Research Article

Mass Spectrum of $1^1P_1$ Meson State and Mixing Angle of Strange Axial-Vector Mesons

Xue-Chao Feng,1 Ke-Wei Wei,2 Jie Wu,1 Yun-Qiang Zhang,1 Ming-Yang Wu,1 and Feng-Chun Jiang1

1 Department of Technology and Physics, Zhengzhou University of Light Industry, Zhengzhou 450002, China
2 College of Physics and Electrical Engineering, Anyang Normal University, Anyang 455002, China

Correspondence should be addressed to Xue-Chao Feng; fxchao@zzuli.edu.cn

Received 29 December 2012; Revised 6 March 2013; Accepted 26 March 2013

Academic Editor: Alexey Petrov

Copyright © 2013 Xue-Chao Feng et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

In the presence of the $h_1(1235)$, $h_1(1170)$, $D_1(2420)$, and $D_{13}(2536)$ being the members of the $1^1P_1$ meson multiplet, we investigate the mass spectrum of the $1^1P_1$ meson state and the mixing angle of strange axial-vector mesons in the framework of meson-meson mass mixing and Regge phenomenology. Also, in the glueball dominance picture, the kaon is phenomenologically determined to have a mass of about 1356 MeV. The results are compared with the values from different phenomenological models and may be useful for the assignment of the $1^1P_1$ meson multiplet in the future.

1. Introduction

Quantum Chromodynamics (QCD) is widely accepted as a successful theory of the strong interactions in particle physics. However, the understanding of the strong interactions is far from complete, and it is difficult to interpret the particle properties for the experimental data from the first principles. To be able to determine the nature of a new experimental resonance, it is necessary to create a template to compare the observed states with theoretical predictions. Therefore, different phenomenological models, such as quenched lattice gauge theory [1], the Dyson-Schwinger formalism [2], and constituent quark model [3], are built to describe the properties of experimental results [4].

In the recent issue of Review of Particle Physics (PDG), the $h_1(1235)$, $h_1(1170)$, $D_1(2420)$, and $D_{13}(2536)$ have been observed in the experiment and assigned as the members of the $1^1P_1$ meson multiplet [5]. However, the assignment for the strange member $s\bar{s}$ state is problematic. On the one hand, based on the decay results in the $^3P_0$ model, Barnes et al. suggest that the $h_1(1380)$ should be a candidate of the $1^1P_1$ meson [6]. On the other hand, the mass of $h_1(1380)$ is inconsistent with the prediction in the potential model, which is about 100 MeV higher than the measured masses of the $h_1(1380)$ [7]. Accordingly, the mass and decays of the $h_1(1380)$ need to be further examined experimentally.

Apart from the $s\bar{s}$ member, the mass of the $K_{1B}$ also remains interesting. Due to the states $K_{1}(1^1P_1)$ and $K_{3}(1^3P_1)$ having no definite C-parity and the SU(3) broken, the $K_{1}(1^1P_1)$ and $K_{3}(1^3P_1)$ can mix via the spin-orbit interaction to form the physical states $K_1(1270)$ and $K_3(1400)$. In order to compare the theoretical prediction with experimental data, the mixing angle of $K_{1A}$ and $K_{1B}$ has been discussed with different theoretical approaches [8–12]. In addition to the light meson states, the charmonium is one of the most exciting areas of high energy physics. In the $1^1P_1$ meson state, the $h_c(1P)$ has been observed in the decay $\psi(2S) \rightarrow \pi^0h_c$ with $h_c \rightarrow \gamma\eta_c$ by CLEO [13] and in the reaction $p\bar{p} \rightarrow h_c \rightarrow \gamma\eta_c \rightarrow \gamma\gamma\gamma$ [14]. The experimental signals about $h_c(1P)$ are still very weak so far.

In allusion to the problems mentioned earlier, the purpose of this work is to investigate the mass spectrum of $1^1P_1$ meson state and mixing angle of strange axial-vector mesons. The results should be useful for exploring the nature and the assignment of $1^1P_1$ meson state. The organization of this paper is as follows. In Section 2, the brief review of the mass mixing matrix of isoscalar states is given (for
the detailed review, see, e.g., [15–17]). In Section 3, the mass spectrum of $1^1P_1$ meson state is presented in the framework of Regge phenomenology and glueball-meson mass relation. The mixing angle of $K_{1A}$ and $K_{1B}$ is predicted in Section 4, and a summary and conclusion are given in Section 5.

