Research Article

Dirac Equation under Scalar, Vector, and Tensor Cornell Interactions

**H. Hassanabadi,¹ E. Maghsoodi,¹
S. Zarrinkamar,² and H. Rahimov³**

¹ Physics Department, Shahrood University of Technology, Shahrood 3619995161, Iran

² Young Researchers Club, Garmsar Branch, Islamic Azad University, Garmsar, Iran

³ Computer Engineering Department, Shahrood University of Technology, Shahrood, Iran

Correspondence should be addressed to E. Maghsoodi, e.maghsoodi184@gmail.com

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Spin and pseudospin symmetries of Dirac equation are solved under scalar, vector, and tensor interactions for arbitrary quantum number via the analytical ansatz approach. The spectrum of the system is numerically reported for typical values of the potential parameters.

1. Introduction

No doubt, the Cornell potential (alternatively called Funnel in the literature) is, if not the best, among the most appealing interactions in particle physics. The Cornell potential contains a confining term besides the Coulomb interaction and has successfully accounted for the particle physics data [1]. Unfortunately, to our best knowledge, the potential does not possess exact solutions under all common equations of quantum mechanics, that is, the nonrelativistic Schrödinger equation, and relativistic Dirac, Klein-Gordon, Proca, and Duffin-Kemmer-Petiau (DKP) equations. Here, we focus on the relativistic symmetries of Dirac equation, that is, spin and pseudospin symmetries, which provide a reliable theoretical basis for hadronic and nuclear spectroscopy [2–12]. This has motivated many studies under various interactions within the past two decades ([13–18] and many references therein). Nevertheless, none of these papers has investigated the symmetry limits under the Cornell potential. This is definitely due to the complicated nature of the resulting differential equation which cannot be solved by common analytical techniques of quantum mechanics such as the supersymmetry quantum mechanics (SUSYQ), Lie groups, Nikiforov-Uvarov (NU) technique, and point canonical transformations. In our study, we make use of the ansatz approach to deal with this

complicated equation. A survey on the application of this technique to other wave equations including Schrödinger, spinless-Salpeter, Dirac, Klein-Gordon, and DKP equations can be found in [19–27]. We organize the study as follows. In the first step, we review the most essential equations of the symmetry limits. We next propose a physical ansatz solution to the equation and, in a systematic manner, calculate the spectrum of the system for any arbitrary state. To provide a better understanding of the solutions, we provide some numerical data for the spectrum as well.

2. Dirac Equation Including Tensor Coupling

In spherical coordinates, Dirac equation with both scalar potential $S(r)$ and vector potential $V(r)$ is expressed as [2, 3]

$$[\vec{\alpha} \cdot \vec{p} + \beta(M + S(r)) - i\beta\vec{\alpha} \cdot \hat{r}U(r)]\psi(\vec{r}) = [E - V(r)]\psi(\vec{r}), \quad (2.1)$$

where E is the relativistic energy of the system; α and β are the 4×4 Dirac matrices and $\vec{p} = -i\vec{\nabla}$ stands for the momentum operator. For a particle in a spherical field, the total angular momentum operator $\vec{J}$ and spin-orbit matrix operator $K = (\vec{\sigma} \cdot \vec{L} + 1)$, where σ and L are, respectively, the Pauli matrix and orbital angular momentum, commute with the Dirac Hamiltonian. The eigenvalues of K are $\kappa = -(j + 1/2)$ for the aligned spin ($s_{1/2}$, $p_{3/2}$, etc.) and $\kappa = (j + 1/2)$ for the unaligned spin ($p_{1/2}$, $d_{3/2}$, etc.). The complete set of the conservative quantities can be chosen as (H, K, J^2, J_z) . As shown in [13], the Dirac spinor is considered as

$$\psi_{nk}(\vec{r}) = \begin{pmatrix} f_{nk}(\vec{r}) \\ g_{nk}(\vec{r}) \end{pmatrix} = \begin{pmatrix} \frac{F_{nk}(\vec{r})}{r} Y_{jm}^{\ell}(\theta, \varphi) \\ i \frac{G_{nk}(\vec{r})}{r} Y_{jm}^{\tilde{\ell}}(\theta, \varphi) \end{pmatrix}, \quad (2.2)$$

where $F_{nk}(\vec{r})$ and $G_{nk}(\vec{r})$ are the radial wave functions of the upper and lower components, respectively, and $Y_{jm}^{\ell}(\theta, \varphi)$ and $Y_{jm}^{\tilde{\ell}}(\theta, \varphi)$, respectively, stand for spin and pseudospin spherical harmonics coupled to the angular momentum j . m is the projection of the angular momentum on the z -axis. The orbital angular momentum quantum numbers ℓ and $\tilde{\ell}$ refer to the upper and lower components, respectively. The quasidegenerate doublet structure can be expressed in terms of pseudospin angular momentum $s = 1/2$ and pseudoorbital angular momentum $\tilde{\ell}$, which is defined as $\tilde{\ell} = \ell + 1$ for aligned spin $j = \tilde{\ell} - 1/2$ and $\tilde{\ell} = \ell - 1$ for unaligned spin $j = \tilde{\ell} + 1/2$. As shown in [2, 3], substitution of (2.2) into (2.1) yields the following two-coupled differential equations:

$$\begin{aligned} \left(\frac{d}{dr} + \frac{\kappa}{r} - U(r) \right) F_{nk}(r) &= (M + E_{nk} - V(r) + S(r)) G_{nk}(r), \\ \left(\frac{d}{dr} - \frac{\kappa}{r} + U(r) \right) G_{nk}(r) &= (M - E_{nk} + V(r) + S(r)) F_{nk}(r), \end{aligned} \quad (2.3)$$

which simply give

$$\left\{ \frac{d^2}{dr^2} - \frac{\kappa(\kappa+1)}{r^2} + \frac{2\kappa}{r}U(r) - \frac{dU(r)}{dr} - U^2(r) + \frac{d\Delta(r)/dr}{M + E_{n\kappa} - \Delta(r)} \left(\frac{d}{dr} + \frac{\kappa}{r} - U(r) \right) \right\} F_{n\kappa}(r) \\ = (M + E_{n\kappa} - \Delta(r))(M - E_{n\kappa} + \Sigma(r))F_{n\kappa}(r), \quad (2.4)$$

$$\left\{ \frac{d^2}{dr^2} - \frac{\kappa(\kappa-1)}{r^2} + \frac{2\kappa}{r}U(r) + \frac{dU(r)}{dr} - U^2(r) - \frac{d\Sigma(r)/dr}{M - E_{n\kappa} + \Sigma(r)} \left(\frac{d}{dr} - \frac{\kappa}{r} + U(r) \right) \right\} G_{n\kappa}(r) \\ = (M + E_{n\kappa} - \Delta(r))(M - E_{n\kappa} + \Sigma(r))G_{n\kappa}(r), \quad (2.5)$$

where, as the notation indicates, $\Delta(r) = V(r) - S(r)$ and $\Sigma(r) = V(r) + S(r)$.

