Review Article

Charge Radii and Quadrupole Moments of the Low-Lying Baryons in the Chiral Constituent Quark Model

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The chiral constituent quark model ($\chi$CQM) with general parameterization method (GPM) has been formulated to calculate the charge radii of the spin $(1/2)^+$ octet and $(3/2)^+$ decuplet baryons and quadrupole moments of the spin $(3/2)^+$ decuplet baryons and spin $(3/2)^+ \rightarrow (1/2)^+$ transitions. The implications of such a model have been investigated in detail for the effects of symmetry breaking and GPM parameters pertaining to the one-, two-, and three-quark contributions. Our results are not only comparable with the latest experimental studies but also agree with other phenomenological models. It is found that the $\chi$CQM is successful in giving a quantitative and qualitative description of the charge radii and quadrupole moments.

1. Introduction

The internal structure of baryons is determined in terms of electromagnetic Dirac and Pauli form factors $F_1(Q^2)$ and $F_2(Q^2)$ or equivalently in terms of the electric and magnetic Sachs form factors $G_E(Q^2)$ and $G_M(Q^2)$ [1]. The electromagnetic form factors are the fundamental quantities of theoretical and experimental interest which are further related to the static low energy observables of charge radii and magnetic moments. One of the main challenges in the theoretical and experimental hadronic physics is to understand the structure of hadrons within the quantum chromodynamics (QCD) in terms of these moments. Although QCD is accepted as the fundamental theory of strong interactions, the direct prediction of these kinds of observables from the first principle of QCD still remains a theoretical challenge as they lie in the nonperturbative regime of QCD.

Following the discoveries that the quarks and antiquarks carry only 30% of the total proton spin [2, 3], the orbital angular momentum of quarks and gluons is expected to make a significant contribution. In addition to this, there is a significant contribution coming from the strange quarks in the nucleon which are otherwise not present in the valence structure. It therefore becomes interesting to discuss the interplay between the spin of nonvalence quark and the orbital angular momentum in understanding the spin structure of baryons. Further, the experimental developments [4–6], providing information on the radial variation of the charge, and magnetization densities of the proton give the evidence for the deviation of the charge distribution from spherical symmetry. On the other hand, it is well known that the quadrupole moment of the nucleon should vanish on account of its spin-1/2 nature. This observation has naturally turned to be the subject of intense theoretical and experimental activity.

The mean square charge radius ($r_B^2$), giving the possible “size” of baryon, has been investigated experimentally with the advent of new facilities at JLAB, SELEX Collaborations [7–13]. Several measurements have been made for the charge radii of $p$, $n$, and $\Sigma^-$ in electron-baryon scattering experiments [13, 14] giving $r_p = 0.877 \pm 0.007$ fm ($r_n^2 = 0.779 \pm 0.025$ fm$^2$ [15]) and $r_n^2 = -0.1161 \pm 0.0022$ fm$^2$ [12]. The recent measurement of $r_{\Sigma^-}^2$ [13, 14] is particularly interesting as it gives the first estimate for the charge form factor of a strange baryon at low momentum transfer.

The $\Delta(1232)$ resonance is the lowest-lying excited state of the nucleon in which the search for quadrupole strength
has been carried out [16–19]. The spin and parity selection rules in the \( \gamma + p \rightarrow \Delta' \) transition allow three contributing photon absorption amplitudes, the magnetic dipole \( G_{M1} \), the electric quadrupole moment \( G_{E2} \), and the charge quadrupole moment \( G_{C2} \). The \( G_{M1} \) amplitude gives us information on magnetic moment, whereas the information on the intrinsic quadrupole moment can be obtained from the measurements of \( G_{E2} \) and \( G_{C2} \) amplitudes [12]. If the charge distribution of the initial and final three-quark states was spherically symmetric, the \( G_{E2} \) and \( G_{C2} \) amplitudes of the multipole expansion would be zero [20]. However, the recent experiments at Mainz, Bates, Bonn, and JLab Collaborations [7–11, 21, 22] reveal that these quadrupole amplitudes are clearly nonzero [12]. The ratio of electric quadrupole amplitude to the magnetic dipole amplitude is at least \( E2/M1 \equiv -0.025 \pm 0.005 \), and a comparable value of same sign and magnitude has been measured for the \( C2/M1 \) ratio [12]. Further, the quadrupole transition moment \( (Q^\Delta)^2 \) measured by LEGS and Mainz collaborations \((-0.108 \pm 0.009 \pm 0.034 \text{ fm}^2 \) [16–18] and \(-0.0846 \pm 0.0033 \text{ fm}^2 \) [9], resp.) also leads to the conclusion that the nucleon and the \( \Delta' \) are intrinsically deformed.

The naive quark model (NQM) [23–27] is one of the simplest model to describe the hadron properties and interactions in the low energy regime. This model is able to provide a simple intuitive picture of the hadron structure in terms of three valence quarks \( (qqq) \) for baryons and quark antiquark \( (q\bar{q}) \) for mesons. It allows the direct calculations of the low energy hadronic matrix elements including their spectra and successfully accounts for many of the low energy properties of the hadrons in terms of the valence quarks [28–32]. Interestingly, with the inclusion of the spin-spin interactions generated configuration mixing [28] between the valence quarks, the NQM has not only given an accurate description of the hadron spectroscopy data but also has been able to describe some subtle features of the data including the \( N - \Delta \) mass difference, photoproduction amplitudes, and baryon magnetic moments [29–36]. However, some major findings on the experimental front discussed below have brought into prominence the inadequacies of the NQM.

The measurements in the deep inelastic scattering (DIS) experiments [2, 3] indicate that the valence quarks of the proton carry only about 30% of its spin and also establishes the asymmetry of the quark distribution functions [37–41]. This is referred to as the “proton spin problem” in NQM. Several effective and phenomenological models have been developed to explain the “proton spin problem” by including spontaneous breaking of chiral symmetry and have been further applied to study the electromagnetic properties of baryons.

For the calculations pertaining to the baryon charge radii, NQM leads to vanishing charge radii for the neutral baryons like \( n, \Sigma^0, \Xi^0 \), and \( \Lambda \). This is in contradiction to the experimental data. The inclusion of quark spin-spin interactions in NQM modify the baryon wavefunction to some extent leading to the breaking of the SU(3) symmetry and a nonvanishing neutron charge mean square radius [29–32]. A likely cause of these dynamical shortcomings is that the NQM does not respect chiral symmetry whose spontaneous breaking leads to the emission of Goldstone bosons (GBs). In this context, it becomes important to incorporate the effect of chiral symmetry breaking (\( \chi \)SB) in the phenomenological models to obtain a reasonable agreement with the data. On the other hand, a wide variety of accurately measured data have been accumulated for the static low energy properties of baryons, for example, masses, electromagnetic moments, charge radii, and low energy dynamical properties such as scattering lengths and decay rates, which has renewed considerable interest in the low energy baryon spectroscopy. The direct calculations of these quantities from the first principle of QCD are extremely difficult as they require the nonperturbative methods. Several effective and phenomenological models such as Lattice QCD, effective field theories, QCD sum rules, and variants of quark models have been developed to explain the failures of the NQM and further applied to study the properties of baryons.

Some of the important models measuring the charge radii of octet baryon are the Skyrme model with bound state approach [42–44], slow-rotor approach [45], semibosonized SU(3) NJL model [46], cloudy bag model [47], variants of constituent quark models [48–54], \( 1/N_c \) expansion approach [55–58], perturbative chiral quark model (P\( \chi \)QM) [59], heavy-baryon chiral perturbation theory (\( HB\chi PT \)) [60], chiral perturbation theory (\( \chi PT \)) [61, 62], Lattice QCD [63], and so forth. The charge radii of decuplet baryons have been studied within the framework of Lattice QCD [64–66], quark model [67], \( 1/N_c \) expansion [55, 56], chiral perturbation theory [68], and so forth. The results for different theoretical models are however not consistent with each other.

There have been a lot of theoretical investigations in understanding the implications of the \( C2/M1 \) and \( E2/M1 \) ratios in finding out the exact sign of deformation in the spin (1/2)\(^+\) octet baryons. However, there is a little consensus between the results even with respect to the sign of the nucleon deformation. Some of the models predict the deformation in nucleon as oblate [69, 70], some predict a prolate nucleon deformation [71–80] whereas others speak about “deformation” without specifying the sign. The quadrupole moment of the (3/2)\(^+\) decuplet baryons has also been studied using the variants of the constituent quark model (CQM) [81–84], chiral quark soliton model (\( \chi QSM \)) [85], spectator quark model [86–88], slow-rotator approach (SRA) [89, 90], skyrme model [91, 92], general parametrization method [93, 94], light cone QCD sum rules (QCDSR) [95, 96], large \( N_c \) [97, 98], chiral perturbation theory (\( \chi PT \)) [99–103], Lattice QCD (LQCD) [104–108], and so forth. In this case also, the results for different theoretical models are not consistent in terms of sign and magnitude with each other.

