Research Article

Radiatively Generated Leptogenesis in S_4 Flavor Symmetry Models

T. Phong Nguyen^{1,2} and P. V. Dong³

¹ Department of Physics, Cantho University, Can Tho, Vietnam

² Institute of Particle and Nuclear Studies, High Energy Accelerator Research Organization (KEK), Tsukuba, Ibaraki 305-0801, Japan

³ Institute of Physics, VAST, P.O. Box 429, Bo Ho, Hanoi 10000, Vietnam

Correspondence should be addressed to T. Phong Nguyen, thanhphong@ctu.edu.vn

Received 13 September 2011; Accepted 17 November 2011

Academic Editor: Vicente Pleitez

Copyright © 2012 T. P. Nguyen and P. V. Dong. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

We study how leptogenesis can be implemented in the seesaw models with S_4 flavor symmetry, which lead to the tri-bimaximal neutrino mixing matrix. By considering renormalization group evolution from a high-energy scale of flavor symmetry breaking (the GUT scale is assumed) to the low-energy scale of relevant phenomena, the off-diagonal terms in a combination of Dirac Yukawa-coupling matrix can be generated and the degeneracy of heavy right-handed neutrino Majorana masses can be lifted. As a result, the flavored leptogenesis is successfully realized. We also investigate how the effective light neutrino mass $|\langle m_{ee} \rangle|$ associated with neutrinoless double beta decay can be predicted along with the neutrino mass hierarchies by imposing the experimental data on the low-energy observables. We find a link between the leptogenesis and the neutrinoless double beta decay characterized by $|\langle m_{ee} \rangle|$ through a high-energy CP phase ϕ , which is correlated with the low-energy Majorana CP phases. It is shown that the predictions of $|\langle m_{ee} \rangle|$ for some fixed parameters of the high-energy physics can be constrained by the current observation of baryon asymmetry.

1. Introduction

The neutrino experimental data can provide an important clue for elucidating the origin of observed hierarchies in the mass matrices of quarks and leptons. The recent experiments of neutrino oscillation have gone into a new phase of precise determination of the mixing angles and squared mass differences [1, 2], which indicate that the tri-bimaximal mixing (TBM) for the three flavors of leptons

$$U_{\text{TB}} = \begin{pmatrix} \frac{\sqrt{2}}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \quad (1.1)$$

can be regarded as the PMNS matrix $U_{\text{PMNS}} \equiv U_{\text{TB}} P_\nu$ [3–6], where P_ν is a diagonal matrix of CP phases. However, properties related to the leptonic CP violation have not been completely known yet. The large mixing angles, which may be suggestive of a flavor symmetry, are completely different from the quark mixing ones. Therefore, it is very important to find a model that naturally leads to those mixing patterns of quarks and leptons with a good accuracy. In recent years there have been a lot of efforts in searching for models which result in the TBM pattern naturally and a fascinating way seems to be the use of some discrete non-Abelian flavor groups added to the gauge groups of the standard model. There is a series of proposals based on groups A_4 [7–16], T' [17–21], and S_4 [22–36]. The common feature of these models is that they are naturally realized at a very-high-energy scale Λ and the groups are spontaneously broken due to a set of scalar multiplets, the flavons.

In addition to the explanation of smallness of observed neutrino masses, the seesaw mechanism [37–39] has another appearing feature so-called leptogenesis mechanism for generation of observed baryon asymmetry of the Universe (BAU), through the decay of heavy right-handed (RH) Majorana neutrinos [40–44]. If this BAU was made via the leptogenesis, then the CP violation in leptonic sector is required. For the Majorana neutrinos of three flavors there are one Dirac-type phase and two Majorana-type phases, one (or a combination) of which in principle can be measured through neutrinoless double beta ($0\nu 2\beta$) decays [45–48]. The exact TBM pattern forbids at low energy the CP violation in neutrino oscillations, due to $U_{e3} = 0$. Therefore, any observation of the leptonic CP violation, for instance, in the $0\nu 2\beta$ decay, can strengthen our belief in the leptogenesis by demonstrating that the CP is not a symmetry of leptons. It is interesting to explore this existence of the CP violation due to the Majorana CP-violating phases by measuring $|\langle m_{ee} \rangle|$ and examine a link between observable low-energy $0\nu 2\beta$ decay and the BAU. The authors in [35, 36] have shown that the TBM pattern can be generated naturally in the framework of the seesaw mechanism with $SU(2)_L \times U(1)_Y \times S_4$ symmetry. The textures of mass matrices as given in [35, 36] also could not generate a lepton asymmetry which is essential for the baryogenesis. In this paper, we investigate possibility of radiative leptogenesis when renormalization group (RG) effects are taken into account. We will show that the leptogenesis can be linked to the $0\nu 2\beta$ decay through the seesaw mechanism.

The rest of this work is organized as follows. In Section 2, we present the low-energy observables in two variants of supersymmetric seesaw model based on flavor symmetry S_4 . We especially focus on the effective neutrino mass governing the $0\nu 2\beta$ decay. In Section 3, we study RG effects on the Yukawa couplings and heavy Majorana neutrino mass matrices so that the leptogenesis becomes available. This leptogenesis in the two models due to the RG effects is studied in detail in Section 4. Finally, Section 5 is devoted to our conclusions.

Table 1: Transformation properties of the lepton sector and all the flavons of the BMM model where $\omega = e^{i2\pi/3}$.

Field	l	e^c	μ^c	τ^c	ν^c	$h_{u,d}$	θ	ψ	η	Δ	φ	ξ'
S_4	3_1	1_2	1_2	1_1	3_1	1_1	1_1	3_1	2	3_1	2	1_2
Z_5	ω^4	1	ω^2	ω^4	ω	1	1	ω^2	ω^2	ω^3	ω^3	1
$U(1)_{\text{FN}}$	0	1	0	0	0	0	-1	0	0	0	0	0

2. Two S_4 Models

In this section we give a review of the main features of Bazzocchi-Merlo-Morisi (BMM) model [35] and Ding model [36]. We simultaneously discuss the $0\nu 2\beta$ decay, leptogenesis, and phenomenological difficulties associated with the models to be solved.

2.1. Bazzocchi-Merlo-Morisi Model

In this model the flavor symmetry is S_4 accompanied with cyclic group Z_5 and Froggatt-Nielsen symmetry $U(1)_{\text{FN}}$ [49], that is, $G_f = S_4 \times Z_5 \times U(1)_{\text{FN}}$. The matter fields and flavons are given in Table 1. The superpotential for the lepton sector reads

$$\begin{aligned}
 w_l &= \sum_{i=1}^4 \frac{\theta}{\Lambda} \frac{y_{e,i}}{\Lambda^3} e^c (lX_i)_{1_2} h_d + \frac{y_\mu}{\Lambda^2} \mu^c (l\psi\eta)_{1_2} h_d + \frac{y_\tau}{\Lambda} \tau^c (l\psi)_{1_1} h_d + h.c. + \dots, \\
 w_\nu &= x(\nu^c l)_{1_1} h_u + x_d(\nu^c \nu^c \varphi)_{1_1} + x_t(\nu^c \nu^c \Delta)_{1_1} + h.c. + \dots,
 \end{aligned} \tag{2.1}$$

where $X_i = \psi\psi\eta$, $\psi\eta\eta$, $\Delta\Delta\xi'$, $\Delta\varphi\xi'$ and the dots denote higher-order contributions. The VEV alignment of flavons is

$$\begin{aligned}
 \langle \psi \rangle &= (0 \ 1 \ 0)^T v_\psi, & \langle \Delta \rangle &= (1 \ 1 \ 1)^T v_\Delta, \\
 \langle \eta \rangle &= (0 \ 1)^T v_\eta, & \langle \varphi \rangle &= (1 \ 1)^T v_\varphi, & \langle \xi' \rangle &= v_{\xi'},
 \end{aligned} \tag{2.2}$$

where all the VEVs are of the same order of magnitude and for this reason being parameterized as $\text{VEVs}/\Lambda = u$. The remaining VEV which originates from a different mechanism is v_θ , denoted by $v_\theta/\Lambda = t$. It is shown in [35] that u and t belong to a well-determined range $0.01 < u, t < 0.05$.

