Research Article

Spin-One DKP Equation in the Presence of Coulomb and Harmonic Oscillator Interactions in (1 + 3)-Dimension

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In this work, we study Duffin-Kemmer-Petiau equation in the presence of coulomb and harmonic oscillator potentials in (1 + 3)-dimension for spin-one particles and we obtain energy eigenvalues and corresponding eigenfunctions.

1. Introduction

The Duffin-Kemmer-Petiau (Duffin, 1938; Kemmer, 1938; Petiau, 1936) equation is a first-order relativistic wave equation for spin-zero and one bosons [1–3]. It is similar with Dirac equation in which we replace the algebra of the gamma matrices by beta matrices. More recently, there have been a renewed interest in DKP equation; particularly it has been applied to QCD (large and short distances) by Gribov [4], to covariant Hamiltonian dynamics by Kanatchikov [5] and has been generalized to curve space-time by Red'kov [6] and Lunardi et al. [7]. In addition, the relativistic model of α -nucleus elastic scattering where they have been treated by the formalism of the DKP theory [8] and covariant Hamiltonian [9] in the casual approach [10, 11] and there has been an increasing interest on the DKP oscillator [12–17]. Recently, many articles have been devoted to investigate DKP theory under different types of potential; hence, we can cite the following [18–29]. Since the wave function includes all the necessary information about considering systems, the energy eigenvalues and corresponding eigenfunctions between interaction systems in relativistic quantum mechanics

and in nonrelativistic quantum mechanics are studied more efficiently in recent years. In this study, we have investigated DKP equation with coulomb and harmonic oscillator potentials in $(1+3)$ -dimension.

2. DKP Equation

The DKP equation in free field is given by (in natural units $\hbar = c = 1$) [1-3]

$$(i\beta^\mu \partial_\mu - m_0)\Psi = 0 \quad (2.1)$$

β^μ are the DKP matrices which are satisfied in this algebra:

$$\beta^\mu \beta^\nu \beta^\lambda + \beta^\lambda \beta^\nu \beta^\mu = g^{\mu\nu} \beta^\lambda + g^{\lambda\nu} \beta^\mu, \quad (2.2)$$

where $g^{\mu\nu} = \text{diag}(1, -1, -1, -1)$ and $(g^{\mu\nu})^2 = 1$, being the metric tensor of Minkowski space-time.

For the spin-one case, β^μ matrices are

$$\beta^0 = \begin{pmatrix} 0 & \bar{0} & \bar{0} & \bar{0} \\ \bar{0}^T & 0 & I & 0 \\ \bar{0}^T & I & 0 & 0 \\ \bar{0}^T & 0 & 0 & 0 \end{pmatrix}, \quad (2.3)$$

$$\beta^i = \begin{pmatrix} 0 & \bar{0} & e_i & \bar{0} \\ \bar{0}^T & 0 & 0 & -iS_i \\ -e_i^T & 0 & 0 & 0 \\ \bar{0}^T & -iS_i & 0 & 0 \end{pmatrix}$$

with S matrices being 3×3 ones, $(S_i)_{jk} = -i\varepsilon_{ijk}$ where ε_{ijk} is 1, -1, 0 for an even permutation, an odd permutation, and repeated indices, respectively. e_i Matrices are 1×3 , $(e_i)_{1j} = \delta_{ij}$, that is,

$$e_1 = (1, 0, 0), \quad e_2 = (0, 1, 0), \quad e_3 = (0, 0, 1) \quad (2.4)$$

I and 0 , respectively, represent unit and null 3×3 matrices and $\bar{0}$ s are 1×3 ones [30].

3. DKP Equation in Three-Dimensional Space-Time

Furthermore, for an elastic scattering, the interaction is [31]

$$U = S(r) + PS_\mu(r) + \beta^0 V(r) + \beta^0 PV_P(r), \quad (3.1)$$

where each term has a specific Lorentz character. Two Lorentz vectors may be written as β^μ and $P\beta^\mu$ by assuming rotational invariance and parity conservation. DKP matrices have three

irreducible representations: one-dimension representation where is trivial, five-dimension representation that is for spin-zero particles, and ten-dimension representation that is for spin-one particles [1–3].