2.1. Pseudospin Symmetry Limit

Under the condition of the pseudospin symmetry, $d\Sigma(r)/dr = 0$ or equivalently $\Sigma(r) = C_{\text{gps}} = \text{Const.}$ [2, 3]. We choose $\Delta(r)$ as the Cornell potential:

$$\Delta(r) = a_{\text{ps}}r + \frac{b_{\text{ps}}}{r}. \quad (2.6)$$

For the tensor term, we consider the Cornell potential:

$$U(r) = Ar + \frac{B}{r}. \quad (2.7)$$

Substitution of these two terms into (2.5) gives

$$\left\{ \frac{d^2}{dr^2} + \frac{1}{r^2} \left(-\kappa(\kappa-1) + 2\kappa B - B - B^2 \right) - A^2 r^2 + \left(a_{\text{ps}}M - a_{\text{ps}}E_{n\kappa}^{\text{ps}} + a_{\text{ps}}C_{\text{ps}} \right) r \right. \\ \left. + \frac{1}{r} \left(b_{\text{ps}}M - b_{\text{ps}}E_{n\kappa}^{\text{ps}} + b_{\text{ps}}C_{\text{ps}} \right) - M^2 - MC_{\text{ps}} + \left(E_{n\kappa}^{\text{ps}} \right)^2 - E_{n\kappa}^{\text{ps}}C_{\text{ps}} + 2\kappa A + A - 2AB \right\} \\ \times G_{n\kappa}^{\text{ps}}(r) = 0, \quad (2.8)$$

where $\kappa = -\tilde{\ell}$ and $\kappa = \tilde{\ell} + 1$ for $\kappa < 0$ and $\kappa > 0$, respectively.

2.2. Spin Symmetry Limit

In the spin symmetry limit $d\Delta(r)/dr = 0$ or $\Delta(r) = C_{gs} = \text{const.}$ [2, 3]. As the previous section, we consider

$$\Sigma(r) = a_s r + \frac{b_s}{r}, \quad (2.9a)$$

$$U(r) = Ar + \frac{B}{r}. \quad (2.9b)$$

Substitution of the latter in (2.4) gives

$$\left\{ \frac{d^2}{dr^2} + \frac{1}{r^2} \left(-\kappa(\kappa+1) + 2\kappa B + B - B^2 \right) - A^2 r^2 + (-a_s M - a_s E_{n\kappa}^s + a_s C_s) r \right. \\ \left. + \frac{1}{r} (-b_s M - b_s E_{n\kappa}^s + b_s C_s) - M^2 + MC_s + \left(E_{n\kappa}^{\text{ps}} \right)^2 - E_{n\kappa}^s C_s + 2\kappa A - A - 2AB \right\} F_{n\kappa}^s(r) = 0, \quad (2.10)$$

where $\kappa = \ell$ and $\kappa = -\ell - 1$ for $\kappa < 0$ and $\kappa > 0$, respectively.

3. The Ansatz Solution

3.1. Solution of the Pseudospin Symmetry Limit

In the previous section, we obtained a Schrödinger-like equation of the form

$$\frac{d^2 G_{n\kappa}^{\text{ps}}(r)}{dr^2} + \left[\varepsilon_{n\kappa}^{\text{ps}} - a_1^{\text{ps}} r^2 - b_1^{\text{ps}} r + \frac{c_1^{\text{ps}}}{r} + \frac{d_1^{\text{ps}} - \kappa(\kappa-1)}{r^2} \right] G_{n\kappa}^{\text{ps}}(r) = 0, \quad (3.1)$$

where

$$\begin{aligned} \varepsilon_{n\kappa}^{\text{ps}} &= -M^2 - MC_{\text{ps}} + \left(E_{n\kappa}^{\text{ps}} \right)^2 - E_{n\kappa}^{\text{ps}} C_{\text{ps}} + 2\kappa A + A - 2AB, \\ a_1^{\text{ps}} &= A^2, \\ b_1^{\text{ps}} &= -a_{\text{ps}} M + a_{\text{ps}} E_{n\kappa}^{\text{ps}} - a_{\text{ps}} C_{\text{ps}}, \\ c_1^{\text{ps}} &= b_{\text{ps}} M - b_{\text{ps}} E_{n\kappa}^{\text{ps}} + b_{\text{ps}} C_{\text{ps}}, \\ d_1^{\text{ps}} &= 2\kappa B - B - B^2. \end{aligned} \quad (3.2)$$

Equation (3.1) fails to admit exact analytical solutions. Therefore, we follow the ansatz approach with the starting square:

$$G_{n\kappa}^{\text{ps}}(r) = f_n^{\text{ps}}(r) \exp(g_\kappa^{\text{ps}}(r)), \quad (3.3)$$

where

$$f_n^{\text{ps}}(r) = \begin{cases} 1, & \text{if } n = 0, \\ \prod_{i=1}^n (r - \alpha_i^{\text{ps}}), & \text{if } n \geq 1, \end{cases} \quad (3.4)$$

$$g_\kappa^{\text{ps}}(r) = -\frac{1}{2}\alpha^{\text{ps}}r^2 - \beta^{\text{ps}}r + \delta^{\text{ps}} \ln r, \quad \alpha^{\text{ps}} > 0, \beta^{\text{ps}} > 0. \quad (3.5)$$

By substitution of $f_n(r)$ and $g_\kappa(r)$ into (3.3), we find

$$G_{n\kappa}^{\text{ps}''}(r) = \left[g_\kappa^{\text{ps}''}(r) + g_\kappa^{\text{ps}/2}(r) + \frac{f_n^{\text{ps}''}(r) + 2g_\kappa^{\text{ps}'}(r)f_n^{\text{ps}'}(r)}{f_n^{\text{ps}}(r)} \right] G_{n\kappa}^{\text{ps}}(r). \quad (3.6)$$

