Research Article

Effects of Flavor Dependence on Weak Decays of J/ψ and Υ

Rohit Dhir,^{1,2} R. C. Verma,³ and Avinash Sharma¹

¹ School of Basic and Applied Sciences, GGSIP University, New Delhi 110075, India

² Department of Physics and IPAP, Yonsei University, Seoul 120-749, Republic of Korea

³ Department of Physics, Punjabi University, Patiala 147001, India

Correspondence should be addressed to Rohit Dhir; dhir.rohit@gmail.com

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We carry out a detailed analysis of effects of flavor dependence of average transverse quark momentum inside a meson on $J/\psi \rightarrow P$ and $\Upsilon \rightarrow B_c$ transition form factors and two-body weak hadronic decays of J/ψ and Υ employing the factorization scheme. We predict the branching ratios of semileptonic and nonleptonic weak decays of J/ψ and Υ mesons in Cabibbo-angle-enhanced and Cabibbo-angle-suppressed modes.

1. Introduction

Due to remarkable improvements of experimental techniques and instrumentation in the recent years, it is expected that more accurate measurements may now be available for rare decays also. The BES collaboration has observed some rare decays including the semileptonic as well as nonleptonic modes [1, 2]. As a result, it has revived the interest in the rare weak decays of $J/\psi(c\bar{c})$ into the light quarks, whose branching ratios are expected to be of the order of 10^{-8} [3–11]. The future experiments [3, 12–14] of Beijing Electron Positron Collider (BES-III) and Large Hadron Collider (LHC) hope to accumulate data for more than 10^{10} events of J/ψ per year, which would make it possible to measure such rare decays. From the theoretical point of view, such weak decays are particularly interesting because these are expected to explore mechanism responsible for hadronic transitions and are also important for the study of nonperturbative QCD effects. Further, decays of a vector meson involve polarization effects that may help in probing the underlying dynamics and hadron structure. Within the standard model framework, the flavor changing decays of J/ψ and Υ states are also possible, though naively these are expected to have rather lower branching ratios in comparison to their conventional hadronic and radiative decays.

Earlier, Verma, Kamal, and Czarnecki (VKC) [6] had given the first estimates of $J/\psi \rightarrow PP/PV$ weak decay

rates using the factorization scheme. VKC employs the Bauer, Stech, and Wirbel (BSW) model to estimate the $J/\psi \rightarrow P$ transition form factors ignoring their q^2 dependence on their predictions. Further, Sharma and Verma [7] reanalyzed the J/ψ decays in the same model by including q^2 dependence and new values of the form factors and decay constants. The predictions in the earlier work [6, 7] are based on s -wave dominance for $J/\psi \rightarrow PV$. The analysis has also been extended to predict branching ratios of weak decays of Υ based on heavy quark effective theory. Recently, Wang et al. [8, 9] and Shen and Wang [10] have calculated $J/\psi \rightarrow P$ transition form factors using the QCD sum rules and employing covariant light-front quark model, respectively, to predict the decay rates of J/ψ meson.

In the present work, we employ BSW model framework [15–17] to investigate the effects of flavor dependence on $J/\psi \rightarrow P$ and $\Upsilon \rightarrow B_c$ transitions form factors and subsequently on $J/\psi \rightarrow PP/PV$ and $\Upsilon \rightarrow B_c P/B_c V$ decays, caused by possible variation of average transverse quark momentum (ω) inside a meson. In the light of Heavy Quark Symmetry (HQS) [18], we use the dipole q^2 dependence for the form factors A_0 , A_2 , and V and monopole q^2 dependence for the form factor A_1 and include contributions from p - and d -waves for $J/\psi \rightarrow PV$ and $\Upsilon \rightarrow B_c P/B_c V$ decays. We also predict branching ratios of weak semileptonic and nonleptonic decays of J/ψ and Υ in Cabibbo-angle-enhanced

and Cabibbo-angle-suppressed modes. In support of flavor dependence of the form factors and corresponding branching ratios, we also perform an alternate QCD *inspired* calculation to obtain ω for heavy quarkonium $c\bar{c}$ and $b\bar{b}$ states.

Order of presentation is as follows. In Sections 2 and 3, we outline the framework employed for analysing semileptonic and nonleptonic weak decays of J/ψ . Form factors and branching ratios in BSW model are presented in Section 4. Section 5 deals with effects of flavor dependence on J/ψ decays. Semileptonic and nonleptonic weak decays of Υ are analyzed in Section 6. Summary and discussion are given in the last section.

2. Semileptonic Weak Decays of J/ψ

The semileptonic decay amplitude $A_{\text{SL}}(J/\psi \rightarrow Pl\bar{\nu})$ can be expressed as

$$A_{\text{SL}}(J/\psi \rightarrow P) = \frac{G_F}{\sqrt{2}} V_{Qq}^* L^\mu H_\mu, \quad (1)$$

where

$$L^\mu = \bar{\nu}(k_\nu) \gamma^\mu (1 - \gamma_5) l(k_l), \quad (2)$$

$$H_\mu = \langle P | J_\mu | J/\psi \rangle,$$

V_{Qq} is the appropriate CKM matrix element for $Q \rightarrow q$ transition and J^μ is the usual weak V - A current. $J/\psi \rightarrow P$ matrix element is given by

$$\begin{aligned} \langle P | J_\mu | J/\psi \rangle = & \frac{1}{m_{J/\psi} + m_P} \varepsilon_{\mu\nu\rho\sigma} \varepsilon_{J/\psi}^\nu (P_{J/\psi} + P_P)^\rho q^\sigma V(q^2) \\ & - i(m_{J/\psi} + m_P) \varepsilon_{J/\psi}^\mu A_1(q^2) \\ & - i \frac{\varepsilon_{J/\psi} \cdot q}{m_{J/\psi} + m_P} (P_{J/\psi} + P_P)^\mu A_2(q^2) \\ & + i \frac{\varepsilon_{J/\psi} \cdot q}{q^2} (2m_{J/\psi}) q^\mu A_3(q^2) \\ & - i \frac{\varepsilon_{J/\psi} \cdot q}{q^2} (2m_{J/\psi}) q^\mu A_0(q^2), \end{aligned} \quad (3)$$

where $\varepsilon_{J/\psi}$ is the polarization vector of J/ψ and $P_{J/\psi}$ and P_P are the four momenta of J/ψ and pseudoscalar meson, respectively, and $q^\mu = (P_{J/\psi} - P_P)^\mu$. $A_3(q^2)$ is related to $A_1(q^2)$ and $A_2(q^2)$ as:

$$A_3(0) = \frac{(m_{J/\psi} + m_P)}{2m_{J/\psi}} A_1(0) + \frac{(m_{J/\psi} - m_P)}{2m_{J/\psi}} A_2(0). \quad (4)$$

The total decay width for $J/\psi \rightarrow Pl\bar{\nu}$ is the sum of longitudinal and transverse decay widths given by

$$\Gamma(J/\psi \rightarrow Pl\bar{\nu}) = \sum_{i=L,\pm} \Gamma_i(J/\psi \rightarrow Pl\bar{\nu}), \quad (5)$$

where the longitudinal decay width (Γ_L) is defined as

$$\begin{aligned} \Gamma_L(J/\psi \rightarrow Pl\bar{\nu}) = & \frac{G_F^2}{(2\pi)^3} |V_{Qq}|^2 \\ & \times \int_{m_l^2}^{q^2} dq^2 \left(\frac{q^2 - m_l^2}{q^2} \right)^2 \\ & \times \frac{\sqrt{\lambda(m_{J/\psi}^2, m_P^2, q^2)}}{24m_{J/\psi}^3} \\ & \times \left[\frac{3m_l^2}{2q^2} \lambda(m_{J/\psi}^2, m_P^2, q^2) A_0^2(q^2) \right. \\ & \left. + \left(1 + \frac{m_l^2}{2q^2} \right) |h_0|^2 \right] \end{aligned} \quad (6a)$$

and the transverse decay width ($\Gamma_T = \Gamma_+ + \Gamma_-$) is expressed as

$$\begin{aligned} \Gamma_\pm(J/\psi \rightarrow Pl\bar{\nu}) = & \frac{G_F^2}{(2\pi)^3} |V_{Qq}|^2 \int_{m_l^2}^{q^2} dq^2 \left(\frac{q^2 - m_l^2}{q^2} \right)^2 \\ & \times \frac{\sqrt{\lambda(m_{J/\psi}^2, m_P^2, q^2)}}{24m_{J/\psi}^3} \left(1 + \frac{m_l^2}{2q^2} \right) |h_\pm|^2. \end{aligned} \quad (6b)$$

