Research Article
Cross Sections of Charged Current Neutrino Scattering off $^{132}$Xe for the Supernova Detection

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The total cross sections as well as the neutrino event rates are calculated in the charged current neutrino and antineutrino scattering off $^{132}$Xe isotope at neutrino energies $E_\nu < 100$ MeV. Transitions to excited nuclear states are calculated in the framework of quasiparticle random-phase approximation. The contributions from different multipoles are shown for various neutrino energies. Flux-averaged cross sections are obtained by convolving the cross sections with a two-parameter Fermi-Dirac distribution. The flux-averaged cross sections are also calculated using terrestrial neutrino sources based on conventional sources (muon decay at rest) or on low-energy beta-beams.

1. Introduction

The detection of neutrinos and their properties is one of the top priorities of modern nuclear and particle physics as well as astrophysics. Among the probes which involve neutrinos, the neutrino-nucleus reactions possess a prominent position. Detailed predictions of neutrino-nucleus cross sections (NNCS) are crucial to detect or distinguish neutrinos of different flavor and explore the basic structure of the weak interactions [1–14]. Measured cross sections for neutrino-nucleus scattering at neutrino energies which are relevant for supernova neutrinos are available in only a few cases, that is, for $^{56}$Fe [15], $^{12}$C [15, 16], and the deuteron [17]. The use of microscopic nuclear structure models is therefore essential, for a quantitative description of neutrino-nucleus reactions. These include the nuclear shell model [18, 19], the random-phase approximation (RPA), relativistic RPA [20, 21], continuum RPA (CRPA) [22], quasiparticle RPA (QRPA) [23–26], projected quasiparticle RPA (PQRPA) [27], hybrid models of CRPA, the shell model [28, 29], and the Fermi gas model [30]. The shell model provides a very accurate description of ground-state wave functions. The description of high-lying excitations, however, necessitates the use of large-model spaces, and this often leads to computational difficulties, making the approach applicable essentially only to light- and medium-mass nuclei. Therefore, for, systematic studies of weak interaction rates for relevant heavy nuclei of mass number around $A = 128–132$, microscopic calculations must be performed using models based on the RPA [23, 25].

The signature of supernova neutrino interaction taking place in various detectors is the observation of electrons, positrons, photons, and other particles which are produced through the charged and neutral current interactions. Two processes that contribute to the total event rates in the detectors are the charged current (CC) reactions

$$\nu_e + ZX_N \rightarrow Z+1X_{N-1}^* + e^-,$$

$$\bar{\nu}_e + Z X_N \rightarrow Z-1X_{N+1}^* + e^+$$

(1)

and the neutral current (NC) reactions

$$\nu_x (\bar{\nu}_x) + Z X_N \rightarrow X_{x^*}, \quad x = e, \mu, \tau.$$  

(2)

The neutrinos $\nu_x$ (or antineutrinos $\bar{\nu}_x$) with $x = \mu, \tau$ do not have sufficient energy to produce corresponding leptons in charged current reactions and interact only through neutral current interactions and therefore have a higher-average energy than $\nu_e$ and $\bar{\nu}_e$, which interact through charged current as well as neutral current interactions. Numerical simulations give the following values of average energy for the different neutrino flavors, that is, $\langle E_{\nu_e} \rangle \sim 10-11$ MeV, $\langle E_{\nu_x} \rangle \sim 15-16$ MeV, and $\langle E_{\nu_x} \rangle \sim 23–25$ MeV, and are consistent with
the supernova neutrino spectrum given by a Fermi-Dirac distribution [31, 32]:
\[
\phi(E_\nu) = \frac{N_2(\alpha)}{T^3} \frac{E_\nu^2}{1 + \exp[(E_\nu/T) - \alpha]},
\]
where \(T\) is the neutrino temperature, \(\alpha\) is a degeneracy parameter taken to be either 0 or 3. \(N_2(\alpha)\) denotes the normalization factor depending on \(\alpha\) given from
\[
N_k(\alpha) = \left( \int_0^\infty \frac{x^k}{1 + e^{\alpha - x}} dx \right)^{-1},
\]
for \(k = 2\). Following [33], the average neutrino energy \(\langle E_\nu \rangle\) can be written in terms of the functions of (4) as
\[
\langle E_\nu \rangle = \frac{N_2(\alpha)T^3}{N_3(\alpha)}. \tag{5}
\]
Most calculations of neutrino-nucleus cross sections have been taken the value \(\alpha = 0\). However, in astrophysical applications, it might be important to perform studies of reaction rates for different values of \(\alpha\) depending on the simulation performed and on the specific supernova phase considered [29, 34]. In our study, the value \(\alpha = 3\) has also been used. The average energy values for the various neutrino species imply that for \(\alpha = 0\), the values of temperature \(T\) are 3.5 MeV (2.75 MeV) for \(\nu_e\), 5 MeV (4 MeV) for \(\overline{\nu}_e\), and 8 MeV (6 MeV) for \(\nu_x\) (\(x = \mu, \tau, \nu_\mu, \nu_\tau\)). The recent theoretical studies predict a smaller value of temperature for \(\nu_x\) which is closer to \(\overline{\nu}_e\) [34–37].

