Research Article

On Current Conversion between Particle Rapidity and Pseudorapidity Distributions in High Energy Collisions

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In high energy collisions, one usually needs to give a conversion between the particle rapidity and pseudorapidity distributions. Currently, two equivalent conversion formulas are used in experimental and theoretical analyses. An investigation in the present work shows that the two conversions are incomplete. Then, we give a revision on the current conversion between the particle rapidity and pseudorapidity distributions.

1. Introduction

High energy collisions are an important research field in particle and nuclear physics. In the collisions, a lot of particles are produced, and the rapidity and/or pseudorapidity distributions can be obtained and studied [1–3]. Usually, one needs to do a conversion between the rapidity and pseudorapidity distributions in the case of only one of the two distributions being obtained. There are two equivalent conversion formulas used in the current literature [4–11]. Naturally, one thinks that the two conversions are perfect in investigations of the rapidity and pseudorapidity distributions.

However, our incidental find shows that the two conversions are incomplete. In obtaining the Jacobian in the current literature [4–11], a nongiven quantity, namely, transverse momentum, is erroneously used as a given one, which renders an incomplete conversion. In this paper, we will give a reanalysis on the Jacobian. A revised conversion between the rapidity and pseudorapidity distributions will be presented.

2. General Definition

We consider a system of high energy projectile-target collisions. The incident projectile direction is defined as oz axis, and the reaction plane is defined as xoz plane. Let E , p , p_L , p_T , m_0 , and θ denote, respectively, the energy, momentum, longitudinal momentum, transverse momentum, rest mass, and emission angle of a concerned particle. According to

general textbooks on particle physics [12, 13], the rapidity (which is in fact the longitudinal rapidity) is defined by

$$y \equiv \frac{1}{2} \ln \left(\frac{E + p_L}{E - p_L} \right), \quad (1)$$

where

$$E = \sqrt{p^2 + m_0^2}, \quad (2)$$

$$p_L = p \cos \theta.$$

In the case of $p \gg m_0$, we have

$$y \approx \frac{1}{2} \ln \left(\frac{p + p_L}{p - p_L} \right) = \frac{1}{2} \ln \left(\frac{1 + \cos \theta}{1 - \cos \theta} \right) = -\ln \tan \left(\frac{\theta}{2} \right) \equiv \eta, \quad (3)$$

where η is the pseudorapidity.

Because the condition of $p \gg m_0$ is not always satisfied, the pseudorapidity distribution (density function) $f_\eta(\eta) = (1/N)(dN/d\eta)$ and the rapidity distribution (density function) $f_y(y) = (1/N)(dN/dy)$ are not approximately equal to each other, where dN denotes the particle number in the pseudorapidity or rapidity bin and N denotes the total number of considered particles.

3. Current Conversion

To give a conversion between $dN/d\eta$ and dN/dy in the case of one of them being obtained, one has two equivalent methods which are currently used in the literature [4–11]. According to [4, 5, 11], the first conversion relation between $dN/d\eta$ and dN/dy can be given by

$$\frac{dN}{d\eta} = \frac{dN}{dy} \frac{dy}{d\eta} = \frac{p}{E} \frac{dN}{dy}, \quad (4)$$

where

$$\begin{aligned} \frac{p}{E} &= \frac{\sqrt{E^2 - m_0^2}}{E} = \sqrt{1 - \frac{m_0^2}{E^2}} \\ &= \sqrt{1 - \left(\frac{m_0}{\sqrt{p_T^2 + m_0^2} \cosh y} \right)^2}. \end{aligned} \quad (5)$$

Then, the first conversion is given by [4–6]

$$\frac{dN}{d\eta} = \frac{p}{E} \frac{dN}{dy} = \sqrt{1 - \left(\frac{m_0}{\sqrt{p_T^2 + m_0^2} \cosh y} \right)^2} \frac{dN}{dy}, \quad (6)$$

where $\sqrt{p_T^2 + m_0^2} \equiv m_T$ is the transverse mass. We see that the first conversion is related to p_T .

