Research Article

On Thermodynamics of Charged and Rotating Asymptotically AdS Black Strings

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In this paper, we study thermodynamics of cylindrically symmetric black holes and calculate the equation of states and heat capacity of charged and rotating black strings. In the process, we treat the cosmological constant as a thermodynamic pressure and its conjugate quantity as a thermodynamic volume. It is shown that, when taking the equivalence between the thermodynamic quantities of black strings and the ones of general thermodynamic system, the isothermal compressibility and heat capacity of black strings satisfy the stability conditions of thermodynamic equilibrium and no divergence points exist for heat capacity. Thus, we obtain the conclusion that the thermodynamic system relevant to black strings is stable and there is no second-order phase transition for AdS black holes in the cylindrically symmetric spacetime.

1. Introduction

Black hole physics, especially black hole thermodynamics, refers to many fields such as theories of gravitation, statistical physics, particle physics, and field theory, which makes the profound and fundamental connection between the theories, and much attention has been paid to the subject. It can be said that black hole physics has become the laboratory of many relevant theories. The pioneering works of Bekenstein and Hawking have opened many interesting aspects of unification of quantum mechanics, gravity, and thermodynamics. These are known for the last forty years [1–5]. The black hole thermodynamics has the similar forms to the general thermodynamics, which attracted great attention. In particular the case with negative cosmological constant (AdS case) has concerned many physicists [6–22]. Asymptotically, AdS black hole spacetimes admit a gauge duality description and are described by dual conformal field theory. Correspondingly, one has a microscopic description of the underlying degrees of freedom at hand. This duality has been recently exploited to study the behavior of quark-gluon plasmas and for the qualitative description of various condensed matter phenomena [12].

Recently, the studies on black hole thermodynamics in spherically symmetric spacetime by considering cosmological constant as the variable have got many attentions [12–16, 23–25]. In the previous works on the AdS black hole, cosmological constant corresponds to pressure in general thermodynamic system, the relation is [12, 13, 15]

$$P = -\frac{1}{8\pi}\Lambda = \frac{3}{8\pi}\frac{1}{l^2}, \quad (1)$$

and the corresponding thermodynamic volume is

$$V = \left(\frac{\partial M}{\partial P} \right)_{S, Q_i, J_k}. \quad (2)$$

In [16], the relation between cosmological constant and pressure is given in the higher dimensional AdS spherically symmetric spacetime, which supplies the basis for the study on the black hole thermodynamics in AdS spherically symmetric spacetime.

Theoretically, if we consider black holes in AdS spacetime as a thermodynamic system, the critical behaviors and phase transitions should also exist. Until now the statistical origin of black hole thermodynamics is still unclear. Therefore,

the search for the connection between kinds of thermodynamic quantities in AdS spacetime is meaningful, which may help to understand the entropy, temperature, and heat capacity of black holes and to build the consistent theory for black hole thermodynamics.

In this paper, we generalize the works of [12–19] and research the charged and rotating cylindrically symmetric spacetime. According to (1), we analyzed the thermodynamic properties of charged and rotating black string, calculated the heat capacity, and discussed the critical behaviors and phase transition of black string.

2. Rotating Charged Black Strings

The asymptotically AdS solution of the Einstein-Maxwell equations with cylindrical symmetry can be written as [26–28]

$$\begin{aligned} ds^2 = & -\Xi^2 \left(f(r) - \frac{a^2 r^2}{\Xi^2 l^4} \right) dt^2 + \frac{dr^2}{f(r)} \\ & - 2 \frac{a \Xi l}{r} \left(b - \frac{l}{r} \lambda^2 \right) dt d\phi \\ & + \left[\Xi^2 r^2 - a^2 f(r) \right] d\phi^2 + \frac{r^2}{l^2} dz^2, \\ A_\mu = & -\Xi \frac{l \lambda}{r} \left(\delta_\mu^0 - \frac{a}{\Xi} \delta_\mu^2 \right), \end{aligned} \quad (3)$$

where

$$f(r) = \frac{r^2}{l^2} - \frac{bl}{r} + \frac{\lambda^2 l^2}{r^2}, \quad \Xi^2 = 1 + \frac{a^2}{l^2}. \quad (4)$$

a , b , and λ are the constant parameters of the metric. The entropy, mass, electric charge, and angular momentum per unit length of black string are

