Research Article

The Other Side of Gravity and Geometry:
Antigravity and Anticurvature

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Gravity is one of the four known fundamental interactions used to study and interpret physical phenomena. It governs diverse phenomena, especially those connected with large-scale structures. From more than one decade, existing gravity theories have suffered from some problems, when confronting their predictions with the results of some experiments and observations. This situation has led to many suggestions, none of which is final, so far. Here, we show that the assumption of existence of another side of gravity, a repulsive gravity or antigravity, together with its attractive side, may give a satisfactory solution to gravity problems. We caught here two pieces of evidence for the existence of antigravity in nature. The first is on the laboratory scale, the COW experiment, and the second is on the cosmic scale, SN type Ia observation. On the other hand, we show how gravity theories can predict antigravity, using a new defined geometric object called Parameterized anticurvature. This shows clearly how Einstein’s geometrization philosophy can solve recent gravity problems in a satisfactory and easy way. Also, it may throw some light on the mystery of physical nature of “Dark Energy.”

1. Introduction

Nowadays, it is well known that most of the phenomena in the Universe can be interpreted successfully using one or more of the following interactions:

(1) gravity;
(2) electromagnetism;
(3) weak force;
(4) strong force.
Although gravity is the most popular among the above-mentioned interactions, one can say that we are now living in an electromagnetic civilization. Most of the tools and equipments of modern civilization depend on electromagnetism. This is mainly due to our deep understanding of the electromagnetic interaction compared with other interactions. However, there are some pieces of evidence forcing one to claim that ancient civilization has been a gravitational one. Among these pieces of evidence, a “null one,” that is, the ignorance of mankind, in that civilization, of any interaction but gravity. Unfortunately, there are no documents found, so far, supporting the above-mentioned claim.

In contrast to other interactions, gravity is the long-range force that cannot be shielded. It affects many, if not all, processes, activities, and phenomenae, starting from biological processes within biological systems, passing through human activities (e.g., lifting liquids, flights, rockets, etc.), ending with the stability of large-scale systems (e.g., atmosphere of the Earth and planets, solar system, stars, galaxies, and the Universe itself). A better understanding of gravity would certainly give a signal for the beginning of a new era of civilization.

Gravity, as we experience on the Earth’s surface and in the solar and similar systems, is associated with an attraction force. Theoretical physicists take this “fact” into consideration when constructing gravity theories. Newton has succeeded to quantify this attractive force using his law of universal gravity. Einstein, in the context of his theory of General Relativity (GR), has interpreted gravity as a geometric property, \textit{space-time curvature}. It has been shown that, using Einstein’s point of view, one can interpret more physical phenomenae than using Newton’s one.

Although GR is the most acceptable theory for gravity, so far, it suffers from several problems, especially those connected with recent observations. None of the existing theories of gravity, including GR, can interpret the results of the following observations, for instance

1. supernova type Ia observation [1];
2. the rotation velocities of stars in spiral galaxies [2];
3. pioneer 10, 11 velocity observation, “Pioneer Anomaly” [3];
4. the mass discrepancy in clusters of galaxies [4].

Such observations indicate that our understanding of gravity is not complete enough. It seems that there is something missing in the theories describing gravity. Such theories should be modified or replaced by others, that take into account the \textit{missing factors}, if any. Many authors have tried to tackle such problems, suggesting different solutions. The most famous candidate used is “Dark Energy” (cf. [5–8]), an exotic term implying the existence of an unknown force, most likely repulsive.

Assuming that \textit{attraction} is one side of gravity and \textit{repulsion} is its other side, many of the recent gravity problems can be analyzed, understood, and solved. This paper discusses briefly some experimental and observational pieces of evidence, also theoretical predictions, for the existence of the other side of gravity, the \textit{repulsive} side. This may illuminate the road towards a more satisfactory theory for gravity and a better understanding of this interaction. Pieces of evidences for attractive gravity are popular and do not need any sophisticated equipments to discover. In contrast, lines of evidence for \textit{repulsive} gravity are not so obvious and need sophisticated technology to be explored. In what follows, we are going to discuss, briefly, two of these lines of evidence. The first evidence is on the very large scale, the cosmic scale, while the second evidence is on the laboratory scale.
2. Observational and Experimental Evidence for Repulsive Gravity

The first evidence emerged from the analysis of the results of Supernovae type Ia observation [1]. These observations need space telescopes and equipments and would not have been carried out without the use of such sophisticated space technology. The objects associated with this type of Supernovae are considered to be standard candles, which can be used to measure long distances in the Universe with high accuracy. On the other hand, radial velocities of such objects can be easily obtained as a result of measurements of their red shifts. Knowing distances and velocities, one can get the rate of expansion of our Universe, using Hubble’s relation. It has been shown [1] that the Universe is in a phase, with an accelerating expansion rate. This result is in contradiction with all accepted theories of gravity, including GR (with vanishing cosmological constant). The increasing rate of expansion indicates very clearly that there is a large scale repulsive force driving the expansion of the Universe.