The second conversion is given in [5, 7–11]. We have

$$\begin{aligned} \frac{dN}{d\eta} &= \frac{\cosh \eta}{\sqrt{1 + m_0^2 p_T^{-2} + \sinh^2 \eta}} \frac{dN}{dy} = \frac{\cosh \eta}{\sqrt{m_T^2 p_T^{-2} + \sinh^2 \eta}} \frac{dN}{dy} \\ &= \frac{\cosh \eta}{\sqrt{\cosh^2 \eta + m_0^2 p_T^{-2}}} \frac{dN}{dy}, \end{aligned} \quad (7)$$

which is also related to p_T . In [11], a similar conversion which uses $m^2 P^{-2}$ instead of $m_0^2 p_T^{-2}$ in (7) is given, where $m = 350$ MeV, $P = 0.13$ GeV + 0.32 GeV $(\sqrt{s}/1 \text{ TeV})^{0.115}$, and $\sqrt{s}$ denotes the center-of-mass energy. The conversion used in [11] is a mutation of the second conversion.

We now give the eduction of the current conversion. According to [12],

$$\begin{aligned} y &= \frac{1}{2} \ln \left(\frac{E + p_L}{E - p_L} \right) = \frac{1}{2} \ln \left(\frac{\sqrt{p^2 + m_0^2} + p_L}{\sqrt{p^2 + m_0^2} - p_L} \right) \\ &= \frac{1}{2} \ln \left(\frac{\sqrt{p_T^2 \cosh^2 \eta + m_0^2} + p_T \sinh \eta}{\sqrt{p_T^2 \cosh^2 \eta + m_0^2} - p_T \sinh \eta} \right). \end{aligned} \quad (8)$$

In the case of p_T being a given quantity, we have

$$\begin{aligned} \frac{dy}{d\eta} &= \frac{1}{2} \cdot \frac{\sqrt{p_T^2 \cosh^2 \eta + m_0^2} - p_T \sinh \eta}{\sqrt{p_T^2 \cosh^2 \eta + m_0^2} + p_T \sinh \eta} \\ &\quad \cdot \frac{d}{d\eta} \left(\frac{\sqrt{p_T^2 \cosh^2 \eta + m_0^2} + p_T \sinh \eta}{\sqrt{p_T^2 \cosh^2 \eta + m_0^2} - p_T \sinh \eta} \right) \\ &= \frac{1}{2} \cdot \left[\frac{1}{\sqrt{p_T^2 \cosh^2 \eta + m_0^2} + p_T \sinh \eta} \right. \\ &\quad \cdot \frac{d}{d\eta} \left(\sqrt{p_T^2 \cosh^2 \eta + m_0^2} + p_T \sinh \eta \right) \\ &\quad \left. - \frac{1}{\sqrt{p_T^2 \cosh^2 \eta + m_0^2} - p_T \sinh \eta} \right. \\ &\quad \left. \cdot \frac{d}{d\eta} \left(\sqrt{p_T^2 \cosh^2 \eta + m_0^2} - p_T \sinh \eta \right) \right] \\ &= \frac{1}{2} \cdot \left[\frac{1}{\sqrt{p_T^2 \cosh^2 \eta + m_0^2} + p_T \sinh \eta} \right. \\ &\quad \times \left(\frac{p_T^2 \cosh \eta \sinh \eta}{\sqrt{p_T^2 \cosh^2 \eta + m_0^2}} + p_T \cosh \eta \right) \\ &\quad \left. - \frac{1}{\sqrt{p_T^2 \cosh^2 \eta + m_0^2} - p_T \sinh \eta} \right. \\ &\quad \times \left(\frac{p_T^2 \cosh \eta \sinh \eta}{\sqrt{p_T^2 \cosh^2 \eta + m_0^2}} - p_T \cosh \eta \right) \left. \right] \\ &= \frac{1}{2} \cdot \frac{1}{p_T^2 + m_0^2} \\ &\quad \cdot \left[\left(\frac{p_T^2 \cosh \eta \sinh \eta}{\sqrt{p_T^2 \cosh^2 \eta + m_0^2}} + p_T \cosh \eta \right) \right. \\ &\quad \times \left(\sqrt{p_T^2 \cosh^2 \eta + m_0^2} - p_T \sinh \eta \right) \\ &\quad \left. - \left(\frac{p_T^2 \cosh \eta \sinh \eta}{\sqrt{p_T^2 \cosh^2 \eta + m_0^2}} - p_T \cosh \eta \right) \right. \\ &\quad \times \left. \left(\sqrt{p_T^2 \cosh^2 \eta + m_0^2} + p_T \sinh \eta \right) \right] \end{aligned}$$