The helicity amplitudes h_0 and $h_\pm$ are given by

$$\begin{aligned} h_0 = & \frac{(m_{J/\psi} + m_P)}{2m_{J/\psi}} \left\{ (m_{J/\psi}^2 - m_P^2 - q^2) A_1(q^2) \right. \\ & \left. - \frac{\lambda(m_{J/\psi}^2, m_P^2, q^2)}{(m_{J/\psi} + m_P)^2} A_2(q^2) \right\}, \\ h_\pm = & \left\{ \frac{\lambda(m_{J/\psi}^2, m_P^2, q^2)}{(m_{J/\psi} + m_P)} V(q^2) \right. \\ & \left. \mp (m_{J/\psi} + m_P) A_1(q^2) \right\}, \end{aligned} \quad (7)$$

where m_l is the mass of the lepton, $0 \leq q^2 \leq q_{\text{max}}^2 = (m_{J/\psi} - m_P)^2$, and $\lambda(m_{J/\psi}^2, m_P^2, q^2) = (m_{J/\psi}^2 + m_P^2 - q^2)^2 - 4m_{J/\psi}^2 m_P^2$ is related to the three-momentum of the daughter meson in the rest frame of J/ψ meson by

$$P_X = \frac{\sqrt{\lambda(m_{J/\psi}^2, m_P^2, q^2)}}{2m_{J/\psi}}. \quad (8)$$

3. Nonleptonic Weak Decays of J/ψ

3.1. Weak Hamiltonian. The QCD modified weak hamiltonian [19] generating the c -quark decays for Cabibbo-angle-enhanced mode ($\Delta C = \Delta S = -1$) is given by

$$H_W = \frac{G_F}{\sqrt{2}} V_{ud} V_{cs}^* \{a_1 (\bar{s}c) (\bar{u}d) + a_2 (\bar{s}d) (\bar{u}c)\} + h.c., \quad (9a)$$

and for Cabibbo-angle-suppressed mode ($\Delta C = -1, \Delta S = 0$)

$$H_W = \frac{G_F}{\sqrt{2}} V_{ud} V_{cd}^* \left[a_1 \{(\bar{u}d) (\bar{d}c) - (\bar{u}s) (\bar{s}c)\} + a_2 \{(\bar{u}c) (\bar{d}d) - (\bar{u}c) (\bar{s}s)\} \right] + h.c., \quad (9b)$$

where $\bar{q}_1 q_2 \equiv \bar{q}_1 \gamma_\mu (1 - \gamma_5) q_2$ represents the color singlet V - A current and V_{ij} denote standard Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix elements. a 's are the undetermined coefficients assigned to the effective charge current, a_1 , and the effective neutral current, a_2 , parts of the weak Hamiltonian. These parameters are related to the QCD coefficients $c_{1,2}$ as follows:

$$a_{1,2} = c_{1,2} + \zeta c_{2,1}, \quad (10)$$

where $\zeta = 1/N_c$, N_c is the number of colors. Usually ζ is treated as a free parameter to be fixed by the experiment. However, we follow the conventional N_c limit to fix the QCD coefficients $a_1 \approx c_1$ and $a_2 \approx c_2$, where [19]

$$a_1 = 1.26, \quad a_2 = -0.51 \quad (11)$$

are obtained on the basis of $D \rightarrow K\pi$ decays.

3.2. Branching Ratios. In the standard factorization scheme, the decay amplitudes of $J/\psi \rightarrow PP/PV$ are obtained by sandwiching the QCD modified weak Hamiltonian (up to the weak scale $G_F/\sqrt{2} \times \text{CKM factor} \times \text{QCD coefficient}$),

$$\langle PX | H_W | J/\psi \rangle \sim \langle X | J^\mu | 0 \rangle \langle P | J_\mu^\dagger | J/\psi \rangle, \quad (12)$$

where $X = P, V$. Matrix elements [15–17] of the weak currents are defined as

$$\langle P(k) | A_\mu | 0 \rangle = -i f_P k_\mu, \quad (13a)$$

$$\langle V(k) | V_\mu | 0 \rangle = \varepsilon_\mu^* m_V f_V. \quad (13b)$$

The decay rate formula for $J/\psi \rightarrow PP$ decays [20, 21] is given by

$$\Gamma(J/\psi \rightarrow PP) = \frac{p_c^3}{24\pi m_{J/\psi}^2} |A(J/\psi \rightarrow PP)|^2, \quad (14)$$

where p_c is the magnitude of the three momenta of final state meson in the rest frame of J/ψ meson and $m_{J/\psi}$ denote its mass. In general, the three-momentum p_c is defined as

$$p_c = \frac{1}{2m_{J/\psi}} \left[m_{J/\psi}^2 - (m_P + m_X)^2 \right] \left[m_{J/\psi}^2 - (m_P - m_X)^2 \right]^{1/2}. \quad (15)$$

Thus the decay amplitude $A(J/\psi \rightarrow PP)$, say for color-enhanced $J/\psi \rightarrow D_s^+ \pi^-$ decay, can be expressed as

$$A(J/\psi \rightarrow D_s^+ \pi^-) = \frac{G_F}{\sqrt{2}} V_{ud} V_{cs}^* a_1 \times (2m_{J/\psi}) f_\pi A_0^{J/\psi D_s}(m_\pi^2). \quad (16)$$

For color-suppressed decays the QCD factor a_1 is replaced by a_2 .

The decay rate of $J/\psi \rightarrow PV$ decays [22, 23], composed of three independent Helicity amplitudes H_0 and $H_{\pm 1}$, is expressed as

$$\Gamma(J/\psi \rightarrow PV) = \frac{p_c}{8\pi m_{J/\psi}^2} (|H_0|^2 + |H_{+1}|^2 + |H_{-1}|^2). \quad (17)$$

The amplitudes H_0 and $H_{\pm 1}$ are defined as follows:

$$H_{\pm 1} = a \pm c(x^2 - 1)^{1/2}, \quad H_0 = -ax - b(x^2 - 1), \quad (18)$$

where

$$x = \frac{m_{J/\psi}^2 - m_P^2 - m_V^2}{2m_{J/\psi} m_V},$$

$$a = m_V f_V (m_{J/\psi} + m_V) A_1(m_V^2),$$

$$b = \frac{-2m_{J/\psi} m_V^2 f_V A_2(m_V^2)}{(m_{J/\psi} + m_P)}, \quad (19)$$

$$c = \frac{-2m_{J/\psi} m_V^2 f_V V(m_V^2)}{m_{J/\psi} + m_P}.$$

The coefficients a , b , and c describe the s -, d -, and p -wave contributions, respectively.

4. $J/\psi \rightarrow P$ Form Factors in BSW Framework

We employ the BSW [15–17] model for evaluating the meson form factors. In this model, the meson wave function is given by

$$\psi_m(\mathbf{p}_T, x) = N_m \sqrt{x(1-x)} \exp\left(-\frac{p_T^2}{2\omega^2}\right)$$

$$\times \exp\left(-\frac{m^2}{2\omega^2} \left(x - \frac{1}{2} - \frac{m_{q_1}^2 - m_{q_2}^2}{2m^2}\right)^2\right), \quad (20)$$

TABLE I: Form factors of $J/\psi \rightarrow P$ transitions.