Systematic neutrino-nucleus interaction measurements could be an ideal tool to explore the weak nuclear response. At present, new experiments on various nuclei are being proposed with a new facility using muon decay at rest [38]. Another possibility could be offered by beta-beams. This is a new method to produce pure and well-known electron neutrino beams, exploiting the beta-decay of boosted radioactive ions [39]. The idea of establishing a low-energy beta-beam facility has been first proposed in [40] and discussed in nuclear structure studies, core-collapse supernova physics, and the study of fundamental interactions [40–49].

A detector whose active target consists of the noble liquid Xenon can offer unique detection capabilities in the field of neutrino physics [47, 50] as well as the ability to detect very low-energy signals in the context of dark matter searches [51, 52]. The new concept of a spherical TPC detector, filled with high-pressure Xenon, has also been proposed as a device able to detect low-energy neutrinos as those coming from a galactic supernova. In particular, a TPC detector can be used to observe coherent NC as well as CC neutrino-nucleus scattering [37, 53–58].

In this paper, we present microscopic calculations of the CC
\[
\nu_e (\overline{\nu}_e) + ^{132}\text{Xe} \rightarrow ^{132}\text{Cs}^* (1^{+}3^+1^+) + e^- (e^+) \tag{6}
\]
reaction cross sections. The corresponding-reduced matrix elements in the low- and intermediate-neutrino energy range have been calculated in the framework of quasiparticle random-phase approximation (QRPA). We present the total neutrino-nucleus cross sections as well as the contribution of the various multipole and discuss how their importance evolves, as a function of neutrino energy. A comparison between the CC cross sections and those involved by the coherent NC ones [37, 55] is also presented. Finally, we give the flux-averaged cross sections associated to the Fermi-Dirac distribution as well as to distributions based on terrestrial neutrino sources such as the low-energy beta-beams or to conventional sources (muon decay at rest).

2. The Formalism for Neutrino-Nucleus Cross Sections Calculations

Let us consider a neutral or charged current neutrino-nucleus interaction in which a low- or intermediate-energy neutrino (or antineutrino) is scattered inelastically from a nucleus \((A, Z)\). The initial nucleus is assumed to be spherically symmetric having ground state a \(|1^+\rangle = |0^+_1\rangle\) state. The corresponding standard model effective Hamiltonian of the current-current interaction can be written as
\[
\mathcal{H} = \frac{G a_{\text{CC,NC}}}{\sqrt{2}} j_\mu (x) J^\mu (x), \tag{7}
\]
where \(G = 1.1664 \times 10^{-5} \text{ GeV}^{-2}\) is the Fermi weak coupling constant, \(a_{\text{CC}} = \cos \theta_W\) for charged current reaction, and \(a_{\text{NC}} = -5\) for neutral current reaction. \(j_\mu\) and \(J^\mu\) denote the leptonic and hadronic currents, respectively. According to V-A theory, the leptonic current takes the form
\[
j_\mu = \overline{\psi}_{\nu_\mu} (x) \gamma_\mu (1 - \gamma_5) \psi_{\nu_\mu} (x), \tag{8}\]
where \(\psi_{\nu_\mu}\) are the neutrino/antineutrino spinors.

From a nuclear physics point of view, only the hadronic current is important. The structure for neutral current (NC) and charged current (CC) processes of both vector and axial-vector components (neglecting the pseudoscalar contributions) is written as
\[
j_\mu^{\text{CC,NC}} = \overline{\psi}_N \left[ F_1^{\text{CC,NC}} (q^2) \gamma_\mu + F_2^{\text{CC,NC}} (q^2) \frac{i \sigma_{\mu\nu} q^\nu}{2M} \right] \psi_N + F_A^{\text{CC,NC}} (q^2) \gamma_\mu \gamma_5 \psi_N, \tag{9}\]
where \(M\) stands for the nucleon mass, and \(\psi_N\) denote the nucleon spinors. The form factors \(F_1^{\text{CC}} (q^2)\) and \(F_A^{\text{CC}} (q^2)\) are defined as
\[
F_{1,2}^{\text{CC}} (q^2) = F_{1,2}^{\text{NC}} (q^2) - F_{1,2}^p (q^2), \tag{10}
\]
\[
F_A^{\text{CC}} (q^2) = F_A^{\text{NC}} (q^2).
\]
and the neutral current form factors $F_{1,2}^{NC}(q^2)$ and $F_A^{NC}(q^2)$ as

\[
F_{1,2}^{NC}(q^2) = \left( \frac{1}{2} - \sin^2 \theta_W \right) \left[ F_{1,2}^P (q^2) - F_{1,2}^n (q^2) \right] \tau_0 \\
- \sin^2 \theta_W \left[ F_{1,2}^P (q^2) + F_{1,2}^n (q^2) \right],
\]

(11)

\[
F_A^{NC}(q^2) = \frac{1}{2} F_A (q^2) \tau_0.
\]

Here, $\tau_0$ represents the nucleon isospin operator, and $\theta_W$ is the Weinberg angle ($\sin^2 \theta_W = 0.2325$). The detailed expressions of nucleonic form factors $F_{1,2}^{p,n}(q^2)$ are given in [59]. The axial-vector form factor $F_A(q^2)$ is given by [60]

\[
F_A = -g_A \left( 1 - \frac{q^2}{M_A^2} \right)^2,
\]

(12)

where $M_A = 1.05$ GeV is the dipole mass, and $g_A = 1.258$ is the static value (at $q = 0$) of the axial form factor.