The DKP equation in the presence of interaction is written as

$$\left(i\beta^\mu\partial_\mu - m_0 - \beta^0 PV(r)\right)\Psi = 0. \quad (3.2)$$

As usual, a solution of the following form removes the time component

$$\Psi(x, y, z, t) = \exp(-iE_{n,l}t)\psi_{n,l}(x, y, z) \quad (3.3)$$

as the problem is considered in one spatial dimension, we consider one quantum number and write the wave function as

$$\psi_{n,l}^T(x, y, z) = \left(\varphi_{n,l}^{(1)}, \varphi_{n,l}^{(2)}, \varphi_{n,l}^{(3)}, \varphi_{n,l}^{(4)}, \varphi_{n,l}^{(5)}, \varphi_{n,l}^{(6)}, \varphi_{n,l}^{(7)}, \varphi_{n,l}^{(8)}, \varphi_{n,l}^{(9)}, \varphi_{n,l}^{(10)}\right)^T. \quad (3.4)$$

So we choose,

$$\varphi_{n,l}^{(1)} = i\phi_{n,l}, \quad \vec{F} = \left(\varphi_{n,l}^{(2)}, \varphi_{n,l}^{(3)}, \varphi_{n,l}^{(4)}\right), \quad \vec{G} = \left(\varphi_{n,l}^{(5)}, \varphi_{n,l}^{(6)}, \varphi_{n,l}^{(7)}\right), \quad \vec{H} = \left(\varphi_{n,l}^{(8)}, \varphi_{n,l}^{(9)}, \varphi_{n,l}^{(10)}\right). \quad (3.5)$$

Substituting of the above relations in (3.6), we have determined ten coupled equations as follows

$$i\left(\frac{\partial F_{n,l}^{(2)}}{\partial x} - \frac{\partial F_{n,l}^{(1)}}{\partial y}\right) = m_0 H_{n,l}^{(3)}, \quad (3.6)$$

$$i\left(\frac{\partial F_{n,l}^{(1)}}{\partial z} - \frac{\partial F_{n,l}^{(3)}}{\partial x}\right) = m_0 H_{n,l}^{(2)}, \quad (3.7)$$

$$i\left(\frac{\partial F_{n,l}^{(3)}}{\partial y} - \frac{\partial F_{n,l}^{(2)}}{\partial z}\right) = m_0 H_{n,l}^{(1)}, \quad (3.8)$$

$$\left(\frac{\partial G_{n,l}^{(3)}}{\partial z} + \frac{\partial G_{n,l}^{(2)}}{\partial y} + \frac{\partial G_{n,l}^{(1)}}{\partial x}\right) = m_0 \phi_{n,l}, \quad (3.9)$$

$$E_{n,l}G_{n,l}^{(1)} + i \left(\frac{\partial H_{n,l}^{(3)}}{\partial y} - \frac{\partial H_{n,l}^{(2)}}{\partial z} \right) = m_0 F_{n,l}^{(1)}, \quad (3.10)$$

$$E_{n,l}G_{n,l}^{(2)} + i \left(\frac{\partial H_{n,l}^{(1)}}{\partial z} - \frac{\partial H_{n,l}^{(3)}}{\partial x} \right) = m_0 F_{n,l}^{(2)}, \quad (3.11)$$

$$E_{n,l}G_{n,l}^{(3)} + i \left(\frac{\partial H_{n,l}^{(2)}}{\partial x} - \frac{\partial H_{n,l}^{(1)}}{\partial y} \right) = m_0 F_{n,l}^{(3)}, \quad (3.12)$$

$$(E_{n,l} - V(r))F_{n,l}^{(1)} + \frac{\partial \varphi}{\partial x} = m_0 G_{n,l}^{(1)}, \quad (3.13)$$

$$(E_{n,l} - V(r))F_{n,l}^{(2)} + \frac{\partial \varphi}{\partial y} = m_0 G_{n,l}^{(2)}, \quad (3.14)$$

$$(E_{n,l} - V(r))F_{n,l}^{(3)} + \frac{\partial \varphi}{\partial z} = m_0 G_{n,l}^{(3)}. \quad (3.15)$$

Combining the above equations, we have

$$\begin{aligned} i\nabla \times \vec{F} &= m_0 \vec{H}, \\ \nabla \cdot \vec{G} &= m_0 \phi_{n,l}, \\ E_{n,l} \vec{G} + i\nabla \times \vec{H} &= m_0 \vec{F}, \\ (E_{n,l} - V(r))\vec{F} + \nabla \phi_{n,l} &= m_0 \vec{G}. \end{aligned} \quad (3.16)$$