The mass matrix for the charged leptons is given by

$$m_l = \begin{pmatrix} y_e^{(1)} u^2 t & y_e^{(2)} u^2 t & y_e^{(2)} u^2 t \\ 0 & y_\mu u & 0 \\ 0 & 0 & y_\tau \end{pmatrix} u v_d. \tag{2.3}$$

The Dirac and RH-Majorana neutrino mass matrices are, respectively, obtained as

$$m_\nu^d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} x v_u, \quad (2.4)$$

$$M_R = B e^{i\alpha_1} \begin{pmatrix} 2re^{i\phi} & 1 - re^{i\phi} & 1 - re^{i\phi} \\ 1 - re^{i\phi} & 1 + 2re^{i\phi} & -re^{i\phi} \\ 1 - re^{i\phi} & -re^{i\phi} & 1 + 2re^{i\phi} \end{pmatrix},$$

where $B = 2|x_d|v_\phi$, $C = 2|x_t|v_\Delta$, and $r = C/B$ are real and positive. The phases α_1 and α_2 are the arguments of $x_{d,t}$, and $\phi \equiv \alpha_2 - \alpha_1$ being the only physical phase remained in M_R . Notice that the M_R can be exactly diagonalized by the TBM matrix:

$$M_R^D = V_R^T M_R V_R = \text{Diag.}(M_1, M_2, M_3),$$

$$M_1 = B|3re^{i\phi} - 1|, \quad M_2 = 2B, \quad M_3 = B|3re^{i\phi} + 1|, \quad (2.5)$$

$$V_R = U_{TB} V_P, \quad V_P = \text{Diag.}(e^{i\gamma_1/2}, 1, e^{i\gamma_2/2}),$$

$$\gamma_{1,2} = -\arg(3re^{i\phi} \mp 1).$$

Integrating out the heavy degrees of freedom, we get the effective light neutrino mass matrix, which is given by the seesaw relation, $m_{\text{eff}} = -(m_\nu^d)^T M_R^{-1} m_\nu^d$ [37–39], and diagonalized by the TBM matrix:

$$U_\nu^T m_{\text{eff}} U_\nu = \text{Diag.}(m_1, m_2, m_3)$$

$$= -\text{Diag.}\left(\frac{x^2 v_u^2}{M_1}, \frac{x^2 v_u^2}{M_2}, \frac{x^2 v_u^2}{M_3}\right), \quad (2.6)$$

$$U_\nu = U_{TB} \text{Diag.}(e^{-i\gamma_1/2}, 1, e^{-i\gamma_2/2}).$$

In order to find the lepton mixing matrix we need to diagonalize the charged-lepton mass matrix:

$$m_l^D = U_l^\dagger m_l U_l = \text{Diag.}(y_e u^2 t, y_\mu u, y_\tau) u v_d, \quad (2.7)$$

where U_l is unity matrix. Therefore we get

$$U_{\text{PMNS}} = U_l^\dagger U_\nu \equiv U_\nu$$

$$= e^{-i\gamma_1/2} U_{TB} \text{Diag.}(1, e^{i\beta_1}, e^{i\beta_2}), \quad (2.8)$$

where $\beta_1 = \gamma_1/2$ and $\beta_2 = (\gamma_1 - \gamma_2)/2$ are Majorana CP violating phases. The phase factored out to the left has no physical meaning, since it can be eliminated by a redefinition of the charged lepton fields. The light neutrino mass eigenvalues are simply the inverse of the heavy

Figure 1: Allowed parameter region by the 1σ experimental constraints (2.9) for the ratio $r = C/B$ as a function of $\cos \phi$. The blue (dark) and red (light) curves correspond to the IH and NH spectra.

neutrino ones, a part from a minus sign and the global factor from m_ν^d , as can be seen in (2.6). There are nine physical parameters consisting of the three light neutrino masses, three mixing angles, and three CP-violating phases in general. The mixing angles are entirely fixed by the G_f symmetry group, predicting TBM and in turn no Dirac CP-violating phase. The remaining five physical parameters, β_1 , β_2 , m_1 , m_2 , and m_3 , are determined by the five real parameters B, C, v_u, x , and ϕ .

The light neutrino mass spectrum can have both normal or inverted hierarchy depending on the sign of $\cos \phi$. If $\cos \phi < 0$, one has normal hierarchy (NH), whereas if $\cos \phi > 0$, one has inverted hierarchy (IH). In order to see how this correlation in the allowed parameter space is constrained, we consider the experimental data at 1σ [1, 2]:

$$\begin{aligned} |\Delta m_{31}^2| &= (2.29 - 2.52) \times 10^{-3} \text{ eV}^2, \\ \Delta m_{21}^2 &= (7.45 - 7.88) \times 10^{-5} \text{ eV}^2. \end{aligned} \quad (2.9)$$

(Hereafter, we always use the experimental data at 1σ for our numerical calculations of low-energy observables.) The correlations between r and $\cos \phi$ for the NH spectrum (red (light) plot) and IH one [blue (dark) plot] are, respectively, presented in Figure 1.

Because there is no Dirac CP-violating phase as mentioned, the only contribution from the Majorana phases to the $0\nu 2\beta$ decay comes from β_1 . The effective neutrino mass governing the $0\nu 2\beta$ decay is given by

$$\begin{aligned} |\langle m_{ee} \rangle| &= \frac{1}{3} |2m_1 + m_2 e^{2i\beta_1}| \\ &= \frac{m_0}{3(1 - 6r \cos \phi + 9r^2)} \sqrt{8.5 + 13.5r^2 + 20.25r^4 - 3r(13 + 12r^2) \cos \phi + 9r^2 \cos^2 \phi}, \end{aligned} \quad (2.10)$$

where $m_0 = x^2 v_u^2 / B$. The behavior of $|\langle m_{ee} \rangle|$ is plotted in Figure 2 as a function of ϕ . The horizontal line (0.2 eV) is the current lower bound sensitivity [50–53] while the dashed line (10^{-2} eV) is a future sensitivity [54, 55].

Figure 2: Prediction of the effective neutrino mass $|\langle m_{ee} \rangle|$ responsible for $0\nu 2\beta$ decay as a function of ϕ by the 1σ experimental constraints (2.9). The blue (dark) and red (light) curves correspond to the IH and NH spectra.

Figure 3: The Majorana CP phase β_1 as a function of ϕ plotted by the 1σ experimental constraints (2.9). The blue (dark) and red (light) curves correspond to the IH and NH spectra.

Using (10) we can obtain the explicit relation between ϕ and β_1 :

$$\sin 2\beta_1 = \frac{-3r \sin \phi}{1 - 6r \cos \phi + 9r^2}. \quad (2.11)$$

Figure 3 represents this relation corresponding to the NH spectrum [red (light) plot] and IH one [blue (dark) plot].

In a basis where the charged current is flavor diagonal and the heavy neutrino mass matrix M_R is diagonal and real, the Dirac mass matrix m_ν^d gets modified to

$$m_\nu^d \longrightarrow Y_\nu u_u = V_R^T m_\nu^d, \quad (2.12)$$

Table 2: Representations of the matter fields of lepton sector and flavons under $S_4 \times Z_3 \times Z_4$.