In the convention we used in the present paper, $q^2$, the square of the momentum transfer, is written as

\[
q^2 = q^a q_a = \omega^2 - q^2 = (\epsilon_i - \epsilon_f)^2 - \left( \mathbf{p}_i - \mathbf{p}_f \right)^2,
\]

(13)

where $\omega = \epsilon_i - \epsilon_f$ is the excitation energy of the nucleus. $\epsilon_i$ denotes the energy of the incoming neutrino and $\epsilon_f$ denotes the energy of the outgoing lepton. $\mathbf{p}_i$, $\mathbf{p}_f$ are the corresponding 3-momenta. In (11), we have not taken into account the strange quark contributions in the form factors. In the scattering reaction considered in our paper, only low-momentum transfers are involved, and the contributions from strangeness can be neglected [61].

The neutrino/antineutrino-nucleus differential cross section, after applying a multipole analysis of the weak hadronic current, is written as

\[
\sigma (\epsilon_i) = \frac{2G^2 (a_{CC,NC})^2}{2I_f + 1} \sum_f |\tilde{B}_f| \epsilon_f \\
\times \int_{-1}^{1} d (\cos \theta) F (\epsilon_f, Z_f) \\
\times \left( \sum_{J=0}^{\infty} \sigma_{CL}^J + \sum_{J=1}^{\infty} \sigma_{T}^J \right),
\]

(14)

where $\theta$ denotes the lepton scattering angle. The summations in (14) contain the contributions $\sigma_{CL}^J$, for the Coulomb $\tilde{M}_f$ and longitudinal $\tilde{F}_f$, and $\sigma_{T}^J$, for the transverse electric $\tilde{E}_f^T$ and magnetic $\tilde{H}_f^T$ multipole operators [62]. These operators include both polar-vector and axial-vector weak interaction components. The contributions $\sigma_{CL}^J$ and $\sigma_{T}^J$ are written as

\[
\sigma_{CL}^J = \left( 1 + a \cos \theta \right) \left| \langle J_f \tilde{M}_f (q) \rangle \right|^2 \\
+ \left( 1 + a \cos \theta - 2b \sin^2 \theta \right) \left| \langle J_f \tilde{M}_f (q) \rangle \right|^2 \\
+ \left[ \frac{\omega}{q} (1 + a \cos \theta) + c \right] \\
\times 2 \Re \left\langle J_f \tilde{M}_f (q) \right| \langle \langle J_f \tilde{M}_f (q) \right| \right. \\
\times \left. \left( \langle \langle \tilde{M}_f (q) \rangle \right|^2 \\
+ \left| \langle J_f \tilde{M}_f (q) \rangle \right|^2 \right] \\
+ \left[ \left( \epsilon_i + \epsilon_f \right) \right. \\
\times \left. \frac{1}{q} (1 - a \cos \theta) - c \right] \\
\times 2 \Re \left\langle J_f \tilde{M}_f (q) \right| \langle \langle J_f \tilde{M}_f (q) \right| \rangle,
\]

(15)

where $b = \epsilon_i \epsilon_f / q^2$, $a = (1 - (m_f c^2 / \epsilon_f) )^{1/2}$, and $c = (m_f c^2) / \epsilon_f$. In (16), the ($-$) sign corresponds to neutrino scattering and the ($+$) sign to antineutrino. The absolute value of the three momentum transfers is given by

\[
q = |q| = \sqrt{\omega^2 + 2 \epsilon_i \epsilon_f (1 - a \cos \theta) - (m_f c^2)^2}.
\]

(17)

For charged current reactions, the cross-sectional equation (14) must be corrected for the distortion of the outgoing lepton wave function by the Coulomb field of the daughter nucleus. The cross section can either be multiplied by the Coulomb correction factor [63], or, at higher energies, the effect of the Coulomb field can be described by the effective momentum approximation (EMA) [63–65]. In this approximation, the lepton momentum $p_f$ and energy $\epsilon_f$ are modified as

\[
p_f \text{eff} = \frac{1}{c} \sqrt{\left( \epsilon_f \text{eff} \right)^2 - \left( m_f c^2 \right)^2}
\]

where $V_C^{\text{eff}}$ is the effective Coulomb potential. In a recent study using exact Dirac wave functions, it has been shown that an accurate approximation for the effective electron momenta is obtained by using the mean value of the Coulomb potential, $V_C = 4V_C(0)/5$, where $V_C(0) = -3Z/4\alpha/(2R)$ corresponds to the electrostatic potential evaluated at the center of the nucleus [66, 67]. $Z_f$ is the charge of the daughter nucleus, and $R$ is its radius assuming spherical charge distribution. $\alpha$ denotes the fine structure constant. In calculations with EMA, the original lepton momentum $p_f$ and energy $\epsilon_f$ appearing in the expression for the cross section are replaced by the above effective quantities.
3. Energies and Wave Functions