Here, we consider the case $n = 0$. From (3.4)–(3.6) we find

$$\begin{aligned} G_{0\kappa}^{\text{ps}''}(r) \\ = \left\{ (\alpha^{\text{ps}})^2 r^2 + 2\alpha^{\text{ps}}\beta^{\text{ps}}r - \alpha^{\text{ps}}[1 + 2(\delta^{\text{ps}} + 0)] + (\beta^{\text{ps}})^2 - \frac{2\beta^{\text{ps}}(\delta^{\text{ps}} + 0)}{r} + \frac{\delta^{\text{ps}}(\delta^{\text{ps}} - 1)}{r^2} \right\} G_{0\kappa}^{\text{ps}}(r). \end{aligned} \quad (3.7)$$

By comparing the corresponding powers of (3.1) and (3.7), we have

$$\begin{aligned} \alpha^{\text{ps}} &= \sqrt{a_1^{\text{ps}}}, \quad a_1^{\text{ps}} > 0, \\ \beta^{\text{ps}} &= \frac{b_1^{\text{ps}}}{2\sqrt{a_1^{\text{ps}}}}, \\ c_1^{\text{ps}} &= 2\beta^{\text{ps}}(\delta^{\text{ps}} + 0), \\ \delta^{\text{ps}} &= \frac{1}{2} \left(1 \pm \sqrt{(2\kappa - 1)^2 - 4d_1^{\text{ps}}} \right) = \frac{1}{2} (1 \pm k_1^{\text{ps}}), \\ \varepsilon_{0\kappa}^{\text{ps}} &= \alpha^{\text{ps}}[1 + 2(\delta^{\text{ps}} + 0)] - (\beta^{\text{ps}})^2, \end{aligned} \quad (3.8)$$

where $k_1^{\text{ps}} = \sqrt{(2\kappa - 1)^2 - 4d_1^{\text{ps}}}$. Actually, to have well-behaved solutions of the radial wave function at boundaries, namely, the origin and the infinity, we need to take δ from (3.8) as

$$\delta^{\text{ps}} = \frac{1}{2} (1 + k_1^{\text{ps}}). \quad (3.9)$$

Form (3.2), (3.8), the ground-state energy satisfies

$$-M^2 - MC_{\text{ps}} + \left(E_{0\kappa}^{\text{ps}}\right)^2 - E_{0\kappa}^{\text{ps}}C_{\text{ps}} + 2\kappa A + A - 2AB - \alpha^{\text{ps}}[1 + 2(\delta^{\text{ps}} + 0)] + \beta^{\text{ps}2} = 0 \quad (3.10a)$$

or

$$\left(E_{0\kappa}^{\text{ps}}\right)^2 - E_{0\kappa}^{\text{ps}}C_{\text{ps}} - M^2 - MC_{\text{ps}} + 2\kappa A + A - 2AB - \sqrt{a_1^{\text{ps}}} \left(2 + \sqrt{(2\kappa - 1)^2 - 4d_1^{\text{ps}}}\right) + \frac{b_1^{\text{ps}2}}{4a_1^{\text{ps}}} = 0, \quad (3.10b)$$

which is more compactly written as

$$E_{0\kappa}^{\text{ps}\pm} = \frac{1}{2} \left\{ C_{\text{ps}} \pm \left[C_{\text{ps}}^2 - 8\kappa A - 4A + 8AB + 4M^2 + 4MC_{\text{ps}} + 8\sqrt{a_1^{\text{ps}}} \right. \right. \\ \left. \left. + 4\sqrt{a_1^{\text{ps}}} \sqrt{4\kappa^2 - 4\kappa + 1 - 4d_1^{\text{ps}}} - 4\beta^{\text{ps}2} \right]^{1/2} \right\}, \quad (3.11)$$

$$+ 4\sqrt{a_1^{\text{ps}}} \sqrt{4\kappa^2 - 4\kappa + 1 - 4d_1^{\text{ps}}} - 4\beta^{\text{ps}2} \Big]^{1/2} \Big\}, \quad (3.35),$$

where the parameter a_{ps} of potential (2.6) from (3.8) should satisfy the following restriction:

$$a_{\text{ps}} = \frac{-2b_{\text{ps}}\sqrt{a_1^{\text{ps}}}}{\left(1 + \sqrt{(2\kappa - 1)^2 - 4d_1^{\text{ps}}}\right)} = \frac{-2b_{\text{ps}}\sqrt{a_1^{\text{ps}}}}{\left(1 + k_1^{\text{ps}}\right)}. \quad (3.12)$$

From (3.3), (3.4), and (3.8), the upper and lower components of the wave function are

$$g_{\kappa}(r) = -\frac{1}{2}\sqrt{a_1^{\text{ps}}}r^2 - \frac{b_1^{\text{ps}}}{2\sqrt{a_1^{\text{ps}}}}r + \frac{1}{2}\left(1 + \sqrt{(2\kappa - 1)^2 - 4d_1^{\text{ps}}}\right)\ln r, \quad (3.13a)$$

$$G_{0\kappa}^{\text{ps}}(r) = N_{0\kappa}r^{(1/2)(1+\sqrt{(2\kappa-1)^2-4d_1^{\text{ps}}})}\exp\left(-\frac{1}{2}\sqrt{a_1^{\text{ps}}}r^2 - \frac{b_1^{\text{ps}}}{2\sqrt{a_1^{\text{ps}}}}r\right), \quad (3.13b)$$

$$F_{0\kappa}^{\text{ps}}(r) = \frac{1}{M - E_{0\kappa}^{\text{ps}} + C_{\text{ps}}}\left(\frac{d}{dr} - \frac{\kappa}{r} + U(r)\right)G_{0\kappa}^{\text{ps}}(r). \quad (3.13c)$$