2. The Review of Mass Mixing Matrix of Isoscalar States

In the quark model, the two isoscalar states with the same $J^{PC}$ quantum number will mix to form the physical states. One can obtain the mass-squared matrix in the $S = s\bar{s}$ and $N = (u\bar{u} + d\bar{d})/\sqrt{2}$ basis [18] as follows:

$$M_s^2 = \begin{pmatrix} M_N^2 + 2A_{nn} & \sqrt{2}A_{ns} \\ \sqrt{2}A_{ns} & M_s^2 + A_{ss} \end{pmatrix},$$

(1)

where $M_N$ and $M_s$ are the masses of bare states $N$ and $S$, respectively; $A_{nn}, A_{ns},$ and $A_{ss}$ are the mixing parameters which describe the $q\bar{q} \rightarrow q\bar{q}$ transition amplitudes. In order to reduce the number of parameters, the following expressions are widely used in [7, 19]:

$$A_{ns} = A_{nn}X, \quad A_{ss} = A_{nn}X^2,$$

(2)

where $X$ is a phenomenological parameter describing the SU(3) broken ratio of the nonstrange and strange quark propagators via the constituent quark mass ratio. In the present work, we take $X = 0.63$, which is determined within the nonrelativistic constituent quark model [17, 20].

In the meson nonet, the physical isoscalar states $\varphi$ and $\varphi'$ are the eigenstates of mass-squared matrix $M_s^2$, and the masses square $M_\varphi^2$ and $M_\varphi'^2$ are the eigenvalues, respectively. The physical states $\varphi$ and $\varphi'$ can be related to the $S = s\bar{s}$ and $N = (u\bar{u} + d\bar{d})/\sqrt{2}$ by

$$\begin{pmatrix} \langle |\varphi\rangle \\ \langle |\varphi'|\rangle \end{pmatrix} = U \begin{pmatrix} |N\rangle \\ |S\rangle \end{pmatrix},$$

(3)

and the unitary matrix $U$ can be described as

$$UMU^\dagger = \begin{pmatrix} M_\varphi^2 & 0 \\ 0 & M_{\varphi'}^2 \end{pmatrix}.$$

(4)

The mix of the physical isoscalar states can also be described as

$$\begin{pmatrix} \langle |\varphi\rangle \\ \langle |\varphi'|\rangle \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} |\varphi_8\rangle \\ |\varphi_1\rangle \end{pmatrix},$$

(5)

with

$$\varphi_1 = \frac{u\bar{u} + d\bar{d} + s\bar{s}}{\sqrt{3}},$$

$$\varphi_8 = \frac{u\bar{u} + d\bar{d} - 2s\bar{s}}{\sqrt{6}},$$

(6)

where $\theta$ is the nonet mixing angle.

From the relations (3), (4), and (5), one will obtain

$$U = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \sqrt{\frac{2}{3}} & -\sqrt{\frac{1}{3}} \\ \sqrt{\frac{1}{3}} & \sqrt{\frac{2}{3}} \end{pmatrix}. $$

(7)

Based on the relations (4) and (7), the following relations are obtained:

$$M_N^2 + 2A = \left( \sqrt{\frac{2}{3}} \cos \theta - \sqrt{\frac{1}{3}} \sin \theta \right) M_\varphi^2$$

$$+ \left( \sqrt{\frac{1}{3}} \cos \theta + \sqrt{\frac{2}{3}} \sin \theta \right) M_{\varphi'}^2,$$

$$M_s^2 + AX^2 = \left( \sqrt{\frac{2}{3}} \cos \theta - \sqrt{\frac{1}{3}} \sin \theta \right) M_\varphi'^2$$

$$+ \left( \sqrt{\frac{1}{3}} \cos \theta + \sqrt{\frac{2}{3}} \sin \theta \right) M_{\varphi'}^2.$$  

(8)

Considering the fact that $N$ is the orthogonal partner of the isovector state of a meson nonet, one can expect that $N$ degenerates with isovector state $M_N = M_{1^+}$, and the masses (8) for the ground pseudoscalar meson, we obtain the pure $s\bar{s}$ member mass of the ground pseudoscalar meson nonet $M_{\varphi}(1^1S_0) = 685$ MeV [17]. Here and in the following, all the masses used as input for our calculation are taken from the PDG [5].

3. Mass Spectrum of $1^1P_1$ Meson State in Regge Phenomenology and Glueball Dominance Picture

Regge theory is cornered with the particle spectrum, the forces between particles, and the high energy behavior of scattering amplitudes. In [21], Li et al. investigate the masses of different meson multiplets in the framework of Regge phenomenology and suggest that the quasilinear Regge trajectories could provide a reasonable description for the meson mass spectrum. Khrushchov predicts the masses spectrum of excited meson states using the phenomenology formulas derived from the Regge trajectories [22]. Anisovich et al. also suggest that the mass of meson states can fit to the quasilinear Regge trajectories with good accuracy [23]. In the last decade, Regge trajectories are reanalysed to make predictions for the masses of states not yet to be discovered in experiment and thus the quantum numbers of the newly discovered states [21–28].