Models		$A_0^{J/\psi D}(0)$	$A_1^{J/\psi D}(0)$	$A_2^{J/\psi D}(0)$	$V^{J/\psi D}(0)$
This work	$\omega = 0.40 \text{ GeV}$	0.40	0.44	-0.23	1.17
	Using flavor dependent ω	0.55 ± 0.02	$0.77^{+0.09}_{-0.07}$	$0.31^{+0.27}_{-0.16}$	$2.14^{+0.15}_{-0.11}$
	QCD inspired $\omega \equiv \alpha_s m$	0.54	0.80	0.47	2.21
	[7]	0.61	0.68	-0.28	1.82
	[8]	0.27	0.27	0.34	0.81
	[10]	0.68	0.68	0.18	1.6
Models		$A_0^{J/\psi D_s}(0)$	$A_1^{J/\psi D_s}(0)$	$A_2^{J/\psi D_s}(0)$	$V^{J/\psi D_s}(0)$
This work	$\omega = 0.40 \text{ GeV}$	0.47	0.55	-0.14	1.26
	Using flavor dependent ω	$0.71^{+0.04}_{-0.02}$	0.94 ± 0.07	$0.33^{+0.33}_{-0.23}$	$2.30^{+0.09}_{-0.06}$
	QCD inspired $\omega \equiv \alpha_s m$	0.69	0.96	0.51	2.36
	[7]	0.66	0.78	-0.10	1.80
	[8]	0.37	0.38	0.35	1.07
	[10]	0.68	0.68	0.13	1.8

where m denotes the meson mass, m_i denotes the i th quark mass, and N_m is the normalization factor. ω is the average transverse quark momentum, $\omega^2 = \langle p_T^2 \rangle$, which is of the order of Λ_{QCD} [18].

Expressing the current J_μ in terms of the annihilation and creation operators, the form factors are given by the following integrals:

$$A_0^{J/\psi P}(0) = A_3^{J/\psi P}(0) = \int d^2 \mathbf{p}_T \quad (21)$$

$$\times \int_0^1 dx \left(\psi_{J/\psi}^{*1,0}(\mathbf{p}_T, x) \sigma_z^{(0)} \psi_P(\mathbf{p}_T, x) \right),$$

$$V(0) = \frac{m_{q_1(J/\psi)} - m_{q_1(P)}}{m_{J/\psi} - m_P} I, \quad (22)$$

$$A_1(0) = \frac{m_{q_1(J/\psi)} + m_{q_1(P)}}{m_{J/\psi} + m_P} I, \quad (23)$$

where

$$I = \sqrt{2} \int d^2 \mathbf{p}_T \int_0^1 \frac{dx}{x} \left(\psi_{J/\psi}^{*1,-1}(\mathbf{p}_T, x) i\sigma_y^{(0)} \psi_P(\mathbf{p}_T, x) \right), \quad (24)$$

$m_{q_1(J/\psi)}$ and $m_{q_1(P)}$ denote masses of the nonspectator quarks participating in the quark decay process. From (21) it is clear that the form factors are sensitive to the choice of ω , which is treated as a free parameter in the model. We wish to remark that in BSW [15–17] model the form factors are calculated by taking same value of $\omega = 0.40 \text{ GeV}$ for initial as well as final states. With the quark masses (in GeV),

$$m_u = m_d = 0.35, \quad m_s = 0.55, \quad (25)$$

$$m_c = 1.7, \quad m_b = 4.9.$$

$J/\psi \rightarrow P$ form factors thus calculated are presented in rows 2 and 10 of Table I.

It has been pointed out in the BSW2 model [18] that consistency with the Heavy Quark Symmetry (HQS) requires certain form factors such as F_1 , A_0 , A_2 , and V to have dipole q^2 dependence, whereas A_1 has monopole q^2 dependence, that is,

$$A_0(q^2) = \frac{A_0(0)}{(1 - q^2/m_P^2)^2}, \quad A_1(q^2) = \frac{A_1(0)}{(1 - q^2/m_A^2)}, \quad (26)$$

$$A_2(q^2) = \frac{A_2(0)}{(1 - q^2/m_A^2)^2}, \quad V(q^2) = \frac{V(0)}{(1 - q^2/m_V^2)^2} \quad (27)$$

with appropriate pole masses.

4.1. Numerical Results. In this section, we present the branching ratios calculated from the form factors (without flavor dependence) obtained in the last section for semileptonic and nonleptonic weak decays of J/ψ meson.

4.1.1. Branching Ratios of Semileptonic Decays. Using these form factors, we obtain the branching ratios of semileptonic weak decays of $J/\psi \rightarrow Pl\bar{\nu}$, which are presented in column 2 of Table 2. The branching ratios BR_0 , BR_- , and BR_+ for various $J/\psi \rightarrow PV$, corresponding to the Helicity amplitudes h_0 and $h_{\pm 1}$, are given in columns 2, 3, and 4 of Table 3. We find that net branching ratios for semileptonic decays, $B(J/\psi \rightarrow D^+ e^- \bar{\nu}_e) = 0.23 \times 10^{-10}$, $B(J/\psi \rightarrow D_s^+ e^- \bar{\nu}_e) = 3.89 \times 10^{-10}$, $B(J/\psi \rightarrow D^+ \mu^- \bar{\nu}_\mu) = 0.22 \times 10^{-10}$, and $B(J/\psi \rightarrow D_s^+ \mu^- \bar{\nu}_\mu) = 3.75 \times 10^{-10}$, are well below the experimental limits.

TABLE 2: Branching ratios (in the units of 10^{-10}) of $J/\psi \rightarrow P\bar{V}$ decays.

Decays	This work			[8]	[10]
	$\omega = 0.40 \text{ GeV}$	Using flavor dependent ω	QCD inspired $\omega \equiv \alpha_s m$		
$J/\psi \rightarrow D^+ e^- \bar{\nu}_e$	0.23	$0.60^{+0.08}_{-0.07}$	0.62	0.073	5.4
$J/\psi \rightarrow D_s^+ e^- \bar{\nu}_e$	3.89	$10.4^{+0.90}_{-0.75}$	10.6	1.8	5.6
$J/\psi \rightarrow D^+ \mu^- \bar{\nu}_\mu$	0.22	$0.58^{+0.08}_{-0.06}$	0.60	0.071	5.1
$J/\psi \rightarrow D_s^+ \mu^- \bar{\nu}_\mu$	3.75	$9.93^{+0.95}_{-0.65}$	10.2	1.7	5.6

TABLE 3: Branching ratios BR_0 , BR_- , and BR_+ corresponding to the Helicity amplitudes H_0 and $H_{\pm 1}$ (in the units of 10^{-10}) of $J/\psi \rightarrow P\bar{V}$ decays.