For the first node ($n = 1$), using $f_1(r) = (r - \alpha_1^{(1)})$ and $g_\kappa(r)$ from (3.5), we arrive at

$$\begin{aligned} & a_1^{\text{ps}} r^2 + b_1^{\text{ps}} r - \frac{c_1^{\text{ps}}}{r} + \frac{\kappa(\kappa - 1) - d_1^{\text{ps}}}{r^2} - \varepsilon_{1\kappa}^{\text{ps}} \\ &= -\alpha^{\text{ps}} - \frac{\delta^{\text{ps}}}{r^2} + (\alpha^{\text{ps}})^2 r^2 + (\beta^{\text{ps}})^2 + \frac{(\delta^{\text{ps}})^2}{r^2} + 2\alpha^{\text{ps}} \beta^{\text{ps}} r - 2\delta^{\text{ps}} \alpha^{\text{ps}} - \frac{2\beta^{\text{ps}} \delta^{\text{ps}}}{r} \\ &+ \frac{-2\alpha^{\text{ps}} r - 2\beta^{\text{ps}} + 2\delta^{\text{ps}}/r}{r - \alpha_1^{(1)}}. \end{aligned} \quad (3.14)$$

Here, the consequent relations between the potential parameters and the coefficients α^{ps} , β^{ps} , δ^{ps} , and $\alpha_1^{(1)}$ are

$$\begin{aligned} \alpha^{\text{ps}} &= \sqrt{a_1^{\text{ps}}}, \quad \beta^{\text{ps}} = \frac{b_1^{\text{ps}}}{2\sqrt{a_1^{\text{ps}}}}, \quad \delta^{\text{ps}} = \frac{1}{2}(1 + k_1^{\text{ps}}), \\ \varepsilon_{1\kappa}^{\text{ps}} &= \alpha^{\text{ps}}[1 + 2(\delta^{\text{ps}} + 1)] - (\beta^{\text{ps}})^2, \\ (c_1^{\text{ps}} - 2\beta^{\text{ps}} \delta^{\text{ps}}) \alpha_1^{(1)} &= 2\delta^{\text{ps}}, \\ c_1^{\text{ps}} - 2\beta^{\text{ps}}(\delta^{\text{ps}} + 1) &= 2\alpha^{\text{ps}} \alpha_1^{(1)}. \end{aligned} \quad (3.15)$$

By solving the above equations one can find c_1^{ps} and $\alpha_1^{(1)}$ as

$$c_1^{\text{ps}} = \frac{b_1^{\text{ps}}}{2\sqrt{a_1^{\text{ps}}}} \left(\sqrt{(2\kappa - 1)^2 - 4d_1^{\text{ps}}} + 2 \right) + \sqrt{\frac{b_1^{\text{ps}2}}{4a_1^{\text{ps}}} + 2\sqrt{a_1^{\text{ps}}} \left(1 + \sqrt{(2\kappa - 1)^2 - 4d_1^{\text{ps}}} \right)}, \quad (3.16a)$$

$$\alpha^{\text{ps}} \alpha_1^{(1)2} + \beta^{\text{ps}} \alpha_1^{(1)} - \delta^{\text{ps}} = 0,$$

$$\longrightarrow \alpha_1^{(1)} = \frac{1 - \beta^{\text{ps}} + \sqrt{(\beta^{\text{ps}})^2 + 4\alpha^{\text{ps}} \delta^{\text{ps}}}}{\alpha^{\text{ps}}} = \frac{-b_1^{\text{ps}}}{4a_1^{\text{ps}}} + \sqrt{\frac{b_1^{\text{ps}2}}{16a_1^{\text{ps}2}} + \frac{1 + \sqrt{(2\kappa - 1)^2 - 4d_1^{\text{ps}}}}{2\sqrt{a_1^{\text{ps}}}}}. \quad (3.16b)$$

The energy eigenvalues therefore are

$$\begin{aligned} E_{1\kappa}^{\text{ps}\pm} &= \frac{1}{2} \left\{ C_{\text{ps}} \pm \left[C_{\text{ps}}^2 - 8\kappa A - 4A + 8AB + 4M^2 + 4MC_{\text{ps}} + 16\sqrt{a_1^{\text{ps}}} \right. \right. \\ &\quad \left. \left. + 4\sqrt{a_1^{\text{ps}}} \sqrt{4\kappa^2 - 4\kappa + 1 - 4d_1^{\text{ps}}} - 4\beta^{\text{ps}2} \right]^{1/2} \right\}, \end{aligned} \quad (3.17)$$

where the parameter a_{ps} from (3.8) is

$$a_{\text{ps}} = \frac{2\sqrt{a_1^{\text{ps}}} \left(2c_1^{\text{ps}} + c_1^{\text{ps}} k_1^{\text{ps}} + \sqrt{c_1^{\text{ps}2} + 6\sqrt{a_1^{\text{ps}}} + 14\sqrt{a_1^{\text{ps}}} k_1^{\text{ps}} + 10\sqrt{a_1^{\text{ps}}} k_1^{\text{ps}2} + 2\sqrt{a_1^{\text{ps}}} k_1^{\text{ps}3} \right)}{-3M - 4Mk_1^{\text{ps}} - Mk_1^{\text{ps}2} + 3E_{1\kappa}^{\text{ps}} + 4E_{1\kappa}^{\text{ps}} k_1^{\text{ps}} + E_{1\kappa}^{\text{ps}} k_1^{\text{ps}2} - 3C_{\text{ps}} - 4C_{\text{ps}} k_1^{\text{ps}} - C_{\text{ps}} k_1^{\text{ps}2}}. \quad (3.18)$$

For the upper and lower components of the wave function we thus have

$$G_{1\kappa}^{\text{ps}}(r) = N_{1\kappa} (r - a_1^{(1)}) r^{(1/2)(1 + \sqrt{(2\kappa-1)^2 - 4d_1^{\text{ps}}})} \exp\left(-\frac{1}{2}\sqrt{a_1^{\text{ps}}} r^2 - \frac{b_1^{\text{ps}}}{2\sqrt{a_1^{\text{ps}}}} r\right), \quad (3.19a)$$

$$F_{1\kappa}^{\text{ps}}(r) = \frac{1}{M - E_{1\kappa}^{\text{ps}} + C_{\text{ps}}} \left(\frac{d}{dr} - \frac{\kappa}{r} + U(r) \right) G_{1\kappa}^{\text{ps}}(r). \quad (3.19b)$$