For neutral-current-neutrino-nucleus-induced reactions, the ground state and the excited states of the even-even nucleus are created using the quasiparticle random-phase approximation (QRPA) including two quasineutron and two quasiproton excitations in the QRPA matrix [68] (hereafter denoted by pp-nn QRPA). We start by writing the A-fermion Hamiltonian $\hat{H}$, in the occupation-number representation, as a sum of two terms. One is the sum of the single-particle energies $\{e_{\alpha}\}$, which runs over all values of quantum numbers $\alpha \equiv \{n_{\alpha}, l_{\alpha}, j_{\alpha}, m_{\alpha}\}$ and the second term which includes the two-body interaction $V$, that is,

$$H = \sum_{\alpha} e_{\alpha} c_{\alpha}^+ c_{\alpha} + \frac{1}{4} \sum_{\alpha \beta \gamma \delta} \nabla_{\alpha \beta \gamma \delta} c_{\alpha}^+ c_{\beta}^+ c_{\gamma}^+ c_{\delta},$$  \hspace{1cm} (19)

where the two-body term contains the antisymmetric two-body interaction matrix element defined by $\nabla_{\alpha \beta \gamma \delta} = \langle \alpha \beta | V | \gamma \delta \rangle - \langle \alpha \beta | V | \delta \gamma \rangle$. The operators $c_{\alpha}^+$ and $c_{\alpha}$ stand for the usual creation and destruction operators of nucleons in the state $\alpha$.

For spherical nuclei with partially filled shells, the most important effect of the two-body force is to produce pairing correlations. The pairing interaction is taken into account by using the BCS theory [69]. The simplest way to introduce these correlations in the wave function is to perform the Bogoliubov-Valatin transformation:

$$\begin{align*}
\bar{a}_{\alpha}^+ &= u_{\alpha} a_{\alpha}^+ - v_{\alpha} a_{\alpha}, \\
\bar{a}_{\alpha} &= u_{\alpha} a_{\alpha}^+ + v_{\alpha} a_{\alpha},
\end{align*}$$  \hspace{1cm} (20)

where $c_{\alpha}^+ = a_{\alpha}^+ (-1)^{l_{\alpha}+m_{\alpha}}$, $\bar{a}_{\alpha}^+ = a_{\alpha}^+ (-1)^{l_{\alpha}+m_{\alpha}}$, and $\alpha \equiv \{n_{\alpha}, l_{\alpha}, j_{\alpha}, m_{\alpha}\}$. The occupation amplitudes $v_{\alpha}$ and $u_{\alpha}$ are determined via variational procedure for minimizing the energy of the BCS ground state for protons and neutrons, separately. In the BCS approach, the ground state of an even-even nucleus is described as a superconducting medium, where all the nucleons have formed pairs that effectively act as bosons. The BCS ground state is defined as

$$\text{[BCS]} = \prod_{\alpha > 0} (\bar{u}_{\alpha} - v_{\alpha} c_{\alpha}^+ c_{\alpha}) \text{[CORE]} ,$$  \hspace{1cm} (21)

where [CORE] represents the nuclear core (effective particle vacuum).

After the transformation (20), the Hamiltonian can be written in its quasiparticle representation as

$$H = \sum_{\alpha} E_{\alpha} a_{\alpha}^+ a_{\alpha} + H_{\text{qp}},$$  \hspace{1cm} (22)

where the first term gives the single-quasiparticle energies $E_{\alpha}$, and the second one includes the different components of the residual interaction.

In the present calculations, we use a renormalization parameter $g_{\text{pair}}$ which can be adjusted solving the BCS equations. The monopole matrix elements $\langle \alpha \alpha; f = 0 | V | \beta \beta; f = 0 \rangle$ of the two-body interaction are multiplied by a factor $g_{\text{pair}}$. The adjustment can be done by comparing the resulting lowest-quasiparticle energy to the phenomenological energy gap $\Delta$ obtained from the separation energies of the neighboring doubly even nuclei for protons and neutrons, separately.

The excited states of the even-even reference nucleus are constructed by use of the QRPA. In the QRPA, the creation operator for an excited state $|\omega; J_f^M\rangle$ has the form

$$\hat{Q}^+ (J_{\omega}^M) = \sum_{\alpha \leq \alpha'} \left[ X_{\alpha \alpha'} J_f^M A^+ (\alpha \alpha'; J M) - Y_{\alpha \alpha'} J_f^M \bar{A} (\alpha \alpha'; J M) \right],$$  \hspace{1cm} (23)

where the quasiparticle pair creation $A^+ (\alpha \alpha'; J M)$ and annihilation $\bar{A} (\alpha \alpha'; J M)$ operators are defined as

$$A^+ (\alpha \alpha'; J M) \equiv (1 + \delta_{\alpha \alpha'})^{-1/2} [a_{\alpha} a_{\alpha'}]^+ J M,$$

$$\bar{A} (\alpha \alpha'; J M) \equiv (-1)^{J_f + M} A (\alpha' J - M),$$  \hspace{1cm} (24)

where $\alpha$ and $\alpha'$ are either proton (p) or neutron (n) indices, $M$ labels the magnetic substates, and $\omega$ numbers the states for particular angular momentum $J$ and parity $\pi$.