$$\begin{aligned}
&= \frac{1}{2} \cdot \frac{1}{p_T^2 + m_0^2} \cdot \frac{2(p_T^2 + m_0^2) p_T \cosh \eta}{\sqrt{p_T^2 \cosh^2 \eta + m_0^2}} \\
&= \frac{p_T \cosh \eta}{\sqrt{p_T^2 \cosh^2 \eta + m_0^2}} \\
&= \frac{\cosh \eta}{\sqrt{\cosh^2 \eta + m_0^2 p_T^{-2}}} = \frac{p}{E} = \beta,
\end{aligned} \tag{9}$$

where β denotes the velocity of the concerned particle. Then, we obtain the current conversion.

However, we would like to point out that the previous conversion is incomplete due to the fact that $p_T = p / \cosh \eta$ is also a function of η , which should be considered in doing the differential treatment. Instead, p and E can be regarded as given quantities.

4. Revised Conversion

In the differential treatment, we think that both the $p_T = p / \cosh \eta$ and $p_L = p \tanh \eta$ are functions of η . Contrarily, p and E have the fixed values for a given particle. Then,

$$y = \frac{1}{2} \ln \left(\frac{E + p_L}{E - p_L} \right) = \frac{1}{2} \ln \left(\frac{E + p \tanh \eta}{E - p \tanh \eta} \right) \tag{10}$$

$$= \frac{1}{2} \ln \left(\frac{1 + \beta \tanh \eta}{1 - \beta \tanh \eta} \right) \equiv h(\eta),$$

$$\begin{aligned}
\frac{dy}{d\eta} &= \frac{1}{2} \cdot \frac{1 - \beta \tanh \eta}{1 + \beta \tanh \eta} \cdot \frac{d}{d\eta} \left(\frac{1 + \beta \tanh \eta}{1 - \beta \tanh \eta} \right) \\
&= \frac{1}{2} \cdot \frac{1 - \beta \tanh \eta}{1 + \beta \tanh \eta} \\
&\quad \cdot \left[\frac{1}{1 - \beta \tanh \eta} + \frac{1 + \beta \tanh \eta}{(1 - \beta \tanh \eta)^2} \right] \cdot \beta \frac{d}{d\eta} (\tanh \eta) \\
&= \frac{1}{2} \cdot \frac{1 - \beta \tanh \eta}{1 + \beta \tanh \eta} \cdot \frac{2\beta}{(1 - \beta \tanh \eta)^2} \cdot \frac{1}{\cosh^2 \eta}
\end{aligned}$$

$$\begin{aligned}
&= \frac{\beta}{1 - \beta^2 \tanh^2 \eta} \cdot \frac{1}{\cosh^2 \eta} \\
&= \frac{\beta}{\cosh^2 \eta - \beta^2 \sinh^2 \eta} = \frac{\beta}{1 + (1 - \beta^2) \sinh^2 \eta} \\
&= \frac{1 - (1 - \beta^2) \cosh^2 \eta}{\beta}.
\end{aligned}$$

(11)

It is different from the first conversion which gives that $dy/d\eta = \beta$. Correspondingly,

$$\begin{aligned}
\eta &= \frac{1}{2} \ln \left(\frac{p + p_L}{p - p_L} \right) = \frac{1}{2} \ln \left(\frac{p + E \tanh y}{p - E \tanh y} \right) \\
&= \frac{1}{2} \ln \left(\frac{\beta + \tanh y}{\beta - \tanh y} \right) \equiv \varphi(y),
\end{aligned} \tag{12}$$

