v} g_{a 1}).
\]

(4.21)

For diagrams (4e) and (4f) in Figure 4 the absorptive part of the amplitude corresponding to the process of \( B^+ \rightarrow D^{*+}(p_1, e_1) \overline{D}^0 (p_2) \rightarrow J/\psi(p_3, e_3) \rho^+(p_4, e_4) \), where \( D^- \) and \( D^{*-} \) mesons are exchanged, respectively, becomes

\[
\text{Abs}(4e) = \frac{G_F}{\sqrt{2}} \int \frac{d^3 p_1}{2 E_1 (2\pi)^3} \frac{d^3 p_2}{2 E_2 (2\pi)^3} (2\pi)^4 \delta^4(p_B - p_1 - p_2)
\]

\[
\times V_{cb} V^*_{cd} m_{D^0} (-i) g_{\mu b} \epsilon_4 (p_1) (p_2) (p_3) (p_4) (\epsilon_4 \cdot q) (\epsilon_1 \cdot p_2)
\]

\[
\times \left[ f_{D^0} F^{BD}_1 \left( m^2_{D^0} \right) + f_{D^0} A^{BD}_0 \left( m^2_{D^0} \right) \right] \frac{F^2(q^2, m^2_{D^0})}{q^2 - m^2_{D^0}}
\]

(4.22)

\[
= - \frac{G_F}{4 \sqrt{2} \pi m_B} \left( f_{D^0} F^{BD}_1 \left( m^2_{D^0} \right) + f_{D^0} A^{BD}_0 \left( m^2_{D^0} \right) \right) V_{cb} V^*_{cd} m_{D^0} g_{\mu b} \epsilon_4 (p_1) (p_2)
\]

\[
\times \int_{-1}^{1} |\vec{p}_1| d(\cos \theta) \frac{F^2(q^2, m^2_{D^0})}{q^2 - m^2_{D^0}} H_5,
\]
where

\[ H_5 = \epsilon_{\rho \sigma \eta} \epsilon_4^\mu p_2^\rho p_3^\sigma (\epsilon_4 \cdot p_2) \left( -p_2^\nu + \frac{(p_1 \cdot p_2)p_1^\nu}{m_D^2} \right), \]

\[ \text{Abs}(4f) = \frac{G_F}{\sqrt{2}} \left( \frac{d^3p_1}{2E_1(2\pi)^3} \frac{d^3p_2}{2E_2(2\pi)^3} (2\pi)^4 \delta^4(p_B - p_1 - p_2) \right. \]
\[ \times V_{cb} V_{cd}^* m_{D^*} (-i) \epsilon_4^\mu \epsilon_2^\nu p_2^\rho p_3^\sigma \left[ 2p_{1\nu}g_{\rho \beta} - (p_1 + p_3)_{\rho} g_{\mu \nu} + 2p_{3\mu}g_{\rho \nu} \right] \]
\[ \times (i) \sqrt{2} g_{D^\nu D^p} \epsilon_{\rho \sigma \eta} \epsilon_4^\mu \epsilon_2^\nu p_2^\rho p_3^\sigma \left[ F_{11}^D(m_{D^*}^2) + f_{D^0 A_0^{BD}}(m_{D^*}^2) \right] \frac{F^2(q^2, m_D^2)}{q^2 - m_D^2}. \]

For diagrams (4g) and (4h) in Figure 4, the absorptive part of the amplitude corresponding to the process of \( B^- \to D^*(p_1) D^0(p_2, e_2) \to J/\psi(p_3, e_3) \rho^+(p_4, e_4) \), where \( D^- \) and \( D^{*-} \) mesons are exchanged, respectively, is given

\[ \text{Abs}(4g) = \frac{G_F}{\sqrt{2}} \left( \frac{d^3p_1}{2E_1(2\pi)^3} \frac{d^3p_2}{2E_2(2\pi)^3} (2\pi)^4 \delta^4(p_B - p_1 - p_2) \right. \]
\[ \times V_{cb} V_{cd}^* m_{D^*} \left( i\sqrt{2} \right) g_{D^0 D^p} \epsilon_{\rho \sigma \eta} \epsilon_4^\mu \epsilon_2^\nu p_2^\rho p_3^\sigma \left( -i \right) g_{D^0 D^0} (e_3 \cdot q) (e_2 \cdot p_1) \]
\[ \times \left[ F_{11}^D(m_{D^*}^2) + f_{D^0 A_0^{BD}}(m_{D^*}^2) \right] \frac{F^2(q^2, m_D^2)}{q^2 - m_D^2}. \]