Decays	$\omega = 0.40 \text{ GeV}$			Using flavor dependent ω			QCD inspired $\omega \equiv \alpha_s m$		
	BR_0	BR_-	BR_+	BR_0	BR_-	BR_+	BR_0	BR_-	BR_+
$J/\psi \rightarrow D^+ e^- \bar{\nu}_e$	0.14	0.076	0.014	0.31 ± 0.04	$0.24^{+0.04}_{-0.03}$	$0.048^{+0.005}_{-0.004}$	0.31	0.26	0.052
$J/\psi \rightarrow D_s^+ e^- \bar{\nu}_e$	2.65	0.11	0.19	6.42 ± 0.40	$3.25^{+0.42}_{-0.20}$	$0.59^{+0.06}_{-0.05}$	6.40	3.54	0.63
$J/\psi \rightarrow D^+ \mu^- \bar{\nu}_\mu$	0.14	0.074	0.013	0.30 ± 0.04	$0.24^{+0.04}_{-0.03}$	0.047 ± 0.005	0.30	0.36	0.050
$J/\psi \rightarrow D_s^+ \mu^- \bar{\nu}_\mu$	2.55	0.10	0.19	6.17 ± 0.41	$3.26^{+0.33}_{-0.25}$	$0.58^{+0.07}_{-0.05}$	6.14	3.44	0.61

4.1.2. Branching Ratios of Nonleptonic Decays

- (a) $J/\psi \rightarrow PP$ Decays: for η and η' emitting decays, we take the following basis:

$$\eta = \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}) \sin \phi_p - (s\bar{s}) \cos \phi_p, \quad (28)$$

$$\eta' = \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}) \cos \phi_p + (s\bar{s}) \sin \phi_p, \quad (29)$$

where $\phi_p = \theta_{\text{ideal}} - \theta_{\text{physical}}$; we take $\theta_{\text{physical}} = -15.4^\circ$ [24]. With the following decay constants (in GeV) [24–26],

$$f_\pi = 0.131, \quad f_K = 0.160, \quad f_\eta = 0.133, \quad f_{\eta'} = 0.126. \quad (30)$$

Branching ratios for the Cabibbo-angle-enhanced decays are $B(J/\psi \rightarrow D_s^+ \pi^-) = 3.32 \times 10^{-10}$ and $B(J/\psi \rightarrow D^0 K^0) = 0.72 \times 10^{-10}$ (column 2 of Table 4).

- (b) $J/\psi \rightarrow PV$ Decays: using the following decay constants (in GeV) [24–26]:

$$\begin{aligned} f_\rho &= 0.221, & f_{K^*} &= 0.220, \\ f_\omega &= 0.195, & f_\phi &= 0.229, \end{aligned} \quad (31)$$

obtained branching ratios for various $J/\psi \rightarrow PV$ decays are $B(J/\psi \rightarrow D_s^+ \rho^-) = 1.77 \times 10^{-9}$ and $B(J/\psi \rightarrow D^0 K^{*0}) = 2.51 \times 10^{-10}$ (column 2 of Table 5).

For the sake of comparison of the relative contributions of the Helicity amplitudes H_0 and $H_{\pm 1}$, we have calculated the corresponding branching ratios BR_0 , BR_- , and BR_+ for these decays, which are given in column 2 of Table 6.

5. Flavor Dependent Effects on J/ψ Decays

Since ω , being a dimensional quantity, may possess some flavor dependence, it may not be justified to take the same value for all the mesons. In our recent work [27, 28], we have investigated such flavor dependence through ω in $B_c \rightarrow P/V$ form factors and consequently in $B_c \rightarrow PP/PV$ decay widths, which may be measured in near future experiments.

5.1. Flavor Dependence on $J/\psi \rightarrow P$ Form Factors. In this section, we investigate effects of the flavor dependence on $J/\psi \rightarrow P/V$ decays. Following the prescription of [27, 28], we estimate ω for different mesons from $|\psi(0)|^2$, that is, wave function overlap at the origin, using the following ansatz:

$$|\psi(0)|^2 = \text{constant} \times \omega^3 \quad (32)$$

which may be justified in light of inverse size of the mesons and dimensionality arguments. $|\psi(0)|^2$ is extracted from the hyperfine splitting for the meson masses [29],

$$|\psi(0)|^2 = \frac{9m_i m_j}{32\alpha_s \pi} (m_V - m_P), \quad (33)$$

where m_V and m_P , respectively, denote masses of the vector and pseudoscalar mesons composed of i and j quarks. Conventionally, meson masses fix quark masses (in GeV) to be $m_u = m_d = 0.31$, $m_s = 0.49$, $m_c = 1.7$, and $m_b = 5.0$ for $\alpha_s(m_b) = 0.19$, $\alpha_s(m_c) = 0.25$, and $\alpha_s = 0.48$ (for light flavors u , d , and s). However, uncertainty of α_s , particularly for the light quark sector, may lead to some variation in the quark masses [25, 26, 30–34]. So, in our analysis, we allow the following range in the quark masses:

$$\begin{aligned} m_u &= m_d = 0.31 \pm 0.05, & m_s &= 0.49 \pm 0.05, \\ m_c &= 1.7 \pm 0.05, & m_b &= 5.0 \pm 0.05. \end{aligned} \quad (34)$$

Calculated values of $|\psi(0)|^2$ for different mesons are given in column 2 of Table 7. We use the well-measured form

TABLE 4: Branching ratios (in the units of 10^{-10}) of $J/\psi \rightarrow PP$ decays.

Decay	This work			[7]	[8]	[10]
	$\omega = 0.40 \text{ GeV}$	Using flavor dependent ω	QCD inspired $\omega \equiv \alpha_s m$			
$\Delta C = \Delta S = +1$						
$J/\psi \rightarrow D_s^+ \pi^-$	3.32	$7.41^{+0.13}_{-0.23}$	7.13	6.12	2.0	2.5
$J/\psi \rightarrow D^0 K^0$	0.72	$1.39^{+0.10}_{-0.14}$	1.34	1.96	0.36	—
$\Delta C = +1, \Delta S = 0$						
$J/\psi \rightarrow D_s^+ K^-$	0.24	0.53 ± 0.02	0.52	0.39	0.16	—
$J/\psi \rightarrow D^+ \pi^-$	0.15	$0.29^{+0.02}_{-0.03}$	0.28	0.39	0.08	—
$J/\psi \rightarrow D^0 \pi^0$	0.012	$0.024^{+0.002}_{-0.003}$	0.023	0.039	—	—
$J/\psi \rightarrow D^0 \eta$	0.036	$0.070^{+0.005}_{-0.007}$	0.067	0.011	—	—
$J/\psi \rightarrow D^0 \eta'$	0.002	$0.004^{+0.0002}_{-0.0005}$	0.004	0.002	—	—
$\Delta C = +1, \Delta S = -1$						
$J/\psi \rightarrow D^+ K^-$	0.012	0.023 ± 0.002	0.022	—	—	0.5
$J/\psi \rightarrow D^0 \bar{K}^0$	0.002	$0.004^{+0.000}_{-0.001}$	0.004	—	—	—

TABLE 5: Branching Ratios (in the units of 10^{-10}) of $J/\psi \rightarrow PV$ decays.

Decay	This Work			[7]	[8]	[10]
	$\omega = 0.40 \text{ GeV}$	Using flavor dependent ω	QCD inspired $\omega \equiv \alpha_s m$			
$\Delta C = \Delta S = +1$						
$J/\psi \rightarrow D_s^+ \rho^-$	17.7	$51.1^{+7.6}_{-6.0}$	53.2	25.41	12.6	28
$J/\psi \rightarrow D^0 K^{*0}$	2.51	$7.61^{+1.6}_{-1.2}$	8.12	7.19	1.54	—
$\Delta C = +1, \Delta S = 0$						
$J/\psi \rightarrow D_s^+ K^{*-}$	0.97	$2.82^{+0.40}_{-0.30}$	2.96	1.48	0.82	—
$J/\psi \rightarrow D^+ \rho^-$	0.72	$2.16^{+0.5}_{-0.3}$	2.28	1.54	0.42	—
$J/\psi \rightarrow D^0 \rho^0$	0.06	0.18 ± 0.03	0.19	0.15	—	—
$J/\psi \rightarrow D^0 \omega$	0.05	$0.16^{+0.03}_{-0.02}$	0.17	0.13	—	—
$J/\psi \rightarrow D^0 \phi$	0.14	$0.42^{+0.08}_{-0.07}$	0.44	0.46	—	—
$\Delta C = +1, \Delta S = -1$						
$J/\psi \rightarrow D^+ K^{*-}$	0.042	$0.13^{+0.02}_{-0.03}$	0.13	—	—	5.5
$J/\psi \rightarrow D^0 \bar{K}^{*0}$	0.007	$0.021^{+0.004}_{-0.003}$	0.022	—	—	—

factor [35–37] $F_0^{DK}(0) = 0.78 \pm 0.04$ to fix $\omega_D = 0.43 \pm 0.03 \text{ GeV}$ which in turn yields ω for other mesons as given in column 3 of Table 7. We find that all the $J/\psi \rightarrow P$ transition form factors get significantly enhanced due to the flavor dependence of parameter ω , particularly, $\omega_{J/\psi} = 0.71 \pm 0.02 \text{ GeV}$. The $J/\psi \rightarrow P$ transition form factors, required for the weak decays under investigation, are found to be increased, which are given in rows 3 and 11 of Table 1. Uncertainties shown in the form factors arise due to variation in the quark masses discussed above.