As the hadron structure is sensitive to the pion cloud in the low energy regime, a coherent understanding is necessary as it will provide a test for the QCD-inspired effective field theories. One of the important nonperturbative approaches which finds its application in the low energy regime is the chiral constituent quark model (\( \chi \)CQM) [109,
It is one of the most convenient languages for the treatment of light hadrons at low energies using the effective interaction Lagrangian of the strong interactions. The \( \chi \) QCM coupled with the "quark sea" generation through the chiral fluctuation of a constituent quark GBs [111–119] successfully explains the "proton spin problem" [117–119], hyperon \( \beta \) decay parameters [120, 121], strangeness content in the nucleon [122–124], magnetic moments of octet and decuplet baryons including their transitions [125–127], magnetic moments of \( (1/2)^- \) octet baryon resonances [128], magnetic moments of \( (1/2)^- \) and \( (3/2)^- \) \( \Lambda \) resonances [129], charge radii [130], quadrupole moment [131], and so forth. The model is successfully extended to predict the important role played by the small intrinsic charm (IC) content in the nucleon [122–124], magnetic moments of octet and spin \( (1/2)^+ \) and spin \( (3/2)^+ \) charm baryons including their radiative decays [135]. In view of the above developments in the \( \chi \) QCM, it becomes desirable to extend the model to calculate the charge radii and quadrupole moment of the spin \( (1/2)^+ \) octet and spin \( (3/2)^+ \) decuplet baryons.

The purpose of the present communication is to calculate the charge radii of the spin \( (1/2)^+ \) octet and spin \( (3/2)^+ \) decuplet baryons and quadrupole moments of the spin \( (3/2)^+ \) decuplet baryons including the spin \( (3/2)^+ \) \( \rightarrow \) \( (1/2)^+ \) transitions within the framework of \( \chi \) QCM using a general parametrization method (GPM) [134–138]. In order to understand the role of pseudoscalar mesons in the baryon charge radii and quadrupole moment, we will compare our results with NQM as well as other phenomenological models. The detailed analysis of SU(3) symmetry breaking would also be carried out in the \( \chi \) QCM. Further, we aim to discuss the implications of GPM parameters by calculating the extent to which the three-quark term contributes.

2. Charge Radii and Quadrupole Moments

The mean square charge radii \( r_0^2 \) and quadrupole moments \( Q_0 \) are the lowest order moments of the charge density \( \rho \) in a low-momentum expansion. The charge radii contain fundamental information about the possible "size" of the baryons whereas the "shape" of a spatially extended particle is determined by its quadrupole moment [139–142].

The mean square charge radius \( r_0^2 \) of a given baryon is a scalar under spatial rotation and is defined as

\[
\langle r^2 \rangle = \int d^3r \rho(r) r^2,
\]

where \( \rho(r) \) is the charge density. The intrinsic quadrupole moment with respect to the body frame of axis is defined as

\[
Q_0 = \int d^3r \rho(r) (3z^2 - r^2).
\]

For the charge density concentrated along the \( z \)-direction, the term proportional to \( 3z^2 \) dominates, \( Q_0 \) is positive, and the particle is prolate shaped. If the charge density is concentrated in the equatorial plane perpendicular to \( z \) axis, the term proportional to \( r^2 \) dominates, \( Q_0 \) is negative, and the particle is oblate shaped.

The most general form of the multipole expansion of the nuclear charge density \( \rho \) in the spin-flavor space can be expressed as

\[
\rho = A' \sum_{i=1}^3 e_i 1 - B' \sum_{i \neq j}^3 e_i [2\sigma_i \cdot \sigma_j - (3\sigma_i \cdot \sigma_j - \sigma_i \cdot \sigma_j)]
\]

\[
- C' \sum_{i \neq j \neq k} e_i [2\sigma_j \cdot \sigma_k - (3\sigma_i \cdot \sigma_j - \sigma_i \cdot \sigma_k)].
\]

The charge radii operator composed of the sum of one-, two-, and three-quark terms is expressed as

\[
\hat{q}^2 = A \sum_{i=1}^3 e_i 1 + B \sum_{i \neq j}^3 e_i \sigma_i \cdot \sigma_j + C \sum_{i \neq j \neq k} e_i \sigma_j \cdot \sigma_k,
\]

whereas the quadrupole moment operator composed of a two- and three-quark term can be expressed as

\[
\hat{Q} = B \sum_{i \neq j}^3 e_i (3\sigma_i \cdot \sigma_j - \sigma_i \cdot \sigma_j)
\]

\[
+ C \sum_{i \neq j \neq k} e_i (3\sigma_j \cdot \sigma_k - \sigma_j \cdot \sigma_k).
\]

The coefficients called GPM parameters of the charge radii and quadrupole moments are related to each other as \( A = A', B = -2B', \) and \( C = -2C' \). These GPM parameters are to be determined from the experimental observations on charge radii and quadrupole moment.

Before calculating the matrix elements corresponding to the charge radii and quadrupole moment, it is essential to simplify various operator terms involved in (4). It can be easily shown that

\[
\sum_{i \neq j} e_i (\sigma_i \cdot \sigma_j) = 2J \cdot \sum_{i} e_i \sigma_i - 3 \sum_i e_i,
\]

\[
\sum_{i \neq j \neq k} e_i (\sigma_j \cdot \sigma_k) = \pm 3 \sum_i e_i - \sum_{i \neq j} e_i (\sigma_i \cdot \sigma_j),
\]

where +ve sign holds for \( J = 3/2 \) and −ve sign for \( J = 1/2 \) states leading to different operators for spin \( (1/2)^+ \) and spin \( (3/2)^+ \) baryons:

<table>
<thead>
<tr>
<th>Operator</th>
<th>( \sum_{i \neq j} e_i (\sigma_i \cdot \sigma_j) )</th>
<th>( \sum_{i \neq j \neq k} e_i (\sigma_j \cdot \sigma_k) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( J = \frac{1}{2} )</td>
<td>( 3 \sum_i e_i \sigma_{ix} - 3 \sum_i e_i )</td>
<td>( -3 \sum_i e_i \sigma_{ix} )</td>
</tr>
<tr>
<td>( J = \frac{3}{2} )</td>
<td>( 5 \sum_i e_i \sigma_{iz} - 3 \sum_i e_i \sigma_{ix} )</td>
<td>( 6 \sum_i e_i - 5 \sum_i e_i \sigma_{ix} )</td>
</tr>
</tbody>
</table>
The charge radii operators for the spin \(1/2^+\) octet and spin \(3/2^+\) decuplet baryons can now be expressed as

\[
\vec{r}_B^2 = (A - 3B) \sum_i e_i + 3 (B - C) \sum_i e_i \sigma_{iz},
\]

(8)

\[
\vec{r}_{B^*}^2 = (A - 3B + 6C) \sum_i e_i + 5 (B - C) \sum_i e_i \sigma_{iz}.
\]

(9)

It is clear from the above equations that the determination of charge radii basically reduces to the evaluation of the charge \(\sum_i e_i\) and spin \(\sum_i e_i \sigma_{iz}\) structure of a given baryon. The charge radii squared \(\vec{r}_B^2(B^*)\) for the octet (decuplet) baryons can now be calculated by evaluating matrix elements corresponding to the operators in (8) and (9) and are given as

\[
r_B^2 = \langle B | \vec{r}_B^2 | B \rangle, \quad r_{B^*}^2 = \langle B^* | \vec{r}_{B^*}^2 | B^* \rangle.
\]

(10)

Here \(|B\rangle\) and \(|B^*\rangle\), respectively, denote the spin-flavor wavefunctions for the spin \(1/2^+\) octet and the spin \(3/2^+\) decuplet baryons.

The quadrupole moment operators for the spin \(1/2^+\), spin \(3/2^+\) baryons, and spin \(3/2^+ \to (1/2)^+\) transitions can be calculated from the operator in (5) and are expressed as

\[
\tilde{Q}_B = B' \left( \sum_{i \neq j} e_j \sigma_{iz} \sigma_{ik} - 3 \sum_i e_i \sigma_{iz} + 3 \sum_i e_i \right),
\]

(11)

\[
\tilde{Q}_{B^*} = B' \left( 3 \sum_{i \neq j} e_j \sigma_{iz} \sigma_{ik} + 3 \sum_i e_i \sigma_{iz} \right),
\]

\[
\tilde{Q}_{B^*} = 3B' \sum_{i \neq j} e_j \sigma_{iz} \sigma_{ik} + 3C' \sum_{i \neq j} e_i \sigma_{iz} \sigma_{ik},
\]

It is clear from the above equations that the determination of quadrupole moment basically reduces to the evaluation of the flavor \(\sum_i e_i\), spin \(\sum_i e_i \sigma_{iz}\), and tensor terms \(\sum_i e_i \sigma_{iz} \sigma_{ik}\) and \(\sum_i e_i \sigma_{iz} \sigma_{ik}\) for a given baryon.