Following the analytic iteration procedures for the second node ($n = 2$) with $f_2(r) = (r - \alpha_1^{(2)})(r - \alpha_2^{(2)})$ and $g_\kappa(r)$ as defined in (3.6), the relations between the potential parameters and the coefficients $\alpha^{\text{ps}}, \beta^{\text{ps}}, \delta^{\text{ps}}, \alpha_1^{(2)}$, and $\alpha_2^{(2)}$ are

$$\begin{aligned} \alpha^{\text{ps}} &= \sqrt{a_1^{\text{ps}}}, \quad \beta^{\text{ps}} = \frac{b_1^{\text{ps}}}{2\sqrt{a_1^{\text{ps}}}}, \quad \delta^{\text{ps}} = \frac{1}{2}(1 + k_1^{\text{ps}}), \\ \varepsilon_{2\kappa}^{\text{ps}} &= \alpha^{\text{ps}}[1 + 2(\delta^{\text{ps}} + 2)] - (\beta^{\text{ps}})^2, \\ c_1^{\text{ps}} - 2\beta^{\text{ps}}(\delta^{\text{ps}} + 2) &= 2\alpha^{\text{ps}} \sum_{i=1}^2 \alpha_i^{(2)}, \quad (c_1^{\text{ps}} - 2\beta^{\text{ps}}\delta^{\text{ps}}) \sum_{i<j}^2 \alpha_i^{(2)} \alpha_j^{(2)} = 2\delta^{\text{ps}} \sum_{i=1}^2 \alpha_i^{(2)}, \\ [c_1^{\text{ps}} - 2\beta^{\text{ps}}(\delta^{\text{ps}} + 1)] \sum_{i=1}^2 \alpha_i^{(2)} &= 4\alpha^{\text{ps}} \sum_{i<j}^2 \alpha_i^{(2)} \alpha_j^{(2)} + 2(2\delta^{\text{ps}} + 1). \end{aligned} \quad (3.20)$$

The coefficients $\alpha_1^{(2)}$ and $\alpha_2^{(2)}$ are found from the constraint relations [20, 27–29]:

$$\alpha^{\text{ps}} \sum_{i=1}^2 \alpha_i^{(2)2} + \beta^{\text{ps}} \sum_{i=1}^2 \alpha_i^{(2)} - (2\delta^{\text{ps}} + 1) = 0, \quad (3.21)$$

$$\delta^{\text{ps}} \sum_{i=1}^2 \alpha_i^{(2)2} - \left(\beta^{\text{ps}} \sum_{i=1}^2 \alpha_i^{(2)} + 1 \right) \sum_{j<k}^2 \alpha_j^{(2)} \alpha_k^{(2)} - 2\alpha^{\text{ps}} \sum_{j<k}^2 \alpha_j^{(2)2} \alpha_k^{(2)2} = 0. \quad (3.22)$$

Therefore, the energy eigenvalue in this case is

$$E_{2\kappa}^{\text{ps}\pm} = \frac{1}{2} \left\{ C_{\text{ps}} \pm \left[C_{\text{ps}}^2 - 8\kappa A - 4A + 8AB + 4M^2 + 4MC_{\text{ps}} + 24\sqrt{a_1^{\text{ps}}} \right. \right. \\ \left. \left. + 4\sqrt{a_1^{\text{ps}}} \sqrt{4\kappa^2 - 4\kappa + 1 - 4d_1^{\text{ps}} - 4\beta^{\text{ps}2}} \right]^{1/2} \right\}, \quad (3.23)$$

and the lower component of the wave function is

$$G_{2\kappa}^{\text{ps}}(r) = N_{2\kappa} \prod_{i=1}^2 (r - \alpha_i^{(2)}) r^{(1/2)(1+\sqrt{(2\kappa-1)^2-4d_1^{\text{ps}}})} \exp \left(-\frac{1}{2} \sqrt{a_1^{\text{ps}}} r^2 - \frac{b_1^{\text{ps}}}{2\sqrt{a_1^{\text{ps}}}} r \right). \quad (3.24)$$

3.2. Solution of the Spin Symmetry Limit

In this case, our ordinary differential equation is

$$\frac{d^2 F_{n\kappa}^s(r)}{dr^2} + \left[\varepsilon_{n\kappa}^s - a_1^s r^2 - b_1^s r + \frac{c_1^s}{r} + \frac{d_1^s - \kappa(\kappa+1)}{r^2} \right] F_{n\kappa}^s(r) = 0, \quad (3.25)$$

with

$$\begin{aligned} \varepsilon_{n\kappa}^s &= -M^2 + MC_s + (E_{n\kappa}^s)^2 - E_{n\kappa}^s C_s + 2\kappa A - A - 2AB, \\ a_1^s &= A^2, \\ b_1^s &= a_s M + a_s E_{n\kappa}^s - a_s C_s, \\ c_1^s &= -b_s M - b_s E_{n\kappa}^s + b_s C_s, \\ d_1^s &= 2\kappa B + B - B^2, \end{aligned} \quad (3.26)$$

which cannot be solved by our common exact analytical techniques. Let us propose the ansatz solution:

$$F_{n\kappa}^s(r) = f_n^s(r) \exp(g_\kappa^s(r)), \quad (3.27)$$

where

$$f_n^s(r) = \begin{cases} 1, & \text{if } n = 0, \\ \prod_{i=1}^n (r - \alpha_i^n), & \text{if } n \geq 1, \end{cases} \quad (3.28)$$

$$g_\kappa^s(r) = -\frac{1}{2} \alpha^s r^2 - \beta^s r + \delta^s \ln r, \quad \alpha^s > 0, \beta^s > 0. \quad (3.29)$$

By substitution of $f_n(r)$ and $g_\kappa(r)$ into (3.27), we find

$$F_{n\kappa}^{s''}(r) = \left[g_\kappa^{s''}(r) + g_\kappa^{s'^2}(r) + \frac{f_n^{s''}(r) + 2g_\kappa^{s'}(r)f_n^{s'}(r)}{f_n^s(r)} \right] F_{n\kappa}^s(r). \quad (3.30)$$

For the case of $n = 0$, from (3.26)–(3.29), we find

$$F_{n\kappa}^{s''}(r) = \left\{ (\alpha^s)^2 r^2 + 2\alpha^s \beta^s r - \alpha^s [1 + 2(\delta^s + 0)] + (\beta^s)^2 - \frac{2\beta^s(\delta^s + 0)}{r} + \frac{\delta^s(\delta^s - 1)}{r^2} \right\} F_{n\kappa}^s(r). \quad (3.31)$$