The second evidence comes from a sophisticated experiment which has been suggested and carried out from more than three decades ago, in an Earth’s laboratory. This experiment is known in the literature as the “COW” experiment. It has been suggested by Colella, Overhauser, and Werner in 1974 and carried out many times starting from 1975 to 1997 [9–13]. The results of this experiment indicate clearly that there is a real discrepancy with existing theories. Before discussing this discrepancy, we give a simple account on the experiment.

The experiment studies quantum interference of thermal neutrons moving in the Earth’s gravitational field. A neutron interferometer is used for this purpose (see Figure 1). A beam of thermal neutrons A is split into two beams A1, A2 at the point a. The beam A1 is reflected at b, while A2 is reflected at d. The two reflected beams A1 and A2 interfere at the point c of the interferometer.

Assuming that the path lengths ab = dc, ad = bc and that the trajectory of the neutrons is affected by the Earth’s gravitational potential, a phase difference between the two beams A1 and A2 is expected. This is due to the difference in the Earth’s gravitational potential affecting the paths ab and dc, (since ab is more close to the Earth’s surface than dc). Using the interference pattern, one can measure the phase difference, and consequently the difference in the Earth’s gravitational potential.

The theories used for calculating the phase shift have been quantum mechanics and Newton’s theory of gravitation (The Earth’s gravitational field is a weak field. Newton’s theory of gravitation is a limiting case of GR in the weak field regime. So both theories will give about the same prediction). It has been found that the experimental results are lower than theoretical predictions by eight parts in one thousand (0.008), while the sensitivity of
the interferometer used is one part in one thousand (0.001). Consequently, there is a real discrepancy between the results of this experiment and theoretical predictions [9].

Now, the results of this experiment show clearly that the Earth gravitational potential measured is different from that predicted by Newton theory of gravity (or by GR), even in the weak field regime. The absolute value (measured) of this potential is less than the corresponding value predicted by known theories of gravity! One probable interpretation is that there is a repulsive force reducing the value of the potential, predicted by theories that take into consideration attraction only.

The above two lines of evidence give a probable indication that there is a repulsive force affecting trajectories of particles, whether long range (photons in the cosmos) or short range (neutrons in the laboratory). Now, we have two possible approaches for interpreting the above evidence. The first is that they can be considered as indicators for the existence of a new interaction, fifth force, different from those given in the introduction. The second is that one, or more, of the four known interactions is not well understood. The first possibility has been extensively examined (cf. [14, 15]). So, let us examine the second one. Weak and strong interactions can be easily ruled out, since they are of very short range (the order of one Fermi). Also, the electromagnetic interaction can be excluded since the two pieces of evidence considered concern trajectories of electrically neutral particles (photons or neutrons). Thus, we are left with the gravitational interaction only. Deep examination of this interaction may lead to a better understanding of gravity.

If we assume that gravity has two sides as mentioned above. The first is the side that is well known on the Earth’s surface and in the solar and similar systems, the side connected with attraction. The second is the side connected with repulsion which is not so obvious in the solar system. Then, two important questions emerge as follows.

(i) What geometric object (Assuming the geometrization philosophy (very successful in dealing with gravity) is being applied) is responsible for repulsion?

(ii) Why repulsion is so small, compared with attraction, in some systems while it is relativity large in others?

In what follows we are going to discuss two theoretical (geometric) features predicting, very naturally, the existence of repulsive gravity. This will give possible answers to the above-mentioned questions and, consequently, a convincing theoretical interpretation for accelerating expansion of the Universe and for the discrepancy in the COW experiment.