Using the three-quark spin-flavor wavefunctions for the spin \(1/2^+\) octet and spin \(3/2^+\) decuplet baryons, the quadrupole moment can now be calculated by evaluating the matrix elements of operators in (11). We now have

\[
Q_B = \langle B | \tilde{Q}_B | B \rangle, \quad Q_{B^*} = \langle B^* | \tilde{Q}_{B^*} | B^* \rangle,
\]

(12)

\[
Q_{B \to B^*} = \langle B^* | \tilde{Q}_{B^*} | B \rangle.
\]

3. Naive Quark Model (NQM)

The appropriate operators for the spin and flavor structure of baryons in NQM are defined as

\[
\sum_i e_i = \sum_{q=\bar{u},d,s} n_q^B q + \sum_{q=\bar{u},d,s} n_q^B \bar{q}
\]

\[
= n_u u + n_d d + n_s s + n_u^b u + n_d^b d + n_s^b s,
\]

(13)

where \(n_q^B\) is the number of quarks with charge \(q\) \(\bar{q}\) and \(n_q^B\) is the number of polarized quarks \(q\) \(\bar{q}\). For a given baryon \(u = -\bar{d}\) and \(u = -\bar{u}\), with similar relations for the \(d\) and \(s\) quarks. The general expression for the charge radii of any of the spin \(1/2^+\) octet baryon in (4) can be expressed as

\[
\vec{r}_B^2 = (A - 3B) \left( \sum_{q=\bar{u},d,s} n_q - \sum_{q=\bar{u},d,s} n_{\bar{q}} \right) q + 3 (B - C) \left( \sum_{q=\bar{u},d,s} n_q - \sum_{q=\bar{u},d,s} n_{\bar{q}} \right) q,
\]

(14)

Before we discuss the details of the charge radii calculations, it is essential to define the octet and decuplet wavefunctions. The "mixed" state octet baryon wavefunction generated by the spin-spin forces [33, 34] which improves the predictions of the various spin-related properties [117–119] is expressed as

\[
|B\rangle \equiv \left| 8, \frac{1}{2}^+ \right\rangle = \cos \theta |56, 0^+ \rangle_{N=0} + \sin \theta |70, 0^+ \rangle_{N=2},
\]

(15)

with

\[
|56, 0^+ \rangle_{N=0} = \frac{1}{\sqrt{2}} \left( \varphi' \chi' + \varphi'' \chi'' \right) \psi'(0^+),
\]

\[
|70, 0^+ \rangle_{N=2} = \frac{1}{2} \left[ \left( \varphi'' \chi' + \varphi' \chi'' \right) \psi'(0^+) + \left( \varphi' \chi' - \varphi'' \chi'' \right) \psi''(0^+) \right]
\]

(16)

Here \(\theta\) is the mixing angle and \(\chi, \varphi, \psi\) are the spin, isospin, and spatial wavefunctions. For the details of the wavefunction, we refer the readers to [33, 34]. Using the
Table 1: Charge radii of octet baryons in NQM_{config} in terms of the GPM parameters. The results in NQM without configuration can easily be calculated by substituting $\theta = 0$.

<table>
<thead>
<tr>
<th>Charge radii</th>
<th>NQM_{config}</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_p^2$</td>
<td>$(A - 3B)[2u + d] + (B - C) [\cos^2 \theta (4u_+ - d_+) + \sin^2 \theta (2u_+ + d_+)]$</td>
</tr>
<tr>
<td>$r_n^2$</td>
<td>$(A - 3B)[u + 2d] + (B - C) [\cos^2 \theta (-u_+ + 4d_+) + \sin^2 \theta (u_+ + 2d_+)]$</td>
</tr>
<tr>
<td>$r_{\Sigma^+}^2$</td>
<td>$(A - 3B)[2u + s] + (B - C) [\cos^2 \theta (4u_+ - s_+) + \sin^2 \theta (2u_+ + s_+)]$</td>
</tr>
<tr>
<td>$r_{\Sigma^-}^2$</td>
<td>$-(A - 3B)[2d + s] - (B - C) [\cos^2 \theta (4d_+ - s_+) + \sin^2 \theta (2d_+ + s_+)]$</td>
</tr>
<tr>
<td>$r_{\Sigma^0}^2$</td>
<td>$(A - 3B)[u + d + s] + (B - C) [\cos^2 \theta (2u_+ + 2d_+ - s_+) + \sin^2 \theta (u_+ + d_+ + s_+)]$</td>
</tr>
<tr>
<td>$r_{\Xi^+}^2$</td>
<td>$(A - 3B)[2u + d] + (B - C) [\cos^2 \theta (4u_+ - d_+) + \sin^2 \theta (2u_+ + d_+)]$</td>
</tr>
<tr>
<td>$r_{\Xi^-}^2$</td>
<td>$-(A - 3B)[d + 2s] + (B - C) [\cos^2 \theta (d_+ + 2s) + \sin^2 \theta (d_+ + 2s)]$</td>
</tr>
<tr>
<td>$r_{\Lambda}^2$</td>
<td>$(A - 3B)[u + d + s] + \sqrt{2}(B - C) [u_+ - d_+]$</td>
</tr>
</tbody>
</table>

Table 2: Charge radii of decuplet baryons in NQM in terms of GPM parameters.

<table>
<thead>
<tr>
<th>Charge radii</th>
<th>NQM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_{\Xi^+}^2$</td>
<td>$\frac{1}{2} [(A - 3B + 6C)(3u) + 5(B - C)(3u)]$</td>
</tr>
<tr>
<td>$r_{\Xi^-}^2$</td>
<td>$(A - 3B + 6C)(2u + d) + 5(B - C)(2u_+ + d_+)$</td>
</tr>
<tr>
<td>$r_{\Sigma^+}^2$</td>
<td>$(A - 3B + 6C)(u) + 5(B - C)(u_+ + 2d_+)$</td>
</tr>
<tr>
<td>$r_{\Sigma^-}^2$</td>
<td>$-(A - 3B + 6C)(3d) + 5(B - C)(3d_+)$</td>
</tr>
<tr>
<td>$r_{\Sigma^0}^2$</td>
<td>$(A - 3B + 6C)(2u + s) + 5(B - C)(2u_+ + s_+)$</td>
</tr>
<tr>
<td>$r_{\Xi^0}^2$</td>
<td>$-(A - 3B + 6C)(u + d + s) + 5(B - C)(u_+ + d_+ + s_+)$</td>
</tr>
<tr>
<td>$r_{\Omega^+}^2$</td>
<td>$(A - 3B + 6C)(3s) - 5(B - C)(3s_+)$</td>
</tr>
</tbody>
</table>

Configuration mixing generated by the spin-spin forces does not affect the spin $(3/2)^+$ decuplet baryons. The wavefunction in this case is given as

$$|B^+\rangle \equiv \begin{pmatrix} 10, 3^- \end{pmatrix} = |56, 0^+\rangle_{N=0} = \chi \phi^\dagger \psi^\dagger (0^+),$$  \hspace{1cm} (18)

Using the baryon wavefunction from the above equation and the charge radii operator from (9), the general expression for the charge radii of spin $(3/2)^+$ baryons can be expressed as

$$\widehat{r}_{B^+} = (A - 3B + 6C) \left( \sum_{u,d,s} n_{\bar{u},\bar{d},\bar{s}} - \sum_{u,d,s} n_{u,d,s} \right) q + 5(B - C) \left( \sum_{u,d,s} n_{\bar{u},\bar{d},\bar{s}} - \sum_{u,d,s} n_{u,d,s} \right) q^+. \hspace{1cm} (19)$$

As an example, the charge radii for $\Delta^+$ baryon can be expressed as

$$r_{\Delta^+}^2 = (A - 3B + 6C)(2u + d) + 5(B - C)(2u_+ + d_+). \hspace{1cm} (20)$$

The expressions for the charge radii of other decuplet baryons in NQM are presented in Table 2. For the spin $(1/2)^+$ octet baryons, the quadrupole moment of $p$ and $\Sigma^+$ in NQM can be expressed as

$$Q_p = 3B'(2u + d - 2u_+ - d_+)$$
$$Q_{\Sigma^+} = 3B'(2u + s - 2u_+ - s_+).$$  \hspace{1cm} (21)
For the spin \((3/2)^+\) decuplet baryons, the quadrupole moment of \(\Delta^+\) and \(\Xi^*^-\) can be expressed as

\[
Q_{\Delta^+} = B' (6u + 3d + 2u_+ + d_+) \\
+ C' (-6u - 3d + 10u_+ + 5d_+),
\]

\[
Q_{\Xi^*-} = B' (3d + 6s + d_+ + 2s_+) \\
+ C' (-3d - 6s + 5d_+ + 10s_+). 
\]

Similarly, for the spin \((3/2)^+ \rightarrow (1/2)^+\) transitions, the quadrupole moment of the \(\Delta^+ p\) and \(\Sigma^+ \Sigma^-\) transitions can be expressed as

\[
Q_{\Delta^+ p} = 2\sqrt{2}B' (u_+ - d_+) + 2\sqrt{2}C' (-u + d),
\]

\[
Q_{\Sigma^+ \Sigma^-} = 2\sqrt{2}B' (d_+ - s_+) + 2\sqrt{2}C' (-d + s). 
\]

The expressions for quadrupole moment of the other \((1/2)^+\) octet and \((3/2)^+\) decuplet baryons and for spin \((3/2)^+ \rightarrow (1/2)^+\) transitions in NQM can similarly be calculated. The results are presented in Tables 7, 8, and 9.