Note that the normally uncertainties lie between 3% and 11% for all the form factors except for $A_2(q^2)$, which is drastically increased from 80% to 100%. These are likely to change theoretical predictions for branching of semileptonic as well as nonleptonic decays. Since the form factors $A_1(q^2)$, $V(q^2)$ and $A_2(q^2)$, describe the s -, p -, and d -wave contributions to the nonleptonic decay amplitudes, respectively, the s - d interference term may acquire significance when both $A_1(q^2)$ and $A_2(q^2)$ become large. Incidentally, $A_2(q^2)$ being small, such contribution turned out to be very small as

compared to the terms proportional to $A_1(q^2)$. Therefore, the variations in $A_2(q^2)$ even of the order of 80% to 100% are not going to affect the branching ratios seriously.

5.2. QCD Inspired Calculation of ω . The wave function for the heavy meson like, for example, B meson is known to have a pronounced peak at $x \approx x_0 = m_b/M_B$ (M_B being the B meson mass). It is thus obvious that the only region close to the peak position contributes to any degree of significance. This implies a good approximation for parameter ω to be of the order of Λ_{QCD} . This may be easily understood since $\langle p_T^2 \rangle \sim \Lambda_{\text{QCD}}^2$, $\psi_m(p_T, x) \rightarrow 0$ if $p_T^2 \gg \Lambda_{\text{QCD}}^2$ and has peak when $x \rightarrow 1$. Also, the average transverse quark momentum of the heavy quark equals to that of heavy meson such that $x \rightarrow 1$. In accord with the heavy quark effective theory, it also implies equal velocities of the b -quark and of the B meson up to corrections of order p_T/M_B . In the light of these arguments, we present another method to determine ω , for instance by looking at the typical inverse size of the system under scrutiny, which is of order Λ_{QCD} for the lighter

TABLE 6: Branching Ratios BR_0 , BR_- and BR_+ corresponding to the Helicity amplitudes H_0 and $H_{\pm 1}$ (in the units of 10^{-10}) of $J/\psi \rightarrow PV$ decays.

Decay	$\omega = 0.40 \text{ GeV}$			Using flavor dependent ω			QCD inspired $\omega \equiv \alpha_s m$		
	BR_0	BR_+	BR_-	BR_0	BR_+	BR_-	BR_0	BR_+	BR_-
$\Delta C = \Delta S = +1$									
$J/\psi \rightarrow D_s^+ \rho^-$	6.26	2.75	8.74	$17.3^{+2.3}_{-1.8}$	$7.55^{+0.10}_{-0.30}$	$26.3^{+3.6}_{-2.7}$	17.7	7.97	27.7
$J/\psi \rightarrow D^0 K^{*0}$	0.75	0.32	1.45	$2.13^{+0.42}_{-0.35}$	$0.93^{+0.18}_{-0.20}$	$4.56^{+0.86}_{-0.67}$	2.25	1.00	4.85
$\Delta C = +1, \Delta S = 0$									
$J/\psi \rightarrow D_s^+ K^{*-}$	0.27	0.18	0.52	$0.74^{+0.10}_{-0.08}$	$0.52^{+0.10}_{-0.08}$	$1.56^{+0.42}_{-0.35}$	0.77	0.54	1.65
$J/\psi \rightarrow D^+ \rho^-$	0.027	0.076	0.37	$0.77^{+0.15}_{-0.12}$	$0.22^{+0.06}_{-0.05}$	$1.17^{+0.22}_{-0.17}$	0.81	0.24	1.25
$J/\psi \rightarrow D^0 \rho^0$	0.022	0.066	0.031	$0.064^{+0.012}_{-0.011}$	$0.018^{+0.005}_{-0.004}$	$0.096^{+0.018}_{-0.014}$	0.067	0.019	0.103
$J/\psi \rightarrow D^0 \omega$	0.020	0.006	0.028	$0.056^{+0.011}_{-0.019}$	$0.017^{+0.005}_{-0.004}$	$0.088^{+0.022}_{-0.013}$	0.059	0.018	0.094
$J/\psi \rightarrow D^0 \phi$	0.032	0.022	0.083	$0.091^{+0.018}_{-0.015}$	$0.063^{+0.018}_{-0.013}$	$0.26^{+0.05}_{-0.04}$	0.096	0.068	0.28
$\Delta C = +1, \Delta S = -1$									
$J/\psi \rightarrow D^+ K^{*-}$	0.012	0.0053	0.024	$0.035^{+0.007}_{-0.006}$	$0.015^{+0.005}_{-0.003}$	$0.075^{+0.014}_{-0.011}$	0.037	0.016	0.080
$J/\psi \rightarrow D^0 \bar{K}^{*0}$	0.002	0.004	9	$0.006^{+0.001}_{-0.001}$	$0.003^{+0.000}_{-0.001}$	$0.012^{+0.003}_{-0.001}$	0.006	0.003	0.013

TABLE 7: $|\psi(0)|^2$ and ω for vector and pseudoscalar mesons.

Meson	$ \psi(0) ^2$ (in GeV^3)	Parameter ω (in GeV)	QCD inspired $\omega \equiv \alpha_s m$ (in GeV)
$\rho(\pi)$	0.011 ± 0.003	0.33 ± 0.03	—
$K^*(K)$	0.011 ± 0.03	0.33 ± 0.03	—
$D^*(D)$	$0.026^{+0.006}_{-0.004}$	0.43 ± 0.03	—
$D_s^*(D_s)$	$0.041^{+0.007}_{-0.005}$	$0.51^{+0.02}_{-0.03}$	—
$J/\psi(\eta_c)$	$0.115^{+0.011}_{-0.008}$	0.71 ± 0.02	0.79
$B^*(B)$	0.033 ± 0.005	$0.47^{+0.02}_{-0.03}$	—
$B_s^*(B_s)$	0.053 ± 0.006	0.55 ± 0.02	—
$B_c^*(B_c)$	$0.281^{+0.0017}_{-0.0011}$	0.96 ± 0.02	—
$\Upsilon(\eta_b)$	$1.850^{+0.03}_{-0.04}$	$1.80^{+0.01}_{-0.02}$	1.83

mesons and the heavy-light ones and is of order meson mass multiplied by strong coupling constant (α_s) for the heavy quarkonium ($q\bar{q}$) states. It is interesting to note that following the relation

$$\omega \equiv \alpha_s \times m \quad \text{for } c\bar{c} \text{ and } b\bar{b} \quad (35)$$

gives larger values of ω , that is, $\omega_{J/\psi} = 0.79$ and $\omega_\Upsilon = 1.83$, ignoring the small uncertainties in α_s for heavier quarks. A plot for ω thus calculated for various mesons (especially for $c\bar{c}$ and $b\bar{b}$ states) has interesting pattern with respect to the physical masses of the mesons, which is shown in Figure 1.