The $X$ and $Y$ forward- and backward-going amplitudes are determined from the QRPA matrix equation

$$(A \cdot B) \left( X \right) = \omega \left( Y \right),$$  \hspace{1cm} (25)

where $\omega$ denotes the excitation energies of the nuclear state $|J_f^M\rangle$. The QRPA matrices, $A \cdot B$, are deduced by the matrix elements of the double commutators of $A^+$ and $A$ with the nuclear Hamiltonian $\hat{H}$ defined as

$$A \cdot \beta \beta' \equiv \langle \text{BCS} || [ A (\alpha \alpha'; J M), \hat{H}, A^+ (\beta \beta'; J M)] || \text{BCS} \rangle ,$$

$$B \cdot \beta \beta' \equiv - \langle \text{BCS} || [ A (\alpha \alpha'; J M), \hat{H}, \bar{A} (\beta \beta'; J M)] || \text{BCS} \rangle ,$$  \hspace{1cm} (26)

where $2[A, B, C] = [A, B, C] + [[A, B], C]$. Finally, the two-body matrix elements of each multipolarity $J^P$, occurring in the QRPA matrices $A \cdot B$ and $B \cdot B$, are multiplied by two phenomenological scaling constants, namely, the particle-hole strength $g_{\text{ph}}$ and the particle-particle strength $g_{\text{pp}}$.

These parameter values are determined by comparing the resulting lowest-phonon energy with the corresponding lowest-collective vibrational excitation of the doubly even nucleus and by reproducing some giant resonances which play crucial role.

For charged current neutrino-nucleus reactions, the excited states $|\omega; J_f^M\rangle$ of the odd-odd nucleus are generated adopting the proton-neutron QRPA(pnQRPA). The QRPA in its proton-neutron form contains phonons made out of proton-neutron pairs as follows:

$$\bar{Q}^+ (J_{\omega}^M) = \sum_{\text{pn}} \left[ X_{\text{pn}} J_f^M A^+ (\text{pn}; J M) - Y_{\text{pn}} J_f^M \bar{A} (\text{pn}; J M) \right].$$  \hspace{1cm} (27)
The matrices $A$ and $B$ defined in the canonical basis are

$$
\mathcal{A}_{pn,p' n'} = \delta_{pn,p' n'} (E_p + E_n) + g_{pp} (u_p \mu_n u_p u_n + v_p v_n v_p v_n) V_{pn,p' n'}^{pp}
$$

$$
+ g_{ph} (u_p \mu_n v_p v_n + v_p v_n u_p u_n) V_{pn,p' n'}^{ph}
$$

$$
\mathcal{B}_{pn,p' n'} = - g_{pp} (u_p \mu_n v_p v_n + v_p v_n u_p u_n) V_{pn,p' n'}^{pp}
$$

$$
+ g_{ph} (u_p \mu_n v_p v_n + v_p v_n u_p u_n) V_{pn,p' n'}^{ph},
$$

(28)

where $E_p$ and $E_n$ are the two-quasiparticle excitation energies, and $V_{pn,p' n'}^{pp}$ and $V_{pn,p' n'}^{ph}$ are the p-h and p-p matrix elements of the residual nucleon-nucleon interaction $V$, respectively. For charged current reactions, the matrix elements of any transition operator $\mathcal{O}_\lambda$ between the ground state $|0^+_g\rangle$ and the excited $|\omega; J\rangle$ can be factored as follows:

$$
\langle 0^+_g | \mathcal{O}_\lambda | \omega; J^p M \rangle = \sum_{pn} \langle p | \mathcal{O}_\lambda | n \rangle \left( X_{pn}^{\omega, p, \lambda} u_p + Y_{pn}^{\omega, p, \lambda} u_n \right),
$$

(29)

where $\langle p | \mathcal{O}_\lambda | n \rangle$ are the reduced matrix elements calculated independently for a given single-particle basis [70, 71].

4. Results and Discussion

Transition matrix elements of the type entering in (15) and (16) can be calculated in the framework of pnQRPA. The initial nucleus $^{132}\text{Xe}$ was assumed to be spherically symmetric having a $0^+$ ground state. Two-oscillator (3\hbar\omega and 4\hbar\omega) major shells, plus the intruder orbital $h_{11/2}$ from the next higher-oscillator major shell, were used for both protons and neutrons as the valence space of the studied nuclei. The corresponding single-particle energies (SPE) were produced by the Coulomb corrected Woods-Saxon potential using the parameters of Bohr and Mottelson [72].