4. Chiral Constituent Quark Model (\(\chi\)CQM)

In light of the recent developments and successes of the \(\chi\)CQM in explaining the low energy phenomenology [117–127], we formulate the quadrupole moments for the \((3/2)^+\) decuplet baryons and spin \((3/2)^+ \rightarrow (1/2)^+\) transitions. The basic process in the \(\chi\)CQM is the Goldstone boson (GB) emission by a constituent quark which further splits into a \(q\bar{q}\) pair as

\[
q_s \rightarrow GB^0 + q'_s \rightarrow (q\bar{q}) + q'_s, 
\]

where \(q\bar{q} + q'_s\) constitute the "quark sea" [111–116]. The effective Lagrangian describing the interaction between quarks and a nonet of GBs is

\[
\mathcal{L} = g_s \bar{q} \phi q, 
\]

with

\[
q = \begin{pmatrix} u \\ d \\ s \end{pmatrix},
\]

\[
\Phi' = \begin{pmatrix} 
\phi_{uu}u\bar{u} + \phi_{ud}d\bar{d} + \phi_{us}s\bar{s} \\
\phi_{dd}d\bar{d} + \phi_{ds}s\bar{d} + \phi_{ss}u\bar{s} \\
\phi_{ud}d\bar{u} + \phi_{ds}s\bar{d} + \phi_{ss}u\bar{s}
\end{pmatrix}, 
\]

where

\[
\phi_{uu} = \phi_{dd} = \frac{1}{2} + \frac{\beta}{6} + \frac{\zeta}{3}, \quad \phi_{ss} = \frac{2\beta}{3} + \frac{\zeta}{3},
\]

\[
\phi_{us} = \phi_{ds} = \phi_{ud} = \phi_{ds} = \frac{-\beta}{3} + \frac{\zeta}{3},
\]

\[
\phi_{ud} = \phi_{uu} = \phi_{ud} = \phi_{ud} = 1,
\]

\[
\psi_{uu} = \phi_{us} = \phi_{ud} = \phi_{ud} = \alpha. 
\]

A redistribution of flavor and spin structure takes place in the interior of baryon due to the chiral symmetry breaking, and the modified flavor and spin content of the baryon can be calculated by substituting for every constituent quark:

\[
q \rightarrow P_q q + |\psi(q)|^2, 
\]

\[
q_s \rightarrow P_q q_s + |\psi(q_s)|^2. 
\]

Here \(P_q = 1 - \sum P_{q_f}\) is the transition probability of no emission of GB from any of the \(q\) quark with

\[
\sum P_{q_s} = a (\phi_{uu}^2 + \phi_{ud}^2 + \phi_{us}^2 + \phi_{uu}^2 + \phi_{ss}^2),
\]

\[
\sum P_{q_d} = a (\phi_{ud}^2 + \phi_{dd}^2 + \phi_{ds}^2 + \phi_{dd}^2 + \phi_{ss}^2),
\]
\[ \sum P_s = a \left( \phi_{su}^2 + \phi_{sd}^2 + \phi_{ss}^2 + \phi_{su}^2 + \phi_{sd}^2 \right), \]

and \(|\psi(q)|^2 (|\psi(q_s)|^2)|^2\) are the transition probabilities of the emission of a \(q(q_s)\) quark:

\[
|\psi(u)|^2 = a \left[ (2\phi_{su}^2 + \phi_{sd}^2 + \phi_{ss}^2 + \phi_{su}^2) u + \phi_{su}^2 \bar{u} \right]
+ \left( \phi_{su}^2 + \phi_{sd}^2 \right) \left( d + \bar{d} \right) + (\phi_{su}^2 + \phi_{sd}^2)(s + \bar{s}) \text{,}
\]

\[
|\psi(d)|^2 = a \left[ (\phi_{du}^2 + 2\phi_{dd}^2 + \phi_{ds}^2 + \phi_{ds}^2 + \phi_{ds}^2) d + \phi_{dd}^2 \bar{d} \right]
+ \left( \phi_{du}^2 + \phi_{dd}^2 \right) (u + \bar{u}) + (\phi_{dd}^2 + \phi_{dd}^2)(s + \bar{s}) \text{,}
\]

\[
|\psi(s)|^2 = a \left[ (\phi_{su}^2 + \phi_{sd}^2 + \phi_{ss}^2 + \phi_{su}^2 + \phi_{sd}^2 + \phi_{ss}^2) s + \phi_{su}^2 \bar{s} \right]
+ \left( \phi_{su}^2 + \phi_{sd}^2 \right) (u + \bar{u}) + (\phi_{su}^2 + \phi_{sd}^2)(d + \bar{d}) \text{,}
\]

\[
|\psi(u_s)|^2 = a \left[ (\phi_{su}^2 + \phi_{sd}^2 + \phi_{ss}^2) u_s + \phi_{su}^2 \bar{u}_s \right]
+ \left( \phi_{su}^2 + \phi_{sd}^2 \right) (d + \bar{d}) \text{,}
\]

\[
|\psi(d_s)|^2 = a \left[ (\phi_{du}^2 + \phi_{dd}^2 + \phi_{ds}^2 + \phi_{ds}^2 + \phi_{ds}^2) d_s + \phi_{dd}^2 \bar{d}_s \right]
+ \left( \phi_{du}^2 + \phi_{dd}^2 \right) (u + \bar{u}) + (\phi_{du}^2 + \phi_{dd}^2)(s + \bar{s}) \text{.}
\]

After the inclusion of “quark sea,” the charge radii for the spin \((1/2)^+\) octet baryons, in \(\chi\)CQM with configuration mixing \((\chi\text{CQM}_{\text{config}})\), can be obtained by substituting (29) for each quark in (17). The charge radii for the case of \(p\) and \(\Sigma^+\) are now expressed as

\[ r_p^2 = (A - 3B)(2P_u u + 2|\psi(u)|^2) + 3(B - C) \left[ \cos^2 \theta \left( \frac{4}{3} P_u u_s + \frac{4}{3} |\psi(u_s)|^2 \right) 
- \frac{1}{3} P_d d_s + \frac{1}{3} |\psi(d_s)|^2 \right]
+ \sin^2 \theta \left( \frac{2}{3} P_u u_s + \frac{2}{3} |\psi(u_s)|^2 \right)
+ \frac{1}{3} P_s s_s + \frac{1}{3} |\psi(s_s)|^2 \right], \]

\[ r_{\Sigma^+}^2 = (A - 3B)(2P_u u + 2|\psi(u)|^2) + 3(B - C) \left[ \cos^2 \theta \left( \frac{4}{3} P_u u_s + \frac{4}{3} |\psi(u_s)|^2 \right) 
- \frac{1}{3} P_s s_s + \frac{1}{3} |\psi(s_s)|^2 \right]
+ \sin^2 \theta \left( \frac{2}{3} P_u u_s + \frac{2}{3} |\psi(u_s)|^2 \right)
+ \frac{1}{3} P_d d_s + \frac{1}{3} |\psi(d_s)|^2 \right]. \]

The charge radii in the \(\chi\text{CQM}_{\text{config}}\) for other spin \((1/2)^+\) octet baryons are presented in Table 3. The results without configuration mixing can easily be obtained by taking the mixing angle \(\theta = 0\).

Similarly, for the spin \((3/2)^+\) decuplet baryons, the charge radii are modified on substituting for each quark from (29). For example, the charge radii for \(\Delta^+\) in \(\chi\)CQM can be expressed as

\[ r_{\Delta^+}^2 = (A - 3B + 6C)(2P_u u + 2|\psi(u)|^2) + 3(B - C)(2P_u u + 2|\psi(u_s)|^2) + 3(B - C)|\psi(d_s)|^2 \]
+ 5(B - C)|\psi(d)|^2 . \]

The charge radii of the other decuplet baryons can be calculated similarly and are detailed in Table 4.

After the inclusion of “quark sea,” the quadrupole moment for the spin \((1/2)^+\) octet baryons vanishes on account of the effective cancelation of contribution coming from the “quark sea” and the orbital angular momentum as observed spectroscopically. For the spin \((3/2)^+\) decuplet baryons, the quadrupole moment in \(\chi\)CQM can be obtained by substituting (29) for each quark in (21). The quadrupole moment of \(\Delta^+\) and \(\Sigma^+\) in \(\chi\)CQM can be expressed as

\[ Q_{\Delta^+} = 4B' + 2C' \left( B' + 5C' \right) a \left( 2 + \frac{\beta^2}{3} + \frac{2\alpha^2}{3} \right), \]

\[ Q_{\Sigma^+} = -4B' - 2C' \left( B' + 5C' \right) a \left( \frac{4\alpha^2}{3} + \beta^2 + \frac{2\alpha^2}{3} \right). \]

Similarly, the quadrupole moment of \(\Delta^+ p\) and \(\Sigma^+ \Sigma^-\) transitions in \(\chi\)CQM can be expressed as

\[ Q_{\Delta^+ p} = 2\sqrt{2} \left( B' \left( 1 - a \left( 1 + \alpha^2 + \frac{\beta^2}{3} + \frac{2\alpha^2}{3} \right) \right) \right) \]
\[ Q_{\Sigma^+ \Sigma^-} = 2\sqrt{2} B' a \left( \frac{\alpha^2}{3} - \frac{\beta^2}{3} \right). \]

The expressions for the quadrupole moment of other \((3/2)^+\) decuplet baryons and spin \((3/2)^+ \rightarrow (1/2)^+\) transitions in \(\chi\)CQM can similarly be calculated. The results are presented in Tables 8 and 9.