It may also be pointed out that Sharma et al. [38] have reported that $|\psi(0)|^2$, which may be extracted from leptonic decays of $c\bar{c}$ and $b\bar{b}$ vector mesons, cannot simultaneously fit with that needed for their masses. Perhaps similar analysis needs to be carried out for $|\psi(0)|^2$ extracted from form factors and hyperfine splitting.

5.3. Branching Ratios Including Flavor Dependent Effects. Different values of ω for initial and final state mesons yield $J/\psi \rightarrow P$ transition form factors, which are given in Table 1. Using these form factors we calculate the branching ratios of semileptonic decays of J/ψ as presented in column 3 of

FIGURE 1: Variation of average transverse quark momentum with respect to physical meson masses.

Table 2. Contributions from the Helicity amplitudes H_0 and $H_{\pm 1}$ are also obtained for the corresponding branching ratios

BR_0 , BR_- , and BR_+ for these decays and are given in column 3 of Table 3.

Further, we calculate branching ratios of various $J/\psi \rightarrow PP/PV$ decays, which are listed in column 3 of Tables 4 and 5. In order to compare flavor dependent effects for different Helicity terms with those obtained at fixed $\omega = 0.40$ GeV, we present branching ratios BR_0 , BR_- , and BR_+ in column 3 of Table 6. For the sake of comparison, we also give results of other works in Tables 2, 4, and 5. The following observations are made.

- (i) Branching ratios of all semileptonic as well as nonleptonic decays of J/ψ meson get significantly enhanced because of inclusion of flavor dependent effects.
- (ii) The enhanced branching ratios for semileptonic decays are $B(J/\psi \rightarrow D_s^+ e^- \bar{\nu}_e) = 10.4_{-0.75}^{+0.90} \times 10^{-10}$ and $B(J/\psi \rightarrow D_s^+ \mu^- \bar{\nu}_\mu) = 9.93_{-0.65}^{+0.95} \times 10^{-10}$. Presently, only experimental [1] upper limits are available,

$$\begin{aligned} B(J/\psi \rightarrow D^+ e^- \bar{\nu}_e + c.c.) &< 1.2 \times 10^{-5}, \\ B(J/\psi \rightarrow D_s^+ e^- \bar{\nu}_e + c.c.) &< 4.9 \times 10^{-5}. \end{aligned} \quad (36)$$

- (iii) Among the Cabibbo-angle-enhanced $J/\psi \rightarrow PP$ decays, we find that branching ratios of dominant decays are $B(J/\psi \rightarrow D_s^+ \pi^-) = 7.41_{-0.23}^{+0.13} \times 10^{-10}$ and $B(J/\psi \rightarrow D^0 K^0) = 1.39_{-0.14}^{+0.10} \times 10^{-10}$.
- (iv) For Cabibbo-angle-suppressed but color-enhanced modes $J/\psi \rightarrow D_s^+ K^-$ and $J/\psi \rightarrow D^+ \pi^-$, we obtain the following ratios based on the naive factorization scheme:

$$\begin{aligned} \frac{B(J/\psi \rightarrow D_s^+ K^-)}{B(J/\psi \rightarrow D_s^+ \pi^-)} &= 0.072 \pm 0.003, \\ \frac{B(J/\psi \rightarrow D^+ \pi^-)}{B(J/\psi \rightarrow D^0 K^0)} &= 0.21 \pm 0.03. \end{aligned} \quad (37)$$

- (v) In case of $J/\psi \rightarrow PV$ decays, for the color-enhanced decay of the Cabibbo-angle-enhanced mode, we calculate $B(J/\psi \rightarrow D_s^+ \rho^-) = 5.11_{-0.60}^{+0.76} \times 10^{-9}$, which is higher than the branching ratio of $J/\psi \rightarrow D_s^+ \pi^-$. While for color-suppressed decay we obtain $B(J/\psi \rightarrow D^0 K^{*0}) = 7.61_{-1.2}^{+1.6} \times 10^{-9}$. Our analysis also yields

$$\begin{aligned} \frac{B(J/\psi \rightarrow D_s^+ K^{*-})}{B(J/\psi \rightarrow D_s^+ \rho^-)} &= 0.055 \pm 0.010, \\ \frac{B(J/\psi \rightarrow D^+ \rho^-)}{B(J/\psi \rightarrow D^0 K^{*0})} &= 0.28 \pm 0.07. \end{aligned} \quad (38)$$

5.4. Branching Ratios Using QCD Inspired ω . Using QCD inspired values of $\omega(\equiv \alpha_s m)$ for heavy quarkonium $c\bar{c}$ and $b\bar{b}$ states, the obtained form factors are given in rows 4 and 12 of Table 1. It is interesting to note all the form factors except $A_0(0)$ get significantly enhanced. Correspondingly, branching ratios of semileptonic and nonleptonic decays are calculated, which are presented in column 4 of Tables 2 to 6. We observe the following.

- (i) Branching ratios of all semileptonic decays of J/ψ meson are nearly the same though marginally enhanced, $B(J/\psi \rightarrow D_s^+ e^- \bar{\nu}_e) = 10.6 \times 10^{-10}$ and $B(J/\psi \rightarrow D_s^+ \mu^- \bar{\nu}_\mu) = 10.2 \times 10^{-10}$ in comparison to those obtained in our work including the flavor dependent effects. This happens due to the larger value ω for $c\bar{c}$ state, leading to increased overlap between the initial and final state wave functions.
- (ii) Branching ratios of all the $J/\psi \rightarrow PP$ decays, involving the $A_0(0)$ form factor only, are slightly decreased in comparison to the flavor dependent branching ratios predicted in our analysis.
- (iii) Branching ratios of all the $J/\psi \rightarrow PV$ decays are comparable to our predictions for the flavor dependent case.

6. Semileptonic and Nonleptonic Weak Decays of Υ

6.1. $\Upsilon \rightarrow B_c$ Form Factors and Branching Ratios. Using the framework described in Sections 4 and 5, we extend our analysis for bottom sector. We obtain the form factor for $\Upsilon \rightarrow B_c$ transition at both $\omega = 0.40$ GeV and using the flavor dependent ω , which are compared in rows 2 and 3 of Table 8. Note that the form factors get enhanced by several orders of magnitude for flavor dependent ω . This happens due to the increased overlap of Υ and the final state wave functions for flavor dependent ω in comparison to that at fixed $\omega = 0.40$ GeV as shown in Figures 2 and 3. We wish to remark that the flavor dependent form factors come close to the expectation [7] based on HQET considerations (row 4 of Table 8). Consequently, the branching ratios of semileptonic and nonleptonic weak decays of Υ get significantly enhanced.

We also use the QCD inspired method to determine ω , that is, $\omega_\Upsilon = 1.83$, which match well with our value 1.80 for Υ . Subsequently obtained form factors are given in row 4 of Table 8, which are largely the same as our results, so we exclude the results for decay rates based on the QCD inspired method for further discussion.

6.2. Semileptonic Weak Decays of Υ . Using $\Upsilon \rightarrow B_c$ form factor, appearing in $b \rightarrow c$ transition, and the decay rate formula given in (5), firstly, we calculate the branching ratios for semileptonic decays of Υ at fixed $\omega = 0.40$ GeV, which are given in column 2 of Table 9. The predicted branching ratios of semileptonic weak decays of Υ using the flavor dependent effects are given in column 3 of Table 9. We find that the branching ratio of dominant semileptonic decay is $B(\Upsilon \rightarrow B_c^+ e^- \bar{\nu}_e) = 1.70 \times 10^{-10}$ that is sufficiently enhanced in comparison to (6.28×10^{-15}) obtained using fixed ω . Here also, we give branching ratios of longitudinal and transverse components (BR_0 , BR_- , and BR_+) separately, for the semileptonic decays in Table 10.