Field	l	e^c	μ^c	τ^c	ν^c	$h_{u,d}$	φ	χ	Φ	η	ϕ	Δ
S_4	3_1	1_1	1_2	1_1	3_1	1_1	3_1	3_2	1_2	2	3_1	1_2
Z_3	ω	ω^2	ω^2	ω^2	1	1	1	1	1	ω^2	ω^2	ω^2
Z_4	1	i	-1	$-i$	1	1	i	i	1	1	1	-1

where $v_u = v \sin \beta$, $v = 176 \text{ GeV}$, and the coupling of N_i with leptons and scalar, Y_ν , is given by

$$Y_\nu = x e^{i\gamma_1/2} \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{-1}{\sqrt{6}} & \frac{-1}{\sqrt{6}} \\ \frac{e^{-i\beta_1}}{\sqrt{3}} & \frac{e^{-i\beta_1}}{\sqrt{3}} & \frac{e^{-i\beta_1}}{\sqrt{3}} \\ 0 & \frac{e^{-i\beta_2}}{\sqrt{2}} & \frac{-e^{-i\beta_2}}{\sqrt{2}} \end{pmatrix}. \quad (2.13)$$

Concerned with the CP violation, we notice that the CP phase ϕ originating from m_ν^d obviously takes part at the low-energy CP violation as the Majorana phases β_1 and β_2 . On the other hand, the leptogenesis is associated with both the Yukawa coupling Y_ν and its combination:

$$H \equiv Y_\nu Y_\nu^\dagger = x^2 \cdot \text{Diag.}(1, 1, 1). \quad (2.14)$$

This directly indicates that all off-diagonal H_{ij} vanish, so the CP asymmetry could not be generated and neither leptogenesis. *For the leptogenesis to be viable, the off-diagonal H_{ij} have to be generated.*

2.2. Ding Model

Ding model, proposed in [36], possesses flavor symmetry group $G_f = S_4 \times Z_3 \times Z_4$, where the three factors play different roles. The S_4 controls the mixing angles, the Z_3 guarantees the misalignment in flavor space between neutrino and charged-lepton eigenstates, and the Z_4 is crucial to eliminating unwanted couplings and reproducing observed mass hierarchies. In this framework the mass hierarchies are controlled by spontaneously breaking of the flavor symmetry instead of the Froggatt-Nielsen mechanism [49]. The matter fields of lepton sector and flavons under G_f are assigned as in Table 2.

The superpotential for the lepton sector reads

$$\begin{aligned} w_l = & \frac{y_{e1}}{\Lambda^3} e^c (l\varphi)_{1_1} (\varphi\varphi)_{1_1} h_d + \frac{y_{e2}}{\Lambda^3} e^c ((l\varphi)_2 (\varphi\varphi)_2)_{1_1} h_d + \frac{y_{e3}}{\Lambda^3} e^c ((l\varphi)_{3_1} (\varphi\varphi)_{3_1})_{1_1} h_d \\ & + \frac{y_{e4}}{\Lambda^3} e^c ((l\chi)_2 (\chi\chi)_2)_{1_1} h_d \\ & + \frac{y_{e5}}{\Lambda^3} e^c ((l\chi)_{3_1} (\chi\chi)_{3_1})_{1_1} h_d + \frac{y_{e6}}{\Lambda^3} e^c (l\varphi)_{1_1} (\chi\chi)_{1_1} h_d + \frac{y_{e7}}{\Lambda^3} e^c ((l\varphi)_2 (\chi\chi)_2)_{1_1} h_d \end{aligned}$$

$$\begin{aligned}
& + \frac{y_{e8}}{\Lambda^3} e^c \left((l\varphi)_{3_1} (\chi\chi)_{3_1} \right)_{1_1} h_d \\
& + \frac{y_{e9}}{\Lambda^3} e^c \left((l\chi)_2 (\varphi\varphi)_2 \right)_{1_1} h_d + \frac{y_{e10}}{\Lambda^3} e^c \left((l\chi)_{3_1} (\varphi\varphi)_{3_1} \right)_{1_1} h_d + \frac{y_\mu}{2\Lambda^2} \mu^c \left(l(\varphi\chi)_{3_2} \right)_{1_2} h_d \\
& + \frac{y_\tau}{\Lambda} \tau^c (l\varphi)_{1_1} h_d + \dots, \\
w_\nu = & \frac{y_{\nu 1}}{\Lambda} ((\nu^c l)_2 \eta)_{1_1} h_u + \frac{y_{\nu 2}}{\Lambda} ((\nu^c l)_{3_1} \phi)_{1_1} h_u + \frac{1}{2} M (\nu^c \nu^c)_{1_1} + \dots,
\end{aligned} \tag{2.15}$$

where the dots denote higher-order contributions.

The VEV alignment of flavons are assumed as follows:

$$\begin{aligned}
\langle \varphi \rangle &= (0, v_\varphi, 0), & \langle \chi \rangle &= (0, v_\chi, 0), & \langle \vartheta \rangle &= v_\vartheta, \\
\langle \eta \rangle &= (v_\eta, v_\eta), & \langle \phi \rangle &= (v_\phi, v_\phi, v_\phi), & \langle \Delta \rangle &= v_\Delta.
\end{aligned} \tag{2.16}$$

The charged-lepton mass matrix is obtained by

$$m_l = \text{Diag.} \left(y_e \frac{v_\varphi^3}{\Lambda^3}, y_\mu \frac{v_\varphi v_\chi}{\Lambda^2}, y_\tau \frac{v_\varphi}{\Lambda} \right) v_d, \tag{2.17}$$

where all the components are assumed to be real. The neutrino sector gives rise to the following Dirac and RH-Majorana mass matrices

$$\begin{aligned}
m_\nu^d &= e^{i\alpha_1} \begin{pmatrix} 2be^{i\phi} & a - be^{i\phi} & a - be^{i\phi} \\ a - be^{i\phi} & a + 2be^{i\phi} & -be^{i\phi} \\ a - be^{i\phi} & -be^{i\phi} & a + 2be^{i\phi} \end{pmatrix} v_u, \\
M_R &= \begin{pmatrix} M & 0 & 0 \\ 0 & 0 & M \\ 0 & M & 0 \end{pmatrix},
\end{aligned} \tag{2.18}$$

where the quantity M is also supposed to be real and positive. The phase $\phi \equiv \alpha_2 - \alpha_1$, where α_1, α_2 are denoted as the arguments of $y_{\nu 1}, y_{\nu 2}$, respectively, is the only physical phase survived because the global phase α_1 can be rotated away. The real and positive components a and b are defined as

$$a = |y_{\nu 1}| \frac{v_\eta}{\Lambda}, \quad b = |y_{\nu 2}| \frac{v_\phi}{\Lambda}. \tag{2.19}$$

After seesawing, the effective light neutrino mass matrix is obtained from $m_{\text{eff}} = -(m_\nu^d)^T M_R^{-1} m_\nu^d$, which can be diagonalized by the TBM matrix:

$$U_\nu^T m_{\text{eff}} U_\nu = \text{Diag.}(m_1, m_2, m_3), \tag{2.20}$$

where

$$\begin{aligned} m_1 &= m_0(1 + 9r^2 - 6r \cos \phi), \\ m_2 &= 4m_0, \\ m_3 &= m_0(1 + 9r^2 + 6r \cos \phi), \end{aligned} \quad (2.21)$$

with $m_0 = v_u^2 a^2 / M$ and $r = b/a$. The lepton mixing matrix is given by

$$U_{\text{PMNS}} = U_\nu = e^{-i\gamma_1/2} U_{\text{TB}} \text{Diag.}(1, e^{i\beta_1}, e^{i\beta_2}), \quad (2.22)$$

where $\beta_1 = \gamma_1/2$, $\beta_2 = (\gamma_1 - \gamma_2)/2$ are Majorana CP-violating phases with

$$\begin{aligned} \gamma_1 &= \arg \left\{ (a - 3be^{i\phi})^2 \right\}, \\ \gamma_2 &= \arg \left\{ -(a + 3be^{i\phi})^2 \right\}. \end{aligned} \quad (2.23)$$

It is clear that the phase factored out to the left has no physical meaning. Moreover, the mixing angles are entirely fixed by the G_f symmetry, predicting TBM and in turn no Dirac CP-violating phase. There remain only five physical quantities, $\beta_1, \beta_2, m_1, m_2$, and m_3 , completely determined by the five parameters M, v_u, a, b , and ϕ .

There are two possible orderings in the masses of effective light neutrinos depending on the sign of $\cos \phi$: the NH corresponding to $\cos \phi > 0$ while the IH to $\cos \phi < 0$, which contrast with the previous model. The relation between r and $\cos \phi$ for the NH spectrum (red plot) and IH one (blue plot) is included in Figure 4. Similarly to the previous model, the contribution to the $0\nu 2\beta$ decay entirely comes from the Majorana phase β_1 . The relevant effective-neutrino mass is given by

$$\begin{aligned} |\langle m_{ee} \rangle| &= \frac{1}{3} \left| 2m_1 + m_2 e^{2i\beta_1} \right| \\ &= m_0 \sqrt{1 - 4r \cos \phi + 2r^2(2 + 3 \cos 2\phi) - 12r^3 \cos \phi + 9r^4}, \end{aligned} \quad (2.24)$$

where $m_0 = a^2 v_u^2 / M$. The behavior of $|\langle m_{ee} \rangle|$ as a function of ϕ is plotted in Figure 5, where the horizontal line and dashed line are the current lower bound and the future one as mentioned. Moreover, the relation between ϕ and β_1 can be obtained from (2.23) as

$$\sin 2\beta_1 = \frac{6r \sin \phi (1 - 3r \cos \phi)}{1 - 6r \cos \phi + 9r^2}, \quad (2.25)$$

which is presented in Figure 6 corresponding to the NH ordering [red (light) plot] and IH one [blue (dark) plot].