### 5. Results and Discussion

The calculations of charge radii and quadrupole moment of octet and decuplet baryons involve two set of parameters the SU(3) symmetry breaking parameters of \(\chi\)CQM and the GPM parameters. The \(\chi\)CQM parameters \(a, \alpha, \alpha^2, \beta \), and \(\alpha^2\) represent, respectively, the probabilities of fluctuations to pions \(K, \eta, \eta'\). A best fit of \(\chi\)CQM parameters can be obtained by carrying out a fine grained analysis of the spin and flavor distribution functions [117–119] leading to

\[ a = 0.12, \quad \alpha = 0.7, \quad \beta = 0.4, \quad \zeta = -0.15. \]
Table 3: Charge radii of octet baryons in $\chi$CQM$_{\text{config}}$ in terms of SU(3) symmetry breaking parameters and GPM parameters. These results are obtained by substituting $q \rightarrow P_q + |\psi(q)|^2$ and $q_\downarrow \rightarrow P_q q_\downarrow + |\psi(q_\downarrow)|^2$ for every constituent quark in NQM. Results in $\chi$CQM without configuration mixing can easily be obtained by substituting the mixing angle $\theta = 0$.

<table>
<thead>
<tr>
<th>Charge radii</th>
<th>$\chi$CQM$_{\text{config}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_p^2$</td>
<td>$A - 3B + (B - C) \left[ \cos^2 \theta \left( 3 - a \left( 4 + 2a^2 + 2\beta^2 + 2\zeta^2 \right) \right) + \sin^2 \theta \left( 1 - \frac{a}{3} \left( 6 + 2\beta^2 + 2\zeta^2 \right) \right) \right]$</td>
</tr>
<tr>
<td>$r_n^2$</td>
<td>$(B - C) \left[ \cos^2 \theta \left( -2 + \frac{a}{3} \left( 3 + 9a^2 + 2\beta^2 + 4\zeta^2 \right) \right) + \sin^2 \theta \left( a \left( -1 + a^2 \right) \right) \right]$</td>
</tr>
<tr>
<td>$r_{\Sigma^+}^2$</td>
<td>$A - 3B + (B - C) \left[ \cos^2 \theta \left( 3 - \frac{a}{3} \left( 12 + 5a^2 + 4\beta^2 + 6\zeta^2 \right) \right) + \sin^2 \theta \left( 1 - \frac{a}{3} \left( 6 + a^2 + 2\zeta^2 \right) \right) \right]$</td>
</tr>
<tr>
<td>$r_{\Sigma^0}^2$</td>
<td>$A - 3B + (B - C) \left[ \cos^2 \theta \left( -1 + \frac{a}{3} \left( 2a^2 + 5\beta^2 + 2\zeta^2 \right) \right) + \sin^2 \theta \left( -1 + \frac{a}{3} \left( 4a^2 + 3\beta^2 + 2\zeta^2 \right) \right) \right]$</td>
</tr>
<tr>
<td>$r_{\Sigma^-}^2$</td>
<td>$(B - C) \left[ \cos^2 \theta \left( -1 + \frac{a}{3} \left( 3a^2 + 4\beta^2 + 2\zeta^2 \right) \right) + \sin^2 \theta \left( \frac{a}{9} \left( -9 + 6\alpha^2 + 7\beta^2 + 2\zeta^2 \right) \right) \right]$</td>
</tr>
<tr>
<td>$r_{\Lambda}^2$</td>
<td>$(B - C) \left[ \sqrt{3} - \frac{a}{\sqrt{3}} \left( 3 + 3a^2 + 2\beta^2 + 2\zeta^2 \right) \right]$</td>
</tr>
</tbody>
</table>

Table 4: Charge radii of decuplet baryons in $\chi$CQM in terms of SU(3) symmetry breaking parameters and GPM parameters. These results are obtained by substituting $q \rightarrow P_q + |\psi(q)|^2$ and $q_\downarrow \rightarrow P_q q_\downarrow + |\psi(q_\downarrow)|^2$ for every constituent quark in NQM.

<table>
<thead>
<tr>
<th>Charge radii</th>
<th>$\chi$CQM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_{\Omega^-}^2$</td>
<td>$A + 2B + C - \frac{5a}{3} (B - C) (9 + 3a^2 + 2\beta^2 + 4\zeta^2)$</td>
</tr>
<tr>
<td>$r_{\Lambda}^2$</td>
<td>$A + 2B + C - \frac{5a}{3} (B - C) (6 + 2\beta^2 + 2\zeta^2)$</td>
</tr>
<tr>
<td>$r_{\Xi^-}^2$</td>
<td>$5a(B - C)(-1 + a^2)$</td>
</tr>
<tr>
<td>$r_{\Xi^0}^2$</td>
<td>$A + 2B + C - \frac{5a}{3} (B - C) (6a^2 + 2\beta^2 + 2\zeta^2)$</td>
</tr>
<tr>
<td>$r_{\Xi^+}^2$</td>
<td>$A + 2B + C - \frac{5a}{3} (B - C) (6a^2 + 2\beta^2 + 2\zeta^2)$</td>
</tr>
<tr>
<td>$r_{\Xi^-}^2$</td>
<td>$A + 2B + C - \frac{5a}{3} (B - C) (5a^2 + 2\beta^2 + 2\zeta^2)$</td>
</tr>
<tr>
<td>$r_{\Xi^0}^2$</td>
<td>$\frac{5a}{3} (B - C)(-3 + 2a^2 + \beta^2)$</td>
</tr>
<tr>
<td>$r_{\Xi^+}^2$</td>
<td>$\frac{5a}{3} (B - C)(-3 + 2a^2 + 2\beta^2)$</td>
</tr>
<tr>
<td>$r_{\Omega^-}^2$</td>
<td>$A + 2B + C - \frac{5a}{3} (B - C) (4a^2 + 3\beta^2 + 2\zeta^2)$</td>
</tr>
<tr>
<td>$r_{\Omega^+}^2$</td>
<td>$A + 2B + C - \frac{5a}{3} (B - C) (3a^2 + 4\beta^2 + 2\zeta^2)$</td>
</tr>
</tbody>
</table>

The mixing angle $\theta$ is fixed from the consideration of neutron charge radius [28]. This set of parameters has already been tested for a wide variety of low energy matrix elements and has been able to give a simultaneous fit to the quantities describing proton spin and flavor structure [117–119], weak vector-axial vector form factors [120, 121], strangeness content in the nucleon [122–124], octet and decuplet baryons magnetic moments [125–127], and so forth.

The order of GPM parameters corresponding to the one-, two-, and three-quark terms decreases with the increasing complexity of terms and obeys the hierarchy $A > B > C$ [143, 144]. These are fitted by using the available experimental values for the charge radii and quadrupole moment of nucleons as input. In the present case, we have used $r_p = 0.877 \pm 0.007$ fm [12], $r_n = -0.1161 \pm 0.0022$ fm$^2$ [12], and $Q_{\Lambda}^2 = -0.0846 \pm 0.0033$ fm$^3$ [19]. The set of GPM parameters obtained after $\chi^2$ minimization are as follows:

$$A = 0.879, \quad B = 0.094, \quad C = 0.016. \quad (37)$$

For the quadrupole moment calculations, best fit set of parameters obtained after $\chi^2$ minimization are as follows:

$$B' = -0.047, \quad C' = -0.008, \quad (38)$$

obeying the hierarchy $B' > C'$ [143, 144] corresponding to the two- and three-quark contribution. Since we also intend to investigate the extent to which the three-quark term contributes, we calculate the charge radii corresponding to the one- and two-quark terms only by taking $C = 0$. Similarly, if we intend to calculate the charge radii corresponding to just the one-quark term, we can take $B = C = 0$.

Using the set of parameters discussed above, we have calculated the numerical values for the charge radii of octet and decuplet baryons in $\chi$CQM$_{\text{config}}$ and presented the results in Tables 5 and 6, respectively. To understand the implications of chiral symmetry breaking and “quark sea,” we have also presented the results of NQM as well as comparing our results with the predictions of other available phenomenological models. Since the calculations in $\chi$CQM have been carried...
out using the GPM, the NQM results have also been presented by including the one-, two-, and three-quark contributions of the GPM parameters. It is clear from Tables 1 and 2 that if we consider the contribution coming from one-quark term only, the charge radii of the charged baryons are equal whereas all neutral baryons have zero charge radii. These predictions are modified on the inclusion of two- and three-quark terms of GPM and are further modified on the inclusion of “quark sea” and SU(3) symmetry breaking effects. Thus, it seems that the GPM parameters alone are able to explain the experimentally observed nonzero charge radii of the neutral baryons. However, NQM is unable to account for the “proton spin problem” and other related quantities; the results have been presented for χCQM. The importance of strange quark mass has been investigated by comparing the χCQM results with and without SU(3) symmetry breaking. The SU(3) symmetry results can be easily derived from Tables 3 and 4 by considering α = β = 1 and ζ = −1. The SU(3) breaking results are in general higher in magnitude than the SU(3) symmetric results, and the values obtained are also in agreement with the other models.