The two-body interaction matrix elements were obtained from the Bonn one-boson-exchange potential applying G-matrix techniques [73]. The strong pairing interaction between the nucleons can be adjusted by solving the BCS equations. The monopole matrix elements of the two-body interaction are scaled by the pairing-strength parameters $g_{pp}^p$ and $g_{pair}^p$, separately, for protons and neutrons. The adjustment can be done by comparing the resulting lowest-quasiparticle energy to reproduce the phenomenological pairing gap [74]. The results of this procedure lead to the pairing-strength parameters $g_{pp}^p = 0.98$ and $g_{pair}^p = 1.3$. The particle-particle matrix elements as well as the particle-hole ones are renormalized by means of the parameters $g_{pp}$ and $g_{ph}$, respectively. These parameters were adjusted for each multipole state separately in order to reproduce few of the experimental known energies of the low-lying states in the $^{132}\text{Cs}$ and $^{132}\text{I}$ nucleus, respectively. The obtained values for the corresponding parameters lie in the range $0.6 \leq g_{pp} \leq 1.2$ and $0.5 \leq g_{ph} \leq 1.0$. Especially, for the $1^-$ multipolarity, the values $g_{pp} = 1.0$ and $g_{ph} = 0.5$ were used, while for the $1^+$ multipolarity, the values $g_{pp} = 1$ and $g_{ph} = 1$ were used. Moreover, for $^{132}\text{I}$, the values $g_{pp} = 1$ and $g_{ph} = 1$ were used with the exception of $4^+$ multipolarity for which $g_{pp} = 0.3$ and $g_{ph} = 1.2$ were used. All the states up to $J = 5^+$ have been included. 

### Table 1: Total cross sections for the indicated neutrino-nucleus charged current reactions as a function of incoming neutrino energy.

<table>
<thead>
<tr>
<th>$E_\nu$ (MeV)</th>
<th>$\nu^{-132}\text{Xe}$</th>
<th>$\overline{\nu}^{-132}\text{Xe}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.0</td>
<td>5.37 (−2)</td>
<td>2.05 (−4)</td>
</tr>
<tr>
<td>10.0</td>
<td>2.09 (+1)</td>
<td>1.63 (−1)</td>
</tr>
<tr>
<td>15.0</td>
<td>1.97 (+2)</td>
<td>2.46 (0)</td>
</tr>
<tr>
<td>20.0</td>
<td>6.27 (+2)</td>
<td>1.75 (+1)</td>
</tr>
<tr>
<td>25.0</td>
<td>1.30 (+3)</td>
<td>4.75 (+1)</td>
</tr>
<tr>
<td>30.0</td>
<td>1.82 (+3)</td>
<td>8.69 (+1)</td>
</tr>
<tr>
<td>40.0</td>
<td>2.76 (+3)</td>
<td>1.77 (+2)</td>
</tr>
<tr>
<td>50.0</td>
<td>3.74 (+3)</td>
<td>3.93 (+2)</td>
</tr>
<tr>
<td>60.0</td>
<td>4.76 (+3)</td>
<td>6.92 (+2)</td>
</tr>
<tr>
<td>70.0</td>
<td>5.75 (+3)</td>
<td>9.83 (+2)</td>
</tr>
<tr>
<td>80.0</td>
<td>6.63 (+3)</td>
<td>1.24 (+3)</td>
</tr>
<tr>
<td>90.0</td>
<td>7.32 (+3)</td>
<td>1.47 (+3)</td>
</tr>
<tr>
<td>100.0</td>
<td>7.78 (+3)</td>
<td>1.69 (+3)</td>
</tr>
</tbody>
</table>

### Table 2: Flux-averaged cross sections ($10^{-40}$ cm$^2$) obtained by convoluting the cross sections of Figure 3 with (3). Different temperatures $T$ (MeV) and $\alpha$ values are considered. The average neutrino energy $\langle E_\nu \rangle$ is given in MeV.

<table>
<thead>
<tr>
<th>$T$, $\alpha$</th>
<th>$\langle E_\nu \rangle$</th>
<th>$\nu^{-132}\text{Xe}$</th>
<th>$\overline{\nu}^{-132}\text{Xe}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3.5, 0)</td>
<td>11.0</td>
<td>1.74</td>
<td>0.05</td>
</tr>
<tr>
<td>(4.0, 0)</td>
<td>12.6</td>
<td>2.65</td>
<td>0.09</td>
</tr>
<tr>
<td>(5.0, 0)</td>
<td>15.7</td>
<td>4.90</td>
<td>0.21</td>
</tr>
<tr>
<td>(6.0, 0)</td>
<td>20.0</td>
<td>7.52</td>
<td>0.40</td>
</tr>
<tr>
<td>(8.0, 0)</td>
<td>25.2</td>
<td>13.27</td>
<td>0.95</td>
</tr>
<tr>
<td>(10.0, 0)</td>
<td>31.5</td>
<td>19.05</td>
<td>1.73</td>
</tr>
<tr>
<td>(2.75, 3)</td>
<td>11.0</td>
<td>1.31</td>
<td>0.03</td>
</tr>
<tr>
<td>(3.5, 3)</td>
<td>14.0</td>
<td>3.03</td>
<td>0.09</td>
</tr>
<tr>
<td>(4.3, 3)</td>
<td>16.0</td>
<td>4.52</td>
<td>0.17</td>
</tr>
<tr>
<td>(5.3, 3)</td>
<td>20.0</td>
<td>8.00</td>
<td>0.37</td>
</tr>
<tr>
<td>(6.3, 3)</td>
<td>24.0</td>
<td>11.83</td>
<td>0.67</td>
</tr>
<tr>
<td>(8.3, 3)</td>
<td>32.0</td>
<td>19.62</td>
<td>1.58</td>
</tr>
<tr>
<td>(10, 3)</td>
<td>40.0</td>
<td>26.95</td>
<td>2.80</td>
</tr>
</tbody>
</table>

### Table 3: Expected event rates for a 3 kT xenon detector for a supernova at 10 kpc corresponding to $\langle E_\nu \rangle = 11$ MeV and $\langle E_\nu \rangle = 16$ MeV.