3. Geometric Predictions of Repulsive Gravity

In the last decade, many attempts have been done suggesting new theories, or modifying the existing theories, of gravity in order to account for the accelerating expansion of the Universe and the repulsive force driving it. These attempts can be classified into two classes: physical and geometrical. The physical class includes suggestions about the existence of types of peculiar matter, having certain equations of state, filling the Universe (e.g., Chapling gas [16], phantom [17], etc.). The geometrical class comprises geometric suggestions to solve the problem (e.g., increasing the number of space-time dimensions [18], increasing the order of the curvature scalar $R$ in the lagrangian, $f(R)$ theories [19], the use of geometries with non-vanishing torsion [20], the increase of order of torsion ($T$) in the lagrangian, $f(T)$ theories [21], etc.). None of the above-mentioned attempts could explain, satisfactorily, the repulsive features of gravity. If one of these attempts is accepted as an interpretation for accelerating
expansion of the Universe, it cannot account for the discrepancy of the COW experiment, discussed above.

In what follows we are going to give a brief account on an attempt, belonging to the geometric class, that can give a convincing interpretation for both large-scale and laboratory scale problems as those given in the above Section. This attempt also gives two geometric properties for the existence of repulsive gravity, and convincing answers to the two questions, raised at the end of Section 2.

Before reviewing the attempt, we are going to give a brief idea, as simple as possible, about the underlying geometry of this attempt. The geometric structure used is called the “Parameterized Absolute Parallelism” (PAP) geometry [22]. In 4-dimensions, the structure of a PAP space is defined completely by a tetrad vector field. The general linear connection characterizing this space is written as (we are using starred symbol, to characterize an object belonging to the PAP-geometry, while the same symbol, unstarred, is used for AP objects (\( b = 1 \))

\[
\Gamma^{\alpha \cdot \beta \sigma} = \left\{ \frac{\alpha}{\beta \sigma} \right\} + bY^{\alpha \cdot \beta \sigma}, \tag{3.1}
\]

where \( \left\{ \rho_{\alpha} \right\} \) is the ordinary Christoffel symbol of the Riemannian space (used to construct GR), \( Y^{\alpha \cdot \beta \sigma} \) is a third order nonsymmetric tensor, called contortion, defined in the PAP-space, and \( b \) is a dimensionless parameter whose importance will be discussed later. An important feature of the PAP-space is that it is more general than both Riemannian and conventional Absolute Parallelism (AP) spaces in the sense that

(i) for \( b = 0 \) the PAP-space covers all the Riemannian structure, without any need for a vanishing contortion;

(ii) for \( b = 1 \) the PAP-space reduces to the conventional AP space.

Among other things, these features facilitate comparison between a theory constructed in the PAP-space and the results of any other theory constructed in the AP space or in the Riemannian one, including GR.
The antisymmetric part of the parameterized linear connection (3.1) is called the torsion $\Lambda^a_{\beta\sigma}$ of the connection:

$$\Lambda^a_{\beta\sigma} = b\Lambda^a_{\beta\sigma},$$

where $\Lambda^a_{\beta\sigma}$ is the torsion of the AP space. The PAP curvature tensor is defined by the fourth order tensor [20] given by

$$B^a_{\mu\nu\sigma} \overset{\text{def.}}{=} R^a_{\mu\nu\sigma} + bQ^a_{\mu\nu\sigma} \neq 0,$$

where

$$R^a_{\mu\nu\sigma} \overset{\text{def.}}{=} \left\{ \frac{\alpha}{\mu\sigma} \right\}_\nu - \left\{ \frac{\alpha}{\mu\nu} \right\}_\sigma + \left\{ \frac{\alpha}{e\nu} \right\}_\mu - \left\{ \frac{\alpha}{e\sigma} \right\}_\mu = 0,$$

is the Riemann-Christoffel curvature tensor and

$$Q^a_{\mu\nu\sigma} \overset{\text{def.}}{=} \left\{ \frac{\gamma^a}{\mu\sigma} \right\}_\nu - \left\{ \frac{\gamma^a}{\mu\nu} \right\}_\sigma + b\left[ \frac{\gamma^e}{\mu\nu} \frac{\gamma^a}{e\sigma} - \frac{\gamma^e}{e\mu} \frac{\gamma^a}{\nu\sigma} \right],$$

is a tensor of type (1, 3), purely made of $\Upsilon^a_{\beta\sigma}$.

The curvature (3.3) is, in general, nonvanishing. Consequently, the PAP-space is of the Riemann-Cartan type, that is having simultaneously non-vanishing torsion (3.2) and curvature (3.3).

Now, the two geometric properties predicting the existence of antigravity, and consequent repulsive force, are given below.