For the case of octet baryons, it can be easily shown from Table 5 that in the SU(3) symmetric limit, octet baryon charge radii can be expressed in terms of the nucleon charge radii leading to the following relations:

\[ r_\Sigma^2 = r_\Xi^2 = 2 r_p^2 + r_n^2, \]

### Table 5: Charge radii of octet baryons calculated in χCQM in comparison with other phenomenological models (in units of fm^2).

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_\Xi^2 )</td>
<td>0.61 ± 0.21</td>
<td>0.138</td>
<td>0.14</td>
<td>0.03 ± 0.01</td>
<td>0.05 ± 0.01</td>
<td>0.128</td>
<td>—</td>
<td>0.043</td>
<td>0.062</td>
<td>0.052</td>
</tr>
<tr>
<td>( r_\Sigma^2 )</td>
<td>0.675</td>
<td>0.49</td>
<td>0.798</td>
<td>0.67 ± 0.03</td>
<td>0.72</td>
<td>0.71 ± 0.07</td>
<td>0.672</td>
<td>0.657</td>
<td>0.646</td>
<td>0.678</td>
</tr>
<tr>
<td>( r_\Lambda^2 )</td>
<td>0.135</td>
<td>−0.12</td>
<td>0.074</td>
<td>0.03 ± 0.01</td>
<td>—</td>
<td>0.0</td>
<td>−0.066</td>
<td>—</td>
<td>0.085</td>
<td>0.132</td>
</tr>
</tbody>
</table>

### Table 6: Charge radii of decuplet baryons calculated in χCQM in comparison with other phenomenological models (in units of fm^2).

<table>
<thead>
<tr>
<th>Charge radii</th>
<th>FTQM [67]</th>
<th>CCQM [52]</th>
<th>1/N_c [57, 58]</th>
<th>Lattice [64–66]</th>
<th>With SU(3) ( A = 0.879 )</th>
<th>With SU(3) symmetry breaking ( A = 0.879 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_{\Xi^-}^2 )</td>
<td>1.084</td>
<td>1.18</td>
<td>0.43</td>
<td>1.011</td>
<td>—</td>
<td>0.938</td>
</tr>
<tr>
<td>( r_{\Sigma^+}^2 )</td>
<td>1.084</td>
<td>0.82</td>
<td>0.43</td>
<td>1.011</td>
<td>0.410 (57)</td>
<td>0.938</td>
</tr>
<tr>
<td>( r_{\Sigma^0}^2 )</td>
<td>0.0</td>
<td>0.16</td>
<td>0.00</td>
<td>0.0</td>
<td>—</td>
<td>0.0</td>
</tr>
<tr>
<td>( r_{\Lambda}^2 )</td>
<td>1.084</td>
<td>0.84</td>
<td>0.43</td>
<td>1.011</td>
<td>−0.410 (57)</td>
<td>0.938</td>
</tr>
<tr>
<td>( r_{\Omega^-}^2 )</td>
<td>1.084</td>
<td>0.97</td>
<td>0.42</td>
<td>1.086</td>
<td>0.399 (45)</td>
<td>0.938</td>
</tr>
<tr>
<td>( r_{\Sigma^+}^2 )</td>
<td>1.084</td>
<td>0.84</td>
<td>0.37</td>
<td>0.845</td>
<td>−0.360 (32)</td>
<td>0.938</td>
</tr>
<tr>
<td>( r_{\Sigma^0}^2 )</td>
<td>0.0</td>
<td>0.34</td>
<td>0.03</td>
<td>0.127</td>
<td>0.020 (7)</td>
<td>0.0</td>
</tr>
<tr>
<td>( r_{\Xi^0}^2 )</td>
<td>0.0</td>
<td>0.49</td>
<td>0.06</td>
<td>0.244</td>
<td>0.043 (10)</td>
<td>0.0</td>
</tr>
<tr>
<td>( r_{\Xi^-}^2 )</td>
<td>1.084</td>
<td>0.82</td>
<td>0.33</td>
<td>0.692</td>
<td>−0.330 (20)</td>
<td>0.938</td>
</tr>
<tr>
<td>( r_{\Sigma^-}^2 )</td>
<td>0.390</td>
<td>0.78</td>
<td>0.29</td>
<td>0.553</td>
<td>—</td>
<td>0.245</td>
</tr>
</tbody>
</table>
Table 7: Quadrupole moments of the octet baryons in NQM using the GPM.

<table>
<thead>
<tr>
<th>Baryons</th>
<th>NQM</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho )</td>
<td>( 3B'(2u + d - 2u - d + s) + C'(-4u + d + 4u - d + s) )</td>
</tr>
<tr>
<td>( \eta )</td>
<td>( 3B'(u + 2d - u + 2d') + C'(u - 4d - u + 4d') )</td>
</tr>
<tr>
<td>( \Sigma^+ )</td>
<td>( 3B'(2u + s - 2u - s - d + s) + C'(-4u + s + 4u - s + d) )</td>
</tr>
<tr>
<td>( \Sigma^- )</td>
<td>( 3B'(2d + s - 2d' - s + s) + C'(-4d + s + 4d - s + d) )</td>
</tr>
<tr>
<td>( \Sigma^0 )</td>
<td>( 3B'(u + d - s + d - s + s - d - s + s - d + s) )</td>
</tr>
<tr>
<td>( \Xi^0 )</td>
<td>( 3B'(u + 2d + u - 2d + s + s - d + s + s - d + s + s) )</td>
</tr>
</tbody>
</table>

The inclusion of SU(3) symmetry breaking changes this pattern considerably, and we get

\[
r^2_{\Sigma^+} > r^2_p, \quad r^2_{\Sigma^-} > r^2_{\Sigma^0} > r^2_p + r^2_n. \tag{40}
\]

Also we have

\[
2r^2_{\Lambda} = -2r^2_{\Sigma^0} = r^2_{\Xi^0} = r^2_n, \tag{41}
\]

which has its importance in the isospin limit where the three-quark core in neutral baryons does not contribute to the charge radii. In the limit of SU(3) symmetry breaking, a nonvanishing value for the neutral baryons charge radii is generated by the “quark sea” through the chiral fluctuations of constituent quarks leading to

\[
r^2_{\Lambda} > -r^2_{\Sigma^0}, \quad r^2_{\Xi^0} > r^2_n. \tag{42}
\]

The exact order of SU(3) symmetry breaking effects can be easily found from Table 3. Since experimental information is not available for some of these octet charge radii, the accuracy of these relations can be tested by the future experiments. It is interesting to note that the relation for the \( \Sigma^+ \) baryon charge radii,

\[
r^2_{\Sigma^+} - 2r^2_{\Sigma^0} - r^2_n = 0, \tag{43}
\]

holds good even after incorporating SU(3) symmetry breaking. Since this relation is independent of SU(3) symmetry breaking parameters, any refinement in the \( \Sigma \) baryon charge radii data would have important implications for SU(3) symmetry breaking. Further, our predicted value \( r^2_{\Sigma^+} = 0.664 \) is clearly of the order of proton charge radius and is also in agreement with the recent SELEX collaboration experimental results [12]. It would be important to mention here that the \( \chi \)CQM parameters play an important role in the SU(3) symmetry breaking effects whereas the assumed parametrization plays a dominant role in the valence quark distributions.

In Table 3, we have presented the results for the case with configuration mixing generated by the spin-spin forces. We have not presented the results without configuration mixing which can easily be obtained by taking the mixing angle \( \theta = 0 \). It has been observed that configuration mixing decreases the overall magnitudes of the charge radii in \( \chi \)CQM, but the change is very small as compared to the other low energy properties like spin distribution function, magnetic moments, and so forth [117–127].

On comparing our results with the other phenomenological models, we find that for the case of charged octet baryons, our results are in fair agreement in sign and magnitude with the other model predictions. However, for the neutral octet baryons \( n, \Sigma^0, \Xi^0, \) and \( \Lambda \), different models show opposite sign. For example, if we consider the charge radii for the \( \Lambda \) baryon, our model prediction \((-0.063)\) is opposite in sign to the predictions of the relativistic constituent quark model (RCQM) [51], covariant constituent quark model (CCQM) [52], \( 1/N_c \) expansion [55, 56], and PCQM [59]. On the other hand, it is in agreement with the sign of HB \( \chi \)PT [60]. A similar trend has been observed for the charge radii of \( \Sigma \Lambda \) transition. The difference in the sign may be due to the chiral fluctuation of a constituent quark leading to the reversal of sign in case of neutral octet baryons. This can perhaps be substantiated by a measurement of charge radii of other baryons.