$$\begin{aligned}
\frac{d\eta}{dy} &= \frac{1}{2} \cdot \frac{\beta - \tanh y}{\beta + \tanh y} \cdot \frac{d}{dy} \left(\frac{\beta + \tanh y}{\beta - \tanh y} \right) \\
&= \frac{1}{2} \cdot \frac{\beta - \tanh y}{\beta + \tanh y} \cdot \left[\frac{1}{\beta - \tanh y} + \frac{\beta + \tanh y}{(\beta - \tanh y)^2} \right] \\
&\quad \cdot \frac{d}{dy} (\tanh y) \\
&= \frac{1}{2} \cdot \frac{\beta - \tanh y}{\beta + \tanh y} \cdot \frac{2\beta}{(\beta - \tanh y)^2} \\
&\quad \cdot \frac{1}{\cosh^2 y} = \frac{\beta}{\beta^2 - \tanh^2 y} \cdot \frac{1}{\cosh^2 y} \\
&= \frac{\beta}{\beta^2 \cosh^2 y - \sinh^2 y} \\
&= \frac{\beta}{1 - (1 - \beta^2) \cosh^2 y} = \frac{1 + (1 - \beta^2) \sinh^2 \eta}{\beta}.
\end{aligned} \tag{13}$$

The expressions after the last equal marks in (11) and (13) are obtained from the expressions before the last equal marks in (13) and (11), respectively. It is obvious that the eduction of the revised conversion is simpler than that of the current conversion.

To use (10)–(13), we have relations

$$\begin{aligned}
f_\eta(\eta) |d\eta| &= f_y(y) |dy| = f_y[h(\eta)] \\
&\quad \cdot \left| \frac{\beta}{1 + (1 - \beta^2) \sinh^2 \eta} \right| \cdot |d\eta|, \\
f_y(y) |dy| &= f_\eta(\eta) |d\eta| = f_\eta[\varphi(y)] \\
&\quad \cdot \left| \frac{\beta}{1 - (1 - \beta^2) \cosh^2 y} \right| \cdot |dy|.
\end{aligned} \tag{14}$$

Then, we have further

$$f_\eta(\eta) = f_y[h(\eta)] \cdot \left| \frac{\beta}{1 + (1 - \beta^2) \sinh^2 \eta} \right|, \tag{15}$$

$$f_y(y) = f_\eta[\varphi(y)] \cdot \left| \frac{\beta}{1 - (1 - \beta^2) \cosh^2 y} \right|. \tag{16}$$

Equations (15) and (16) translate the rapidity distribution to pseudorapidity one and the pseudorapidity distribution to rapidity one, respectively.

In the previous discussions,

$$\beta = \sqrt{1 - \left(\frac{m_0}{\sqrt{p_T^2 + m_0^2} \cosh y} \right)^2} = \frac{\cosh \eta}{\sqrt{\cosh^2 \eta + m_0^2 p_T^{-2}}} \tag{17}$$

which can be used in the conversion. Then, the conversion is related to p_T and m_0 . To do a conversion, we need to know p_T and m_0 for each particle.

5. Conclusion and Discussion

We have given a revision on the current conversion between the particle rapidity and pseudorapidity distributions. It is shown that, comparing to the current first conversion, the revised one ((15) or (16)) has an additional term $(1 - \beta^2)\sinh^2\eta$ or $-(1 - \beta^2)\cosh^2 y$ in the denominator. In central rapidity region, $\sinh\eta \approx 0$ and $\cosh y \approx 1$; then, (15) and (16) change to the current conversion. However, in forward rapidity region, the difference between the revised conversion and current one is obvious.

Our conclusion does not mean that the current conversion between the unit-density functions $d^2N/dp_T d\eta$ and $d^2N/dp_T dy$, that is,

$$\frac{d^2N}{dp_T d\eta} = \sqrt{1 - \left(\frac{m_0}{\sqrt{p_T^2 + m_0^2} \cosh y} \right)^2} \frac{d^2N}{dp_T dy}, \quad (18)$$

is also erroneous or incomplete [12]. In fact, the conversion between the two unit-density functions is correct due to p_T being a series of fixed values in (18). To use (18), we also need to know p_T and m_0 for each particle.

Because the conversion between rapidity and pseudorapidity distributions is not simpler than a direct calculation based on the definitions of rapidity and pseudorapidity, we would rather use the direct calculation in modeling analysis. In fact, in the epoch of high energy collider, the dispersion between rapidity and pseudorapidity distributions is small [7]. This means that we would also like to not distinguish strictly rapidity and pseudorapidity distributions in general modeling analysis.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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