(1) Since the PAP geometry covers, at least the domains of both Riemannian and AP-geometries as limiting cases, we are going to use the advantages and properties of these two limiting cases to discuss relation (3.3). The tensor $R^a_{\beta\sigma\delta}$, in Riemannian geometry, measures the curvature of the space, that is, the deviation of the space from being flat. It is completely made of Christoffel symbols. Its vanishing is a necessary and sufficient condition for the space to be flat. Einstein’s idea has been to use this tensor as a measure of the gravitational field of the system.

Let us now examine the curvature in the case of the AP space. It can be written, using (3.3) with $b = 1$, as

$$B^a_{\beta\sigma\delta} = R^a_{\beta\sigma\delta} + Q^a_{\beta\sigma\delta} \equiv 0,$$

where $Q^a_{\beta\sigma\delta}$ is the limiting case of $\frac{Q^a}{\beta\sigma\delta}$ for $b = 1$. It is to be considered that neither $R^a_{\beta\sigma\delta}$ nor $Q^a_{\beta\sigma\delta}$ vanishes, while their sum, that is, the curvature $B^a_{\beta\sigma\delta}$ of the AP space, vanishes identically. This implies an interesting property, that is, the non-vanishing tensors $R^a_{\beta\sigma\delta}$ and $Q^a_{\beta\sigma\delta}$ compensate (balance) each other in such a way that the total curvature of the space is zero. This compensation gives rise to the flatness of the AP space. Now, as $R^a_{\beta\sigma\delta}$ measures
the curvature of the space, $Q^a_{\cdot \beta \sigma \delta}$ represents the additive inverse of this curvature. For this reason, we call $Q^a_{\cdot \beta \sigma \delta}$ the “anticurvature” tensor [23], and consequently the tensor defined by (3.5) is the “parameterized antigravity” tensor.

An interesting physical result can now be obtained. Einstein has used curvature $R^a_{\cdot \beta \sigma \delta}$ as a geometric object representing gravity, in his theory of GR. Similarly, we can use the antigravity $Q^a_{\cdot \beta \sigma \delta}$ as a geometric object representing antigravity, in any suggested theory. But, the complete balance between $R^a_{\cdot \beta \sigma \delta}$ and $Q^a_{\cdot \beta \sigma \delta}$ gives rise to a flat space, that is, balance of the two (basic features) sides of gravity. Observationally, this is not the case, at least in the solar and similar systems, in which gravity dominates over antigravity, that is, curvature dominates over antigravity. Thus, one needs a certain parameter, to be adjusted, in order to fine tune the ratio between the curvature and antigravity in any theory dealing with both sides of gravity. This is ready and clarifies the importance of the parameter $b$, which appears in (3.5), in the PAP-geometry. The presence of this parameter in (3.3) makes the PAP-curvature non-vanishing (in general $b \neq 1$).

This represents the first geometric feature which shows how antigravity can be predicted in the context of the geometrization philosophy. It gives the first geometric feature predicting the existence of antigravity, on theoretical basis.

(2) In the context of the geometrization philosophy, path equations in any appropriate geometry are used to represent trajectories of test particles. For example, the geodesic equation, of Riemannian geometry, is used as equation of motion of a test particle (e.g., planet) in the solar system, in the context of GR. Now, for the PAP-geometry, the path equation can be written in the following form [20]:

$$\frac{d^2 x^\mu}{d \tau^2} + \left\{ \mu \frac{dx^a}{d \tau} \frac{dx^\beta}{d \tau} \right\} = -b^\Lambda_{(ab)} \frac{dx^a}{d \tau} \frac{dx^\beta}{d \tau}.$$  \hspace{2cm} (3.7)

where $\tau$ is the parameter characterizing the path. If $b = 0$, (3.7) reduces to the ordinary geodesic of Riemannian geometry. Equation (3.7) can be considered as a geodesic equation modified by a torsion term. For any field theory written in the PAP-geometry, (3.7) can be used as an equation of motion of a neutral test particle moving in the field, in the domain of this theory.

