Research Article

Essay on Kolmogorov Law of Minus 5 over 3 Viewed with Golden Ratio

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Received 14 July 2013; Accepted 7 August 2013

Academic Editor: Carlo Cattani

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The golden ratio is an astonishing number in high-energy physics, neutrino physics, and cosmology. The Kolmogorov $-5/3$ law plays a role in describing energy transfer of random data or random functions. The contributions of this essay are in twofold. One is to express the Kolmogorov $-5/3$ law by using the golden ratio. The other is to represent the fractal dimension of random data following the Kolmogorov $-5/3$ law with the golden ratio. It is our hope that this essay may be helpful to provide a new outlook of the Kolmogorov $-5/3$ law from the point of view of the golden ratio.

1. Instruction

Let φ be the golden ratio. It equals to $(1 + \sqrt{5})/2$. Its inverse $1/\varphi = (\sqrt{5}-1)/2$ is called the golden mean. Both are irrational numbers. Approximately, they are

$$\varphi \approx 1.618, \quad \frac{1}{\varphi} \approx 0.618. \quad (1)$$

The number φ has wide applications to various fields, ranging from physics (Wurm and Martini [1], King [2], Ding et al. [3], Feruglio and Paris [4]), to cosmology (Livio [5], Boeyens [6]).

In addition to the golden ratio, fractal is a mathematical model attracting interests of scientists and physicists in the field of high-energy physics (Ghosh et al. [7]). While studying energy transfer, the Kolmogorov $-5/3$ law introduced by Kolmogorov [8] plays a role in the field (Qian [9], Brun et al. [10, Chapter 7], Hillebrandt and Kupka [11, Chapter 4], Gomes-Fernandes et al. [12, page 309], Lumley and Yaglom [13], Warhaft [14], Geipel et al. [15]). Motivated by those, this essay aims at exhibiting the Kolmogorov $-5/3$ law and the self-similarity, which is an important fractal property (Mandelbrot [16, 17], Cattani et al. [18, 19]), from the point of the golden ratio. It is our expectation that this essay may be helpful to describe the nature of random data that follows the Kolmogorov $-5/3$ law.

The rest of paper is organized as follows. The preliminaries are briefed in Section 2. The results that the Kolmogorov $-5/3$ law and the self-similarity are explained from a view of the golden ratio are given in Section 3, which is followed by conclusions.

2. Preliminaries

2.1. Golden Ratio. Consider a straight line in Figure 1. Arrange three points A, B, and C, there such that the ratio below

$$\frac{AC}{CB} = \frac{AB}{AC} \quad (2)$$

equals to φ . Then, we say that the straight line is cut in extreme and mean ratio ([5], Ackermann [20], Kaygn et al. [21]).

There are various ways to synthesize the number φ . According to the definition of extreme and mean ratio, one has

$$x = \frac{x+1}{x}. \quad (3)$$

Multiplying x on the both sides of the above yields

$$x^2 - x - 1 = 0. \quad (4)$$

FIGURE 1: Illustration of a straight line for the golden ratio.

Solving (4) produces

$$x_1 = \frac{1 + \sqrt{5}}{2}, \quad x_2 = \frac{1 - \sqrt{5}}{2}. \quad (5)$$

Conventionally, x_1 is denoted by φ , and x_2 is denoted by $-1/\varphi$.

2.2. Fractal Dimension. Let the autocorrelation function (ACF) of a random function $x(t)$ be $r_{xx}(\tau)$, where $r_{xx}(\tau) = E[x(t)x(t+\tau)]$. Then, $r_{xx}(\tau)$ for $\tau \rightarrow 0$ represents the small-scaling phenomenon of $x(t)$. Following Davies and Hall [22], if $r_{xx}(\tau)$ is sufficiently smooth on $(0, \infty)$ and if

$$r_{xx}(0) - r_{xx}(\tau) \sim c_1 |\tau|^\alpha \quad \text{for } |\tau| \rightarrow 0, \quad (6)$$

where c_1 is a constant and $0 < \alpha \leq 2$ is the fractal index of $x(t)$, the fractal dimension, which is denoted by D , of $x(t)$ is in the form

$$D = 2 - \frac{\alpha}{2}. \quad (7)$$

Note that fractal dimension $1 \leq D < 2$ is a measure to characterize the local self-similarity or local roughness of $x(t)$ (Mandelbrot [17], Gneiting and Schlather [23]).