Figure 4: Allowed parameter region by the 1σ experimental constraints (2.9) for the ratio $r = b/a$ as a function of $\cos \phi$. The blue (dark) and red (light) curves correspond to the IH and NH ordering.

Figure 5: Prediction of the effective mass $|\langle m_{ee} \rangle|$ responsible for $0\nu 2\beta$ as a function of ϕ by the 1σ experimental constraints (2.9). The blue (dark) and red (light) curves correspond to the IH and NH ordering.

Figure 6: Relation between the phase β_1 and ϕ as given by the 1σ experimental constraints (2.9). The blue (dark) and red (light) curves correspond to the IH and NH ordering.

In a basis where the charged current is flavor diagonal, we diagonalize M_R in order to go into the physical mass basis of the RH neutrinos:

$$V_R^T M_R V_R = \text{Diag.}(M, M, -M), \quad (2.26)$$

where

$$V_R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \quad (2.27)$$

In this basis, the Dirac mass matrix m_ν^d gets the form

$$m_\nu^d \longrightarrow Y_\nu v_u = V_R^T m_\nu^d, \quad (2.28)$$

where $v_u = v \cdot \sin \beta$, $v = 176 \text{ GeV}$ and the coupling of N_i with leptons and scalar, Y_ν , is given by

$$Y_\nu = \begin{pmatrix} 2be^{i\phi} & a - be^{i\phi} & a - be^{i\phi} \\ \sqrt{2}(a - be^{i\phi}) & \frac{a + be^{i\phi}}{\sqrt{2}} & \frac{a + be^{i\phi}}{\sqrt{2}} \\ 0 & \frac{-(a + 3be^{i\phi})}{\sqrt{2}} & \frac{(a + 3be^{i\phi})}{\sqrt{2}} \end{pmatrix}. \quad (2.29)$$

Again, the CP phase ϕ which comes from m_ν^d also takes part at the low-energy CP violation as the Majorana phases β_1 and β_2 . On the other hand, the leptogenesis is associated with both the Yukawa coupling Y_ν and its combination:

$$H \equiv Y_\nu Y_\nu^\dagger = \begin{pmatrix} 2a^2 + 6b^2 - 4ab \cos \phi & \sqrt{2}(a^2 - 3b^2 + 2ab \cos \phi) & 0 \\ \sqrt{2}(a^2 - 3b^2 + 2ab \cos \phi) & 3a^2 + 3b^2 - 2ab \cos \phi & 0 \\ 0 & 0 & a^2 + 9b^2 + 6ab \cos \phi \end{pmatrix}, \quad (2.30)$$

which directly indicates that all $\text{Im}[H_{ij}]$ vanish and in turn unflavored leptogenesis could not take place. *However, flavored leptogenesis can work if the degeneracy of the heavy Majorana neutrino masses is lifted.*

3. Relevant RG Equations

In both models, the CP asymmetries due to the decay of heavy RH Majorana neutrinos at leading order vanish; therefore the leptogenesis could not take place. The radiative effects due to RG running from a high to low scale can naturally lead not only to a degenerate splitting of heavy Majorana masses (for Ding model), but also to an enhancement in vanished off-diagonal terms of $H = Y_\nu Y_\nu^\dagger$ (for BMM model), which are necessary ingredients for a successful leptogenesis mechanism.

The radiative behavior of heavy RH-Majorana mass matrix $\mathbf{M}_R$ is dictated by the following RG equation [56–60]:

$$\frac{d\mathbf{M}_R}{dt} = 2 \left[\left(\mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger \right) \mathbf{M}_R + \mathbf{M}_R \left(\mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger \right)^T \right], \quad (3.1)$$

where $t = (1/16\pi^2) \ln(M/\Lambda')$ and M is an arbitrary renormalization scale. The cutoff scale Λ' can be regarded as the G_f breaking scale $\Lambda' = \Lambda$ and assumed to be in order of the GUT scale, $\Lambda' \sim 10^{16}$ GeV.

The RG equation for the Dirac neutrino Yukawa coupling can be written as

$$\frac{d\mathbf{Y}_\nu}{dt} = \mathbf{Y}_\nu \left[\left(T - 3g_2^2 - \frac{3}{5}g_1^2 \right) + \mathbf{Y}_l^\dagger \mathbf{Y}_l + 3\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu \right], \quad (3.2)$$

where $T = \text{Tr}(3Y_u^\dagger Y_u + Y_\nu^\dagger Y_\nu)$, Y_u and Y_l are the Yukawa couplings of up-type quarks and charged leptons, and $g_{2,1}$ are the $SU(2)_L$ and $U(1)_Y$ gauge coupling constants, respectively.

Let us first reformulate (3.1) in the basis where $\mathbf{M}_R$ is diagonal. Since $\mathbf{M}_R$ is symmetric, it can be diagonalized by a unitary matrix V_R as mentioned:

$$V_R^T \mathbf{M}_R V_R = \text{Diag.}(M_1, M_2, M_3). \quad (3.3)$$

As the structure of $\mathbf{M}_R$ changes with the evolution of the scale, the V_R depends on the scale too. The RG evolution of $V_R(t)$ can be written as

$$\frac{d}{dt} V_R = V_R A, \quad (3.4)$$

where A is an anti-Hermitian matrix $A^\dagger = -A$ due to the unitarity of V_R . Differentiating (3.3) we obtain

$$\frac{dM_i \delta_{ij}}{dt} = A_{ij}^T M_j + M_i A_{ij} + 2 \left\{ V_R^T \left[\left(\mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger \right) \mathbf{M}_R + \mathbf{M}_R \left(\mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger \right)^T \right] V_R \right\}_{ij}. \quad (3.5)$$

Absorbing the unitary factor into the Dirac Yukawa coupling $Y_\nu \equiv V_R^T \mathbf{Y}_\nu$, the real diagonal part of (3.5) becomes

$$\frac{dM_i}{dt} = 4M_i \left(Y_\nu Y_\nu^\dagger \right)_{ii}. \quad (3.6)$$

The RG equation for Y_ν in the basis of diagonal $\mathbf{M}_R$ is given by

$$\frac{dY_\nu}{dt} = Y_\nu \left[\left(T - 3g_2^2 - \frac{3}{5}g_1^2 \right) + \mathbf{Y}_l^\dagger \mathbf{Y}_l + 3Y_\nu^\dagger Y_\nu \right] + A^T Y_\nu. \quad (3.7)$$

Finally, we obtain the RG equation for H responsible for the leptogenesis:

$$\begin{aligned} \frac{dH}{dt} = & 2 \left(T - 3g_2^2 - \frac{3}{5}g_1^2 \right) H + 2Y_\nu (\mathbf{Y}_l^\dagger \mathbf{Y}_l) Y_\nu^\dagger \\ & + 6H^2 + A^T H + H A^*. \end{aligned} \quad (3.8)$$

The heavy Majorana mass splitting generated through the relevant RG evolution is thus given by

$$\delta_N^{ij} = 4(H_{ii} - H_{jj})t, \quad (3.9)$$

where H is defined in (2.30). Neglecting the RG evolution of Y_ν and its combination $H = Y_\nu Y_\nu^\dagger$, all the necessary components for the flavored leptogenesis in Ding model are available. The flavored CP asymmetries ε_i^α can be obtained from (2.29), (2.30), (3.9), and (4.3).