$$\begin{aligned} S = \frac{\pi \Xi r_+^2}{2l}, \quad M = \frac{1}{8} (3\Xi^2 - 1) b, \\ Q = \frac{\Xi \lambda}{2}, \quad J = \frac{3}{8\Xi} ba, \end{aligned} \quad (5)$$

where r_+ is the location of the event horizon of black hole, which satisfies $f(r_+) = 0$. The Hawking temperature, angular velocity, and electric potential of black string are

$$T = \frac{3r_+^4 - \lambda^2 l^4}{4\pi \Xi l^2 r_+^3}, \quad \Omega_+ = \frac{a}{\Xi l^2}, \quad \Phi = \frac{\lambda l}{\Xi r_+}. \quad (6)$$

Expressing the mass per unit length of black string as the function of entropy S , angular momentum J , electric charge

Q , and pressure P (cosmological constant l), from (4) and (5), we have

$$\begin{aligned} Q^2 = \frac{l^2 S^2}{\pi^2 r_+^4} \left(\frac{br_+}{l} - \frac{r_+^4}{l^4} \right), \quad \frac{Q^2 \pi^2}{l S^2} + \frac{1}{l^3} = \frac{b}{r_+^3}, \\ b = \frac{r_+^3 (S^2 + Q^2 \pi^2 l^2)}{l^3 S^2}, \\ \frac{4J\pi r_+^2}{3} = \frac{(Q^2 \pi^2 l^2 + S^2)}{l^2 S} r_+^3 a, \end{aligned} \quad (7)$$

$$\begin{aligned} r_+^2 = \frac{2lS}{9\pi(S^2 + Q^2 \pi^2 l^2)^2} \\ \times \left[\sqrt{16J^4 S^2 l^2 \pi^6 + 81(S^2 + Q^2 \pi^2 l^2)^4} - 4J^2 S l \pi^3 \right] \\ = \frac{2lS}{9\pi(S^2 + Q^2 \pi^2 l^2)^2} Y, \end{aligned} \quad (8)$$

where $Y = \sqrt{16J^4 S^2 l^2 \pi^6 + 81(S^2 + Q^2 \pi^2 l^2)^4} - 4J^2 S l \pi^3$. From this we can get

$$\begin{aligned} M = \frac{1}{8} \left[\frac{12l^2 S^2}{\pi^2 r_+^4} - 1 \right] b \\ = \frac{1}{8} \left[\frac{12l^2 S^2 - \pi^2 r_+^4}{\pi^2 r_+^4} \right] \frac{(S^2 + Q^2 \pi^2 l^2)}{l^3 S^2} r_+^3 \\ = \frac{1}{8\pi^2 S^2 l^3} (S^2 + Q^2 \pi^2 l^2) \left[\frac{12l^2 S^2 - \pi^2 r_+^4}{r_+} \right] \\ = \frac{3}{\sqrt{8\pi^3 l^3 S}} \left[\frac{3 \times 81 (S^2 + Q^2 \pi^2 l^2)^4 - Y^2}{81 (S^2 + Q^2 \pi^2 l^2)^2 \sqrt{Y}} \right] \\ = \frac{3}{\sqrt{8\pi^3 l^3 S}} \left[\frac{2 \times 81 (S^2 + Q^2 \pi^2 l^2)^4 + 8J^2 S l \pi^3 Y}{81 (S^2 + Q^2 \pi^2 l^2)^2 \sqrt{Y}} \right], \end{aligned} \quad (9)$$

where $Y^2 = -8J^2 S l \pi^3 Y + 81(S^2 + Q^2 \pi^2 l^2)^4$. From (9), we can find that the thermodynamic quantities of black string satisfy the first law of thermodynamics as

$$dM = TdS + \Phi dQ + \Omega dJ + VdP. \quad (10)$$

From (10), one can deduce

$$\begin{aligned} T = \left(\frac{\partial M}{\partial S} \right)_{J,Q,P}, \quad \Omega = \left(\frac{\partial M}{\partial J} \right)_{S,Q,P}, \\ \Phi = \left(\frac{\partial M}{\partial Q} \right)_{J,S,P}, \quad V = \left(\frac{\partial M}{\partial P} \right)_{J,Q,S}, \end{aligned}$$