In obtaining the last result as follows, we combine the above equations:

$$\begin{aligned} (E_{n,l}(E_{n,l} - V(r)) - m_0^2)\vec{F} - \nabla \times (\nabla \times \vec{F}) + \nabla(\nabla \cdot \vec{F}) &= 0, \\ \nabla \times (\nabla \times \vec{F}) &= \nabla(\nabla \cdot \vec{F}) - \nabla^2 \vec{F}. \end{aligned} \quad (3.17)$$

Then,

$$(E_{n,l}(E_{n,l} - V(r)) - m_0^2)\vec{F} + \nabla^2 \vec{F} = 0. \quad (3.18)$$

Moreover,

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r} - \frac{L^2}{r^2}, \quad (3.19)$$

So we have,

$$\left(\frac{d^2}{dr^2} + \frac{2}{r} \frac{d}{dr} + E_{n,l}^2 - E_{n,l} V(r) - m_0^2 - \frac{l(l+1)}{r^2} \right) \vec{F} = 0. \quad (3.20)$$

Thus, if we suppose that $V(r) = 0$, this equation reduces to $P^2 = E_{n,l}^2 - m_0^2$.

4. Exact Solutions of DKP Equation under Coulomb Potential

Now for deriving eigenvalues of energy and wave functions of (3.20), we have

$$\frac{d^2 \vec{F}}{dr^2} + \frac{2}{r} \frac{d \vec{F}}{dr} + \frac{1}{r^2} \left(E_{n,l}^2 r^2 - E_{n,l} V_0 r - m_0^2 r^2 - l(l+1) \right) \vec{F} = 0. \quad (4.1)$$

We determine the energy eigenvalues from (4.2) as follows:

$$(2n+1) \sqrt{m_0^2 - E_{n,l}^2} + E_{n,l} V_0 + 2 \sqrt{\left(\frac{1}{4} + l(l+1) \right) (m_0^2 - E_{n,l}^2)} = 0. \quad (4.2)$$

So

$$E_{n,l} = \pm \frac{2m_0(n+l+1)}{\sqrt{V_0^2 + 4(n+l+1)^2}}, \quad (4.3)$$

$$\varphi_{n,l}^{(2)}(\vec{r}) = N_{n,l} r^l e^{-\sqrt{m_0^2 - E_{n,l}^2} r} L_n^{2l+1} \left(2\sqrt{m_0^2 - E_{n,l}^2} r \right).$$

We have plotted wave function versus r in Figure 1. The energy eigenvalues for different l are reported in Table 1 to give a better view of the obtained results. Also, we have displayed the energy eigenvalues versus V_0 in Figure 2. We can now demonstrate that the spectra given in Table 1 present the pattern appearing in supersymmetry quantum mechanics [32] where the levels of energy $E_{n,l} = E_{n+1,l-1}$ are degenerate.

5. Exact Solutions of DKP Equation under Harmonic Oscillator Potential

In this section, we study (3.20) with harmonic oscillator potential,

$$\left(\frac{d^2}{dr^2} + \frac{2}{r} \frac{d}{dr} + E_{n,l}^2 - E_{n,l} \alpha r^2 - m_0^2 - \frac{l(l+1)}{r^2} \right) \vec{F} = 0. \quad (5.1)$$

Figure 1: Wave functions for coulomb interaction versus r for $(m_0 = 1, V_0 = -0.1, \text{ and } l = 0)$.

Table 1: Energy eigenvalues for coulomb interaction with $(m_0 = 1, V_0 = -0.1)$.

$ n, l\rangle$	$E_{n,l}$	$ n, l\rangle$	$E_{n,l}$	$ n, l\rangle$	$E_{n,l}$	$ n, l\rangle$	$E_{n,l}$
$ 0, 0\rangle$	0.998752	$ 0, 1\rangle$	0.999688	$ 0, 2\rangle$	0.999861	$ 0, 3\rangle$	0.999922
$ 1, 0\rangle$	0.999688	$ 1, 1\rangle$	0.999861	$ 1, 2\rangle$	0.999922	$ 1, 3\rangle$	0.999950
$ 2, 0\rangle$	0.999861	$ 2, 1\rangle$	0.999922	$ 2, 2\rangle$	0.999950	$ 2, 3\rangle$	0.999965
$ 3, 0\rangle$	0.999922	$ 3, 1\rangle$	0.999950	$ 3, 2\rangle$	0.999965	$ 3, 3\rangle$	0.999974
$ 4, 0\rangle$	0.999950	$ 4, 1\rangle$	0.999965	$ 4, 2\rangle$	0.999974	$ 4, 3\rangle$	0.999980
$ 5, 0\rangle$	0.999965	$ 5, 1\rangle$	0.999974	$ 5, 2\rangle$	0.999980	$ 5, 3\rangle$	0.999985
$ 6, 0\rangle$	0.999974	$ 6, 1\rangle$	0.999980	$ 6, 2\rangle$	0.999985	$ 6, 3\rangle$	0.999988
$ 7, 0\rangle$	0.999980	$ 7, 1\rangle$	0.999985	$ 7, 2\rangle$	0.999988	$ 7, 3\rangle$	0.999990
$ 8, 0\rangle$	0.999985	$ 8, 1\rangle$	0.999988	$ 8, 2\rangle$	0.999990	$ 8, 3\rangle$	0.999991