Notice however that in BMM model a nonvanishing CP asymmetry requires $\text{Im}[H_{ij}(Y_\nu)_{i\alpha}(Y_\nu)_{j\alpha}^*] \neq 0$ with Y_ν defined in (2.13). Therefore, to have a viable radiative leptogenesis we need to induce a nonvanishing $H_{ij}(i \neq j)$ at the leptogenesis scale. Indeed, this is possible since the RG effects due to the τ -Yukawa coupling contribution imply at the leading order yields [56–60]

$$H_{ij}(t) = 2y_\tau^2(Y_\nu)_{i3}(Y_\nu)_{j3}^* \times t. \quad (3.10)$$

The flavored CP asymmetries ε_i^α can then be obtained from (2.13), (2.14), (3.10), and (4.1).

4. Radiatively Induced Flavored Leptogenesis

As already noticed, the leptogenesis cannot be realized in the S_4 models at the leading order, so this section is devoted to study the flavored leptogenesis with the effects of RG evolution.

The lepton asymmetries, which are produced by out-of-equilibrium decays of heavy RH neutrinos in early Universe at temperatures above $T \sim (1 + \tan^2 \beta) \times 10^{12}$ GeV, do not distinguish among lepton flavors, called conventional or unflavored leptogenesis. However, if the scale of heavy RH neutrino masses is about $M \leq (1 + \tan^2 \beta) \times 10^{12}$ GeV, we need to take into account lepton flavor effects, called flavored leptogenesis.

In this case, the CP asymmetry as generated by the decay of i th heavy RH neutrino far from almost degenerate is given by [61–71]

$$\varepsilon_i^\alpha = \frac{1}{8\pi H_{ii}} \sum_{j \neq i} \text{Im} [H_{ij}(Y_\nu)_{i\alpha}(Y_\nu)_{j\alpha}^*] g \left(\frac{M_j^2}{M_i^2} \right), \quad (4.1)$$

where Y_ν and $H = Y_\nu Y_\nu^\dagger$ are in the basis where M_R is real and diagonal. Here the loop function $g(M_j^2/M_i^2)$ is

$$g\left(\frac{M_j^2}{M_i^2}\right) \equiv g_{ij}(x) = \sqrt{x} \left[\frac{2}{1-x} - \ln \frac{1+x}{x} \right]. \quad (4.2)$$

This function depends strongly on the hierarchy of light neutrino masses.

For an almost degenerate heavy Majorana mass spectrum, the leptogenesis can be naturally implemented through the resonant leptogenesis [72, 73]. In this case, the CP asymmetry is generated by the i th heavy RH neutrino (N_i) when decaying into a lepton flavor α ($= e, \mu, \tau$) and dominated by the one-loop self-energy contributions [74],

$$\varepsilon_i^\alpha = \sum_{j \neq i} \frac{\text{Im}[H_{ij}(Y_\nu)_{i\alpha}(Y_\nu)_{j\alpha}^*]}{16\pi H_{ii}\delta_N^{ij}} \left(1 + \frac{\Gamma_j^2}{4M_j\delta_N^{ij2}} \right), \quad (4.3)$$

where $\Gamma_j = H_{jj}M_j/8\pi$ is the decay width of j th RH neutrino and δ_N^{ij} is mass splitting parameter defined as

$$\delta_N^{ij} = 1 - \frac{M_j}{M_i}. \quad (4.4)$$

As reminded in the previous section, by properly taking into account the RG effects, the nonzero flavored CP asymmetries ε_i^α as given above can be obtained.

Once the initial values of ε_i^α are fixed, the final result of BAU, η_B , can be given by solving a set of flavor-dependent Boltzmann equations including the decay, inverse decay, and scattering processes as well as the nonperturbative sphaleron interaction. In order to estimate the washout effects, we introduce parameters K_i^α which are the wash-out factors due to the inverse decay of Majorana neutrino N_i into the lepton flavor α . The explicit form of K_i^α is given by

$$K_i^\alpha = \frac{\Gamma_i^\alpha}{H(M_i)} = \left(Y_\nu^\dagger \right)_{\alpha i} (Y_\nu)_{i\alpha} \frac{v_u^2}{m_* M_i}, \quad (4.5)$$

where Γ_i^α is the partial decay width of N_i into the lepton flavors and Higgs scalars, $H(M_i) \simeq (4\pi^3 g_*/45)^{1/2} M_i^2/M_{\text{Pl}}$ with the Planck mass $M_{\text{Pl}} = 1.22 \times 10^{19}$ GeV and the effective number of degrees of freedom $g_* \simeq 228.75$ is the Hubble parameter at temperature $T = M_i$, and the equilibrium neutrino mass $m_* \simeq 10^{-3}$. From (2.13), (2.29), and (4.5) we can obtain the washout parameters corresponding to the two models.

Each lepton asymmetry for a single flavor ε_i^α is weighted differently by the corresponding washout parameter K_i^α , appearing with a different weight in the final formula for the baryon asymmetry [75, 76]:

$$\eta_B \simeq -10^{-2} \sum_{N_i} \left[\varepsilon_i^e \kappa \left(\frac{93}{110} K_i^e \right) + \varepsilon_i^\mu \kappa \left(\frac{19}{30} K_i^\mu \right) + \varepsilon_i^\tau \kappa \left(\frac{19}{30} K_i^\tau \right) \right], \quad (4.6)$$

provided that the scale of heavy RH neutrino masses is about $M \leq (1 + \tan^2 \beta) \times 10^9$ GeV where the μ and τ Yukawa couplings are in equilibrium and all the flavors are to be treated separately. And

$$\eta_B \simeq -10^{-2} \sum_{N_i} \left[\varepsilon_i^2 \kappa \left(\frac{541}{761} K_i^2 \right) + \varepsilon_i^\tau \kappa \left(\frac{494}{761} K_i^\tau \right) \right] \quad (4.7)$$

is given if $(1 + \tan^2 \beta) \cdot 10^9$ GeV $\leq M_i \leq (1 + \tan^2 \beta) \cdot 10^{12}$ GeV where only the τ Yukawa coupling is in equilibrium and treated separately while the e and μ flavors are indistinguishable. Here $\varepsilon_i^2 = \varepsilon_i^e + \varepsilon_i^\mu$, $K_i^2 = K_i^e + K_i^\mu$.

The wash-out factors are given by

$$\kappa_i^\alpha \simeq \left(\frac{8.25}{K_i^\alpha} + \left(\frac{K_i^\alpha}{0.2} \right)^{1.16} \right)^{-1}. \quad (4.8)$$

4.1. Bazzocchi-Merlo-Morisi Model

In this model, the RH neutrino masses are strongly hierarchical. For the NH case, the lightest RH neutrino mass is M_3 , then the leptogenesis is governed by the decay of M_3 neutrino. The explicit form of flavored CP asymmetries ε_3^α is given from (2.13), (2.14), (3.10), and (4.1):

$$\begin{aligned} \varepsilon_3^e &\simeq 0, \\ \varepsilon_3^\mu &\simeq \varepsilon_3^\tau \simeq \frac{y_\tau^2 x^2}{24\pi} \left(\frac{1}{2} \sin 2\beta_2 \cdot g_{31} - \sin 2(\beta_1 - \beta_2) \cdot g_{32} \right) \cdot t. \end{aligned} \quad (4.9)$$

The corresponding washout parameters are

$$K_3^e = 0, \quad K_3^{\mu,\tau} \simeq \frac{3}{4} K_1^e. \quad (4.10)$$

For the IH case, the lightest RH neutrino is of M_1 , then the leptogenesis is governed by the decay of M_1 neutrino. The flavored CP asymmetries ε_1^α are obtained as

$$\begin{aligned} \varepsilon_1^e &\simeq \frac{-y_\tau^2 x^2}{36\pi} \sin 2\beta_1 \cdot g_{12} \cdot t, \\ \varepsilon_1^\mu &\simeq \frac{y_\tau^2 x^2}{24\pi} \left(\frac{1}{3} \sin 2\beta_1 \cdot g_{12} - \frac{1}{2} \sin 2\beta_2 \cdot g_{13} \right) \cdot t, \\ \varepsilon_1^\tau &\simeq \frac{y_\tau^2 x^2}{24\pi} \left(\frac{1}{3} \sin 2\beta_1 \cdot g_{12} + \frac{1}{2} \sin 2\beta_2 \cdot g_{13} \right) \cdot t, \end{aligned} \quad (4.11)$$

Figure 7: Prediction of η_B as a function of $|\langle m_{ee} \rangle|$ for the NH case (a) and IH case (b). The horizontal solid and dashed lines correspond to the experimental central value and phenomenologically allowed region.

with corresponding washout parameters

$$K_1^e \simeq \frac{2m_0}{3m_*(1 - 6r \cos \phi + 9r^2)}, \quad (4.12)$$

$$K_1^{\mu,\tau} \simeq \frac{1}{4} K_1^e.$$

Applying (4.6), (4.7), and (4.8), the BAU for two cases are then obtained. Notice also that in the NH case the leptogenesis has no contribution from the electron flavor decay channel which makes the scale of the heavy RH neutrino mass for a successful leptogenesis higher than that of the IH case.