$$\begin{aligned}
& \left(\frac{\partial M}{\partial l} \right)_{s,Q,J} \\
&= -\frac{9}{2l\sqrt{8\pi^3 l^3 S}} \\
&\quad \times \left[\frac{2 \times 81(S^2 + Q^2 \pi^2 l^2)^4 + 8J^2 S l \pi^3 Y}{81(S^2 + Q^2 \pi^2 l^2)^2 \sqrt{Y}} \right] \\
&\quad + \frac{3(S^2 + Q^2 \pi^2 l^2)}{\sqrt{Y} \sqrt{8\pi^3 l^3 S}} \\
&\quad \times \left[8Q^2 \pi^2 l - \frac{(S^2 + Q^2 \pi^2 l^2)}{Y} \frac{\partial Y}{\partial l} \right. \\
&\quad \left. + \frac{4J^2 S \pi^3}{81(S^2 + Q^2 \pi^2 l^2)^3} \right. \\
&\quad \left. \times \left(2Y + l \frac{\partial Y}{\partial l} \right) - \frac{32J^2 S l^2 \pi^5 Q^2 Y}{81(S^2 + Q^2 \pi^2 l^2)^4} \right], \quad (11)
\end{aligned}$$

where $\partial Y / \partial l = (81 \times 4Q^2 \pi^2 l (S^2 + Q^2 \pi^2 l^2)^3 - 4J^2 S \pi^3 Y) / \sqrt{Y}$. From (1), one can derive the corresponding “thermodynamic” volume of black string as

$$V = \left(\frac{\partial M}{\partial P} \right)_{s,Q,J} = \frac{b\pi}{2} l^2 \Xi^2 - \frac{2\pi l^2 l^3}{3r_+}. \quad (12)$$

From (12), one can get

$$\begin{aligned}
\Xi^2 &= \frac{IV}{\pi r_+^3} \left(V - \frac{2\pi l^3 Q^2}{r_+} \right) \\
&\quad \pm \frac{l}{r_+^2} \sqrt{\frac{1}{\pi^2 r_+^2} \left(V - \frac{2\pi l^3 Q^2}{r_+} \right)^2 + \frac{16l^2 Q^2}{3}}. \quad (13)
\end{aligned}$$

Because of $Q = 0$, only the plus sign is kept, namely,

$$\begin{aligned}
\Xi^2 &= \frac{IV}{\pi r_+^3} \left(V - \frac{2\pi l^3 Q^2}{r_+} \right) \\
&\quad + \frac{l}{r_+^2} \sqrt{\frac{1}{\pi^2 r_+^2} \left(V - \frac{2\pi l^3 Q^2}{r_+} \right)^2 + \frac{16l^2 Q^2}{3}}. \quad (14)
\end{aligned}$$

From $\Xi^2 = 1 + a^2 / l^2$, we can derive

$$\begin{aligned}
a^2 &= \left[\frac{IV}{\pi r_+^3} \left(V - \frac{2\pi l^3 Q^2}{r_+} \right) \right. \\
&\quad \left. + \frac{l}{r_+^2} \sqrt{\frac{1}{\pi^2 r_+^2} \left(V - \frac{2\pi l^3 Q^2}{r_+} \right)^2 + \frac{16l^2 Q^2}{3}} - 1 \right] l^2. \quad (15)
\end{aligned}$$

From $J = (3/8)\Xi b a$, we get

$$J^2 = \frac{9}{64} \Xi^2 \left(\frac{r_+^3}{l^3} + \frac{4Q^2 l}{r_+ \Xi^2} \right)^2 a^2. \quad (16)$$

3. Thermodynamics of Charged Black String

In this section, we discuss thermodynamics of static charged black string. When $a = 0$, $\Xi = 1$. From (12), one can get

$$\begin{aligned}
V &= \frac{\pi r_+^3}{2l} - \frac{2\pi Q^2 l^3}{3r_+}, \\
T &= \frac{3r_+}{4\pi l^2} - \frac{l^2 Q^2}{\pi r_+^3}. \quad (17)
\end{aligned}$$