By comparing the corresponding powers of (3.27) and (3.33), we have

$$\begin{aligned} \alpha^s &= \sqrt{a_1^s}, \\ \beta^s &= \frac{b_1^s}{2\sqrt{a_1^s}}, \quad a_1^s > 0, \\ c_1^s &= 2\beta^s(\delta^s + 0), \\ \delta^s &= \frac{1}{2} \left(1 \pm \sqrt{(2\kappa + 1)^2 - 4d_1^s} \right) = \frac{1}{2} (1 \pm k_1^s), \\ \varepsilon_{0\kappa}^s &= \alpha[1 + 2(\delta^s + 0)] - (\beta^s)^2, \end{aligned} \quad (3.32)$$

where $k_1^s = \sqrt{(2\kappa + 1)^2 - 4d_1^s}$. To have physically acceptable solutions, we pick up the value

$$\delta^s = \frac{1}{2} (1 + k_1^s). \quad (3.33)$$

By considering (3.26), (3.32), the first node eigenvalue satisfies

$$-M^2 + MC_s + (E_{n\kappa}^s)^2 - E_{n\kappa}^s C_s + 2\kappa A - A - 2AB - \alpha^s [1 + 2(\delta^s + 0)] + (\beta^s)^2 = 0, \quad (3.34)$$

or equivalently

$$\begin{aligned} E_{0\kappa}^{s\pm} &= \frac{1}{2} \left\{ C_s \pm \left[C_s^2 + 4M^2 - 4MC_s - 8\kappa A + 4A + 8AB + 8\sqrt{a_1^s} \right. \right. \\ &\quad \left. \left. + 4\sqrt{a_1^s} \sqrt{4\kappa^2 + 4\kappa + 1 - 4d_1^s} - 4\beta^{s2} \right]^{1/2} \right\}, \end{aligned} \quad (3.35)$$

where the parameter a_s of potential (2.9a) should satisfy the restriction

$$a_s = \frac{-2b_s\sqrt{a_1^s}}{(1+k_1^s)}. \quad (3.36)$$

From (3.27), (3.28), and (3.32), the upper and lower components of the wave function are

$$g_\kappa(r) = -\frac{1}{2}\sqrt{a_1^s}r^2 - \frac{b_1^s}{2\sqrt{a_1^s}}r + \frac{1}{2}\left(1 + \sqrt{(2\kappa+1)^2 - 4d_1^s}\right)\ln r, \quad (3.37)$$

$$F_{0\kappa}^s(r) = N_{0\kappa}r^{(1/2)(1+\sqrt{(2\kappa+1)^2-4d_1^s})}\exp\left(-\frac{1}{2}\sqrt{a_1^s}r^2 - \frac{b_1^s}{2\sqrt{a_1^s}}r\right), \quad (3.38)$$

$$G_{0\kappa}^s(r) = \frac{1}{M + E_{0\kappa}^s - C_s}\left(\frac{d}{dr} + \frac{\kappa}{r} - U(r)\right)F_{0\kappa}^s(r). \quad (3.39)$$

Secondly, for the first node ($n = 1$), using $f_1(r) = (r - \alpha_1^{(1)})$ and $g_\kappa(r)$ from (3.28)–(3.30), our resulting equation is

$$\begin{aligned} & a_1^s r^2 + b_1^s r - \frac{c_1^s}{r} + \frac{-d_1^s + \kappa(\kappa+1)}{r^2} - \varepsilon_{n\kappa}^s \\ &= -\alpha^s - \frac{\delta^s}{r^2} + (\alpha^s)^2 r^2 + (\beta^s)^2 + \frac{(\delta^s)^2}{r^2} + 2\alpha^s \beta^s r - 2\delta^s \alpha^s - \frac{2\beta^s \delta^s}{r} + \frac{-2\alpha^s r - 2\beta^s + 2\delta^s/r}{r - \alpha_1^{(1)}}. \end{aligned} \quad (3.40)$$

The relations between the potential parameters and the coefficients $\alpha^s, \beta^s, \delta^s$, and $\alpha_1^{(1)}$ are

$$\begin{aligned} \alpha^s &= \sqrt{a_1^s}, \quad \beta^s = \frac{b_1^s}{2\sqrt{a_1^s}}, \quad a_1^s > 0, \quad \delta^s = \frac{1}{2}\left(1 + \sqrt{(2\kappa+1)^2 - 4d_1^s}\right) = \frac{1}{2}(1 + k_1^s), \\ \varepsilon_{1\kappa}^s &= \alpha^s[1 + 2(\delta^s + 1)] - (\beta^s)^2, \\ (c_1^s - 2\beta^s \delta^s)\alpha_1^{(1)} &= 2\delta^s, \\ c_1^s - 2\beta^s(\delta^s + 1) &= 2\alpha^s \alpha_1^{(1)}, \end{aligned} \quad (3.41)$$

where c_1^s and $\alpha_1^{(1)}$ are found from (3.41) as [20, 27–29]

$$c_1^s = \frac{b_1^s}{2\sqrt{a_1^s}} \left(\sqrt{(2\kappa+1)^2 - 4d_1^s} + 2 \right) + \sqrt{\frac{b_1^{s2}}{4a_1^s} + 2\sqrt{a_1^s} \left(1 + \sqrt{(2\kappa+1)^2 - 4d_1^s} \right)}, \quad (3.42a)$$

$$\alpha^s \alpha_1^{(1)2} + \beta^s \alpha_1^{(1)} - \delta^s = 0, \quad (3.42b)$$

or

$$\alpha_1^{(1)} = \frac{1}{2} \frac{-\beta^s + \sqrt{(\beta^s)^2 + 4\alpha^s \delta^s}}{\alpha^s} = \frac{-b_1^s}{4a_1^s} + \sqrt{\frac{b_1^{s2}}{16a_1^{s2}} + \frac{1 + \sqrt{(2\kappa+1)^2 - 4d_1^s}}{2\sqrt{a_1^s}}}, \quad (3.42c)$$

which determine the corresponding energy as

$$E_{1\kappa}^{\pm} = \frac{1}{2} \left\{ C_s \pm \left[C_s^2 + 4M^2 - 4MC_s - 8\kappa A + 4A + 8AB + 16\sqrt{a_1^s} \right. \right. \\ \left. \left. + 4\sqrt{a_1^s} \sqrt{4\kappa^2 + 4\kappa + 1 - 4d_1^s - 4\beta^{s2}} \right]^{1/2} \right\}, \quad (3.43)$$

with

$$a_s = \frac{2\sqrt{a_1^s} \left(2c_1^s + c_1^s k_1^s + \sqrt{c_1^{s2} + 6\sqrt{a_1^s} + 14\sqrt{a_1^s} k_1^s + 10\sqrt{a_1^s} k_1^{s2} + 2\sqrt{a_1^s} k_1^{s3}} \right)}{3M + 4Mk_1^s + Mk_1^{s2} + 3E_{1\kappa}^s + 4E_{1\kappa}^s k_1^s + E_{1\kappa}^s k_1^{s2} - 3C_s - 4C_s k_1^s - C_s k_1^{s2}}. \quad (3.44)$$