The prediction for η_B as a function of $|\langle m_{ee} \rangle|$ is shown in Figure 7 where we have used $B = 10^{13}$ GeV for the NH case, $B = 10^{12}$ GeV for the IH case, and $\tan \beta = 30$ as inputs. The horizontal solid and dashed lines correspond to the central value of BAU experimental data $\eta_B^{\text{CMB}} = 6.1 \times 10^{-10}$ [77–79] and phenomenologically allowed region $2 \times 10^{-10} \leq \eta_B \leq 10^{-9}$, respectively. As shown in Figure 7, the current observation of η_B^{CMB} can narrowly constrain the value of $|\langle m_{ee} \rangle|$ for the NH and IH spectrum, respectively. Combining the results presented in Figures 2 and 3 with those from the leptogenesis, we can pin down the Majorana CP phase β_1 via the parameter ϕ .

4.2. Ding Model

In this model, all the heavy RH neutrinos are exactly degenerate. By considering the RG effects, their masses get a tiny splitting (almost degenerate), which lead to a resonant leptogenesis as contributed from all these heavy RH neutrinos. However, if we neglect the RG effects on the H matrix, the contribution of N_3 to lepton asymmetries ε_i^α can be negligible due to $H_{13(31)} = H_{23(32)} = 0$. (Actually, this is also correct if we take into account the RG effects on

the H matrix since the radiative generation of $H_{13(31)}, H_{23(32)}$ is very small.) Combined with (2.29), (2.30), (3.9), and (4.3), the flavor-dependent CP asymmetries ε_i^α are obtained as

$$\begin{aligned}\varepsilon_1^e &\simeq \frac{a^2 r \sin \phi}{32\pi(1 - 2r \cos \phi + 3r^2)t}, & \varepsilon_1^\mu &\simeq \varepsilon_1^\tau \simeq \frac{-a^2 r \sin \phi}{64\pi(1 - 2r \cos \phi + 3r^2)t}, \\ \varepsilon_2^e &\simeq \frac{-a^2 r \sin \phi}{16\pi(3 - 2r \cos \phi + 3r^2)t}, & \varepsilon_2^\mu &\simeq \varepsilon_2^\tau \simeq \frac{a^2 r \sin \phi}{32\pi(3 - 2r \cos \phi + 3r^2)t}.\end{aligned}\quad (4.13)$$

Here the mass slitting parameter δ_N^{12} which can be calculated from (2.30) and (3.9),

$$\delta_N^{12} = -\delta_N^{21} = -4a^2(1 - 3r^2 + 2r \cos \phi), \quad (4.14)$$

has been used. The explicit form of K_i^α is found as

$$\begin{aligned}K_1^e &\simeq 4r^2 \frac{m_0}{m_*}, & m_0 &= \frac{a^2 v_u^2}{M}, \\ K_1^{\mu,\tau} &\simeq \frac{m_0}{m_*}(1 - 2r \cos \phi + r^2), \\ K_2^e &\simeq \frac{2m_0}{m_*}(1 - 2r \cos \phi + r^2), \\ K_2^{\mu,\tau} &\simeq \frac{m_0}{2m_*}(1 + 2r \cos \phi + r^2).\end{aligned}\quad (4.15)$$

With the help of (4.6), the BAU is obtained then.

The prediction for η_B as a function of $|\langle m_{ee} \rangle|$ is shown in Figure 8 where we have used $M = 10^3$ GeV and $\tan \beta = 1$. The horizontal solid and dashed lines correspond to the central value of the BAU experiment result $\eta_B^{\text{CMB}} = 6.1 \times 10^{-10}$ [77–79] and phenomenologically allowed region $2 \times 10^{-10} \leq \eta_B \leq 10^{-9}$, respectively. As seen in Figure 8, the current observation of η_B^{CMB} can narrowly constrain the value of $|\langle m_{ee} \rangle|$ for the NH and IH spectrum, respectively. Again, combining the results in Figures 5 and 6 with those from the leptogenesis, we can pin down the Majorana CP phase β_1 via the parameter ϕ .

5. Conclusions

We have studied the S_4 models in the context of a supersymmetric seesaw model which naturally lead to the TBM form for the lepton mixing matrix. In BMM model, the combination $Y_\nu Y_\nu^\dagger$ is proportional to unity whereas in Ding model the heavy RH Majorana masses are exactly degenerate. This would forbid the desirable leptogenesis to occur in each model. Therefore, for a viable leptogenesis the off-diagonal terms of $Y_\nu Y_\nu^\dagger$ in BMM model have to be generated, while in Ding model the degeneracy of heavy RH Majorana masses has to be lifted. We have shown that these can be easily achieved by the RG effects from a high-energy scale to the low-energy scale which result in the successful leptogenesis.

Figure 8: Prediction of η_B as a function of $|\langle m_{ee} \rangle|$ or the NH case (a) and IH case (b). The horizontal solid and dashed lines correspond to the experimental central value and phenomenologically allowed region.

We have also studied implications to the low-energy observables such as the $0\nu\beta\beta$ decay. It gives the definite predictions for $0\nu2\beta$ decay parameter $|\langle m_{ee} \rangle|$. Interestingly we have found a link between the leptogenesis and amplitude of $0\nu2\beta$ decay $|\langle m_{ee} \rangle|$ through a high-energy CP phase ϕ . We have shown how the high-energy CP phase ϕ is correlated to the low-energy Majorana CP phase and examined how the leptogenesis can be related with the $0\nu2\beta$ decay. It is pointed out that the predictions of $|\langle m_{ee} \rangle|$ for the NH and IH spectra can be constrained by the current observation of the baryon asymmetry of the universe as 6.1×10^{-10} .

Acknowledgments

T. P. Nguyen is grateful to Exchange Program for East Asia Young Researchers, JSPS for financial support. He thanks Professor Yoshimasa Kurihara and Members of Theory Division at KEK for support. This work is supported in part by National Foundation for Science and Technology Development of Vietnam (NAFOSTED) under Grant no. 103.03-2011.35.

References

- [1] T. Schwetz, M. Tórtola, and J. W. F. Valle, "Three-flavour neutrino oscillation update," *New Journal of Physics*, vol. 10, Article ID 113011, 2008.
- [2] M. Maltoni and T. Schwetz, "Three-flavour neutrino oscillation update and comments on possible hints for a non-zero θ_{13} ," arXiv:0812.3161 [hep-ph], 2008.
- [3] P. F. Harrison, D. H. Perkins, and W. G. Scott, "Tri-bimaximal mixing and the neutrino oscillation data," *Physics Letters, Section B*, vol. 530, no. 1-4, pp. 167-173, 2002.
- [4] P. F. Harrison and W. G. Scott, "Symmetries and generalisations of tri-bimaximal neutrino mixing," *Physics Letters, Section B*, vol. 535, no. 1-4, pp. 163-169, 2002.
- [5] P. F. Harrison and W. G. Scott, " μ - τ reflection symmetry in lepton mixing and neutrino oscillations," *Physics Letters, Section B*, vol. 547, no. 3-4, pp. 219-228, 2002.
- [6] P. F. Harrison and W. G. Scott, "Permutation symmetry, tri-bimaximal neutrino mixing and the S_3 group characters," *Physics Letters. B*, vol. 557, no. 1-2, pp. 76-86, 2003.
- [7] E. Ma and G. Rajasekaran, "Softly broken A_4 symmetry for nearly degenerate neutrino masses," *Physical Review D*, vol. 64, no. 11, Article ID 113012, 2001.
- [8] K. S. Babu, E. Ma, and J. W. F. Valle, "Underlying A_4 symmetry for the neutrino mass matrix and the quark mixing matrix," *Physics Letters, Section B*, vol. 552, no. 3-4, pp. 207-213, 2003.