From this, we obtain

$$\begin{aligned}
& \left(\frac{\partial P}{\partial V} \right)_{T,Q} \\
&= \left(\frac{3}{4\pi l^2} + \frac{3l^2 Q^2}{\pi r_+^4} \right) \\
&\quad \times \left(\left(\frac{\pi r_+^3}{2l^2} + \frac{2\pi Q^2 l^2}{r_+} \right) \times \left(\frac{3}{4\pi l^2} + \frac{3l^2 Q^2}{\pi r_+^4} \right) \right. \\
&\quad \left. - \left(\frac{3\pi r_+^2}{2l} + \frac{2\pi Q^2 l^3}{3r_+^2} \right) \times \left(\frac{3r_+}{2\pi l^3} + \frac{2Q^2 l}{r_+^3} \right) \right)^{-1} \\
&\quad \times \frac{3}{4\pi l^3} \\
&= \frac{3/4\pi l^2 + 3l^2 Q^2 / \pi r_+^4}{14Q^4 l^4 / 3r_+^5 - (15r_+^3 / 8l^4 + Q^2 / r_+)} \frac{3}{4\pi l^3}. \quad (18)
\end{aligned}$$

From (18), when $14Q^4 l^4 / 3r_+^5 > 15r_+^3 / l^4 + Q^2 / r_+$, $(\partial P / \partial V)_{T,Q} > 0$, the thermodynamic system is unstable. When

$$\frac{14Q^4 l^4}{3r_+^5} < \frac{15r_+^3}{l^4} + \frac{Q^2}{r_+}, \quad (19)$$

$(\partial P / \partial V)_{T,Q} < 0$, the thermodynamic system is stable. When $Q \rightarrow 0$, $(\partial P / \partial V)_{T,Q} < 0$, the thermodynamic system is stable. Heat capacity at constant pressure is

$$C_{P,Q} = T \left(\frac{\partial S}{\partial T} \right)_{P,Q} = T \left[\frac{\pi r_+}{3/4\pi l + 3l^3 Q^2 / \pi r_+^4} \right]. \quad (20)$$

According to (20), if $T > 0$, namely, $3r_+ / 4l^2 > Q^2 l^2 / r_+^3$, $C_{P,Q}$ will be greater than zero, which fulfills the stable condition

FIGURE 1: Plots of the heat capacity at constant V and Q versus the r_+^4 . The horizontal axis started from $4/3$, because of $T > 0$. The intersection point of $C_{V,Q}$ with the horizontal axis is $20/3$, which shows that when $r_+^4 > (20/3)Q^2l^4$ holds up, $C_{V,Q} > 0$.

of thermodynamic equilibrium. The heat capacity at constant volume is

$$\begin{aligned}
 C_{V,Q} &= T \left(\frac{\partial S}{\partial T} \right)_{V,Q} \\
 &= T \left(\left(\frac{\pi r_+}{l} \left(\frac{3r_+^5 + 12Q^2l^4r_+}{9lr_+^4 + 4Q^2l^5} \right) - \frac{\pi r_+^2}{2l^2} \right) \right. \\
 &\quad \times \left(\left(\frac{3}{4\pi l^2} + \frac{3Q^2l^2}{\pi r_+^4} \right) \times \left(\frac{3r_+^5 + 12Q^2l^4r_+}{9lr_+^4 + 4Q^2l^5} \right) \right. \\
 &\quad \left. \left. - \left(\frac{3r_+}{2\pi l^3} + \frac{2Q^2l}{\pi r_+^3} \right) \right) \right)^{-1} \\
 &= T \frac{10\pi Q^2l^3r_+^2 - 3\pi r_+^6/2l}{28Q^4l^6/\pi r_+^3 - 45r_+^5/4\pi l^2 - 6Q^2r_+l^2/\pi} \\
 &= \left(\frac{3r_+}{4l^2} - \frac{Q^2l^2}{r_+^3} \right) \frac{10\pi Q^2l^3r_+^2 - 3\pi r_+^6/2l}{28Q^4l^6/r_+^3 - 45r_+^5/4l^2 - 6Q^2r_+l^2}.
 \end{aligned} \quad (21)$$

We can plot the curve of $C_{V,Q}$, which shows that only when the condition $r_+^4 > (20/3)Q^2l^4$ holds up, $C_{V,Q} > 0$ will work. See Figure 1.

