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Research Article

$B \to J/\psi K(1270)$ and $B \to J/\psi K(1400)$ Decays in QCD Factorization Approach

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The hadronic $B \to J/\psi K(1270)$ and $B \to J/\psi K(1400)$ has been analyzed in "QCD factorization" approach and generalized factorization. The effective coefficients a_i^h have been calculated for three helicity states $h = 0, +, -$ which give three different contribution of amplitudes. We consider that J/ψ behaves as a light meson in compared to B meson. For $B^+ \to J/\psi K^+(1270)$ and $B^+ \to J/\psi K^+(1400)$, experimental data of branching ratios are $(1.8 \pm 0.5) \times 10^{-3}$ and $<5 \times 10^{-4}$, respectively. Our best obtained results are $(1.79 \pm 0.01) \times 10^{-3}$ at $\theta = 58°$ and $(1.42 \pm 0.01) \times 10^{-3}$ at $\theta = 32°$ for $B^+ \to J/\psi K^+(1270)$ decay. And we have 4.27×10^{-4} at $\theta = 58°$ and 3.45×10^{-4} at $\theta = 32°$ for $B^+ \to J/\psi K^+(1400)$, which are in agreement with experiment.

1. Introduction

Recent experimental results sobtained by BABAR, Belle, and CLEO have opened an interesting area of research about production of axial-vector mesons in B decays. Two body B decays look for CP violation and overconstrain the CKM parameters in the Standard Model. Exclusive modes containing $B \to PP$, PV, and VV, which have been extensively discussed in the literature have confirmed such expectation [1]. In this research, the two-body hadronic decays of B meson into VA (V: vector, A: axial-vector meson) are studied in QCD factorization method. First we introduce the structure of axial-vector meson. Then we study the hadronic decays $B \to J/\psi K(1270)$ and $B \to J/\psi K(1400)$, particularly. There are two distinct types of axial-vector meson, namely, $^3P_1$ and $^1P_1$. In the quark model, two nonets of $J^P = 1^+$ axial-vector meson are expected as the orbital excitation of the $q\bar{q}$ system. In terms of the spectroscopic notation $^{2S+1}L_J$, there are two types of p-wave mesons ($^3P_1$ and $^1P_1$). These two nonets have distinctive C quantum numbers, $C = +$ and $C = -$, respectively. Experimentally, the $J^{PC} = 1^{++}$ nonet consists of $a_1(1260)$, $f_1(1285)$, $f_1(1420)$, and

K_{1A}, while the 1^{+-} nonet has $b_1(1235)$, $h_1(1170)$, $h_1(1380)$, and K_{1B}. There are two mixing effects for axial-vector meson: one is the mixing between $^3P_1$ and $^1P_1$ states, for example, K_{1A} and K_{1B}, and the other is mixing among $^3P_1$ or $^1P_1$ states themselves [2]. The $^3P_1$ meson behaves similarly to the vector meson this is not the case for the $^1P_1$ meson. For the latter, its decay constant vanishes in $SU(3)$ limit. Their light-cone distribution amplitudes are given by using the QCD sum rule method, and the chiral-even two-parton light-cone distribution amplitudes of the $^3P_1$ ($^1P_1$) meson are symmetric (antisymmetric) under the exchange of quark and antiquark momentum fractions in the $SU(3)$ limit due to G-parity. The B decays involving an axial-vector meson and vector meson in final state have three polarization states. We have studied the two body B decays involving axial-vector meson $K(1270)$ or $K(1400)$, and a vector meson J/ψ in final state. The simplest approach to obtain the hadronic matrix elements in decay amplitudes is naive factorization, where the matrix elements have been parameterized into the product of decay constant and form factors. In generalized factorization, it is well established that nonfactorizable contributions must be present in the matrix elements in order to cancel the energy scale μ and renormalization scheme dependence of Wilson coefficients. In QCD factorization approach, the nonfactorizable effects such as vertex corrections, hard spectator interactions, and annihilation contributions are calculable in hard scattering approach, and they have been studied in [3, 4] for the decays $B \to a_1(1260)\pi$, $a_1(1260)K$, $B \to h_1(1235)K^*$, $b_1(1235)K^*$ and $b_1(1235)\rho$ [5].

Using the product expansion, the low-energy effective Hamiltonian for $B \to M_1M_2$ decays can be written generally as [6, 7]

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p^D \left(C_1 O_1^p + C_2 O_2^p + \sum_{i=3}^{10} C_i O_i \right) + \text{h.c.}, \tag{1.1}$$

where $\lambda_p^D = V_{pb}V_{pD}^*$ $(D = d, s)$ are the product of CKM elements and C_i's are Wilson coefficients which have been evaluated at next leading order (NLO). The expressions of local operators are [8].

Current-current operators:

$$\begin{aligned} O_1^p &= (\bar{p}_\alpha b_\beta)_{V-A} \left(\bar{q}_\beta p_\alpha\right)_{V-A}, \\ O_2^p &= (\bar{p}_\alpha b_\alpha)_{V-A} \left(\bar{q}_\beta p_\beta\right)_{V-A}. \end{aligned} \tag{1.2}$$

QCD penguin operators:

$$\begin{aligned} O_3 &= (\bar{q}_\alpha b_\alpha)_{V-A} \sum_{\acute{q}} \left(\bar{\acute{q}}_\beta q_\beta\right)_{V-A}, \\ O_4 &= (\bar{q}_\alpha b_\beta)_{V-A} \sum_{\acute{q}} \left(\bar{\acute{q}}_\beta q_\alpha\right)_{V-A}, \end{aligned}$$

$$O_5 = (\overline{q}_\alpha b_\alpha)_{V-A} \sum_{\acute{q}} \left(\overline{\acute{q}}_\beta q_\beta\right)_{V+A},$$
$$O_6 = (\overline{q}_\alpha b_\beta)_{V-A} \sum_{\acute{q}} \left(\overline{\acute{q}}_\beta q_\alpha\right)_{V+A}. \tag{1.3}$$

Electroweak penguin operators:

$$O_7 = (\overline{q}_\alpha b_\alpha)_{V-A} \sum_{\acute{q}} \frac{3}{2} e_{\acute{q}} \left(\overline{\acute{q}}_\beta q_\beta\right)_{V-A},$$
$$O_8 = (\overline{q}_\alpha b_\beta)_{V-A} \sum_{\acute{q}} \frac{3}{2} e_{\acute{q}} \left(\overline{\acute{q}}_\beta q_\alpha\right)_{V+A},$$
$$O_9 = (\overline{q}_\alpha b_\alpha)_{V-A} \sum_{\acute{q}} \frac{3}{2} e_{\acute{q}} \left(\overline{\acute{q}}_\beta q_\beta\right)_{V-A},$$
$$O_{10} = (\overline{q}_\alpha b_\beta)_{V-A} \sum_{\acute{q}} \frac{3}{2} e_{\acute{q}} \left(\overline{\acute{q}}_\beta q_\alpha\right)_{V+A}. \tag{1.4}$$