The upper and lower components of the wave function are then simply found to be

$$F_{1\kappa}^s(r) = N_{1\kappa} \left(r - \alpha_1^{(1)} \right) r^{(1/2)(1+\sqrt{(2\kappa+1)^2 - 4d_1^s})} \exp \left(-\frac{1}{2} \sqrt{a_1^s} r^2 - \frac{b_1^s}{2\sqrt{a_1^s}} r \right), \quad (3.45a)$$

$$G_{1\kappa}^s(r) = \frac{1}{M + E_{1\kappa}^s - C_s} \left(\frac{d}{dr} + \frac{\kappa}{r} - U(r) \right) F_{1\kappa}^s(r). \quad (3.45b)$$

For the second node ($n = 2$), we choose $f_2(r) = (r - \alpha_1^{(2)})(r - \alpha_2^{(2)})$ and $g_\kappa(r)$ as defined in (3.28) and (3.29). The relations between the potential parameters and the coefficients $\alpha^s, \beta^s, \delta^s, \alpha_1^{(2)}$, and $\alpha_2^{(2)}$ are

$$\begin{aligned}\alpha^s &= \sqrt{a_1^s}, & \beta^s &= \frac{b_1^s}{2\sqrt{a_1^s}}, & \delta^s &= \frac{1}{2}(1 + k_1^s), \\ \varepsilon_{2\kappa}^s &= \alpha^s[1 + 2(\delta^s + 2)] - (\beta^s)^2, \\ c_1^s - 2\beta^s(\delta^s + 2) &= 2\alpha^s \sum_{i=1}^2 \alpha_i^{(2)}, & (c_1^s - 2\beta^s\delta^s) \sum_{i<j}^2 \alpha_i^{(2)} \alpha_j^{(2)} &= 2\delta^s \sum_{i=1}^2 \alpha_i^{(2)}, \\ [c_1^s - 2\beta^s(\delta^s + 1)] \sum_{i=1}^2 \alpha_i^{(2)} &= 4\alpha^s \sum_{i<j}^2 \alpha_i^{(2)} \alpha_j^{(2)} + 2(2\delta^s + 1),\end{aligned}\tag{3.46}$$

where

$$\alpha^s \sum_{i=1}^2 \alpha_i^{(2)2} + \beta^s \sum_{i=1}^2 \alpha_i^{(2)} - (2\delta^s + 1) = 0,\tag{3.47}$$

$$\delta^s \sum_{i=1}^2 \alpha_i^{(2)2} - \left(\beta^s \sum_{i=1}^2 \alpha_i^{(2)} + 1 \right) \sum_{j<k}^2 \alpha_j^{(2)} \alpha_k^{(2)} - 2\alpha^s \sum_{j<k}^2 \alpha_j^{(2)2} \alpha_k^{(2)2} = 0.\tag{3.48}$$

The energy eigenvalue therefore is

$$\begin{aligned}E_{2\kappa}^{s\pm} &= \frac{1}{2} \left\{ C_s \pm \left[C_s^2 + 4M^2 - 4MC_s - 8\kappa A + 4A + 8AB + 24\sqrt{a_1^s} \right. \right. \\ &\quad \left. \left. + 4\sqrt{a_1^s} \sqrt{4\kappa^2 + 4\kappa + 1 - 4d_1^s - 4\beta^{s2}} \right]^{1/2} \right\}.\end{aligned}\tag{3.49}$$

For the upper component of the wave function, we have

$$F_{2\kappa}^s(r) = N_{2\kappa} \prod_{i=1}^2 (r - \alpha_i^{(2)}) r^{(1/2)(1 + \sqrt{(2\kappa+1)^2 - 4d_1^s})} \exp\left(-\frac{1}{2}\sqrt{a_1^s}r^2 - \frac{b_1^s}{2\sqrt{a_1^s}}r\right).\tag{3.50}$$

We have given some numerical values of the energy eigenvalues in Tables 1, 2, 3, 4, 5, and 6 for various states. For the final point, we wish to emphasize on the degeneracy-removing role of the (Cornell) tensor potential. As we already know, for vanishing tensor interaction ($A = B = 0$), the pseudospin doublets, that is, states with quantum numbers $(n, l, j = l + 1/2)$ and $(n - 1, l + 2, j = l + 3/2)$ are degenerate. The degenerate states in the spin doublets are those with quantum numbers $(n, \ell, j = \ell + 1/2)$ and $(n, \ell, j = \ell - 1/2)$, where n, l , and j

Table 1: Bound state for the pseudospin symmetry $b = 1$, $M = 1 \text{ fm}^{-1}$, $C_{ps} = 0 \text{ fm}^{-1}$.

$\tilde{\ell}$	n, κ	(ℓ, j)	$E_{n\kappa}^{\text{ps}^-} (\text{fm}^{-1})$	
			$A = B = 0$	$A = 0.5, B = 0.2$
1	0, -1	0S _{1/2}	-0.882352941	-1.984891569
2	0, -2	0P _{3/2}	-0.945945946	-2.471004354
3	0, -3	0d _{5/2}	-0.969230769	-2.861584264
4	0, -4	0f _{7/2}	-0.980198020	-3.199522676
1	1, -1	1S _{1/2}	-0.945945946	-2.302587881
2	1, -2	1p _{3/2}	-0.969230769	-2.710867336
3	1, -3	1d _{5/2}	-0.980198020	-3.060956677
4	1, -4	1f _{7/2}	-0.986206897	-3.373431922
1	1, 2	0d _{3/2}	-0.945945946	-1.396649362
2	1, 3	0f _{5/2}	-0.969230769	-1.411440323
3	1, 4	0g _{7/2}	-0.980198020	-1.414060440
4	1, 5	0h _{9/2}	-0.986206897	-1.414104092

Table 2: Bound state for the spin symmetry $b = 1$, $M = 1 \text{ fm}^{-1}$, $C_s = 0 \text{ fm}^{-1}$.