(4.25)
where

\[ H_7 = \epsilon_{\mu\nu\rho\sigma}^{\epsilon} p_2^\mu p_3^\nu \epsilon_3 \cdot p_1 \left( -p_1^\nu + \frac{(p_1 \cdot p_2)p_2^\nu}{m_{D'}^2} \right) , \]

\[
\text{Abs}(4h) = \frac{G_F}{\sqrt{2}} \int \frac{d^4p_1}{2E_1(2\pi)^3} \frac{d^4p_2}{2E_2(2\pi)^3} (2\pi)^4 \delta^4(p_B - p_1 - p_2) 
\times V_{cb} V_{cd}^* m_{D'}^2 (i) g_{D^+ D'\rho} e_2^{\epsilon} e^\beta [2p_2 \mu g_{\alpha\nu} - (p_2 + p_4)_{\alpha} g_{\mu\nu} + 2p_4 \nu g_{\alpha\nu}] 
\times (-i) g_{\psi D^+ D} e_{\rho\lambda\eta} e_3^{\epsilon} e^\gamma p_1^\lambda p_3^\eta [F_{D^0} F_{D^+} (m_{D'}^2) + F_{D} A_{D'} (m_{D'}^2)] \frac{F^2(q^2, m_{D'}^2)}{q^2 - m_{D'}^2} 
\times \frac{G_F}{8\sqrt{2} m_B} (f_{D^0} F_{D^0} (m_{D'}^2) + f_{D} A_{D'} (m_{D'}^2)) V_{cb} V_{cd}^* m_{D'}^2 g_{\psi D^+ D} D^0 D^+ D', \]

\[ (4.26) \]

where

\[ H_8 = \epsilon_{\rho\lambda\eta\mu}^{\epsilon} e_3^{\epsilon} e_4^{\beta} p_1^\lambda p_3^\eta [2p_2 \mu g_{\alpha\nu} - (p_2 + p_4)_{\alpha} g_{\mu\nu} + 2p_4 \nu g_{\alpha\nu}] \]

\[ \times \left(-g_{\beta\nu} q_1^\rho q_2^\gamma \frac{q^2}{m_{D'}^2} \right) \left(-p_1^\nu + \frac{(p_1 \cdot p_2)p_2^\nu}{m_{D'}^2} \right). \]

\[ (4.27) \]

As the bridge between the dispersive part of FSI amplitude and the absorptive part, the dispersion relation is

\[
\text{Dis}^4 \left( m_B^2 \right) = \frac{1}{\pi} \int_0^{\infty} \frac{\text{Abs}(4a)(s') + \text{Abs}(4b)(s') + \text{Abs}(4c)(s') \cdots \text{Abs}(4h)(s')}{s' - m_B^2} ds'. \]

\[ (4.28) \]

According to the FSI, the amplitude of \( B^+ \to J/\psi\rho^+ \) decay is given by

\[ A_{\text{FSI}} \left( B^+ \to D^{(*)} D^{0(*)} \to J/\psi\rho^+ \right) = \text{Abs}(4a) + \text{Abs}(4b) + \text{Abs}(4c) + \text{Abs}(4d) + \text{Abs}(4e) \]

\[ + \text{Abs}(4f) + \text{Abs}(4g) + \text{Abs}(4h) + \text{Dis}^4 \left( m_B^2 \right). \]

\[ (4.29) \]

Concerning the QCDF amplitude and FSI correction, then the total decay amplitude turns to

\[ A \left( B^+ \to J/\psi\rho^+ \right) = A_{\text{QCDF}} + i A_{\text{FSI}}. \]

\[ (4.30) \]
5. Numerical Results

Numerical values of effective coefficients $a_i$ for $b \rightarrow d$ transition at $N_c = 3$ are given by [22]

\[
\begin{align*}
a_1 &= 1.05, & a_2 &= 0.053, \\
a_3 &= 0.0048, & a_4 &= -0.0461 - 0.0124i, \\
a_5 &= -0.0045, & a_6 &= -0.0597 - 0.0124i, \\
a_7 &= 0.00003 - 0.00018i, & a_8 &= 0.00045 - 0.00006i, \\
a_9 &= -0.0095 - 0.00018i, & a_{10} &= -0.0014 - 0.00006i.
\end{align*}
\]