Here $(\overline{q_1} q_2)_{V\pm A} = \overline{q}_1 (1 \pm \gamma_5) q_2$. For hadronic weak decays, the short distance effects can be calculated by perturbative theory but the computation of nonperturbative long distance effects is difficult. The fundamental problem in computation of hadronic matrix elements is due to nonperturbative effects arising from the strong interactions. The simplest approach to hadronic matrix elements is naive factorization hypothesis, where the hadronic matrix elements have been parameterized into the product of the decay constants and the form factors. Beneke et al. suggested QCD formula to compute the hadronic matrix elements in the heavy quark limit, combining the hard scattering approach with power counting in $1/m_b$ [9, 10]. In heavy quark limit $m_b \gg \Lambda_{\mathrm{QCD}}$, up to power corrections of order of $\Lambda_{\mathrm{QCD}}/m_b$, the QCD formula for $B \to M_1 M_2$ can be written as

$$\langle M_1 M_2 | O_i | B \rangle = \sum_j F_j^{B\to M_1}\left(m_2^2\right) \int_0^1 du T_{ij}^I(u) \phi_{M_2} + (M_1 \longleftrightarrow M_2)$$
$$+ \int_0^1 d\xi du dv T_i^{II}(\xi, u, v) \phi_B(\xi) \phi_{M_1}(v) \phi_{M_2}(u). \tag{1.5}$$

M_1 and M_2 are both light. Which $F^{B\to M}$ is form factor for B to M transition and ϕ_M is the light cone distribution amplitude for meson. ξ, u, v are momentum fractions of constituent quarks in B, M_1 and M_2 mesons, respectively. T_i and T_{ij} are hard-scattering kernels arising from hard gluon exchanges. Hence, the hadronic matrix elements can be separated into short distance part (hard-scattering kernels) and long distance part. In QCD factorization, nonfactorizable loop effects and spectator scattering contribution have been considered. At

leading order, there is a single diagram with no hard gluon interaction, and the contribution to $T_{ij}(u)$ in (1.5) is independent of u, and u-integral reduces to the normalization condition for the M_2 wave function. consequently, the factorization formula (1.5) reproduces naive factorization, if we neglect gluon exchange. In naive factorization, one neglect all corrections of order $\Lambda_{\mathrm{QCD}}/m_b$ and of order α_s. In QCD factorization, one computes systematically corrections to higher order in α_s, but still neglects power correction of order $\Lambda_{\mathrm{QCD}}/m_b$. In factorization, we have factorizable and non factorizable contributions. There are soft gluons in the factorizable contributions which are absorbed into the physical form factor. The diagrams also have hard contributions, which go into the short distance coefficient. The diagrams containing gluon exchanges which do not belong to the form factor are called nonfactorizable contributions. At order α_s, these contributions can be divided into four groups: vertex corrections, penguin diagrams, hard spectator interactions, and annihilation diagrams. A detailed discussion of QCD factorization approach can be found in [8–10].

This method works well for the case with two light mesons in which the final-state mesons carry large momenta. When there is a heavy quark in the final state such as $\overline{B} \to D^+\pi^-$, this method still works when a spectator quark of B meson is absorbed by D meson. However, when the spectator quark is absorbed by a light quark, for example, in $\overline{B} \to D\pi^0$, nonfactorizable contributions are infrared divergent, and the factorization breaks down. When we consider $B \to J/\psi K(1270)$ and $B \to J/\psi K(1400)$, it looks ambiguous at first sight whether we can apply the same method used in $B \to \pi\pi$, or $\overline{B} \to D^+\pi^-$, since the spectator quark in the B meson goes into light $K(1270)$ or $K(1400)$ meson. However, what is special about J/ψ is that the size of the charmonium is so small ($\sim 1/\alpha_s m_{J/\psi}$) that the charmonium has a negligible overlap with the (B, K) system [11].

2. Input Parameters

2.1. Light-Cone Distribution Amplitude (LCDA)

The QCD corrections can change the local quark-anti quark operators into a series of nonlocal operators. If we assume that the J/ψ behave as a light meson, we can describe the light cone distribution. Note that in QCDF, the LCDAs of the light vector meson are written as [12]

$$\begin{aligned}
&\langle J/\psi(p,\varepsilon)|\bar{c}_{1\alpha}(y)c_{2\beta}(x)|0\rangle \\
&\quad = -\frac{i}{4}\int_0^1 du \exp\left[i(up\cdot y + \bar{u}p\cdot x)\right] \\
&\qquad \times \left\{ f_{J/\psi} m_{J/\psi}\left(\not{p}\frac{\varepsilon^* z}{pz}\phi_{\parallel}^{J/\psi}(u) + \not{\varepsilon}_{\perp}^{*} g_{\perp}^{J/\psi(v)}(u) + \frac{1}{4}\varepsilon_{\mu\nu\rho\sigma}\varepsilon^{*\mu}p^{\rho}z^{\sigma}\gamma^{\nu}\gamma_5 g_{\perp}^{J/\psi(a)}(u)\right) \right. \\
&\qquad \left. + f_{J/\psi}^{\perp}\left(\not{\varepsilon}_{\perp}\not{p}\phi_{\perp}^{J/\psi}(u) - i\frac{m_{J/\psi}^2(\varepsilon^* z)}{(p\cdot z)^2}\sigma_{\mu\nu}p^{\mu}z^{\nu}h_{\parallel}^{J/\psi(t)}(u) - i m_{J/\psi}^2(\varepsilon^* z)\frac{h_{\parallel}^{J/\psi(s)}(u)}{2}\right)\right\}_{\alpha\beta},
\end{aligned} \tag{2.1}$$

where $z = y - x$ with $z^2 = 0$. $\phi_\parallel(u)$, $\phi_\perp(u)$ are twist-2 DAs; $g_\perp^{(v)}(u)$, $g_\perp^{(a)}(u)$, $h_\parallel^{(s)}(u)$ and $h_\parallel^{(t)}(u)$ are twist-3 ones. u is momentum fraction of c quark in J/ψ, $\overline{u} = 1 - u$. $f_{J/\psi}$, $f_{J/\psi}^\perp$ are vector and tensor decay constants, respectively. Similarly, for axial-vector meson ${}^{3(1)}P_1$, we have

$$\begin{aligned}
&\left\langle {}^{3(1)}P_1(p,\varepsilon)\left|\overline{d}_{1\alpha}(y)u_{2\beta}(x)\right|0\right\rangle \\
&= -\frac{i}{4}\int_0^1 du \exp\left[i(vp\cdot y + \overline{v}p\cdot x)\right] \\
&\quad\times\left\{f_{3(1)_{P1}}m_{3(1)_{P1}}\left(\not{p}\gamma_5\frac{\varepsilon^* z}{pz}\phi_\parallel^{3(1)_{P1}}(v) + \not{\varepsilon}_\perp^*\gamma_5 g_\perp^{3(1)_{P1}(v)}(v) + \frac{1}{4}\varepsilon_{\mu\nu\rho\sigma}\varepsilon^{*\mu}p^\rho z^\sigma\gamma^\nu g_\perp^{3(1)_{P1}(a)}(v)\right) + f_{3(1)}^\perp{}_{P_1}\right. \\
&\qquad\left.\times\left(\not{\varepsilon}_\perp\not{p}\gamma_5\phi_\perp^{3(1)_{P1}}(v) - i\frac{m_{3(1)_{P1}}^2(\varepsilon^* z)}{(p\cdot z)^2}\sigma_{\mu\nu}p^\mu z^\nu\gamma_5 h_\parallel^{3(1)_{P1}(t)}(v) - i m_{3(1)_{P1}}^2(\varepsilon^* z)\gamma_5\frac{h_\parallel^{3(1)_{P1}(p)}(v)}{2}\right)\right\}_{\alpha\beta}.
\end{aligned} \tag{2.2}$$