The spin \( (3/2)^+ \) decuplet baryon charge radii, presented in Table 6, are in general higher than the octet baryon charge radii which are in line with the trend followed by the octet and decuplet baryons for the other low energy hadronic matrix elements such as magnetic moments. In this case also, the inclusion of SU(3) symmetry breaking increases the predictions of charge radii. It can be easily shown that SU(3) symmetry results in the following relations for the decuplet baryons:

\[
r^2_{\Delta^{++}} = r^2_{\Delta^+} = r^2_{\Delta^0} = r^2_{\Sigma^+} = r^2_{\Xi^0} = r^2_{\Sigma^-}. \tag{44}
\]

These results are affected by the inclusion of SU(3) symmetry breaking and give

\[
r^2_{\Sigma^+} > r^2_{\Sigma^-} > r^2_{\Delta^+} > r^2_{\Delta^0} > r^2_{\Delta^0} > r^2_{\Xi^0} > r^2_{\Sigma^-}. \tag{45}
\]

Some relations, derived in \( 1/N_c \) expansion of QCD [55–58], are found to be independent of SU(3) symmetry breaking parameters in \( \chi \)CQM. Even though the individual charge radii are affected by SU(3) symmetry breaking, the effects are canceled exactly for the following relations:

\[
2r^2_{\Delta^+} - r^2_{\Delta^0} - r^2_{\Delta^0} = 0, \quad 2r^2_{\Delta^0} - 3r^2_{\Lambda} + 3r^2_{\Xi^0} + r^2_{\Sigma^0} = 0, \tag{46}
\]

In this case also, SU(3) symmetry breaking is expected to reduce the charge radii with increasing strangeness content. As a consequence, \( \Delta^+, \Sigma^+, \) and \( \Xi^0 \) should have successively decreasing charge radii. However, this suppression disappears in \( \chi \)CQM due to the effect of “quark sea,” and the charge radii of \( \Delta^+, \Sigma^+, \) and \( \Xi^0 \) are of almost the same order as that of \( \Sigma^+, \Xi^+, \) and \( \Sigma^0 \), respectively. Again, the sign and magnitude of the decuplet baryon charge radii in \( \chi \)CQM are in fair agreement with the other phenomenological models with the exception for neutral baryons. One of the important predictions in \( \chi \)CQM is a nonzero \( \Delta^0 \) charge radii which
Table 8: Quadrupole moments of the decuplet baryons in NQM and χ CQM using the GPM.

<table>
<thead>
<tr>
<th>Baryon</th>
<th>NQM</th>
<th>χ CQM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δ⁺⁺</td>
<td>B’ (9u + 3u_3) + C’ (-9u + 15u_3)</td>
<td>8B’ + 4C’ - (B’ + 5C’) a/3 (9 + 3α² + 2β² + 4ζ²)</td>
</tr>
<tr>
<td>Δ⁺</td>
<td>B’ (3(2u + d) + 2u_3 + d_3) + C’ (-3(2u + d) + 5(2u_3 + d_3))</td>
<td>4B’ + 2C’ - (B’ + 5C’) a/3 (6 + β² + 2ζ²)</td>
</tr>
<tr>
<td>Δ₀</td>
<td>B’ (3(u + 2d) + u_3 + 2d_3) + C’ (-3(u + 2d) + 5(u_3 + 2d_3))</td>
<td>(B’ + 5C’) a(-1 + α²)</td>
</tr>
<tr>
<td>Δ⁻</td>
<td>B’ (9d + 3d_3) + C’ (-9d + 15d_3)</td>
<td>-4B’ - 2C’ + (B’ + 5C’) a/3 (6α² + β² + 2ζ²)</td>
</tr>
<tr>
<td>Σ⁰⁺⁺</td>
<td>B’ (3(2u + s) + 2u_3 + s_3) + C’ (-3(2u + s) + 5(2u_3 + s_3))</td>
<td>4B’ + 2C’ - (B’ + 5C’) a/3 (6 + α² + 2ζ²)</td>
</tr>
<tr>
<td>Σ⁰⁻⁻</td>
<td>B’ (3(2d + s) + 2d_3 + s_3) + C’ (-3(2d + s) + 5(2d_3 + s_3))</td>
<td>-4B’ - 2C’ + (B’ + 5C’) a/3 (5α² + 2β² + 2ζ²)</td>
</tr>
<tr>
<td>Σ⁺⁺</td>
<td>B’ (3(u + d + s) + u_3 + d_3 + s_3) + C’ (-3(3u + d + s) + 5(u_3 + d_3 + s_3))</td>
<td>(B’ + 5C’) a/3 (-3 + 2α² + β²)</td>
</tr>
<tr>
<td>Σ⁻⁻</td>
<td>B’ (3(u + 2s) + u_3 + s_3) + C’ (-3(u + 2s) + 5(u_3 + 2s_3))</td>
<td>(B’ + 5C’) a/3 (-3 + α² + 2β²)</td>
</tr>
<tr>
<td>Ω⁻</td>
<td>B’ (9s + 3s_3) + C’ (-9s + 15s_3)</td>
<td>-4B’ - 2C’ + (B’ + 5C’) a/3 (3α² + 4β² + 2ζ²)</td>
</tr>
</tbody>
</table>

Table 9: Quadrupole moments of the spin (3/2)⁺ → (1/2)⁺ transitions in NQM and χ CQM using the GPM.

<table>
<thead>
<tr>
<th>Baryon</th>
<th>NQM</th>
<th>χ CQM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δ⁺ p</td>
<td>2√2B’ (u_3 - d_3) + 2√2C’ (-u - d)</td>
<td>2√2B’ (1 - a/3 (3 + 3α² + 2β² + 2ζ²) - 2√2C’</td>
</tr>
<tr>
<td>Σ⁺⁺ Σ⁻⁻</td>
<td>2√2B’ (u_3 - s_3) + 2√2C’ (-u - s)</td>
<td>2√2B’ (1 - a/3 (3 + 2α² + 2β² + 2ζ²) - 2√2C’</td>
</tr>
<tr>
<td>Σ⁺⁻ Σ⁻⁻</td>
<td>2√2B’ (d_3 - s_3) + 2√2C’ (-d - s)</td>
<td>2√2B’ a (α² - β²)</td>
</tr>
<tr>
<td>Σ⁺⁺ Λ⁺⁰</td>
<td>√2B’ (u_3 - d_3) + √2C’ (-u - d)</td>
<td>√2B’ (1 - a/3 (3 + 3α² + 2β² + 2ζ²) - √2C’</td>
</tr>
<tr>
<td>Σ⁻⁻ Λ⁺⁰</td>
<td>√2B’ (u_3 - s_3) + √2C’ (-u - s)</td>
<td>√2B’ (1 - a/3 (3 + 2α² + 2β² + 2ζ²) - √2C’</td>
</tr>
<tr>
<td>Σ⁺⁺ Ξ⁺⁻</td>
<td>√2B’ (d_3 - s_3) + √2C’ (-d - s)</td>
<td>√2B’ a (α² - β²)</td>
</tr>
</tbody>
</table>

vanishes in NQM as well as in some other models. This is further endorsed by the predictions of the field theoretical quark model (FTQM) calculations [67]. The contribution of the three-quark term in the case of decuplet baryons is exactly opposite to that for the octet baryons. Unlike the octet baryon case, the inclusion of the three-quark term increases the value of the baryon charge radii.

For the sake of completeness, certain relations between the octet and decuplet baryon charge radii can also be tested for the spacing between the levels. In NQM, we have

\[ r_{Σ⁺⁺}^2 - r_{Σ⁻⁻}^2 = r_{Σ⁺}^2 - r_{Σ⁻}^2 = r_{Σ⁺}^2 - r_{Σ⁻⁺}^2 = r_{Σ⁻}^2 - r_{Σ⁺⁺}^2 = r_n^2. \]  (47)

In χ CQM, the inclusion of SU(3) symmetry breaking effects creates a spacing between the octet and decuplet baryon charge radii as

\[ r_p^2 - r_{Δ⁺⁺}^2 = r_{Σ⁺⁺}^2 - r_{Σ⁻⁻}^2 = -0.48, \]
\[ r_n^2 - r_{Δ⁺⁺}^2 = r_{Σ⁺⁺}^2 - r_{Σ⁻⁻}^2 = -0.09. \]  (48)

We have calculated the numerical values for the quadrupole moment for the (3/2)⁺ decuplet baryons in χ CQM and presented the results in Table 10. The results of the spin (3/2)⁺ → (1/2)⁺ transitions have been presented in Table 11. To understand the implications of chiral symmetry breaking and “quark sea,” we have also presented the results of NQM. Since the calculations in χ CQM have been carried out using the GPM, the NQM results have also been presented by including the two- and three- quark term contributions of the GPM parameters so that the contribution of the “quark sea” effects can be calculated explicitly. For the case of spin (1/2)⁺ octet baryons, we find that the quadrupole moments are zero for all the cases in NQM. Even if we consider the contribution coming from two-quark terms with the
### Table 10: Quadrupole moments of the spin \((3/2)^+\) decuplet baryons in \(\chi\)CQM using GPM and SU(3) symmetry breaking.