- [9] G. Altarelli and F. Feruglio, "Tri-bimaximal neutrino mixing from discrete symmetry in extra dimensions," *Nuclear Physics B*, vol. 720, no. 1-2, pp. 64–88, 2005.
- [10] G. Altarelli and F. Feruglio, "Tri-bimaximal neutrino mixing, A_4 and the modular symmetry," *Nuclear Physics B*, vol. 741, no. 1-2, pp. 215–235, 2006.
- [11] G. Altarelli, F. Feruglio, and Y. Lin, "Tri-bimaximal neutrino mixing from orbifolding," *Nuclear Physics B*, vol. 775, no. 1-2, pp. 31–44, 2007.
- [12] F. Bazzocchi, S. Kaneko, and S. Morisi, "A SUSY A_4 model for fermion masses and mixings," *Journal of High Energy Physics*, vol. 2008, no. 03, p. 063, 2008.
- [13] G. Altarelli, F. Feruglio, and C. Hagedorn, "A SUSY SU(5) grand unified model of tri-bimaximal mixing from A_4 ," *Journal of High Energy Physics*, vol. 2008, no. 3, 2008.
- [14] Y. Lin, "A predictive A_4 model, charged lepton hierarchy and tri-bimaximal sum rule," *Nuclear Physics B*, vol. 813, no. 1-2, pp. 91–105, 2009.
- [15] M. Hirsch, S. Morisi, and J. W. F. Valle, "Tribimaximal neutrino mixing and neutrinoless double beta decay," *Physical Review D*, vol. 78, no. 9, Article ID 093007, 2008.
- [16] Y. H. Ahn and C. S. Chen, "Nonzero U_{e3} and TeV leptogenesis through A_4 symmetry breaking," *Physical Review D*, vol. 81, no. 10, Article ID 105013, 2010.
- [17] F. Feruglio, C. Hagedorn, Y. Lin, and L. Merlo, "Tri-bimaximal neutrino mixing and quark masses from a discrete flavour symmetry," *Nuclear Physics B*, vol. 775, no. 1-2, pp. 120–142, 2007.
- [18] M. C. Chen and K. T. Mahanthappa, "CKM and tri-bimaximal MNS matrices in a $SU(5) \times {}^{(d)}T$," *Physics Letters, Section B*, vol. 652, no. 1, pp. 34–39, 2007.
- [19] P. H. Frampton and T. W. Kephart, "Flavor symmetry for quarks and leptons," *Journal of High Energy Physics*, vol. 2007, no. 9, article 110, 2007.
- [20] G. J. Ding, "Fermion mass hierarchies and flavor mixing from T' symmetry," *Physical Review D*, vol. 78, no. 3, Article ID 036011, 2008.
- [21] P. H. Frampton and S. Matsuzaki, " T' predictions of PMNS and CKM angles," *Physics Letters, Section B*, vol. 679, no. 4, pp. 347–349, 2009.
- [22] S. Pakvasa and H. Sugawara, "Mass of the t-quark in $SU(2) \times U(1)$," *Physics Letters B*, vol. 82, no. 1, pp. 105–107, 1979.
- [23] T. Brown, N. Deshpande, S. Pakvasa, and H. Sugawara, "CP nonconservation and rare processes in an S_4 model of permutation symmetry," *Physics Letters B*, vol. 141, no. 1-2, pp. 95–99, 1984.
- [24] T. Brown, S. Pakvasa, H. Sugawara, and Y. Yamanaka, "Neutrino masses, mixing, and oscillations in the S_4 model of permutation symmetry," *Physical Review D*, vol. 30, no. 1, pp. 255–257, 1984.
- [25] D. G. Lee and R. N. Mohapatra, "An $SO(10) \times S_4$ scenario for naturally degenerate neutrinos," *Physics Letters, Section B*, vol. 329, no. 4, pp. 463–468, 1994.
- [26] E. Ma, "Neutrino mass matrix from S_4 symmetry," *Physics Letters, Section B*, vol. 632, no. 2-3, pp. 352–356, 2006.
- [27] C. Hagedorn, M. Lindner, and R. N. Mohapatra, " S_4 flavor symmetry and fermion masses: towards a grand unified theory of flavor," *Journal of High Energy Physics*, vol. 2006, no. 06, article 042, 2006.
- [28] Y. Cai and H. B. Yu, "SO(10) grand unification model with S_4 flavor symmetry," *Physical Review D*, vol. 74, no. 11, Article ID 115005, 2006.
- [29] F. Caravaglios and S. Morisi, "Gauge boson families in grand unified theories of fermion masses: $E_6 \times S_4$," *International Journal of Modern Physics A*, vol. 22, no. 14-15, pp. 2469–2491, 2007.
- [30] H. Zhang, "Flavor $S_4 \otimes Z_2$ symmetry and neutrino mixing," *Physics Letters, Section B*, vol. 655, no. 3-4, pp. 132–140, 2007.
- [31] Y. Koide, " S_4 flavor symmetry embedded into SU(3) and lepton masses and mixing," *Journal of High Energy Physics*, vol. 2007, no. 8, article 086, 2007.
- [32] H. Ishimori, Y. Shimizu, and M. Tanimoto, " S_4 flavor symmetry of quarks and leptons in SU(5) GUT," *Progress of Theoretical Physics*, vol. 121, no. 4, pp. 769–787, 2009.
- [33] F. Bazzocchi, L. Merlo, and S. Morisi, "Fermion masses and mixings in a S_4 based model," *Nuclear Physics B*, vol. 816, no. 1-2, pp. 204–226, 2009.
- [34] F. Bazzocchi and S. Morisi, " S_4 as a natural flavor symmetry for lepton mixing," *Physical Review D*, vol. 80, no. 9, Article ID 096005, 2009.
- [35] F. Bazzocchi, L. Merlo, and S. Morisi, "Phenomenological consequences of the seesaw mechanism in S_4 based models," *Physical Review D*, vol. 80, no. 5, Article ID 053003, 2009.
- [36] G. J. Ding, "Fermion masses and flavor mixings in a model with S_4 flavor symmetry," *Nuclear Physics B*, vol. 827, no. 1-2, pp. 82–111, 2010.
- [37] P. Minkowski, " $\mu \rightarrow e\gamma$ at a rate of one out of 109 muon decays?" *Physics Letters B*, vol. 67, no. 4, pp. 421–428, 1977.