Applying equation of motion to LCDAs, one can obtain the Wandzura-Wilczek relations in which twist-3 LCDAs related to the twist-2 ones [2]. Also $\phi_\pm(x)$ are defined as

$$\begin{aligned}
\phi_+(x) &= \frac{g_\perp^{'(a)}(x)}{4} + g_\perp^{(a)}(x) = \int_x^1 \frac{\phi_\parallel(u)}{u}du, \\
\phi_-(x) &= -\left(\frac{g_\perp^{'(a)}(x)}{4} - g_\perp^{(a)}(x)\right) = \int_0^x \frac{\phi_\parallel(u)}{1-u}du,
\end{aligned} \tag{2.3}$$

which are transverse components of twist-3 LCDAs. While $\phi_{v(a)}$ are longitudinal components of twist-3 ones for vector (axial-vector) meson. We specify the light-cone distribution amplitudes (LCDAs) as

$$\begin{aligned}
\phi_V(x,\mu) &= 6x(1-x)\left[1 + \sum_{n=1}^\infty a_n^V(\mu)C_n^{3/2}(2x-1)\right], \\
\phi_v(x,\mu) &= 3\left[2x - 1 + \sum_{n=1}^\infty a_n^{\perp,V}(\mu)P_{n+1}(2x-1)\right],
\end{aligned} \tag{2.4}$$

for the vector meson, where $C_n(x)$ are Gegenbauer polynomials and $P_n(x)$ are Legendre polynomials. The normalization of LCDAs for vector meson is

$$\begin{aligned}
\int_0^1 dx\phi_V(x) &= 1, \\
\int_0^1 dx\phi_v(x) &= 0.
\end{aligned} \tag{2.5}$$

Also,

$$\begin{aligned}
\phi_{\parallel}^{A}(x) &= 6x(1-x)\left[1+3a_{1}^{\parallel,{}^{3}P_{1}}(2x-1)\right],\\
\phi_{\perp}^{A}(x) &= 6x(1-x)\left[a_{0}^{\perp,{}^{3}P_{1}}+3a_{1}^{\perp,{}^{3}P_{1}}(2x-1)\right],\\
\phi_{a}^{A}(x,\mu) &= 3\left[a_{0}^{\perp,{}^{3}P_{1}}(2x-1)+\sum_{n=1}^{\infty}a_{n}^{\perp,{}^{3}P_{1}}(\mu)P_{n+1}(2x-1)\right],
\end{aligned} \tag{2.6}$$

for ${}^{3}P_{1}$ axial-vector meson and

$$\begin{aligned}
\phi_{\parallel}^{A}(x) &= 6x(1-x)\left[a_{0}^{\parallel,{}^{1}P_{1}}+3a_{1}^{\parallel,{}^{1}P_{1}}(2x-1)\right],\\
\phi_{\perp}^{A}(x) &= 6x(1-x)\left[1+3a_{1}^{\perp,{}^{1}P_{1}}(2x-1)\right],\\
\phi_{a}^{A}(x,\mu) &= 3\left[(2x-1)+\sum_{n=1}^{\infty}a_{n}^{\perp,{}^{1}P_{1}}(\mu)P_{n+1}(2x-1)\right],
\end{aligned} \tag{2.7}$$

for ${}^{1}P_{1}$ axial-vector meson. The normalization conditions are

$$\begin{aligned}
\int_{0}^{1}dx\phi_{\parallel}(x) &= 1,\\
\int_{0}^{1}dx\phi_{\perp}(x) &= a_{0}^{\perp,{}^{3}P_{1}},\\
\int_{0}^{1}dx\phi_{a}(x) &= 0,
\end{aligned} \tag{2.8}$$

for ${}^{3}P_{1}$ axial-vector meson and

$$\begin{aligned}
\int_{0}^{1}dx\phi_{\parallel}(x) &= a_{0}^{\parallel,{}^{1}P_{1}},\\
\int_{0}^{1}dx\phi_{\perp}(x) &= 1,\\
\int_{0}^{1}dx\phi_{a}(x) &= 0,
\end{aligned} \tag{2.9}$$

for ${}^{1}P_{1}$ axial-vector meson.

Here ϕ_{V}, $\phi_{\parallel}^{A}$, and $\phi_{\perp}^{A}$ are twist-2 LCDAs; likewise ϕ_{v} and ϕ_{a}^{A} are twist-3 ones for vector and axial-vector mesons, respectively. The parameters $a_{2}^{\parallel}$ and $a_{2}^{\perp}$ are defined by the matrix element of a twist-2 conformal operator with conformal spin 3 in [13]. These parameters are unknown for J/ψ; then we consider asymptotic forms of amplitudes for J/ψ. Also, the Gegenbauer moments for K_{1A} and K_{1B} are given in [14, 15].

For the B meson, we can write the projection as

$$\left\langle 0\left|\bar{b}_\alpha(x)q_\beta(0)\right|B(p)\right\rangle = -\frac{if_B}{4}\phi_B(\rho)\left[(\not{p}+m_b)\gamma_5\right]_{\beta\gamma}. \tag{2.10}$$

A detailed dissection of wave function of B meson can be found [9, 10, 16–18]

$$\begin{aligned}\left\langle 0\left|\bar{b}_\alpha(x)q_\beta(0)\right|B(p)\right\rangle\Big|_{(x_+=x_\perp=0)} &= -i\frac{f_B}{4}\left[(\not{p}+m_b)\gamma_5\right]_{\beta\gamma} \\ &\quad\times\int_0^1 d\bar{\rho}e^{-\bar{\rho}p_+x_-}\left[\phi_1^B(\bar{\rho})+\not{n}_-\phi_2^B(\bar{\rho})\right]_{\gamma\alpha},\end{aligned} \tag{2.11}$$

with $n_- = (1,0,0,-1)$, and the normalization conditions are

$$\begin{aligned}\int_0^1 d\bar{\rho}\phi_1^B(\bar{\rho}) &= 1, \\ \int_0^1 d\bar{\rho}\phi_2^B(\bar{\rho}) &= 0.\end{aligned} \tag{2.12}$$

The B wave function corresponds to λ_B defined by $\int_0^1 d\bar{\rho}\phi_1^B(\bar{\rho})/\bar{\rho} \equiv m_B/\lambda_B$. λ_B is the first inverse moment of the B meson's distribution which is of order Λ_{QCD}.