So the energy eigenvalues can be derived from the below equation:

$$\left(n + \frac{3}{4} + \frac{l}{2}\right)\sqrt{\alpha E_{n,l}} + \frac{m_0^2 - E_{n,l}^2}{4} = 0, \quad (5.2)$$

and the wave function is

$$\varphi_{n,l}^{(2)} = N_{n,l} r^l e^{-(\sqrt{\alpha E_{n,l}}/2)r^2} L_n^{(l+1/2)}\left(\sqrt{\alpha E_{n,l}} r^2\right). \quad (5.3)$$

Figure 2: $E_{n,l}$ versus V_0 . Figure 3: Wave functions for harmonic oscillator potential versus r for ($m_0 = 1$, $\alpha = 0.1$, and $l = 0$).

We have plotted wave functions versus r in Figure 3. The energy eigenvalues for different l are shown in Table 2, and in Figure 4 we have plotted energy eigenvalues versus α .

6. Conclusion

We have investigated DKP equation in the presence of coulomb and harmonic oscillator potentials in three-dimensional of space-time for spin-one particles. Thus, we have derived

Figure 4: Energy eigenvalues versus α .

Table 2: Energy eigenvalues for harmonic oscillator with ($m_0 = 1, \alpha = 0.1$).

$ n, l\rangle$	$E_{n,l}$	$ n, l\rangle$	$E_{n,l}$	$ n, l\rangle$	$E_{n,l}$	$ n, l\rangle$	$E_{n,l}$
$ 0, 0\rangle$	1.465801	$ 0, 1\rangle$	1.760006	$ 0, 2\rangle$	2.040014	$ 0, 3\rangle$	2.307153
$ 1, 0\rangle$	2.040014	$ 1, 1\rangle$	2.307153	$ 1, 2\rangle$	2.562975	$ 1, 3\rangle$	2.808890
$ 2, 0\rangle$	2.562975	$ 2, 1\rangle$	2.808890	$ 2, 2\rangle$	3.046099	$ 2, 3\rangle$	3.275606
$ 3, 0\rangle$	3.046099	$ 3, 1\rangle$	3.275606	$ 3, 2\rangle$	3.498248	$ 3, 3\rangle$	3.714727
$ 4, 0\rangle$	3.498248	$ 4, 1\rangle$	3.714727	$ 4, 2\rangle$	3.925637	$ 4, 3\rangle$	4.131483
$ 5, 0\rangle$	3.925637	$ 5, 1\rangle$	4.131483	$ 5, 2\rangle$	4.332698	$ 5, 3\rangle$	4.529657
$ 6, 0\rangle$	4.332698	$ 6, 1\rangle$	4.529657	$ 6, 2\rangle$	4.722685	$ 6, 3\rangle$	4.912070
$ 7, 0\rangle$	4.722685	$ 7, 1\rangle$	4.912070	$ 7, 2\rangle$	5.098064	$ 7, 3\rangle$	5.280890
$ 8, 0\rangle$	5.098064	$ 8, 1\rangle$	5.280890	$ 8, 2\rangle$	5.460749	$ 8, 3\rangle$	5.637821

energy eigenvalues and wavefunctions where we have plotted the wavefunctions versus r also the energy eigenvalues have been determined in Tables 1 and 2. In effect with increasing the quantum numbers, the values of energy increased. In order to describe the behavior of the energy versus V_0 and α , we have displayed in Figures 2 and 4 that with decreasing of the α and V_0 the values of energy tend to one point. Furthermore we have discussed the solutions of the DKP equation. Hence, our results are useful in the study of relativistic spin-one particles.

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