From (20) and $T > 0$, when $r_+^4 > (4/3)Q^2l^4$, $C_{P,Q}$ will be greater than zero, which fulfills the stable condition of thermodynamic equilibrium. When $r_+^4 = (4/3)Q^2l^4$, the Hawking temperature of heat capacity is zero which corresponds to the extreme case. However, for the $C_{V,Q}$ only the condition $T > 0$ is not enough. One needs more strict condition $r_+^4 > (20/3)Q^2l^4$, under which the $C_{V,Q}$ is greater than zero. This suggests that the thermodynamic system of charged black strings does not have the first-order phase transition only when $r_+^4 > (20/3)Q^2l^4$. On the other hand, the second-order phase transition points of the thermodynamic system of charged black strings turn up when heat capacities diverge. In this charged black string spacetime, under the given condition $r_+^4 > (20/3)Q^2l^4$, $C_{V,Q}$ and $C_{P,Q}$ are always

greater than zero, which suggests that the second-order phase transition of black string will not happen. Whether the phase transition exists when the condition breaks out will be discussed later.

4. Thermodynamics of Rotating Black String

In this section, we discuss thermodynamics of stationary rotating black string. When $Q = 0$, from (12) and (14), we have

$$\Xi^2 = \frac{2lV}{\pi r_+^3}, \quad V \left(\frac{2lV}{\pi r_+^3} - 1 \right) = \frac{32\pi J^2 l^3}{9r_+^3}, \quad (22)$$

$$\begin{aligned}
 V &= \frac{\pi r_+^3}{4l} + \frac{\pi}{2} \sqrt{\frac{r_+^6}{4l^2} + \frac{64J^2 l^2}{9}}, \\
 T &= \frac{3r_+^2 \sqrt{r_+}}{4l^2 \sqrt{2\pi lV}}.
 \end{aligned} \quad (23)$$

From (22) and (23), we deduce

$$\left(\frac{\partial l}{\partial V} \right)_{T,J} = \frac{3(20lV - 8\pi r_+^3)l}{5\pi(9Vr_+^3 + 32\pi J^2 l^3) - 30lV^2} = \frac{5lV - 2\pi r_+^3}{5V}. \quad (24)$$

Thus,

$$\left(\frac{\partial P}{\partial V} \right)_{T,J} = \frac{3(2\pi r_+^3 - 5lV)}{20\pi l^3 V}. \quad (25)$$

From (22), we have $2lV > \pi r_+^3$, so $(\partial P/\partial V)_{T,J} < 0$, which satisfies the condition of thermodynamic equilibrium. We can derive the heat capacities of rotating string at constant pressure and constant volume as follows:

$$\begin{aligned}
 C_{V,J} &= T \left(\frac{\partial S}{\partial T} \right)_{V,J} \\
 &= T \left(\left(\frac{1}{6r_+^2 V} \left(\frac{\pi V}{2lr_+} \right)^{1/2} \left(\frac{2V^2}{\pi} - \frac{32J^2 l^2}{3} \right) \right. \right. \\
 &\quad \left. \left. - \frac{1}{2l} \left(\frac{\pi V r_+}{2l} \right)^{1/2} \right) \right. \\
 &\quad \times \left(\frac{5}{8l^2 V \sqrt{2\pi lV r_+}} \left(\frac{2V^2}{\pi} - \frac{32J^2 l^2}{3} \right) \right. \\
 &\quad \left. \left. - \frac{15r_+^{5/2}}{8l^3 \sqrt{2\pi lV}} \right)^{-1} \right) \\
 &= T \frac{2\pi lV \sqrt{2\pi lV r_+}}{15r_+^3} = \frac{\pi}{10} \frac{V}{l},
 \end{aligned}$$

$$\begin{aligned}
C_{P,J} &= T \left(\frac{\partial S}{\partial T} \right)_{P,J} \\
&= T \left(\left(\frac{1}{6r_+^2 V} \left(\frac{\pi V}{2lr_+} \right)^{1/2} \left(\frac{4lV}{\pi} - r_+^3 \right) + \frac{1}{2} \left(\frac{\pi r_+}{2lV} \right)^{1/2} \right) \right. \\
&\quad \times \left(\frac{5}{8l^2 V \sqrt{2\pi l V r_+}} \left(\frac{4lV}{\pi} - r_+^3 \right) \right. \\
&\quad \left. \left. - \frac{3r_+^{5/2}}{8l^2 V \sqrt{2\pi l V}} \right)^{-1} \right) \\
&= \frac{1}{2} \left(\frac{\pi r_+ V}{2l} \right)^{1/2} \left(\frac{2lV - \pi r_+^3}{5lV - 2\pi r_+^3} \right). \tag{26}
\end{aligned}$$

From (22), we have $lV > \pi r_+^3/2$, so $C_{P,J} > 0$, which satisfies the condition of thermodynamic equilibrium. The second-order phase transition points of thermodynamic systems will appear when heat capacities diverge. According to (26), the heat capacities do not have divergent points; therefore, the second-order phase transition of rotating black string also cannot happen.