2.2. Decay Constant

Decay constants of vector and axial-vector mesons are defined as

$$\begin{aligned}\langle V(p,\varepsilon)|\bar{q}_2\gamma_\mu q_1|0\rangle &= -if_V m_V\varepsilon_\mu^*, \\ \langle A(p,\varepsilon)|\bar{q}_2\gamma_\mu\gamma_5 q_1|0\rangle &= if_A m_A\varepsilon_\mu^*.\end{aligned} \tag{2.13}$$

Also, the transverse decay constants are defined via the tensor current as

$$\begin{aligned}\langle V(p,\varepsilon)|\bar{q}_2\sigma_{\mu\nu}q_1|0\rangle &= -f_V^\perp\left(\varepsilon_\mu^* p^\nu - \varepsilon_\nu^* p^\mu\right), \\ \langle A(p,\varepsilon)|\bar{q}_2\gamma_\mu\gamma_5 q_1|0\rangle &= if_A m_A\varepsilon_\mu^*.\end{aligned} \tag{2.14}$$

In general, the decay constants $f_{1_{p1}}$ and $f_{3_{p1}}^\perp$ are zero in the $SU(3)$ limit.

2.3. Form Factor

The form factors of $\overline{B} \to A$ transitions are defined as [2]

$$\begin{aligned}
\left\langle A(p,\varepsilon)\middle|A_\mu\middle|\overline{B}(p_B)\right\rangle &= i\frac{2}{m_B - m_A}\varepsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}p_B^\alpha p^\beta A^{BA}\left(q^2\right),\\
\left\langle A(p,\varepsilon)\middle|V_\mu\middle|\overline{B}(p_B)\right\rangle &= -\left\{(m_B - m_A)\varepsilon_\mu^* V_1^{BA}\left(q^2\right) - (\varepsilon^*\cdot p_B)(p_B + p)_\mu \frac{V_2^{BA}(q^2)}{m_B - m_A}\right.\\
&\quad \left. -2m_A\frac{\varepsilon^*\cdot p_B}{q^2}q^\mu\left[V_3^{BA}\left(q^2\right) - V_0^{BA}\left(q^2\right)\right]\right\},
\end{aligned} \tag{2.15}$$

and for $\overline{B} \to V$ transitions, we have

$$\begin{aligned}
\langle V(p,\varepsilon)|V_\mu|\overline{B}(p_B)\rangle &= \frac{2}{m_B + m_V}\varepsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}p^\alpha p_B^\beta V^{BV}\left(q^2\right),\\
\langle V(p,\varepsilon)|A_\mu|\overline{B}(p_B)\rangle &= i\left\{(m_B + m_V)\varepsilon_\mu^* A_1^{BV}\left(q^2\right) - \frac{\varepsilon^*\cdot p_B}{m_B + m_V}(p_B + p)_\mu A_2^{BV}\left(q^2\right)\right.\\
&\quad \left. -2m_V\frac{\varepsilon^*\cdot p_B}{q^2}q_\mu\left[A_3^{BV}\left(q^2\right) - A_0^{BV}\left(q^2\right)\right]\right\},
\end{aligned} \tag{2.16}$$

where $q = p_B - p$, and for $q^2 = 0$ we have $A_3^{BV} = A_0^{BV}$ and $V_3^{BA} = V_0^{BA}$. ε_μ is the polarization four-vector, and V, A_1, A_2, A_3, and A_0 are form factors and

$$\begin{aligned}
V_3^{BA}\left(q^2\right) &= \frac{m_B - m_A}{2m_A}V_1^{BA}\left(q^2\right) - \frac{m_B + m_A}{2m_A}V_2^{BA}\left(q^2\right),\\
A_3^{BV}\left(q^2\right) &= \frac{m_B + m_V}{2m_V}A_1^{BV}\left(q^2\right) - \frac{m_B - m_V}{2m_V}A_2^{BV}\left(q^2\right).
\end{aligned} \tag{2.17}$$

The form factors of $B \to K_{1A}$ and $B \to K_{1B}$ transitions have been calculated in the light-cone sum rule (LCSR) [19]. The momentum dependence of form factors is calculated in the LCSR approach which is parameterized in three-parameter form:

$$F\left(q^2\right) = \frac{F(0)}{1 - a\left(q^2/m^2\right)_B + b\left(q^4/m_B^4\right)}, \tag{2.18}$$

which $q^2 = m_{J/\psi}^2$, and two parameters a and b are given in Table 1.

2.4. $B \to J/\psi K(1270)$ and $B \to J/\psi K(1400)$ Decays in QCD Factorization

Now, we want to calculate the branching ratios of $B \to J/\psi K(1270)$ and $B \to J/\psi K(1400)$ decays in QCD factorization method. First, we have calculated the branching ratios in generalized factorization which the effective Wilson coefficients are not dependent on

Table 1: Form factors for $B \to K_{1A}$ and $B \to K_{1B}$ transitions which are obtained in light-cone sum rule models.

	$F(0)$	a	b
$A^{BK_{1A}}$	0.27 ± 0.05	1.60	0.974
$V_0^{BK_{1A}}$	0.22 ± 0.04	2.40	1.78
$V_1^{BK_{1A}}$	0.56 ± 0.11	0.635	0.211
$V_2^{BK_{1A}}$	0.25 ± 0.05	1.51	1.18
$A^{BK_{1B}}$	$-0.22^{+0.06}_{-0.04}$	1.72	0.912
$V_0^{BK_{1B}}$	$-0.45^{+0.12}_{-0.08}$	1.34	0.690
$V_1^{BK_{1B}}$	$-0.48^{+0.13}_{-0.08}$	0.729	0.074
$V_2^{BK_{1B}}$	$-0.01^{+0.03}_{-0.02}$	0.919	0.855

renormalization scale μ. To solve the issue of scale μ dependence, but not the renormalization scheme dependence [20], it is proposed in [21, 22], to isolate from the matrix element of four operators the μ dependence and link with the μ dependence in the Wilson coefficients $C_i(\mu)$ to form c_i^{eff}, effective Wilson coefficients independence of μ. The formula for effective Wilson coefficients and their numerical values has been given explicitly in [22]. For $b \to s$ transition (in naive dimensional regularization): $c_1^{\text{eff}} = 1.1680$, $c_2^{\text{eff}} = -0.3652$, $c_3^{\text{eff}} = 0.0233 + 0.0043i$, $c_4^{\text{eff}} = -0.0482 - 0.0129i$, $c_5^{\text{eff}} = 0.0140 + 0.0043i$, $c_6^{\text{eff}} = -0.0504 - 0.0129i$, $c_7^{\text{eff}}/\alpha = -0.0312 - 0.0357i$, $c_8^{\text{eff}}/\alpha = 0.0551$, $c_9^{\text{eff}}/\alpha = -1.4277 - 0.0357i$, $c_{10}^{\text{eff}}/\alpha = 0.4804$ [1]. α is the fine structure constant. Wilson coefficients appear in decay amplitudes as linear combinations. So, the a_i^{eff} coefficients are

$$\begin{aligned} a_i^{\text{eff}} &\equiv c_i^{\text{eff}} + \frac{c_{i+1}^{\text{eff}}}{N_c} \quad (i = \text{odd}), \\ a_i^{\text{eff}} &\equiv c_i^{\text{eff}} + \frac{c_{i-1}^{\text{eff}}}{N_c} \quad (i = \text{even}). \end{aligned} \tag{2.19}$$