<table>
<thead>
<tr>
<th>$^{132}\text{Xe}(\nu^{-CC} e^{-})^{132}\text{Cs}$</th>
<th>$^{132}\text{Xe}(\overline{\nu}^{-CC} e^{+})^{132}\text{I}$</th>
<th>Total event rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>592</td>
<td>50</td>
<td>642</td>
</tr>
<tr>
<td>445</td>
<td>40</td>
<td>485</td>
</tr>
</tbody>
</table>
Table 4: Fraction (in %) of the flux-averaged cross section associated to states of a given multipolarity with respect to the total flux-averaged cross section, that is, \( \sigma / \langle \sigma \rangle \). The first column tells if the results correspond to low-energy beta-beams or to a conventional source (DAR is for the decay at rest of muons). The neutrino fluxes are those produced by boosted \(^{130}\)Ne ions. The Lorentz ion boost parameter \( \gamma \) takes the values 6, 10, and 14. The last column gives the total flux-averaged cross sections \( (10^{-40}\text{ cm}^2) \).

\[
\begin{array}{cccccccccc}
\gamma & 0^+ & 1^+ & 2^+ & 3^+ & 4^+ & 0^- & 1^- & 2^- & 3^- & \langle \sigma \rangle (10^{-40}\text{ cm}^2) \\
6 & 13.3 & 69.4 & 1.45 & 0.47 & 0.004 & 0.00002 & 10.52 & 4.23 & 0.19 & 0.10 & 8.17 \\
10 & 14.4 & 49.8 & 5.54 & 1.96 & 0.08 & 0.00002 & 19.0 & 6.58 & 1.66 & 0.90 & 19.14 \\
14 & 13.1 & 37.9 & 9.40 & 3.56 & 0.46 & 0.00002 & 20.7 & 7.09 & 5.05 & 2.67 & 29.46 \\
\text{DAR} & 14.9 & 52.6 & 4.45 & 1.54 & 0.03 & 0.00002 & 18.4 & 6.42 & 1.02 & 0.55 & 19.47 \\
\end{array}
\]

Table 5: Same as Table 4, but for the antineutrino-nucleus cross sections and the antineutrino fluxes produced by boosted \(^{6}\)He ions.

\[
\begin{array}{cccccccccc}
\gamma & 0^+ & 1^+ & 2^+ & 3^+ & 4^+ & 0^- & 1^- & 2^- & 3^- & \langle \sigma \rangle (10^{-40}\text{ cm}^2) \\
6 & 32.0 & 53.8 & 0.76 & 0.18 & 0.0007 & 0.00003 & 9.05 & 4.12 & 0.07 & 0.04 & 0.37 \\
10 & 34.4 & 44.3 & 2.00 & 0.51 & 0.008 & 0.00004 & 12.88 & 5.28 & 0.34 & 0.21 & 1.57 \\
14 & 33.6 & 38.1 & 3.54 & 1.18 & 0.06 & 0.00002 & 15.17 & 6.25 & 1.12 & 0.91 & 3.69 \\
\end{array}
\]

In Figure 1, we present the numerical results of the total scattering cross section \( \sigma(E_\nu) \) (14) as a function of the incoming neutrino energy \( E_\nu \) for the reactions \(^{132}\)Xe(\( \nu_e, e^- \))\(^{132}\)Cs and \(^{132}\)Xe(\( \bar{\nu}_e, e^- \))\(^{132}\)I, respectively. The Q values of the reactions are 2.12 MeV and 3.58 MeV, respectively. Here, we have considered a hybrid prescription already used in previous calculations [19, 75, 76], where the Fermi function for the Coulomb correction is used below the energy region on which both approaches predict the same values, while EMA is adopted above this energy region.

The contribution of the different multipoles to the total cross section for the impinging neutrino energies \( E_\nu = 20, 60, \) and 80 MeV is shown in Figure 2. When \( E_\nu = 20 \) MeV, the total cross section \( \sigma_{NC} \) is mainly ascribed to the Gamow-Teller (1\(^+\)) and the Fermi (0\(^+\)) transitions. Other transitions contribute only a few percent to the total cross section. As the neutrino energy increases, the multipole states \( J^π = 1^+, 2^+, \) and \( 2^- \) become important as well. Finally, beyond 80 MeV, all states contribute, and the cross section is being spread over many multipoles.