The Cabibbo-Kobayashi-Maskawa (CKM) matrix is a $3 \times 3$ unitary matrix as [9]

\[
V = \begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix}.
\]

The elements of the CKM matrix can be parameterized by three mixing angles $A, \lambda, \rho$ and a CP-violating phase $\eta$

\[
\begin{align*}
V_{ud} &= 1 - \frac{\lambda^2}{2}, & V_{us} &= \lambda, & V_{ub} &= A\lambda^3(\rho - i\eta), \\
V_{cd} &= -\lambda, & V_{cs} &= 1 - \frac{\lambda^2}{2}, & V_{cb} &= A\lambda^2, \\
V_{td} &= A\lambda^3(1 - \rho - i\eta), & V_{ts} &= -A\lambda^2, & V_{tb} &= 1.
\end{align*}
\]

The results for the Wolfenstein parameters are

\[
\lambda = 0.2257 \pm 0.001, \quad A = 0.814 \pm 0.02, \quad \bar{\rho} = 0.135 \pm 0.023, \quad \bar{\eta} = 0.349 \pm 0.016.
\]

We use the central values of the Wolfenstein parameters and obtain

\[
\begin{align*}
V_{ud} &= 0.9745, & V_{us} &= 0.2257, & V_{ub} &= 0.0035, \\
V_{cd} &= 0.2257, & V_{cs} &= 0.9745, & V_{cb} &= 0.0415, \\
V_{td} &= 0.0087, & V_{ts} &= 0.0415, & V_{tb} &= 1.
\end{align*}
\]
The meson masses and decay constants needed in our calculations are taken as (in units of Mev)

\[ m_{B^*} = 5278 \pm 0.3, \quad m_{D^*} = 1869 \pm 0.2, \]
\[ m_{D_0} = 1864.84 \pm 0.17, \quad m_{D^{*0}} = 2010.27 \pm 0.17, \]
\[ m_{D_s^+} = 2006.97 \pm 0.19, \quad m_{J/\psi} = 3096.916 \pm 0.011, \]
\[ m_B = 4.20 \pm 0.12, \quad m_{\rho} = 775.49 \pm 0.34, \]
\[ f_{B^*} = 176 \pm 42, \quad f_{J/\psi} = 0.416, \]
\[ f_{J/\psi}^{(L)} = 0.405, \quad f_{D^*} = f_{D^{*+}} = 222.6 \pm 19.5, \]
\[ f_{D^{*0}} = f_{D^{*+}} = 230 \pm 20. \]

The Borel mass squares \( M_1^2 \) and \( M_2^2 \) and continuum thresholds \( s_0 \) and \( s_0' \) are auxiliary parameters, hence the physical quantities should be independent of them. The parameters \( s_0 \) and \( s_0' \) are determined from the conditions that guarantee the sum rules for form factors to have the best stability in the allowed \( M_1^2 \) and \( M_2^2 \) region. The working regions for \( M_1^2 \) and \( M_2^2 \) as well as the values for continuum thresholds are determined in [13]. They choose the values \( s_0 = 35 \pm 5 \) GeV\(^2 \), \( s_0' = 7 \pm 1 \) GeV\(^2 \), \( M_1^2 = 17.0 \pm 2.5 \) GeV\(^2 \), \( M_2^2 = 7 \pm 1 \) GeV\(^2 \) from those working values for auxiliary parameters. The values of the form factors are given by

\[ F_0^{BD} \left( m_D^2 \right) = 0.86 \pm 0.22, \quad F_1^{BD} \left( m_{D^*}^2 \right) = 0.95 \pm 0.23, \]
\[ A_0^{BD} \left( m_D^2 \right) = 2.50 \pm 0.60, \quad A_1^{BD} \left( m_{D^*}^2 \right) = 1.09 \pm 0.26, \]
\[ A_2^{BD} \left( m_{D^*}^2 \right) = 1.82 \pm 0.46[13], \quad A_1^{B^*} = A_2^{B^*} = 0.26, \]
\[ V^{B^*} = 0.33[10]. \]

Nonuniversal annihilation phases are [23]

\[ \phi = -55(PP), \quad \phi = -20^\circ(PV), \quad \phi = -70^\circ(VP). \]

The signs of these phases are not predicted. As a result in [23], their predictions for the signs of CP asymmetries must be taken with caution. We use other input parameters as follows:

\[ \Lambda_{QCD} = 0.225 \text{ GeV}, \quad C_F = \frac{4}{3}, \quad N = 3, \]
\[ G_F = 1.166 \times 10^{-5}, \quad \alpha_s = 0.2244, \quad \rho = 0.5, \quad \rho_D = 1, \]
\[ g_{DDP} = g_{D^0} D, \quad g_{DD} = g_{D^*} D^*, \quad g_{DD} = 7.71, \]
\[ g_{D^0} D^* = 2.82, \quad g_{D^0} D = 8.64[5]. \]
Table 1: The branching ratio of $B^+ \rightarrow J/\psi \rho^+$ decay with $\eta = 1 \sim 2$ and experimental data (in units of $10^{-5}$).