Based on the assumption that the hadrons with identical $J^{PC}$ quantum numbers obey quasilinear form of Regge trajectories, one has the following relations [21]:

$$l \propto \alpha_{ij}^{(0)} + \alpha_{ij}^{(1)} M_{ij}^2,$$

$$l \propto \alpha_{ij}^{(1)} + \alpha_{ij}^{(2)} M_{ij}^2,$$

$$l \propto \alpha_{ij}^{(2)} + \alpha_{ij}^{(3)} M_{ij}^2,$$

(9)
Table 1: Comparison of predictions for the masses of the \(1^1 P_1\) meson states with other references. (All in MeV.)

<table>
<thead>
<tr>
<th>Present work</th>
<th>[5]</th>
<th>[15]</th>
<th>[7]</th>
<th>[31]</th>
<th>[32]</th>
<th>[33]</th>
</tr>
</thead>
<tbody>
<tr>
<td>(M_{π})</td>
<td>1499</td>
<td>1386</td>
<td>1495.18</td>
<td>1470</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(M_{K})</td>
<td>1375</td>
<td>1370.03</td>
<td>1340</td>
<td>1368</td>
<td>1356</td>
<td></td>
</tr>
<tr>
<td>(M_{π})</td>
<td>3509</td>
<td>3525.93</td>
<td>3520</td>
<td>3526</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

where \(i\) denotes the quark and antiquark flavors, and \(l\) and \(M_{π}\) are the orbital momentum and mass of the \(i\) \(i\) meson. The parameters \(\alpha_{ij}\) and \(\alpha_{ij}(0)\) are, respectively, the slope and intercept of the Regge trajectory. The intercepts can be described as the following relation by [21, 24]:

\[
\alpha_{ii}(0) + \alpha_{jj}(0) = 2\alpha_{ij}(0).
\]  

(10)

The relation (10) is satisfied in two-dimensional QCD [29] and the quark bremsstrahlung model [30].

With the aid of the parity partners trajectories having the same slopes, we can apply relations (9)-(10) to the \(1^1 S_0\) and \(1^1 P_1\) meson multiplets to predict the following meson masses:

\[
\begin{align*}
\alpha_{s}^M M_s^2(1^1 S_0)_s + \alpha_{s}^M M_s^2(1^1 S_0)_s &= 2\alpha_{s}^M M_s^2(1^1 S_0)_s, \\
\alpha_{s}^M M_s^2(1^1 S_0)_s + \alpha_{s}^M M_s^2(1^1 S_0)_s &= 2\alpha_{s}^M M_s^2(1^1 S_0)_s, \\
\alpha_{s}^M M_s^2(1^1 S_0)_s + \alpha_{s}^M M_s^2(1^1 S_0)_s &= 2\alpha_{s}^M M_s^2(1^1 S_0)_s, \\
\alpha_{s}^M M_s^2(1^1 S_0)_s + \alpha_{s}^M M_s^2(1^1 S_0)_s &= 2\alpha_{s}^M M_s^2(1^1 S_0)_s, \\
\alpha_{s}^M M_s^2(1^1 S_0)_s + \alpha_{s}^M M_s^2(1^1 S_0)_s &= 2\alpha_{s}^M M_s^2(1^1 S_0)_s, \\
\alpha_{s}^M M_s^2(1^1 S_0)_s + \alpha_{s}^M M_s^2(1^1 S_0)_s &= 2\alpha_{s}^M M_s^2(1^1 S_0)_s.
\end{align*}
\]  

(11)

In order to eliminate the Regge slopes from the previous formulas, we adopt the following relation [24]:

\[
\frac{1}{\alpha_{ij}} + \frac{1}{\alpha_{jj}} = \frac{2}{\alpha_{ij}}.
\]  

(12)

For the \(1^1 S_0\) meson multiplet, all members are established well. For the \(1^1 P_1\) meson multiplet, only the \(b_1(1235)\) and \(h_1(1170)\) are established well, and the strange meson \(K_{1B}\) and the charmed mesons \(D_1(2420)\) and \(D_1(2536)\) are the mixtures of spin-singlet \(1^1 P_1\) and spin-triplet \(1^3 P_1\) states. In the present work, based on the small mass splitting between charmed mesons, we conclude that the effect of mixing on the charmed meson masses will be small. Therefore, inserting the masses of the corresponding mesons states and \(M(1^1 P_1)_{kij} = M_{D_1(2536)}\) as well as \(M(1^1 P_1)_{kij} = M_{D_1(2420)}\) into relations (11)-(12), the masses of the \(s\) \(s\) member, \(K_{1B}\) and \(h_1(1P)\) for the \(1^1 P_1\) meson multiplet are obtained, and the results are shown in Table 1.