5. Thermodynamics of Charged and Rotating Black String

In this section, we discuss thermodynamics of static charged and rotating black string. The location r_+ of event horizon satisfies

$$\frac{r_+^2}{l^2} - \frac{bl}{r_+} + \frac{\lambda^2 l^2}{r_+^2} = 0, \tag{27}$$

where

$$\begin{aligned}
r_+ &= \frac{1}{2} \left\{ \gamma^{1/2} + \left[-\gamma + 2(\gamma^2 - 4\lambda^2 l^4)^{1/2} \right]^{1/2} \right\}, \\
\gamma &= \left\{ \frac{b^2 l^6}{2} + \left[\left(\frac{b^2 l^6}{2} \right)^2 + \left(\frac{4\lambda^2 l^4}{3} \right)^2 \right]^{1/2} \right\}^{1/3} \\
&\quad + \left\{ \frac{b^2 l^8}{2} - \left[\left(\frac{b^2 l^8}{2} \right)^2 + \left(\frac{4\lambda^2 l^4}{3} \right)^2 \right]^{1/2} \right\}^{1/3}. \tag{28}
\end{aligned}$$

For discussion purpose and without loss of generality, we take $a(J)$ and Q to be small quantities relative to M or l and r_+ , namely, $a^2/l^2 \ll 1$, $r_+^2 \gg a^2$. From (16), we have

$$J \approx \frac{3}{8} \left(\frac{r_+^3}{l^3} + \frac{4Q^2 l}{r_+} \right) a, \tag{29}$$

and when Q is small

$$a \approx \frac{8l^3}{3r_+^3} J. \tag{30}$$

From (12), we can get the approximate value of volume

$$V \approx \frac{\pi r_+^3}{2l} - \frac{2\pi Q^2 l^3}{3r_+} + \frac{32\pi l^6}{9r_+^6} J^2. \tag{31}$$

According to (6), we can obtain the approximate Hawking temperature

$$T \approx \frac{3r_+}{4\pi l^2} \left(1 - \frac{32l^4}{r_+^6} J^2 \right) - \frac{Q^2 l^2}{\pi r_+^3}. \tag{32}$$

From this, we can deduce

$$\begin{aligned}
\left(\frac{\partial P}{\partial V} \right)_{T,J,Q} &\approx \frac{3/4\pi l^2 + 120l^2 J^2/\pi r_+^6 + 3Q^2 l^2/\pi r_+^4}{16l^3 J^2/r_+^6 - (15r_+^3/8l^4 + Q^2/r_+ + 12J^2/r_+^3)} \\
&\quad \times \frac{3}{4\pi l^3}. \tag{33}
\end{aligned}$$

From (32), when requiring $T > 0$, the following equation should be satisfied:

$$\frac{3r_+}{4l^2} > \frac{24l^2 J^2}{r_+^5} + \frac{Q^2 l^2}{r_+^3}. \tag{34}$$

From (33), when

$$\frac{15r_+^3}{8l^4} > \frac{16l^3 J^2}{r_+^6} - \left(\frac{12J^2}{r_+^3} + \frac{Q^2}{r_+} \right), \tag{35}$$

we have $(\partial P/\partial V)_{T,J,Q} < 0$, which satisfies the condition of thermodynamic equilibrium. Substituting (34) into (33), we can get $15r_+^3/8l^4 > 16l^3 J^2/r_+^6 - 3r_+^3/4l^4$, or

$$\frac{21r_+^3}{8l^4} > \frac{16l^3 J^2}{r_+^6} \approx \frac{9a^2}{4l^3}. \tag{36}$$

From (28), one can deduce $r_+/l \propto (4M)^{1/3} > a$, $r_+^2 \gg a^2$; thus, (36) is satisfied.