Denote the power spectrum density (PSD) function of $x(t)$ by $S_{xx}(\omega)$. Denote by F the operator of Fourier transform. Then,

$$S_{xx}(\omega) = F[r_{xx}(t)] = \int_{-\infty}^{\infty} r_{xx}(t) e^{-j\omega t} dt, \quad j = \sqrt{-1}. \quad (8)$$

In the domain of generalized functions, we have (Kanwal [24], Gelfand and Vilenkin [25], Li and Lim [26])

$$F(|\tau|^\alpha) = -2 \sin\left(\frac{\pi\alpha}{2}\right) \Gamma(1+\alpha) |\omega|^{-(\alpha+1)}, \quad (9)$$

where Γ is the Gamma function. Therefore, we have an analogy of (6) in the frequency domain in the form

$$F(|\tau|^\alpha) \sim c_2 |\omega|^{-(\alpha+1)} \quad \text{for } \omega \rightarrow \infty, \quad (10)$$

where c_2 is a constant (Chan et al. [27]).

3. Results

The Kolmogorov $-5/3$ law implies that the PSD of a random function has the asymptotic behavior expressed by

$$S_{xx}(\omega) \sim c |\omega|^{-5/3} \quad \text{for } \omega \rightarrow \infty, \quad (11)$$

where c is a constant (Monin and Yaglom [28, 29]). The above well characterizes the local irregularity of turbulence function $x(t)$ (Troepe et al. [30]).

In order to connect the Kolmogorov $-5/3$ law with the golden ratio, we write

$$\frac{5}{3} + e = \varphi. \quad (12)$$

From the previous discussions, we have

$$e = \varphi - \frac{5}{3} = \frac{1 + \sqrt{5}}{2} - \frac{5}{3} = \frac{\sqrt{5} - 1}{2} - \frac{2}{3} = \frac{1}{\varphi} - \frac{2}{3}. \quad (13)$$

Thus, the Kolmogorov $-5/3$ law expressed by (11) may be rewritten by

$$S_{xx}(\omega) \sim c |\omega|^{-(\varphi-e)} \quad \text{for } \omega \rightarrow \infty. \quad (14)$$

From (10), we see that the fractal index may be expressed by using the golden ratio as

$$\alpha = \varphi - e - 1. \quad (15)$$

Thus, we attain the fractal dimension from the point of view of the golden ratio in the form

$$D = 2 - \frac{\varphi - e - 1}{2}. \quad (16)$$

From the previous discussions, we have the following theorems.

Theorem 1. Let $x(t)$ be the random function that obeys the Kolmogorov $-5/3$ law expressed by (11). Then, its PSD may be expressed using the golden ratio in (14).

Theorem 2. Let $x(t)$ be the random function that obeys the Kolmogorov $-5/3$ law expressed by (11). Then, its fractal dimension may be represented by using the golden ratio as that in (16). The concrete value of D in (16) is $5/3$.

Finally, we note that the significance of the present results lies in expressing the Kolmogorov $-5/3$ law, which plays a role in energy transfer, by using the golden ratio, which attracts interests of researchers and scientists in high-energy physics, neutrino physics, cosmology, and many others.

4. Conclusions

We have explained our results in expressing the Kolmogorov $-5/3$ law with the golden ratio. In addition, we have expressed the fractal dimension of random data obeying the Kolmogorov $-5/3$ law based on the golden ratio.

Acknowledgments

This work was supported in part by the National Natural Science Foundation of China under the Project Grant nos. 61272402, 61070214, and 60873264, and by the 973 plan under the Project Grant no. 2011CB302800.

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