- [38] M. Gell-Mann, P. Ramond, and R. Slansky, "Supergravity," in *Proceedings of the Supergravity Stony Brook Workshop*, P. Van Nieuwenhuizen and D. Freedman, Eds., New York, NY, USA, 1979.
- [39] T. Yanagida, "Horizontal Symmetry and Masses of Neutrinos," in *Proceedings of the Workshop on Unified Theories and Baryon Number in the Universe*, A. Sawada and A. Sugamoto, Eds., Tsukuba, Japan, 1979.
- [40] R. N. Mohapatra and G. Senjanović, "Neutrino mass and spontaneous parity nonconservation," *Physical Review Letters*, vol. 44, no. 14, pp. 912–915, 1980.
- [41] M. Fukugita and T. Yanagida, "Baryogenesis without grand unification," *Physics Letters B*, vol. 174, no. 1, pp. 45–47, 1986.
- [42] G. F. Giudice, A. Notari, M. Raidal, A. Riotto, and A. Strumia, "Towards a complete theory of thermal leptogenesis in the SM and MSSM," *Nuclear Physics B*, vol. 685, no. 1-3, pp. 89–149, 2004.
- [43] W. Buchmüller, P. Di Bari, and M. Plümacher, "Leptogenesis for pedestrians," *Annals of Physics*, vol. 315, no. 2, pp. 305–351, 2005.
- [44] A. Pilaftsis and T. E. J. Underwood, "Electroweak-scale resonant leptogenesis," *Physical Review D*, vol. 72, no. 11, Article ID 113001, pp. 1–27, 2005.
- [45] S. M. Bilenky, S. Pascoli, and S. T. Petcov, "Majorana neutrinos, neutrino mass spectrum, CP violation, and neutrinoless double β decay: the three-neutrino mixing case," *Physical Review D*, vol. 64, no. 5, Article ID 053010, 2001.
- [46] S. Pascoli, S. T. Petcov, and L. Wolfenstein, "Searching for the CP-violation associated with Majorana neutrinos," *Physics Letters, Section B*, vol. 524, no. 3-4, pp. 319–331, 2002.
- [47] S. Pascoli, S. T. Petcov, and W. Rodejohann, "On the CP violation associated with Majorana neutrinos and neutrinoless double-beta decay," *Physics Letters, Section B*, vol. 549, no. 1-2, pp. 177–193, 2002.
- [48] S. T. Petcov, "Neutrino masses, mixing, Majorana CP-violating phases and $(\beta\beta)_{\text{ov}}$ decay," *New Journal of Physics*, vol. 6, article 109, pp. 1–23, 2004.
- [49] C. D. Froggatt and H. B. Nielsen, "Hierarchy of quark masses, cabibbo angles and CP violation," *Nuclear Physics, Section B*, vol. 147, no. 3-4, pp. 277–298, 1979.
- [50] H. V. Klapdor-Kleingrothaus, A. Dietz, L. Baudis et al., "Latest results from the HEIDELBERG-MOSCOW double beta decay experiment," *European Physical Journal A*, vol. 12, no. 2, pp. 147–154, 2001.
- [51] H. V. Klapdor-Kleingrothaus, I. V. Krivosheina, A. Dietz, and O. Chkvorets, "Search for neutrinoless double beta decay with enriched ^{76}Ge in Gran Sasso 1990–2003," *Physics Letters, Section B*, vol. 586, no. 3-4, pp. 198–212, 2004.
- [52] C. Arnaboldi, D. R. Artusa, F. T. Avignone III et al., "Results from a search for the $0\nu\beta\beta$ -decay of ^{130}Te ," *Physical Review C*, vol. 78, no. 3, Article ID 035502, 2008.
- [53] J. Wolf, "The KATRIN neutrino mass experiment," *Nuclear Instruments and Methods in Physics Research, Section A*, vol. 623, no. 1, pp. 442–444, 2010.
- [54] C. Aalseth et al., "Neutrinoless double beta decay and direct searches for neutrino mass," arXiv:hep-ph/0412300, 2004.
- [55] I. Abt et al., "A new ^{76}Ge double beta decay experiment at LNGS," arXiv:hep-ex/0404039, 2004.
- [56] R. González Felipe, F. R. Joaquim, and B. M. Nobre, "Radiatively induced leptogenesis in a minimal seesaw model," *Physical Review D*, vol. 70, no. 8, Article ID 085009, 2004.
- [57] K. Turzyński, "Degenerate minimal see-saw and leptogenesis," *Physics Letters, Section B*, vol. 589, no. 3-4, pp. 135–140, 2004.
- [58] G. C. Branco, R. González Felipe, F. R. Joaquim, and B. M. Nobre, "Enlarging the window for radiative leptogenesis," *Physics Letters, Section B*, vol. 633, no. 2-3, pp. 336–344, 2006.
- [59] J. A. Casas, J. R. Espinosa, A. Ibarra, and I. Navarro, "General RG equations for physical neutrino parameters and their phenomenological implications," *Nuclear Physics B*, vol. 573, no. 3, pp. 652–684, 2000.
- [60] J. A. Casas, J. R. Espinosa, A. Ibarra, and I. Navarro, "Nearly degenerate neutrinos, supersymmetry and radiative corrections," *Nuclear Physics B*, vol. 569, no. 1–3, pp. 82–106, 2000.
- [61] L. Covi, E. Roulet, and F. Vissani, "CP violating decays in leptogenesis scenarios," *Physics Letters, Section B*, vol. 384, no. 1–4, pp. 169–174, 1996.
- [62] A. Pilaftsis, "Heavy Majorana neutrinos and baryogenesis," *International Journal of Modern Physics A*, vol. 14, no. 12, pp. 1811–1857, 1999.
- [63] T. Endoh, T. Morozumi, and Z. Xiong, "Primordial lepton family asymmetries in seesaw model," *Progress of Theoretical Physics*, vol. 111, no. 1, pp. 123–149, 2004.

- [64] T. Fujihara, S. Kaneko, S. Kang, D. Kimura, T. Morozumi, and M. Tanimoto, "Cosmological family asymmetry and CP violation," *Physical Review D*, vol. 72, no. 1, Article ID 016006, pp. 1–12, 2005.
- [65] A. Abada, S. Davidson, A. Ibarra, F. X. Josse-Michaux, M. Losada, and A. Riotto, "Flavour matters in leptogenesis," *Journal of High Energy Physics*, vol. 2006, no. 09, article 010, 2006.
- [66] S. Blanchet and P. Di Bari, "Flavour effects on leptogenesis predictions," *Journal of Cosmology and Astroparticle Physics*, vol. 2007, no. 03, article 018, 2007.
- [67] S. Antusch, S. F. King, and A. Riotto, "Flavour-dependent leptogenesis with sequential dominance," *Journal of Cosmology and Astroparticle Physics*, vol. 2006, no. 11, article 011, 2006.
- [68] S. Pascoli, S. T. Petcov, and A. Riotto, "Connecting low energy leptonic CP violation to leptogenesis," *Physical Review D*, vol. 75, no. 8, Article ID 083511, 2007.
- [69] S. Pascoli, S. T. Petcov, and A. Riotto, "Leptogenesis and low energy CP-violation in neutrino physics," *Nuclear Physics B*, vol. 774, no. 1–3, pp. 1–52, 2007.
- [70] G. C. Branco, R. González Felipe, and F. R. Joaquim, "A new bridge between leptonic CP violation and leptogenesis," *Physics Letters, Section B*, vol. 645, no. 5–6, pp. 432–436, 2007.
- [71] G. C. Branco, A. J. Buras, S. Jäger, S. Uhlig, and A. Weiler, "Another look at minimal lepton flavour violation, leptogenesis and the ratio M_ν/Λ_{LFV} ," *Journal of High Energy Physics*, vol. 2007, no. 09, article 004, 2007.
- [72] A. Pilaftsis and T. E. J. Underwood, "Electroweak-scale resonant leptogenesis," *Physical Review D*, vol. 72, no. 11, Article ID 113001, pp. 1–27, 2005.
- [73] Z. Z. Xing and S. Zhou, "Tri-bimaximal neutrino mixing and flavor-dependent resonant leptogenesis," *Physics Letters, Section B*, vol. 653, no. 2–4, pp. 278–287, 2007.
- [74] S. Pascoli, S. T. Petcov, and A. Riotto, "Leptogenesis and low energy CP-violation in neutrino physics," *Nuclear Physics B*, vol. 774, no. 1–3, pp. 1–52, 2007.
- [75] A. Abada, S. Davidson, F. X. Josse-Michaux, M. Losada, and A. Riotto, "Flavour issues in leptogenesis," *Journal of Cosmology and Astroparticle Physics*, no. 4, article 004, pp. 47–67, 2006.
- [76] S. Antusch, S. F. King, and A. Riotto, "Flavour-dependent leptogenesis with sequential dominance," *Journal of Cosmology and Astroparticle Physics*, vol. 2011, no. 11, article 011, 2006.
- [77] D. N. Spergel, L. Verde, H. V. Peiris et al., "First-year Wilkinson Microwave Anisotropy Probe (WMAP) observations: determination of cosmological parameters," *Astrophysical Journal*, vol. 148, no. 1, pp. 175–194, 2003.
- [78] M. Tegmark, M. A. Strauss, M. R. Blanton et al., "Cosmological parameters from SDSS and WMAP," *Physical Review D*, vol. 69, no. 10, Article ID 103501, 2004.
- [79] C. L. Bennett, M. Halpern, G. Hinshaw et al., "First-year Wilkinson Microwave Anisotropy Probe (WMAP) observations: preliminary maps and basic results," *Astrophysical Journal*, vol. 148, no. 1, pp. 1–27, 2003.