<table>
<thead>
<tr>
<th>$\eta$</th>
<th>1</th>
<th>1.2</th>
<th>1.4</th>
<th>1.6</th>
<th>1.8</th>
<th>2</th>
<th>EXP [9]</th>
</tr>
</thead>
<tbody>
<tr>
<td>BR</td>
<td>$1.88^{+0.37}_{-0.24}$</td>
<td>$2.33^{+0.37}_{-0.24}$</td>
<td>$3.05^{+0.36}_{-0.24}$</td>
<td>$4.00^{+0.36}_{-0.24}$</td>
<td>$5.39^{+0.37}_{-0.24}$</td>
<td>$7.32^{+0.36}_{-0.24}$</td>
<td>$5.80^{+0.80}_{-0.80}$</td>
</tr>
</tbody>
</table>

Figure 5: The variation of the branching ratio of $B^+ \rightarrow J/\psi \rho^+$ with $\eta = 1 \sim 2.2$.

By using the input parameters and according to the QCDF method of $B^+ \rightarrow J/\psi \rho^+$ decay in Section 2, we get

$$\text{BR}(B^+ \rightarrow J/\psi \rho^+) = (1.42 \pm 0.36) \times 10^{-5}. \quad (5.10)$$

We note that our estimate of branching ratio of $B^+ \rightarrow J/\psi \rho^+$ decay according to QCDF method seems less than the experimental result. Before calculating the $B^+ \rightarrow J/\psi \rho^+$ decay amplitude via FSI, we have to compute the intermediate state amplitude, for the $B^+ \rightarrow D^+ D^0$ decay amplitude we get

$$A(B^+ \rightarrow D^+ D^0) = (2.31 \pm 0.27) \times 10^{-7}. \quad (5.11)$$

Now, using the above amplitude and according to the FSI, we are able to calculate the branching ratio of $B^+ \rightarrow J/\psi \rho^+$ decay with different values of $\eta$ which are shown in Table 1 and Figure 5.

6. Conclusion

In this work, we have calculated the contribution of the $t$-channel FSI, that is, inelastic rescattering processes to the branching ratio of $B^+ \rightarrow J/\psi \rho^+$ decay. For evaluating the FSI effects, we have only considered the absorptive part of the HLL because both hadrons which produced via the weak interaction are on their mass shells.
We have calculated the branching ratio of \( B^+ \to J/\psi \rho^+ \) decay by using FSI. The experimental result of this decay is \( BR(B^+ \to J/\psi \rho^+) = (5 \pm 0.8) \times 10^{-5} \) and according to QCDF method and assuming the FSI corrections into account, our results of the branching ratio are \((1.42 \pm 0.36) \times 10^{-5}\) and \((4.2 \sim 5.8) \times 10^{-5}\), respectively.

There exist some phenomenological parameters in our calculations on FSI, such as, \( \eta \) in (4.16). We have introduced the phenomenological parameter \( \eta \) that its value in the form factor is expected to be of order unity and can be determined from the measured rates. For a given exchanged particle, we have used \( \eta = 1 - 2\), and the branching ratios are 1.88 – 7.32. We have considered that the factor of \( \eta \) has important role to obtain of branching ratio. Note that [6] for the exchange particles \( D^* \) and \( D \) they have fixed \( \eta = 2.2 \) and according to Figure 5, we have found that if \( \eta = 1.58 \sim 1.83 \) is selected (which is in good agreement with the reference [6] selection), the numbers of the branching ratio of the \( B^+ \to J/\psi \rho^+ \) decay are placed in the experimental range.

References


[13] K. Azizi, R. Khosravi, and F. Falahati, “Analysis of the \( B_q \to D_q(D_q*)P \) and \( B_q \to D_q(D_q*)v \) decays within the factorization approach in QCD,” *International Journal of Modern Physics A*, vol. 24, no. 31, pp. 5845–5860, 2009.