6.3. Nonleptonic Weak Decays of Υ . In this section, the analysis is extended to $\Upsilon \rightarrow PP/PV$ decays. The effective

TABLE 8: Form factors of $\Upsilon \rightarrow B_c$ transitions.

Models		$A_0^{YB_c}(0)$	$A_1^{YB_c}(0)$	$A_2^{YB_c}(0)$	$V^{YB_c}(0)$
$\omega = 0.40$ GeV		0.003	0.003	0.003	0.008
This work	Using flavor dependent ω	0.46 ± 0.01	$0.62^{+0.02}_{-0.01}$	$0.38^{+0.06}_{-0.09}$	1.61 ± 0.01
	QCD inspired $\omega \equiv \alpha_s m^*$	0.46	0.62	0.38	1.62
	[7]	0.98	1.01	1.01	1.01

*The form factors here have shown no significant change in the values, therefore, the BRs for the decays involving $\Upsilon \rightarrow B_c$ form factors for QCD inspired ω remain the same as for flavor dependent ω .

TABLE 9: Branching ratios (in the units of 10^{-10}) of $\Upsilon \rightarrow Pl\bar{\nu}$ decays.

Decays	This Work	
	$\omega = 0.40$ GeV	Using flavor dependent ω
$\Upsilon \rightarrow B_c^+ e^- \bar{\nu}_e$	6.28×10^{-5}	$1.70^{+0.03}_{-0.02}$
$\Upsilon \rightarrow B_c^+ \mu^- \bar{\nu}_\mu$	6.24×10^{-5}	$1.69^{+0.04}_{-0.02}$
$\Upsilon \rightarrow B_c^+ \tau^- \bar{\nu}_\tau$	9.86×10^{-5}	$0.29^{+0.05}_{-0.02}$

FIGURE 2: The overlap of wave functions for $\Upsilon \rightarrow B_c$ decays at $\omega_\Upsilon = \omega_{B_c} = 0.40$ GeV. FIGURE 3: The overlap of wave functions for $\Upsilon \rightarrow B_c$ decays at $\omega_\Upsilon = 1.80$ GeV and $\omega_{B_c} = 0.96$ GeV.

weak Hamiltonian generating the dominant b quark decays involving $b \rightarrow c$ transition is given by

$$H_W^{\Delta b=1} = \frac{G_F}{\sqrt{2}} \left\{ V_{cb} V_{ud}^* \left[a_1 (\bar{c}b) (\bar{d}u) + a_2 (\bar{d}b) (\bar{c}u) \right] + V_{cb} V_{cs}^* \left[a_1 (\bar{c}b) (\bar{s}c) + a_2 (\bar{s}b) (\bar{c}c) \right] \right\} + h.c., \quad (39)$$

for the CKM-favored mode. In our analysis we use $a_1 = 1.12$, $a_2 = -0.26$.

Similar to $J/\psi \rightarrow PP$ decays, the factorization scheme expresses weak decay amplitudes as a product of matrix elements of the weak currents (up to the scale $(G_F/\sqrt{2}) \times$ (CKM factor $\times$ QCD factor)) as

$$A(\Upsilon \rightarrow PP) \sim \langle P | J^\mu | 0 \rangle \langle P | J_\mu | \Upsilon \rangle. \quad (40)$$

For instance, the decay amplitude for the color-enhanced mode $\Upsilon \rightarrow B_c^+ \pi^-$ of the CKM-favored decays is given by

$$A(\Upsilon \rightarrow B_c^+ \pi^-) = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* a_1 f_\pi (2m_\Upsilon) A_0^{Y \rightarrow B_c}(m_\pi^2). \quad (41)$$

6.3.1. $\Upsilon \rightarrow PP$ Decays. Following the procedure employed in Sections 3 and 4, we calculate the branching ratios for CKM-favored mode both for fixed $\omega = 0.40$ GeV and for flavor dependent ω , which are presented in columns 2 and 3, respectively, of Table II. In addition to the decay constants given in (30) we use $f_D = 0.208$ GeV and $f_{D_s} = 0.273$ GeV. The dominant decays in this mode are found to be $B(\Upsilon \rightarrow B_c^+ D_s^-) = 0.47 \times 10^{-10} (2.25 \times 10^{-17})$ and $B(\Upsilon \rightarrow B_c^+ \pi^-) = 0.16 \times 10^{-10} (7.90 \times 10^{-16})$, where values in the parentheses are calculated at fixed $\omega = 0.40$ GeV. It may be noted that our branching ratios including the flavor dependent effects compare well with the earlier results, $B(\Upsilon \rightarrow B_c^+ D_s^-) = 0.76 \times 10^{-10}$ and $B(\Upsilon \rightarrow B_c^+ \pi^-) = 0.33 \times 10^{-10}$, obtained by Sharma and Verma [7] using HQET considerations.

TABLE 10: Branching ratios BR_0 , BR_- , and BR_+ corresponding to the Helicity amplitudes h_0 and $h_{\pm 1}$ (in the units of 10^{-10}) of $Y \rightarrow P\bar{V}$ decays.

Decays	$\omega = 0.40 \text{ GeV}$			Using flavor dependent ω		
	BR_0	BR_-	BR_+	BR_0	BR_-	BR_+
$Y \rightarrow B_c^+ e^- \bar{\nu}_e$	5.50×10^{-5}	0.46×10^{-5}	3.10×10^{-5}	$1.40_{-0.02}^{+0.03}$	$0.18_{-0.00}^{+0.01}$	$0.13_{-0.00}^{+0.01}$
$Y \rightarrow B_c^+ \mu^- \bar{\nu}_\mu$	5.47×10^{-5}	0.45×10^{-5}	3.08×10^{-5}	$1.39_{-0.02}^{+0.03}$	$0.18_{-0.00}^{+0.01}$	$0.12_{-0.00}^{+0.01}$
$Y \rightarrow B_c^+ \tau^- \bar{\nu}_\tau$	0.86×10^{-5}	0.81×10^{-6}	0.45×10^{-6}	$0.24_{-0.01}^{+0.00}$	0.03	0.018

TABLE 11: Branching ratios (in the units of 10^{-10}) of $Y \rightarrow B_c P$ decays.

Decays	This Work		[7]
	$\omega = 0.40 \text{ GeV}$	Using flavor dependent ω	
	$\Delta b = 1, \Delta C = 1, \Delta S = 0$		
$Y \rightarrow B_c^+ \pi^-$	7.90×10^{-6}	0.16 ± 0.01	0.33
	$\Delta b = 1, \Delta C = 0, \Delta S = -1$		
$Y \rightarrow B_c^+ D_s^-$	2.25×10^{-5}	0.47 ± 0.01	0.76
	$\Delta b = 1, \Delta C = 1, \Delta S = -1$		
$Y \rightarrow B_c^+ K^-$	6.06×10^{-7}	0.013	0.024
	$\Delta b = 1, \Delta C = 0, \Delta S = 0$		
$Y \rightarrow B_c^+ D^-$	7.25×10^{-7}	$0.015_{-0.00}^{+0.01}$	0.031

6.3.2. $Y \rightarrow PV$ Decays. Employing the decay rate formula for such decays as discussed in Section 3 and following the similar procedure used for $J/\psi \rightarrow PV$ decays, we determine the decay amplitudes for various $Y \rightarrow B_c V$ decays for the CKM-favored modes. We use [25, 26] $f_{D^*} = 0.245 \text{ GeV}$ and $f_{D_s^*} = f_{D_s} = 0.273 \text{ GeV}$ to calculate the branching ratios which are given in Table 12. We observe that the dominant mode is $B(Y \rightarrow B_c^+ D_s^{*-}) = 1.79 \times 10^{-10}$ (5.01×10^{-15}), following by $B(Y \rightarrow B_c^+ \rho^-) = 0.65 \times 10^{-10}$ (2.12×10^{-15}). Here also, the values in the parentheses are those obtained for fixed $\omega = 0.40 \text{ GeV}$. For comparison of the contributions of the Helicity amplitudes H_0 and $H_{\pm 1}$ involved, we have calculated the corresponding branching ratios BR_0 , BR_- , and BR_+ for various $Y \rightarrow PV$ decays both for $\omega = 0.40 \text{ GeV}$ and for flavor dependent ω , which are presented in Table 13.