As it known the physical states $K_1(1270)$ and $K_1(1400)$ are mixture of states ${}^1P_1$ and ${}^3P_1$, whose relations could be parameterized by

$$\begin{aligned} K_1(1270) &= K_{1A} \sin\theta + K_{1B} \cos\theta, \\ K_1(1400) &= K_{1A} \cos\theta - K_{1B} \sin\theta, \end{aligned} \tag{2.20}$$

where K_{1A} and K_{1B} are ${}^3P_1$ and ${}^1P_1$ axial-vector mesons, respectively, and θ is the mixing angle. From experimental date on masses and partial ratios of $K_1(1270)$ and $K_1(1400)$, it is found two solutions for the mixing angle with a two-fold ambiguity, $\theta = \pm 32°$ and $\theta = \pm 58°$ [1]. Hence, the physical decaying amplitudes are given by

$$\begin{aligned} A(B \longrightarrow J/\psi K_1(1270))_p &= A(B \longrightarrow J/\psi K_{1A}) \sin\theta + A(B \longrightarrow J/\psi K_{1B}) \cos\theta, \\ A(B \longrightarrow J/\psi K_1(1400))_p &= A(B \longrightarrow J/\psi K_{1A}) \cos\theta - A(B \longrightarrow J/\psi K_{1B}) \sin\theta. \end{aligned} \tag{2.21}$$

Since the final states of $B \to VA$ carry spin degrees of freedom, we have calculated the helicity amplitudes, separately. In our work, the longitudinal amplitude $h = 0$ and the transverse amplitudes $h = \pm$ are calculated for $B \to J/\psi K_{1A}$ and $B \to J/\psi K_{1B}$ decays [22]. If we assume that J/ψ behaves as a light meson, due to its size, we can describe the light-cone distribution. For $B \to J/\psi K(1270)$ and $B \to J/\psi K(1400)$ decays, we have colour suppressed contribution a_2 and penguin contributions a_3, a_5, a_7, and a_9. Then, the contributions of vertex corrections for these decays are only for $i = 2,3,5,7,9$. Hence, three different helicity amplitudes have been written as

$$\begin{aligned} A^h(B \longrightarrow J/\psi K_{1A}) &= \left[V_{cb}V_{cs}^* a_2^h - V_{tb}V_{ts}^*\left(a_3^h + a_5^h + a_7^h + a_9^h\right)\right] X_h^{(BK_{1A},J/\psi)}, \\ A^h(B \longrightarrow J/\psi K_{1B}) &= \left[V_{cb}V_{cs}^* a_2^h - V_{tb}V_{ts}^*\left(a_3^h + a_5^h + a_7^h + a_9^h\right)\right] X_h^{(BK_{1B},J/\psi)}, \end{aligned} \tag{2.22}$$

where the factorizable amplitude is written as

$$X^{(BK_{1A(1B)},J/\psi)} \equiv \langle J/\psi|(\bar{c}c)_{V-A}|0\rangle \left\langle K_{1A(1B)}\left|\left(\bar{b}s\right)_{V-A}\right|B\right\rangle. \tag{2.23}$$

The helicity dependencies read as

$$\begin{aligned} X_0^{(BK,J/\psi)} &= \frac{if_{J/\psi}}{2m_K}\left[\left(m_B^2 - m_K^2\right) - m_{J/\psi}^2(m_B - m_K)V_1^{BK}\left(q^2\right) - \frac{4m_B^2 p_c^2}{m_B - m_K}V_2^{BK}\left(q^2\right)\right], \\ X_\pm^{(BK,J/\psi)} &= -if_{J/\psi}m_B m_{J/\psi}\left[\left(1 - \frac{m_K}{m_B}\right)V_1^{BK}\left(q^2\right) \mp \frac{2p_c}{m_B - m_K}A^{BK}\left(q^2\right)\right], \end{aligned} \tag{2.24}$$

where p_c is $c.m.$ momentum of decays in the B rest frame

$$p_c = \frac{\sqrt{\left(m_B^2 - (m_1 + m_2)^2\right)\left(m_B^2 - (m_1 - m_2)^2\right)}}{2m_B}, \tag{2.25}$$

m_1 and m_2 are the mass of final state mesons. The branching ratio is given by [18]

$$BR(B \longrightarrow VA) = \frac{p_c}{8\pi m_B^2 \tau_B}\left(\frac{G_F}{\sqrt{2}}\right)^2 |A(B \to VA)|^2, \tag{2.26}$$

where G_F is Fermi constant and τ_B is life time of B meson. If the final states are two light vector mesons, it is expected that $|A_0| > |A_+| > |A_-|$ for B meson decays. (For $\overline{B}$ decays, exchange $- \leftrightarrow +$.) It is because of that the amplitude $|A_+|$ is suppressed by a factor of $\sqrt{2}m_2/m_B$, while the $|A_-|$ amplitude is further suppression in m_1/m_B. But for B to heavy-light final states, $\sqrt{2}m_{\text{heavy}}/m_B$ is of order unity [23]. In QCD factorization, the effective coefficients a_i^h basically are Wilson coefficients in conjunction with short distance

nonfactorizable corrections such as vertex corrections and hard spectator interactions. Then, we have [2]

$$a_i^h(J/\psi K_{1A}(K_{1B})) = \left(C_i + \frac{C_{i\pm1}}{N_c}\right) N_i(J/\psi) \int_0^1 \phi^{J/\psi,h}(x)dx + \frac{C_{i\pm1}}{N_c}\frac{C_F\alpha_s}{4\pi}\left[V_i^h(J/\psi) + \frac{4\pi^2}{N_c}H_i^h(J/\psi K_{1A}(K_{1B}))\right], \tag{2.27}$$

where $i = 1,\ldots,10$, the lower (upper) signs apply when i is even (odd), C_i are Wilson coefficients (we use the next to leading order coefficients, NLO, obtained in the naive dimensional regularization scheme, NDR, at the energy scale $\mu = m_b(m_b)$.), $C_F = (N_c^2 - 1)/(2N_c)$ with $N_c = 3$, J/ψ is emitted meson and $K_{1A}(K_{1B})$ have the spectator quark of the meson. $V_i^h(J/\psi)$ is the vertex corrections and $H_i^h(J/\psi K_{1A}(K_{1B}))$ is the hard-scattering interactions, obtained due to exchange soft and hard gluons. The superscript h denotes the helicity of the final-state meson, that is, $h = 0,\pm$. The LCDA $\phi^{J/\psi,h}$ is $\phi_\parallel^{J/\psi}$ for $h = 0$ and $\phi_\perp^{J/\psi}$ for $h = \pm$. And $N_i^h(J/\psi) = 1$. We can take the infinite mass limit of the b quark in which m_b goes to infinity while $m_{J/\psi}$ is fixed ($m_{J/\psi}/m_b \to 0$). The vertex corrections are given by

$$V_i^0(J/\psi) = \begin{cases} \displaystyle\int_0^1 dx\phi_\parallel^{J/\psi}(x)\left[12Ln\frac{m_b}{\mu} - 18 + g(x)\right], & i = 2,3,9, \\ \displaystyle\int_0^1 dx\phi_\parallel^{J/\psi}(x)\left[-12Ln\frac{m_b}{\mu} + 6 - g(1-x)\right], & i = 5,7, \end{cases} \tag{2.28}$$

$$V_i^\pm(J/\psi) = \begin{cases} \displaystyle\int_0^1 dx\phi_\pm^{J/\psi}(x)\left[12Ln\frac{m_b}{\mu} - 18 + g_T(x)\right], & i = 2,3,9, \\ \displaystyle\int_0^1 dx\phi_\mp^{J/\psi}(x)\left[-12Ln\frac{m_b}{\mu} + 6 - g_T(1-x)\right], & i = 5,7. \end{cases} \tag{2.29}$$