In order to show the relation between P and V clearly, we plot the V - P curve. According to (1), (31), and (32), we can depict the V - P curve of charged and rotating black strings (Figure 2).

From this figure, we know that the V - P curves of charged and rotating black strings are smooth and continuous; therefore, under the condition of isothermality the first-order and second-order phase transitions caused by the variation of pressure or volume do not exist.

The approximate expression of entropy is

$$S = \frac{\pi \Xi r_+^2}{2l} \approx \frac{\pi r_+^2}{2l} \left(1 + \frac{a^2}{2l^2} \right) \approx \frac{\pi r_+^2}{2l} \left(1 + \frac{32l^4}{9r_+^6} J^2 \right). \tag{37}$$

FIGURE 2: Plots of the pressure P versus the volume V . The curves correspond to the parameters $T = 1$, $J = 0.1$, and $Q = 0.0, 0.5, 1.0$. The three curves roughly coincide.

The heat capacity of charged and rotating black strings at constant pressure and constant volume is

$$\begin{aligned}
 C_{V,J,Q} &= T \left(\frac{\partial S}{\partial T} \right)_{V,J,Q} \\
 &\approx T \left(\left(\frac{5\pi^2 Q^2 l}{3} - \frac{\pi^2 r_+^4}{l^3} - \frac{22\pi^2 l^4 J^2}{3r_+^5} + \frac{40\pi^2 l J^2}{9r_+^2} \right) \right. \\
 &\quad \left. \times \left(-\frac{33r_+^3}{8l^4} - \frac{Q^2}{r_+} + \frac{16l^3 J^2}{r_+^6} - \frac{12J^2}{r_+^3} \right)^{-1} \right), \\
 C_{P,J,Q} &= T \left(\frac{\partial S}{\partial T} \right)_{P,J,Q} \\
 &= T \frac{\pi r_+ / l - (64\pi l^3 / 9 r_+^5) J^2}{3/4\pi l^2 + 120l^2 J^2 / \pi r_+^6 + 3Q^2 l^2 / \pi r_+^4}.
 \end{aligned} \tag{38}$$

From (35), one can deduce $\pi^2 r_+^4 / l^3 > 22l^4 J^2 / 3r_+^5 - (40l J^2 / 9r_+^2 + 5Q^2 l / 3)$, $33r_+^3 / 8l^4 > +16l^3 J^2 / r_+^6 - (12J^2 / r_+^3 + Q^2 / r_+)$; therefore,

$$C_{V,J,Q} > 0, \quad C_{P,J,Q} > 0. \tag{39}$$

Thus we can consider the charged and rotating black strings as a thermodynamic system and the system can satisfy the stable conditions of equilibrium under the assumption of small a and Q , because the second order phase transition points of the thermodynamic system turn up when heat capacities diverge. According to (39), the heat capacities are always greater than zero, which suggests that the second-order phase transition of black string will not happen when a and Q are small quantities.

6. Conclusion

In this paper, we study the thermodynamic properties of charged and rotating black strings in cylindrically symmetric AdS spacetime. Like the spherically symmetric case for the

charged and rotating black strings we take the cosmological constant to correspond to the pressure in general thermodynamic system. The relation is (1). We consider the identification, because when solving Einstein equations the cosmological constant l is independent of the symmetry of spacetime under consideration and the pressure in thermodynamic system also has nothing to do with the surface morphology. Thus the relation (1) should also be appropriate to the charged and rotating cylindrically symmetric spacetime.

On the basis of (1), we analyze the corresponding thermodynamic quantities for charged and rotating black strings. We find that, under some conditions, the heat capacities are greater than zero and $(\partial P / \partial V)_{T,J,Q} < 0$, which satisfy the stable condition of thermodynamic equilibrium. Thus, when the system is perturbed slightly and deviates from equilibrium, some process will appear automatically and makes the system restore equilibrium.

Compared with the works of [12–14], it is found that the thermodynamic properties of black holes in spherically symmetric spacetime are different from the ones of black holes in cylindrically symmetric spacetime, specially that the heat capacities of black holes in cylindrically symmetric spacetime do not have divergent points; thus, no second-order phase transition occurs and no critical phenomena similar to Van der Waals gas occur. At present, the problem cannot be explained logically and it deserves further discussion.

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