<table>
<thead>
<tr>
<th>Baryon</th>
<th>NQM (\text{fm}^2)</th>
<th>CQM [84] (\text{fm}^2)</th>
<th>(\chi)PT [99–103] (\text{fm}^2)</th>
<th>SRA [89,90] (\text{fm}^2)</th>
<th>Skyrme [91,92] (\text{fm}^2)</th>
<th>GPM [93,94] (\text{fm}^2)</th>
<th>Symmetry</th>
<th>Symmetry breaking</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta^-)</td>
<td>-0.409</td>
<td>-9.3</td>
<td>-0.8 (\pm) 0.5</td>
<td>-0.87</td>
<td>-8.8</td>
<td>-0.12</td>
<td>-0.3437</td>
<td>-0.3695</td>
</tr>
<tr>
<td>(\Delta^+)</td>
<td>-0.204</td>
<td>-4.6</td>
<td>-0.3 (\pm) 0.2</td>
<td>-0.31</td>
<td>-2.9</td>
<td>-0.06</td>
<td>-0.1719</td>
<td>-0.1820</td>
</tr>
<tr>
<td>(\Delta^0)</td>
<td>0.0</td>
<td>0.0</td>
<td>0.12 (\pm) 0.05</td>
<td>0.24</td>
<td>2.9</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0055</td>
</tr>
<tr>
<td>(\Delta^-)</td>
<td>0.204</td>
<td>4.6</td>
<td>0.6 (\pm) 0.3</td>
<td>0.80</td>
<td>8.8</td>
<td>0.06</td>
<td>0.1719</td>
<td>0.1930</td>
</tr>
<tr>
<td>(\Sigma^{**})</td>
<td>-0.204</td>
<td>-5.4</td>
<td>-0.7 (\pm) 0.3</td>
<td>-0.42</td>
<td>-7.1</td>
<td>-0.069</td>
<td>-0.1719</td>
<td>-0.1808</td>
</tr>
<tr>
<td>(\Sigma^{*+})</td>
<td>0.204</td>
<td>4.0</td>
<td>4.0 (\pm) 0.2</td>
<td>0.52</td>
<td>7.1</td>
<td>0.039</td>
<td>0.1719</td>
<td>0.1942</td>
</tr>
<tr>
<td>(\Sigma^{*0})</td>
<td>0.0</td>
<td>-0.7</td>
<td>-0.13 (\pm) 0.07</td>
<td>0.05</td>
<td>0.0</td>
<td>0.014</td>
<td>0.0</td>
<td>0.0067</td>
</tr>
<tr>
<td>(\Xi^{**})</td>
<td>0.0</td>
<td>-1.3</td>
<td>-0.35 (\pm) 0.2</td>
<td>-0.07</td>
<td>-4.6</td>
<td>-0.1719</td>
<td>0.0</td>
<td>0.0079</td>
</tr>
<tr>
<td>(\Xi^{*+})</td>
<td>0.204</td>
<td>3.4</td>
<td>0.2 (\pm) 0.1</td>
<td>0.35</td>
<td>4.6</td>
<td>0.024</td>
<td>0.1719</td>
<td>0.1954</td>
</tr>
<tr>
<td>(\Omega^-)</td>
<td>0.204</td>
<td>2.8</td>
<td>0.09 (\pm) 0.05</td>
<td>0.24</td>
<td>0.0</td>
<td>0.014</td>
<td>0.1719</td>
<td>0.1966</td>
</tr>
</tbody>
</table>

This is because if flavor symmetry is exact, U-spin conservation forbids such transitions. The exact order of SU(3) symmetry breaking effects can be easily found from Tables 10 and 11. Since there is no experimental or phenomenological information available for any of these quadrupole moments, the accuracy of these relations can be tested by the future experiments.

For the \((3/2)^+\) decuplet baryons presented in Table 10, quadrupole moments results in NQM using the GPM predict an oblate shape for all positively charged baryons (\(\Delta^{++}\), \(\Delta^+\), and \(\Sigma^{**}\)), prolate shape for negatively charged baryons (\(\Delta^-\), \(\Sigma^{*+}\), \(\Xi^{*+}\), and \(\Omega^-\)). It is important to mention here that the NQM is unable to explain the deformation in neutral baryons (\(\Delta^0\), \(\Sigma^{*0}\), and \(\Xi^{*0}\)). On incorporating the effects of chiral symmetry breaking and “quark sea” in the \(\chi\)CQM, a small amount of prolate deformation in neutral baryons (\(\Delta^0\), \(\Sigma^{*0}\), and \(\Xi^{*0}\)) is observed. The trend of deformations is however the same for the positively and negatively charged baryons in \(\chi\)CQM and NQM. The other phenomenological models also observe a similar trend, for example, light cone QCD sum rules \([95,96]\), spectator quark model \([86–88]\), Lattice QCD \([104–108]\), \(\chi\)PT \([99–103]\), chiral quark soliton model (\(\chi\)QSM) \([85]\), and so forth.

For the case of spin \((3/2)^+\) \(\rightarrow\) \((1/2)^+\) transitions in Table II, it is observed that quadrupole moments of all the transitions are oblate in shape. This result is further endorsed by the predictions of Skyrme model \([91,92]\). The effects of chiral symmetry breaking can further be substantiated by a measurement of the other transition quadrupole moments.

### 6. Summary and Conclusion

To summarize, \(\chi\)CQM is able to provide a fairly good description of the charge radii of spin \((1/2)^+\) octet and spin \((3/2)^+\) decuplet baryons and quadrupole moments of spin \((3/2)^+\) decuplet baryons and spin \((3/2)^+\) \(\rightarrow\) \((1/2)^+\) transitions using general parameterization method (GPM). The most significant prediction of the model for the charge
Table II: Quadrupole moments of the spin $(3/2)^+ \rightarrow (1/2)^+$ decuplet to octet transitions in χCQM using GPM and SU(3) symmetry breaking.

<table>
<thead>
<tr>
<th>Baryon</th>
<th>NQM fm$^2$</th>
<th>Skyrme [91, 92] 10$^{-2}$ fm$^2$</th>
<th>GPM [93, 94] fm$^2$</th>
<th>χCQM with SU(3) Symmetry fm$^2$</th>
<th>Symmetry breaking fm$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta^+ p$</td>
<td>$-0.110$</td>
<td>$-5.2$</td>
<td>$-0.082$</td>
<td>$-0.0608$</td>
<td>$-0.0846$</td>
</tr>
<tr>
<td>$\Sigma^<em>\Sigma^</em>$</td>
<td>$-0.110$</td>
<td>$-0.93$</td>
<td>$-0.076$</td>
<td>$-0.0608$</td>
<td>$-0.0864$</td>
</tr>
<tr>
<td>$\Sigma^<em>\Sigma^</em>$</td>
<td>$0.0$</td>
<td>$0.93$</td>
<td>$0.014$</td>
<td>$0.0608$</td>
<td>$0.0018$</td>
</tr>
<tr>
<td>$\Sigma^0\Sigma^0$</td>
<td>$-0.055$</td>
<td>$0.0$</td>
<td>$-0.031$</td>
<td>$0.0$</td>
<td>$-0.0441$</td>
</tr>
<tr>
<td>$\Sigma^0\Xi^0$</td>
<td>$-0.110$</td>
<td>$2.91$</td>
<td>$-0.031$</td>
<td>$-0.0608$</td>
<td>$-0.0864$</td>
</tr>
<tr>
<td>$\Xi^-\Xi^-$</td>
<td>$0.0$</td>
<td>$-2.91$</td>
<td>$0.007$</td>
<td>$0.0$</td>
<td>$0.0018$</td>
</tr>
<tr>
<td>$\Sigma^0\Lambda$</td>
<td>$-0.096$</td>
<td>$-4.83$</td>
<td>$-0.041$</td>
<td>$-0.0526$</td>
<td>$-0.0733$</td>
</tr>
</tbody>
</table>

radii is the nonzero value pertaining to the neutral octet baryons ($n$, $\Sigma^0$, $\Xi^0$, and $\Lambda$) and decuplet baryons ($\Delta^0$, $\Sigma^*$, and $\Xi^*$). For the quadrupole moment, prolate shape is observed for the spin $(3/2)^+$ neutral decuplet baryons ($\Delta^0$, $\Sigma^*$, and $\Xi^*$). The effects of SU(3) symmetry breaking have also been investigated, and the results show considerable improvement over the SU(3) symmetric case. We have also studied the implications of GPM parameters, particularly, the contribution of the three-quark term in the octet and decuplet baryon. We find that the sign of the three-quark term contribution is opposite in the case of octet and decuplet baryons charge radii. New experiments aimed at measuring the charge radii and quadrupole moment of the other baryons are needed for a profound understanding of the hadron structure in the nonperturbative regime of QCD.

In conclusion, we would like to state that at-the-leading-order constituent quarks and the weakly interacting Goldstone bosons constitute the appropriate degrees of freedom in the nonperturbative regime of QCD. The SU(3) symmetry breaking parameters pertaining to the strangeness contribution and the GPM parameters pertaining to the one-, two-, and three-quark contributions are the key in understanding the octet and decuplet baryon charge radii and quadrupole moment.

Acknowledgments

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