The mass of the \(K_{1B}\) is important for the calculation of the mixing angle of the \(K_{1A}\) and \(K_{1B}\) states. In order to check consistence of the results in Table 1, we investigate the mass of kaon in the glueball dominante picture in the next section [18]. And the result is in good agreement with the prediction in the Regge phenomenology.

Table 2: Masses (in GeV) of the ground tensor and pseudovector glueball from different theoretical models.

<table>
<thead>
<tr>
<th>Mass</th>
<th>[34]</th>
<th>[35]</th>
<th>[36]</th>
<th>[37]</th>
<th>[38]</th>
<th>[39]</th>
<th>[40]</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>(M_{ten})</td>
<td>2.59</td>
<td>2.40</td>
<td>2.39</td>
<td>2.15</td>
<td>2.26</td>
<td>2.4</td>
<td>2.417</td>
<td>2.37</td>
</tr>
<tr>
<td>(M_{pse})</td>
<td>2.94</td>
<td>2.98</td>
<td>2.67</td>
<td>2.71</td>
<td></td>
<td></td>
<td>2.83</td>
<td></td>
</tr>
</tbody>
</table>

In the mass matrix, the quark mixing amplitudes in (1) can be expressed as

\[
A_{qq'} = \frac{\langle qq' | H_{PC}^q | k \rangle}{M_{qq'}^2 - M_{kk}^2} \langle k | H_{PC}^q | qq' \rangle,
\]

(13)

where \(H_{PC}^q\) is the quark pair creation operator for the flavor \(q\) and \(|k\rangle\) is a complete set of the intermediate states. Based on the assumption that there is no direct quarkonium-quarkonium mixing and the \(qq' \leftrightarrow q' q\) transitions are dominated by the ground glueball with the corresponding quantum numbers, we obtain the following relations:

\[
\begin{align*}
A_{nn} &= \frac{f_{nG}^2}{M_{nn}^2 - M_{G}^2}, \\
A_{ss} &= \frac{f_{sG}^2}{M_{ss}^2 - M_{G}^2},
\end{align*}
\]

(14)

where \(f_{nG}^q\) and \(f_{sG}^q\) in QCD to be a constant approximately independent of the quantum numbers of a meson nonet; namely [18],

\[
f_{nG}^q f_{sG}^q = \text{Const.}
\]

Following the relations (1), (4), and (14)-(15), we have

\[
\begin{align*}
\left( M_{b_1(1235)}^2 - M_{pse}^2 \right) \left( 2M_{K_{1B}}^2 - M_{b_1(1235)}^2 - M_{pse}^2 \right) &= A_{1^1 P_1} \left( M_{b_1(1235)}^2 - M_{pse}^2 \right) \\
\left( M_{a_1(1320)}^2 - M_{ten}^2 \right) \left( 2M_{K_{1B}}^2 - M_{a_1(1320)}^2 - M_{ten}^2 \right) &= A_{1^1 P_1} \left( M_{a_1(1320)}^2 - M_{ten}^2 \right)
\end{align*}
\]

(16)

with

\[
\begin{align*}
A_{1^1 P_1} &= M_{b_1(1170)}^2 + M_{h_1(1380)}^2 - 2M_{K_{1B}}^2, \\
A_{1^1 P_1} &= M_{b_1(1170)}^2 + M_{h_1(1380)}^2 - 2M_{K_{1B}}^2
\end{align*}
\]

(17)

where \(M_{ten}\) and \(M_{pse}\) are the masses of ground tensor and pseudovector glueball, respectively. In order to extract the mass of \(K_{1B}\) by using relation (16), we should determine the approximate masses of ground tensor and pseudovector glueballs. Some predictions on the masses of the glueball given by different theoretical models are shown in Table 2. In the present work, taking the masses of corresponding mesons in the relation (16) and the average values of glueball masses in Table 2 as input, we obtain the mass of the \(K_{1B}\). Our predicted result 1356 MeV is in agreement with our previous work.
Table 3: Predicted mixing angle $\phi$ of the $K_1(1270)$ and $K_1(1400)$.