With

$$g(x) = 3\left(\frac{1-2x}{1-x}\ln x - i\pi\right) + \left[2Li_2(x) - Ln^2x + \frac{2Lnx}{1-x} - (x \longleftrightarrow 1-x)\right], \quad g_T(x) = g(x) + \frac{Lnx}{1-x}, \tag{2.30}$$

we consider asymptotic function for $\phi_\parallel^{J/\psi}(x) = 6x(1-x)$, and also we have $\phi_+(x) = 3(1-x)^2$ and $\phi_-(x) = 3x^2$. Hard spectator corrections are given by

$$H_i^0(J/\psi K_{1A(1B)}) = \frac{if_B f_{J/\psi} f_{K_{1A(1B)}}}{X_0^{(BK_{1A(1B)},J/\psi)}}\frac{m_B}{\lambda_B} \times \int_0^1 dy\,dx\left(\frac{\phi_\parallel^{K_{1A(1B)}}(y)\phi_\parallel^{J/\psi}(x)}{(1-y)(1-x)} - r_\chi^{K_{1A(1B)}}\frac{\phi^{K_{1A(1B)}}(y)\phi_\parallel^{J/\psi}(x)}{(1-y)x}\right), \tag{2.31}$$

for $i = 2, 3, 9$, and

$$H_i^0\left(J/\psi K_{1A(1B)}\right) = -\frac{i f_B f_{J/\psi} f_{K_{1A(1B)}}}{X_0^{(BK_{1A(1B)}, J/\psi)}} \frac{m_B}{\lambda_B} \times \int_0^1 dy\, dx \left(\frac{\phi_\parallel^{K_{1A(1B)}}(y) \phi_\parallel^{J/\psi}(x)}{(1-y)x} - r_\chi^{K_{1A(1B)}} \frac{\phi^{K_{1A(1B)}}(y) \phi_\parallel^{J/\psi}(x)}{(1-y)(1-x)} \right), \tag{2.32}$$

for $i = 5, 7$. $r_\chi^{K_{1A(1B)}} = 2(m_{K_{1A(1B)}} f_{K_{1A(1B)}}^\perp)/(m_b(\mu) f_{K_{1A(1B)}})$ are chirally enhanced parameters. The transverse hard spectator terms are

$$\begin{aligned} H_i^-\left(J/\psi K_{1A(1B)}\right) &= -\frac{2 i f_B f_{K_{1A(1B)}}^\perp f_{J/\psi} m_{J/\psi}}{m_B X_-^{(BK_{1A(1B)}, J/\psi)}} \frac{m_B}{\lambda_B} \times \int_0^1 dy dx \frac{\phi_\perp^{K_{1A(1B)}}(y) \phi_-^{J/\psi}(x)}{(1-y)^2 x}, \\ H_i^+\left(J/\psi K_{1A(1B)}\right) &= -\frac{2 i f_B f_{K_{1A(1B)}} f_{J/\psi} m_{J/\psi} m_{K_{1A(1B)}}}{m_B^2 X_+^{(BK_{1A(1B)}, J/\psi)}} \frac{m_B}{\lambda_B} \times \int_0^1 dy dx \frac{(1-y-x) \phi_+^{K_{1A(1B)}}(y) \phi_+^{J/\psi}(x)}{(1-y)^2 (1-x)^2}, \end{aligned} \tag{2.33}$$

for $i = 2, 3, 9$, and

$$H_i^-\left(J/\psi K_{1A(1B)}\right) = \frac{2 i f_B f_{K_{1A(1B)}}^\perp f_{J/\psi} m_{J/\psi}}{m_B X_-^{(BK_{1A(1B)}, J/\psi)}} \frac{m_B}{\lambda_B} \times \int_0^1 dy dx \frac{\phi_\perp^{K_{1A(1B)}}(y) \phi_+^{J/\psi}(x)}{(1-y)^2 (1-x)}, \tag{2.34}$$

$$H_i^+\left(J/\psi K_{1A(1B)}\right) = -\frac{2 i f_B f_{K_{1A(1B)}} f_{J/\psi} m_{J/\psi} m_{K_{1A(1B)}}}{m_B^2 X_+^{(BK_{1A(1B)}, J/\psi)}} \frac{m_B}{\lambda_B} \times \int_0^1 dy dx \frac{(y-x) \phi_+^{K_{1A(1B)}}(y) \phi_-^{J/\psi}(x)}{(1-y)^2 x^2}, \tag{2.35}$$

for $i = 5, 7$. In our calculations, the transverse hard spectator term $H_i^+(J/\psi K_{1A})$ is zero. There are some divergences in other hard spectator terms. We have used the unknown parameter X_H to eliminate these divergences, defined as

$$X_H = \int_0^1 \frac{dx}{1-x} = Ln\left(\frac{m_B}{\Lambda_{\mathrm{QCD}}}\right)(1+r), \tag{2.36}$$

where Λ_{QCD} is hadronic parameter and $r = \rho_H e^{i\varphi_H}$ which ρ_H and φ_H are phases parameters ($0 \le \rho_H \le 1$, $-180 \le \varphi_H \le 180$). We consider $r = 0$ in these decays and $B \to J/\psi K$ [24]. This parameter phenomenologically is significant. Hence, the non factorizable contributions (vertex corrections and hard spectator interactions) have been calculated by (2.28)–(2.35).