ℓ	n, κ	(ℓ, j)	$E_{n\kappa}^{s^+} (\text{fm}^{-1})$	
			$A = B = 0$	$A = 0.5, B = 0.2$
1	0, -2	0p _{3/2}	0.882352941	2.408312542
2	0, -3	0d _{5/2}	0.945945946	2.835633655
3	0, -4	0f _{7/2}	0.969230769	3.186164523
4	0, -5	0g _{9/2}	0.980198020	3.494741008
1	1, -2	1p _{3/2}	0.945945946	2.557131582
2	1, -3	1d _{5/2}	0.969230769	2.960686798
3	1, -4	1f _{7/2}	0.980198020	3.301543491
4	1, -5	1g _{9/2}	0.986206897	3.603991399
1	0, 1	0p _{1/2}	0.882352941	1.577184026
2	0, 2	0d _{3/2}	0.945945946	1.665371674
3	0, 3	0f _{5/2}	0.969230769	1.695353106
4	0, 4	0g _{7/2}	0.980198020	1.708910609
1	1, 1	1p _{1/2}	0.945945946	1.798483171
2	1, 2	1d _{3/2}	0.969230769	1.873601141
3	1, 3	1f _{5/2}	0.980198020	1.910316883
4	1, 4	1g _{7/2}	0.986206897	1.931627027

Table 3: Energies in the pseudospin symmetry Limit for $A = 0.5$, $B = 0.2$, $b = 1$, $M = 1 \text{ fm}^{-1}$.

C_{ps}	$E_{n\kappa}^{\text{ps}^-} (\text{fm}^{-1})$			
	1P _{3/2}	1f _{7/2}	0d _{3/2}	0g _{7/2}
-4	-4.720070702	-5.340430189	-3.401741077	-3.398579611
-2	-3.528809436	-4.224630585	-1.997983133	-1.989883697
0	-2.710867336	-3.373431922	-1.396649362	-1.414060440
2	-2.197514516	-2.788106698	-1.161286022	-1.224585997
4	-1.872532099	-2.315650253	-1.021036465	-1.131350750

Table 4: Energies in the pseudospin symmetry limit for $A = 0.5$, $B = 0.2$, $b = 1$, $C_{ps} = 0 \text{ fm}^{-1}$.

$M \text{ (fm}^{-1}\text{)}$	$E_{nk}^{ps-} \text{ (fm}^{-1}\text{)}$			
	$1P_{3/2}$	$1f_{7/2}$	$0d_{3/2}$	$0g_{7/2}$
0	-2.528809436	-3.224630585	-0.997983133	-0.989883697
2	-3.197514516	-3.788106698	-2.161286022	-2.224585997
4	-4.653565405	-5.108745157	-3.914496549	-4.070243046
6	-6.373746671	-6.675285047	-5.75746618	-5.983999509

Table 5: Energies in the spin symmetry limit for $A = 0.5$, $B = 0.2$, $b = 1$, $M = 1 \text{ fm}^{-1}$.

C_s	$E_{nk}^{s+} \text{ (fm}^{-1}\text{)}$			
	$0f_{7/2}$	$1d_{5/2}$	$0P_{1/2}$	$1f_{5/2}$
-4	2.202940158	1.991277770	0.885667514	1.319214847
-2	2.599395981	2.379236118	1.159816872	1.527861114
0	3.186164523	2.960686798	1.577184026	1.910316883
2	4.044444399	3.826180645	2.362620095	2.672956317
4	5.214113098	5.019180712	3.720450502	3.966069464

Table 6: Energies in the spin symmetry limit for $A = 0.5$, $B = 0.2$, $b = 1$, $C_s = 0 \text{ fm}^{-1}$.

$M \text{ (fm}^{-1}\text{)}$	$E_{nk}^{s+} \text{ (fm}^{-1}\text{)}$			
	$0f_{7/2}$	$1d_{5/2}$	$0P_{1/2}$	$1f_{5/2}$
0	3.044444399	2.826180645	1.362620095	1.672956317
2	3.599395981	3.379236118	2.159816872	2.527861114
4	4.926530873	4.721303277	3.669041728	4.186965577
6	6.569688910	5.960686798	5.304646313	6.017810527

are the radial, the orbital, and the total angular momentum quantum numbers, respectively (see Tables 1 and 2). Our numerical data reveals that, in the pseudospin symmetry limit, the degenerate states for $(A = B = 0)$ are $n s_{1/2}, (n - 1)d_{3/2}$ for $\tilde{\ell} = 1$ ($\ell = 0$), $n p_{3/2}, (n - 1)f_{5/2}$ for $\tilde{\ell} = 2$ ($\ell = 1$), $n d_{5/2}, (n - 1)g_{7/2}$ for $\tilde{\ell} = 3$ ($\ell = 2$), $n f_{7/2}, (n - 1)h_{9/2}$ for $\tilde{\ell} = 4$ ($\ell = 3$), and so forth. For spin symmetry limit and $(A = B = 0)$ one can clearly see that the degeneracy occurs in $(np_{1/2}, np_{3/2})$ for $\ell = 1$, $(nd_{3/2}, nd_{5/2})$ for $\ell = 2$, $(nf_{5/2}, nf_{7/2})$ for $\ell = 3$, $(ng_{7/2}, ng_{9/2})$ for $\ell = 4$, and so forth.

4. Conclusion

Because of the established roles of the Cornell potential and spin, and pseudospin symmetries in nuclear and hadrons spectroscopy, we solved the Dirac equation under these symmetry limits for vector, scalar, and tensor interactions of Cornell-type. In our calculations, on the one hand, due to the failure of other common analytical techniques, and, on the other hand, the better insight which analytical techniques provide us in comparison with their numerical counterparts, we used the quasianalytical ansatz approach. By proposing novel physical solutions and after lengthy calculations, we could find the arbitrary-state solutions. Our results clearly show the degeneracy-removing role of the tensor term and provide the requisite understanding of the solutions for possible further studies. Both the

energy spectrum and the eigenfunctions can be used in related systems after the proper phenomenological fits done.

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