<table>
<thead>
<tr>
<th>Refs</th>
<th>Present work</th>
<th>[15]</th>
<th>[12]</th>
<th>[41]</th>
<th>[42]</th>
<th>[43]</th>
<th>[44]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>\phi</td>
<td>$</td>
<td>61.6°</td>
<td>(59.55 ± 2.81)°</td>
<td>59.29°</td>
<td>54.7° or 35.3°</td>
<td>62°</td>
</tr>
</tbody>
</table>

4. The Mixing Angle of $K_{1A}$ and $K_{1B}$

According to the introduction in Section I, the $K_{1A}$ and $K_{1B}$ in the $1^1P_1$ and $1^3P_1$ meson states can mix to produce the physical states $K_1(1270)$ and $K_1(1400)$. In the $|1^1P_1\rangle$ and $|1^3P_1\rangle$ basis, the physical states $K_1(1270)$ and $K_1(1400)$ can be parameterized as [11]

$$K_1(1270) = \sin \phi |1^1P_1\rangle + \cos \phi |1^3P_1\rangle,$$
$$K_1(1400) = \cos \phi |1^1P_1\rangle - \sin \phi |1^3P_1\rangle. \tag{18}$$

In this work, $\phi$ is defined as the $K_{1A}$ and $K_{1B}$ mixing angle. To obtain the mixing angle $\phi$, we use the formula [31]

$$\tan^2 (2\phi) = \left( \frac{M^2_{K_{1A}} - M^2_{K_{1B}}}{M^2_{K_1(1400)} - M^2_{K_1(1270)}} \right)^2 - 1. \tag{19}$$

The relation (19) can also be rewritten as

$$\cos (2\phi) = \frac{M^2_{K_1(1400)} - M^2_{K_1(1270)}}{M^2_{K_{1A}} - M^2_{K_{1B}}}. \tag{20}$$

Moreover, using the following relation which is independent of the mixing angle, the mass of $K_{1A}$ is determined to be 1303 MeV. Consider

$$M^2_{K_{1A}} + M^2_{K_{1B}} = M^2_{K_1(1400)} + M^2_{K_1(1270)}. \tag{21}$$

With the help of the masses of $K_{1A}$, $K_{1B}$, $K_1(1270)$, and $K_1(1400)$, the mixing angle is determined to be $\pm 61.6°$.

5. Results and Conclusions

Based on the fact that the $b_1(1235)$, $h_1(1170)$, $D_1(2420)$, and $D_{s1}(2536)$ are assigned as the $1^3P_1$ meson multiplet, we obtain the masses of the $K_{1B}$, $h_1(1380)$, and $h_1(1P)$ in the $1^3P_1$ meson state. The mass of the $h_1(1380)$ is determined to be 1492 MeV, which is consistent with the value 1470 MeV predicted by Godfrey and Isgur in the potential model [7]. The value is about 100 MeV higher than that of the measured mass of the $h_1(1380)$ in the experiment. Besides, experimental information on the $h_1(1380)$ is rather restricted, which has been reported only by two collaborations, LASS Collaboration [45] (with mass 1380 ± 20 MeV and width 80 ± 30 MeV) and Crystal Barrel Collaboration [46] (with mass 1440 ± 60 MeV and width 170 ± 80 MeV). Therefore, we suggest that the assignment for this state needs to be further tested in the new experiments in the future.

The mass of $K_{1B}$ is useful for determining the mixing angle of the $K_1(1270)$ and $K_1(1400)$. In the present work, the mass for $K_{1B}$ is determined to be 1375 MeV, which is in agreement with the value from the meson-glueball mass relation and other theoretical approaches. The mixing angle of $K_1(1270)$ and $K_1(1400)$ is compared with other different predictions in Table 3. Our predictions may open a window for the assignment of the $1^3P_1$ meson multiplet in the new experiment in the future.

Acknowledgments

This project is supported by the Zhengzhou University of Light Industry Foundation of China (Grant nos. 2009XJ001 and 2012XJ008), in part by the National Natural Science Foundation of China (Grant nos. 10975018 and 1147197), and the Key Project of Scientific and Technological Research of the Education Department of Henan Province (Grant nos. 12BI40001 and 13BI40332).

References

[14] M. Andreotti, S. Bagnasco, W. Baldini et al., "Results of a search for the \( h_1^{(')} (1^{P_1}) \) state of charmonium in the \( \eta_c \gamma \) and \( J/\psi \pi \) decay modes," Physical Review D, vol. 72, no. 3, Article ID 032001, 11 pages, 2005.


[45] A. Abel, J. Adomeit, C. Amsler et al., "Antiproton-proton annihilation at rest into \( K_0 \overline{K_0} \pi^0 \pi^0 \)," Physics Letters B, vol. 415, no. 3, pp. 280–288, 1997.