7. Summary and Discussion

In this paper, we have predicted the rare semileptonic and nonleptonic weak decays of J/ψ and Y mesons. It may be mentioned that the present work differs from the previous ones [6, 7] based on the BSW framework in three aspects.

- (i) Firstly, in the light of HQS based BSW2 model [12], we use the dipole q^2 dependence for the form factors A_0, A_2 , and V , while the earlier work [6, 7] used the monopole q^2 dependence for these form factors.
- (ii) Secondly, the results given in [7] are based on s -wave dominance, while our results take into account the contributions from p - and d -waves also.
- (iii) Lastly, we incorporate flavor dependent effects on $J/\psi \rightarrow P$ and $Y \rightarrow B_c$ form factors in our analysis through the different values of ω for initial and final state mesons.

Also, to support the case of flavor dependence of the form factors, we use an alternate QCD inspired approach to determine ω for heavy quarkonium $c\bar{c}$ and $b\bar{b}$ states. The following conclusions are readily drawn out of our analysis.

(A) Weak decays of J/ψ .

- (i) Initially, we calculate $J/\psi \rightarrow P$ transition form factors at $\omega = 0.40 \text{ GeV}$ for all the mesons: among the Cabibbo-angle-enhanced $J/\psi \rightarrow PP$ decays, branching ratios of dominant decays are calculated to be $B(J/\psi \rightarrow D_s^+ \pi^-) = 3.32 \times 10^{-10}$ and $(J/\psi \rightarrow D^0 K^0) = 0.72 \times 10^{-10}$ for Cabibbo-angle-enhanced mode $J/\psi \rightarrow PV$ decays, branching ratios of dominant decays are obtained to be $B(J/\psi \rightarrow D_s^+ \rho^-) = 1.77 \times 10^{-9}$ and $B(J/\psi \rightarrow D^0 K^{*-}) = 2.51 \times 10^{-10}$.
- (ii) We investigate the effects of possible flavor dependence of ω by determining $|\psi(0)|^2$ from meson masses to fix ω for all the mesons through an ansatz; we, then, calculated $J/\psi \rightarrow P$ transition form factors which get significantly enhanced as compared to those at fixed $\omega = 0.40$; as a result of this, branching ratios of all semileptonic as well as nonleptonic decays are enhanced.
- (iii) Among the Cabibbo-angle-enhanced $J/\psi \rightarrow PP/PV$ decays, the dominant decays are predicted as $B(J/\psi \rightarrow D_s^+ \pi^-) = 7.41_{-0.23}^{+0.13} \times 10^{-10}$, $B(J/\psi \rightarrow D^0 K^0) = 1.39_{-0.14}^{+0.10} \times 10^{-10}$, $B(J/\psi \rightarrow D_s^+ \rho^-) = 5.11_{-0.60}^{+0.76} \times 10^{-9}$, and $B(J/\psi \rightarrow D^0 K^{*-}) = 7.61_{-1.2}^{+1.6} \times 10^{-10}$.

TABLE 12: Branching ratios (in the units of 10^{-10}) of $\Upsilon \rightarrow B_c V$ decays.

Decays	This Work		[7]
	$\omega = 0.40 \text{ GeV}$	Using flavor dependent ω	
	$\Delta b = 1, \Delta C = 1, \Delta S = 0$		
$\Upsilon \rightarrow B_c^+ \rho^-$	2.12×10^{-5}	0.65 ± 0.03	0.88
	$\Delta b = 1, \Delta C = 0, \Delta S = -1$		
$\Upsilon \rightarrow B_c^+ D_s^{*-}$	5.01×10^{-5}	$1.79^{+0.09}_{-0.04}$	2.57
	$\Delta b = 1, \Delta C = 1, \Delta S = -1$		
$\Upsilon \rightarrow B_c^+ K^{*-}$	1.14×10^{-6}	0.035 ± 0.02	0.050
	$\Delta b = 1, \Delta C = 0, \Delta S = 0$		
$\Upsilon \rightarrow B_c^+ D^{*-}$	2.06×10^{-6}	$0.073^{+0.003}_{-0.002}$	0.11

TABLE 13: Branching ratios BR_0 , BR_- , and BR_+ corresponding to the Helicity amplitudes H_0 and $H_{\pm 1}$ (in the units of 10^{-10}) of $\Upsilon \rightarrow B_c V$ decays.

Decays	$\omega = 0.40 \text{ GeV}$			Using flavor dependent ω		
	BR_0	BR_-	BR_+	BR_0	BR_-	BR_+
	$\Delta b = 1, \Delta C = 1, \Delta S = 0$					
$\Upsilon \rightarrow B_c^+ \rho^-$	1.82×10^{-5}	2.48×10^{-6}	6.53×10^{-7}	$0.53^{+0.02}_{-0.01}$	$0.09^{+0.01}_{-0.00}$	$0.024^{+0.02}_{-0.01}$
	$\Delta b = 1, \Delta C = 0, \Delta S = -1$					
$\Upsilon \rightarrow B_c^+ D_s^{*-}$	1.94×10^{-5}	2.30×10^{-5}	7.73×10^{-6}	$0.62^{+0.02}_{-0.01}$	$0.88^{+0.04}_{-0.02}$	$0.29^{+0.01}_{-0.01}$
	$\Delta b = 1, \Delta C = 1, \Delta S = -1$					
$\Upsilon \rightarrow B_c^+ K^{*-}$	9.20×10^{-5}	1.71×10^{-8}	4.55×10^{-8}	$0.027^{+0.001}_{-0.000}$	$0.007^{+0.001}_{-0.001}$	0.017
	$\Delta b = 1, \Delta C = 0, \Delta S = 0$					
$\Upsilon \rightarrow B_c^+ D^{*-}$	8.75×10^{-7}	2.28×10^{-7}	2.97×10^{-7}	$0.027^{+0.001}_{-0.001}$	$0.035^{+0.001}_{-0.001}$	$0.010^{+0.002}_{-0.001}$

(B) Weak decays of Υ :

- (i) It is observed that inclusion of flavor dependent effects through ω significantly enhances the form factors and, consequently, the branching ratios of semileptonic and nonleptonic weak decays of Υ .
- (ii) Branching ratio of the dominating semileptonic mode is predicted to be $B(\Upsilon \rightarrow B_c^+ e^- \bar{\nu}_e) = 1.70 \times 10^{-10}$.
- (iii) Among the Cabibbo-angle-enhanced $\Upsilon \rightarrow B_c P / B_c V$ decay modes, the dominant ones are predicted as $B(\Upsilon \rightarrow B_c^+ D_s^-) = 0.47 \times 10^{-10}$, $B(\Upsilon \rightarrow B_c^+ \pi^-) = 0.16 \times 10^{-10}$, $B(\Upsilon \rightarrow B_c^+ D_s^{*-}) = 1.79 \times 10^{-10}$, and $B(\Upsilon \rightarrow B_c^+ \rho^-) = 0.65 \times 10^{-10}$.
- (iv) Though the predicted branching ratios in the QCD inspired method of obtaining ω are marginally changed, the alternate calculation of ω for heavy quarkonium $c\bar{c}$ and $b\bar{b}$ states seems to support the flavor dependent effects on rare weak decays of J/ψ and Υ mesons.
- (v) Our predictions also agree well with those obtained by Sharma and Verma [7] using the HQET considerations.

It is hoped that these branching ratios would lie in the detectable range and may be measured in future experiments.

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