In the other hand, we know that J/ψ is a heavy vector meson, although in [10] has been supposed that quarkonium is light meson relative to the B meson. If we consider other limit in which m_b goes to infinity, $m_{J/\psi}/m_b$ is held fixed. In this scenario, the amplitude wave function of J/ψ has represented like B meson. Also, to the leading order in $1/m_c$, the J/ψ wave function is [18]

$$\langle J/\psi(p,\varepsilon)|\bar{c}_\alpha(x)c_\beta(0)|0\rangle|_{(x_+=x_\perp=0)} = \frac{f_{J/\psi}}{4}\left[\varepsilon^*(p+m_b)\gamma_5\right]_{\beta\gamma}\int_0^1 d\bar{\xi}e^{-i\xi p_+ x_-}\left[\phi_1^{J/\psi}(\xi)+n_-\phi_2^{J/\psi}(\xi)\right]_{\gamma\alpha}. \tag{2.37}$$

Using [18], the effect of higher twist for J/ψ may not converge enough. Because the charmed quark carries a momentum fraction of order $\sim m_c/m_{J/\psi}$, the distribution amplitudes of J/ψ vanish in the end point region. $\phi_\parallel$ is adapted as the DA of nonlocal vector current of J/ψ rather $g_\perp^v$ as the DA of $\varepsilon_\perp$ component since the latter does not vanish at the end point. Comparing (2.1) with (2.37) the leading twist DA of J/ψ is

$$\begin{gathered}\phi_1^{J/\psi}(u) = \phi_\perp^{J/\psi}(u) \sim 6u\bar{u}, \\ f_{J/\psi} = f_{J/\psi}^\perp.\end{gathered} \tag{2.38}$$

Then, for vertex corrections [18], we have

$$\begin{aligned}
V_i^0(J/\psi) &= \begin{cases} \displaystyle\int_0^1 dx\phi_\parallel^{J/\psi}(x)\left[12Ln\frac{m_b}{\mu} - 18 + f(x,z)\right] + \int_0^1 dx\phi_\perp^{J/\psi}(x)g(x,z), & i = 2,3,9, \\ \displaystyle\int_0^1 dx\phi_\parallel^{J/\psi}(x)\left[-12Ln\frac{m_b}{\mu} + 6 - f(x,z)\right] + \int_0^1 dx\phi_\perp^{J/\psi}(x)g(x,z), & i = 5,7, \end{cases} \\
V_i^\pm(J/\psi) &= \begin{cases} \displaystyle\int_0^1 dx\phi_\parallel^{J/\psi}(x)\left[12Ln\frac{m_b}{\mu} - 18 + h(x,z)\right] + \int_0^1 dx\phi_\perp^{J/\psi}(x)I(x,z), & i = 2,3,9, \\ \displaystyle\int_0^1 dx\phi_\parallel^{J/\psi}(x)\left[-12Ln\frac{m_b}{\mu} + 6 - h(x,z)\right] + \int_0^1 dx\phi_\perp^{J/\psi}(x)I(x,z), & i = 5,7, \end{cases}
\end{aligned} \tag{2.39}$$

with

$$
\begin{aligned}
f(x,z) &= \frac{2zx}{1-z(1-x)} + (3-2x)\frac{\ln x}{1-x} \\
&\quad + \left(-\frac{3}{1-zx} + \frac{1}{1-z(1-x)} - \frac{2zx}{(1-z(1-x))^2}\right) zx \ln zx \\
&\quad + \left(3(1-z) + 2zx + \frac{2z^2x^2}{1-z(1-x)}\right)\frac{\ln(1-z) - i\pi}{1-z(1-x)} + (1-z) \\
&\quad \times \left\{ \frac{-4x}{(1-z)(1-x)} \ln x + \frac{zx}{(1-z(1-x))^2} \ln(1-z) \right. \\
&\qquad + \left(\frac{1}{(1-zx)^2} - \frac{1}{(1-z(1-x))^2} + \frac{2(1+z-2zx)}{(1-z)(1-zx)^2}\right) zx \ln zx \\
&\qquad \left. -i\pi \frac{zx}{(1-z(1-x))^2} \right\}, \\
g(x,z) &= -4r\frac{\ln x}{1-x} + \frac{4zr \ln zx}{1-z(1-x)} - 4zr\frac{\ln(1-z) - i\pi}{1-z(1-x)} \\
&\quad + (1-z)\left\{ \frac{4r}{(1-z)(1-x)} \ln x - \frac{4rz}{(1-z)(1-zx)} \ln zx \right\}, \\
h(x,z) &= \frac{2zx}{1-z(1-x)} + (3-2x)\frac{\ln x}{1-x} \\
&\quad + \left(-\frac{3}{1-zx} + \frac{1}{1-z(1-x)} - \frac{2zx}{(1-z(1-x))^2}\right) zx \ln zx \\
&\quad + \left(3(1-z) + 2zx + \frac{2z^2x^2}{1-z(1-x)}\right)\frac{\ln(1-z) - i\pi}{1-z(1-x)}, \\
I(x,z) &= -4r\frac{\ln x}{1-x} + \frac{4zr \ln zx}{1-z(1-x)} - 4zr\frac{\ln(1-z) - i\pi}{1-z(1-x)},
\end{aligned}
\tag{2.40}
$$

where $z = m_{J/\psi}^2/m_B^2$ and $r = f_{J/\psi}^{\perp} m_c / f_{J/\psi} m_{J/\psi}$. In [11] by contracting (2.14) for J/ψ with p_ν and applying the equation of motion and (2.13) for J/ψ, it finds that $f_{J/\psi}^{\perp} m_{J/\psi} = 2 f_{J/\psi} m_c$. Therefore

$$
\frac{f_{J/\psi}^{\perp} m_c}{f_{J/\psi} m_{J/\psi}} = 2\left(\frac{m_c}{m_{J/\psi}}\right)^2. \tag{2.41}
$$

In [25], the charmed quark carries a momentum fraction of order $\sim m_c/m_{J/\psi}$, which is the same fraction momentum x, then $r = 2x^2$. The contributions of hard spectator interaction have been written as

$$H_i^0(J/\psi K_{1A(1B)}) = \frac{if_B f_{J/\psi} f_{K_{1A(1B)}}}{X_0^{(BK_{1A(1B)},J/\psi)}} \frac{m_B}{\lambda_B} \times \int_0^1 dydx \left(\frac{\phi_\parallel^{K_{1A(1B)}}(y)\phi_\parallel^{J/\psi}(x)}{(1-y)(1-x)} - r_\chi^{K_{1A(1B)}} \frac{\phi_{K_{1A(1B)}}(y)\phi_\parallel^{J/\psi}(x)}{(1-y)x} \right), \tag{2.42}$$

for $i = 2, 3, 9$,

$$H_i^0(J/\psi K_{1A(1B)}) = -\frac{if_B f_{J/\psi} f_{K_{1A(1B)}}}{X_0^{(BK_{1A(1B)},J/\psi)}} \frac{m_B}{\lambda_B} \times \int_0^1 dydx \left(\frac{\phi_\parallel^{K_{1A(1B)}}(y)\phi_\parallel^{J/\psi}(x)}{(1-y)x} - r_\chi^{K_{1A(1B)}} \frac{\phi_{K_{1A(1B)}}(y)\phi_\parallel^{J/\psi}(x)}{(1-y)(1-x)} \right), \tag{2.43}$$

for $i = 5, 7$. The transverse hard spectator terms are

$$\begin{aligned} H_i^-(J/\psi K_{1A(1B)}) &= -\frac{2if_B f_{K_{1A(1B)}}^\perp f_{J/\psi} m_{J/\psi}}{m_B X_-^{(BK_{1A(1B)},J/\psi)}} \frac{m_B}{\lambda_B} \frac{1}{1-z} \times \int_0^1 dydx \frac{\phi_\perp^{K_{1A(1B)}}(y)\phi_-^{J/\psi}(x)}{(1-y)^2 x}, \\ H_i^+(J/\psi K_{1A(1B)}) &= -\frac{2if_B f_{K_{1A(1B)}} f_{J/\psi} m_{J/\psi} m_{K_{1A(1B)}}}{m_B X_+^{(BK_{1A(1B)},J/\psi)}} \frac{m_B}{\lambda_B} \frac{1}{1-z} \times \int_0^1 dydx \frac{(1-y-x)\phi_+^{K_{1A(1B)}}(y)\phi_+^{J/\psi}(x)}{(1-y)^2(1-x)^2}, \end{aligned} \tag{2.44}$$