In order to understand (3.7), physically, let us analyze it using Newton’s terminology. The first term of (3.7) can be considered as the generalized acceleration of a test particle. The other two terms can be viewed as representing two forces driving the motion of this test particle. On one hand, the first force is related to the Christoffel symbols $\{ \cdot_{\beta \sigma} \cdot_{\delta} \}$, which is the only geometric object forming the curvature $R^a_{\cdot \beta \sigma \delta}$. This force is the gravity force, since it is connected with the curvature of the space time. On the other hand, the second force is connected with the torsion (or contortion) [24] in such a way that the vanishing of one is a necessary and sufficient condition for the vanishing of the other. So, in principle, any function of the contortion can be easily written in terms of the torsion and vice versa.) of space-time. Since the contortion (or torsion) is the only ingredient forming the antigravity $Q^a_{\cdot \beta \sigma \delta}$, then by similarity, we can call this force the “antigravity force”. So, this equation can be written in the following block equation:

$$\text{Acceleration} + \text{Gravity force} = \text{Antigravity force}.$$  \hspace{2cm} (3.8)
Consequently, a complete balance between gravity and antigravity forces would result in the vanishing of acceleration.

In order to explore the quantitative nature of these two forces, let us examine the consequences of linearizing (3.7). It has been shown [25] that the potential $\phi$ resulting from the existence of the two forces (two sides of gravity) is given by

$$\phi = \phi_N - b\phi_N. \quad (3.9)$$

The first term, on the R.H.S., is the Newtonian potential due to gravity and the second term is the potential due to antigravity, written in terms of $\phi_N$, for simplicity. Recalling the classical relation between potential and force, and knowing that $b \geq 0$ appear in due course as will, then we can easily conclude that as gravity force is attractive, antigravity force is necessarily repulsive (due to the negative sign on the R.H.S. of (3.9)).

Now, we come to the dimensionless parameter $b$ of (3.7). As stated above, the function of this parameter is to adjust a certain ratio between gravity and antigravity (i.e., between attraction and repulsion) in a certain system. This parameter can be decomposed as follows [23]:

$$b = \frac{n}{2}a\gamma = \frac{\text{antigravity}}{\text{gravity}}, \quad (3.10)$$

where $n$ is a natural number taking the values 0, 1, 2, ..., for particles with quantum spin 0, 1/2, 1, ..., respectively; $a$ is the fine structure constant ($\sim 1/137$) and $\gamma$ is a dimensionless parameter depending on the size of the system under consideration, to be fixed by experiment or observation. The vanishing of $b$ switches off antigravity in any system and reduces any suggested theory, constructed in the PAP-geometry, to a conventional gravity theory (e.g., orthodox GR). Also, in this case (3.7) reduces to the geodesic equation, in which attraction is the only force affecting the trajectory of any test particle.

The discussion given above represents the second geometric feature which shows the quantity and quality of the repulsive force predicted in the context of the geometrization philosophy.

From the discussion given in (1) and (2), one can outlines the main features of a geometric theory predicting and dealing with the two sides of gravity. Applying such theory, a satisfactory interpretation can be achieved to the discrepancy in the COW experiment [26] and for the accelerating expansion of the Universe [27]. The values of the parameter $\gamma$ are found to be of order unity for the Earth’s system and which more greater than unity for the Universe.

For the two questions raised at the end of the previous Section, we have now the following answers.

(1) For the first question: the geometric object responsible for repulsion is the anticurvature tensor (3.5) (or the torsion tensor) of the background geometry characterizing the field.

(2) For the second question: the strength of the repulsive force depends on the value of the parameter $\gamma$ which characterizes the size of the system under consideration.
4. Concluding Remarks

(1) Two main philosophies are used in the 20th century, to solve the emerged physical problems, quantization and geometrization.

(2) Gravity problems are successfully solved using the geometrization philosophy, not the quantization one.

(3) The two main objects characterizing any geometry are curvature and torsion (giving rise to anticurvature).

(4) Einstein has used the curvature in constructing the theory of General Relativity, solving the problems of attractive gravity in the Solar and comparative systems, in the context of the geometrization philosophy.

(5) Some experiments and recent observations give strong evidence for the existence of repulsive gravity together with the attractive one.

(6) In the present paper it is shown that using both curvature and the parameterized anticurvature, one can account for both sides of gravity, attraction and repulsion, and consequently give a satisfactory interpretation for accelerating expansion of the Universe and the discrepancy of COW experiment.

(7) The modified geodesic equation (3.7), in its linearized form, shows clearly that the force resulting from the torsion term (R.H.S. of (3.7)) is a repulsive force (see (3.9)).

The schematic diagram shown in Figure 2 summarizes the advantages of using a complete geometry, that is, a geometry with simultaneously non-vanishing curvature and torsion, in constructing field theories for gravity. The left-hand branch of this diagram gives the geometrization scheme to be used to construct GR (attractive gravity only). The right-hand branch gives a geometrization scheme to be used to construct a theory for both sides of gravity.

References