Figure 3 shows the cross sections of coherent neutral and charged current processes as a function of neutrino energy. As it is seen, the coherent neutral current (\( \nu - NC \)) process [55] presents cross sections which are an order of magnitude greater than the electron neutrino charged current cross sections (\( \nu_e - CC \)). Both of them are even bigger than those from electron antineutrino charge current cross section (\( \bar{\nu}_e - CC \)) events. At \( E_\nu = 80 \) MeV, the difference between \( \nu_e - CC \) and \( \bar{\nu}_e - CC \) turns out to be a factor of 5. This can be understood in terms of the energy threshold and nuclear effects of the reactions. Since the Q value for the \( \bar{\nu}_e \) reactions is 1.46 MeV greater than \( \nu_e \) one, it decreases the incident neutrino energy as \( E_\nu \rightarrow E_{\bar{\nu}_e} - Q \) and therefore reduces the \( \bar{\nu}_e \) cross section for a given energy. In Table 1, the total (anti-) neutrino cross sections are listed in units of \( 10^{-42}\text{ cm}^2 \).

The flux-averaged total cross sections \( \langle \sigma \rangle \) can be calculated by folding the cross sections shown in Figure 3 with the Fermi-Dirac spectrum given by (3) as follows:

\[
\langle \sigma \rangle = \int_0^\infty \sigma(E_\nu) \Phi(E_\nu) dE_\nu.
\]
Figure 1: (Color online). Total cross section as a function of the incoming neutrino energy $E_{\nu}$, in the CC reactions $^{132}\text{Xe}(\nu_e,e^-)^{132}\text{Cs}$ (a) and $^{132}\text{Xe}(\overline{\nu}_e,e^+)^{132}\text{I}$ (b).

Figure 2: (Color online). Partial multipole distributions to the total cross sections for $^{132}\text{Xe}(\nu_e,e^-)^{132}\text{Cs}$, at the incoming neutrino energies $E_{\nu} = 20, 60$, and $80$ MeV.

Several experiments to be done at low-energy beta-beam have been proposed. Throughout our calculations, we have assumed that the boosted ions are storage in a ring similar to that used in [77]. Its total length is $L = 450$ m with straight section length $150$ m, while the detector is located $10$ m away from the straight section. The radius of the cylindrical detector is $2.13$ m with thickness $5$ m. As seen in Figure 5, the DAR spectrum has quite similar shape to the low-energy beta-beam spectrum with $\gamma = 10$. Note that, in principle, since the cross sections approximately grow as the neutrino energy square, the flux-averaged cross sections can show differences due to the high energy part of the neutrino spectrum.

Table 4 presents the contribution of the different states to the flux-averaged cross section. One can see that the results for $\gamma = 10$ are similar to the DAR case. The neutrino-Xenon cross section is dominated by the $0^+$, $1^+$, and $1^-$ multipoles. When the ion boost parameter $\gamma$ increases, the relative contribution of the $1^+$ decreases in favor of all other multipoles except $0^+$ which seems to be almost constant. For $\gamma = 14$, the contribution of all states becomes important in agreement with previously published results [76]. Table 5 presents the results for the antineutrino scattering, where the antineutrino fluxes are produced by the decay of boosted...
6 He ions. As it can be seen, the contribution of both 0\textsuperscript{+} and 1\textsuperscript{+} transitions to the flux-averaged cross sections is lying between 86\% for boosted ions at \( \gamma = 6 \) and 72\% for \( \gamma = 14 \).

5. Conclusions

Detailed microscopic calculations of charged current and neutral current neutrino-nucleus reaction rates are of crucial importance for models of neutrino oscillations, detection of supernova neutrinos, and studies of the \( r \)-process nucleosynthesis. In this paper we have calculated charged-current-neutrino-induced reactions on \(^{132}\text{Xe}\) by including multipole transitions up to \( J = 5 \). Excited states up to a few tens of MeV are taken into account. The ground state of \(^{132}\text{Xe}\) is described with the BCS model, and the neutrino-induced transitions to excited nuclear states are computed in the quasiparticle random-phase approximation.

In addition to the total neutrino-nucleus cross sections, we have also analyzed the evolution of the contributions of different multipole excitations as a function of neutrino energy. It has been shown that except at relatively low-neutrino energies \( E_\nu \leq 30 \) MeV for which the reactions are dominated by the transitions to 0\textsuperscript{+} and 1\textsuperscript{+} states, at higher energies, the inclusion of spin-dipole transitions, as well as excitations of higher multipolarities, is essential for a quantitative description of neutrino-nucleus cross sections. It is found that the \( \nu_e \) cross section on \(^{132}\text{Xe}\) is about 5 times greater than the \( \bar{\nu}_e \) one. This difference is anticipated because of (i) the different \( Q \) values of the corresponding reactions, (ii) the fact that there are less excited states that one can populate in the \( \bar{\nu}_e \) channel with respect to the \( \nu_e \) one and (iii) the different sign (minus for neutrino plus for antineutrino) of the interference term of magnetic and electric transitions introduced in (16).

Finally, we have given the contribution of the different states to the flux-averaged cross section considering low energy neutrino beams.

These are either based on conventional sources (muon decay at rest) or on low-energy beta-beams. We found that the Gamow-Teller (1\textsuperscript{+}) and the Fermi (0\textsuperscript{+}) transitions are the main components. When the Lorentz ion boost parameter \( \gamma \)
Advances in High Energy Physics

increases, the relative contribution of $1^+$ decreases in favor of all other multipole states except $0^+$ which seems to be almost constant, while the contribution of other states like $1^-, 2^+, 3^-$, and $3^+$ become important as well.

References


10 Advances in High Energy Physics