for $i = 2, 3, 9$, and

$$H_i^-(J/\psi K_{1A(1B)}) = \frac{2if_B f_{K_{1A(1B)}}^\perp f_{J/\psi} m_{J/\psi}}{m_B X_-^{(BK_{1A(1B)},J/\psi)}} \frac{m_B}{\lambda_B} \frac{1}{1-z} \times \int_0^1 dydx \frac{\phi_\perp^{K_{1A(1B)}}(y)\phi_+^{J/\psi}(x)}{(1-y)^2(1-x)}, \tag{2.45}$$

$$H_i^+(J/\psi K_{1A(1B)}) = -\frac{2if_B f_{K_{1A(1B)}} f_{J/\psi} m_{J/\psi} m_{K_{1A(1B)}}}{m_B X_+^{(BK_{1A(1B)},J/\psi)}} \frac{m_B}{\lambda_B} \frac{1}{1-z} \times \int_0^1 dydx \frac{(y-x)\phi_+^{K_{1A(1B)}}(y)\phi_-^{J/\psi}(x)}{(1-y)^2 x^2}, \tag{2.46}$$

for $i = 5,7$. Hence, the nonfactorizable contributions which are the same vertex corrections and hard spectator interactions have calculated by (2.28)–(2.35) for first scenario and by (2.39)–(2.46) for second one. Finally, the branching ratios of $B \to J/\psi K(1270)$ and $B \to J/\psi K(1400)$ have calculated at $\mu = m_b$ and at mixing angles $\theta = 32°, 58°$ in Tables 2 and 3.

3. Input Quantities

In this section, we have been introduced the essential input quantities [26]:

$$\begin{gathered} m_B = 5.28\,\text{GeV}, \quad m_{J/\psi} = 3.1\,\text{GeV}, \quad m_{K_{1A}} = 1.367\,\text{GeV}, \quad m_{K_{1B}} = 1.310\,\text{GeV}, \\ f_B = 210 \pm 20\,\text{MeV}, \quad f_{J/\psi} = 405\,\text{MeV}, \quad f_{K_{1A}} = 0.250\,\text{GeV}, \\ f_{K_{1A}}^{\perp}(\mu = 1\,\text{GeV}) = 0.065\,\text{GeV}, \\ a_{0K_{1A}}^{\perp} = 0.26, \quad a_{1K_{1A}}^{\perp} = -1.08, \quad \lambda_B(1\,\text{GeV}) = 225\,\text{MeV}, \\ f_{K_{1B}} = -0.028\,\text{GeV}, \quad f_{K_{1B}}^{\perp}(\mu = 1\,\text{GeV}) = 0.190\,\text{GeV}, \quad a_{1K_{1B}}^{\parallel} = -1.95, \\ C_F = \frac{N^2-1}{2N}, \quad G_F = 1.166 \times 10^{-5}, \quad N = 3, \\ |V_{cb}V_{cs}^*| = 0.039, \quad |V_{tb}V_{ts}^*| = 0.041, \quad \tau_{B^\pm} = (1.638 \pm 0.011) \times 10^{-12}\,\text{S}. \end{gathered} \tag{3.1}$$

From (2.19), the magnitudes of factorizable amplitudes are obtained as

$$\begin{gathered} X_0^{(BK_{1A},J/\psi)} = 3.54i, \quad X_+^{(BK_{1A},J/\psi)} = -1.25i, \quad X_-^{(BK_{1A},J/\psi)} = -5.56i, \\ X_0^{(BK_{1B},J/\psi)} = -5.36i, \quad X_+^{(BK_{1B},J/\psi)} = 1.21i, \quad X_-^{(BK_{1B},J/\psi)} = 5.09i. \end{gathered} \tag{3.2}$$

4. Discussion

In generalized factorization, the branching ratios of $B \to J/\psi K(1270)$ and $B \to J/\psi K(1400)$ decays are small in order to experiment, although they are in range of experimental data for these decays in mixing angles as shown in Tables 2 and 3. In this work, we have been tried to analyze these decays within "QCD factorization" method. Hence, the factorizable amplitudes in (2.24) have been calculated by using the input data of form factors, decay constants, and the non-factorizable contributions corresponding to the vertex corrections, and hard spectator interactions in a_i^h have been calculated at two scenarios. First, J/ψ behaves as a light meson in compared to B meson ($z \to 0$). And in second, we have considered mass

Table 2: Branching ratios of $B \to J/\psi K(1270)$ in generalized factorization and QCD Factorization at $\mu = m_b$.

Factorization	Mixing angle $\theta°$	$BR(B \to J/\psi K(1270))$
General	32	$(0.44 \pm 0.02) \times 10^{-4}$
	58	$(0.42 \pm 0.01) \times 10^{-4}$
QCDF ($z \to 0$)	32	$(1.42 \pm 0.01) \times 10^{-3}$
	58	$(1.79 \pm 0.01) \times 10^{-3}$
QCDF ($z \neq 0$)	32	$(3.18 \pm 0.001) \times 10^{-3}$
	58	$(4.08 \pm 0.01) \times 10^{-3}$
Exp.		$(1.8 \pm 0.5) \times 10^{-3}$

Table 3: Branching ratios of $B \to J/\psi K(1400)$ in generalized factorization and QCD Factorization at $\mu = m_b$.

Factorization	Mixing angle $\theta°$	$BR(B \to J/\psi K(1400))$
General	32	1.27×10^{-6}
	58	3.35×10^{-6}
QCDF ($z \to 0$)	32	3.45×10^{-4}
	58	4.27×10^{-4}
QCDF ($z \neq 0$)	32	5.06×10^{-4}
	58	8.51×10^{-4}
Exp.		$< 5 \times 10^{-4}$

of J/ψ in which $m_{J/\psi}/m_b$ is held fixed. The effective coefficients a_i^h have been calculated at three helicity states $h = 0, +, -$, which give three different contributions of amplitude. The branching ratios are calculated at these conditions and for the mixing angles $\theta = 32, 58$, where given in Tables 2 and 3. As it is shown, the obtained results in QCD factorization for these decays are not suitable assumption for all of the mixing angles. For $B \to J/\psi K(1270)$ and $B \to J/\psi K(1400)$, the experimental branching ratios are $(1.8 \pm 0.5) \times 10^{-3}$ and $< 5 \times 10^{-4}$, respectively [26]. In first scenario, the best obtained results for $B \to J/\psi K(1270)$ decay are 1.79×10^{-3} at $\theta = 58°$ and 1.42×10^{-3} at $\theta = 32°$; for $B \to J/\psi K(1400)$, these are 1.79×10^{-3} at $\theta = 58°$ and 1.42×10^{-3} at $\theta = 32°$, while in second one, as shown in Tables 2 and 3.

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