Research Article

Gauss-Bonnet Braneworld Cosmology with Modified Induced Gravity on the Brane

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Received 21 May 2013; Accepted 19 August 2013

Academic Editor: Jose Edgar Madriz Aguilar

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We analyze the background cosmology for an extension of the DGP gravity with Gauss-Bonnet term in the bulk and $f(R)$ gravity on the brane. We investigate implications of this setup on the late-time cosmic history. Within a dynamical system approach, we study cosmological dynamics of this setup focusing on the role played by curvature effects. Finally, we constrain the parameters of the model by confrontation with recent observational data.

1. Introduction

One of the most significant astronomical observations in the last decade is the accelerated expansion of the universe [1–14]. One way to explain this accelerating phase of the universe expansion is invoking a dark energy component in the matter sector of the Einstein field equations [15–31]. However, it is possible also to modify the geometric part of the field equations to achieve this goal [32–45]. In the spirit of the second viewpoint, the braneworld model proposed by Dvali, Gabadadze, and Porrati (DGP) provides a natural explanation of late-time accelerated expansion in its self-accelerating branch of the solutions [46–51]. Unfortunately, the self-accelerating branch of this scenario suffers from ghost instabilities [52, 53], and, therefore, it is desirable to invoke other possibilities in this braneworld setup. An amazing feature of the DGP setup is that the normal branch of this scenario, which is not self-accelerating, has the capability to realize phantom-like behavior without introducing any phantom field neither in the bulk nor on the brane [54–60]. By the phantom-like behavior one means that an effective energy density, which is positive, grows with time and its equation of state parameter ($\omega_{\text{eff}} = p_{\text{eff}}/\rho_{\text{eff}}$) stays always less than -1 . The phantom-like prescription breaks down if this effective energy density becomes negative. An interesting extension of the DGP setup is possible modification of

the induced gravity on the brane (We call the $f(R)$ term on the brane the *modified induced gravity* since this braneworld scenario is an extension of the DGP model. In the DGP model, the gravity is induced on the brane through interaction of the bulk graviton with loops of matter on the brane. So the phrase “induced gravity” is coming from the DGP character of our model.) on the brane. This can be achieved by treating the induced gravity in the framework of $f(R)$ gravity [61–66]. This extension can be considered as a manifestation of the scalar-tensor gravity on the brane since $f(R)$ gravity can be reconstructed as a general relativity plus a scalar field [32–45]. Some features of this extension such as self-acceleration in the normal branch of the scenario are studied recently [61–68]. Here, we generalize this viewpoint to the case that the Gauss-Bonnet curvature effect is also taken into account. We consider a DGP-inspired braneworld model where the induced gravity on the brane is modified in the spirit of $f(R)$ gravity, and the bulk action contains the Gauss-Bonnet term to incorporate higher-order curvature effects. Our motivation is to study possible influences of the curvature corrections on the cosmological dynamics on the normal branch of the DGP setup. We analyze the background cosmology and possible realization of the phantom-like behavior in this setup. By introducing a *curvature fluid* that plays the role of dark energy component, we show that this model realizes phantom-like behavior on the normal branch

of the scenario in some subspaces of the model parameter space, without appealing to phantom fields neither in the bulk nor on the brane and by respecting the null energy condition in the phantom-like phase of expansion. We show also that in this setup there is smooth crossing of the phantom divide line by the equation of state parameter and the universe transits smoothly from a quintessence-like phase to a phantom-like phase. We present a detailed analysis of cosmological dynamics in this setup within a dynamical system approach in order to reveal some yet unknown features of these kinds of models in their phase space. Finally, confrontation of our model with recent observational data leads us to some constraint on model parameters.

2. The Setup

2.1. Gauss-Bonnet Braneworld with Induced Gravity on the Brane. The action of a GBIG (the Gauss-Bonnet term in the bulk and induced gravity on the brane) braneworld scenario can be written as follows [69–80]:

$$S = S_{\text{bulk}} + S_{\text{brane}}, \quad (1)$$

where, by definition,

$$S_{\text{bulk}} = \frac{1}{16\pi G^{(5)}} \int d^5x \sqrt{-g} \left[\mathcal{R}^{(5)} - 2\Lambda^{(5)} + \alpha \mathcal{L}_{\text{GB}} \right], \quad (2)$$

with

$$\begin{aligned} \mathcal{L}_{\text{GB}} &= \left(\mathcal{R}^{(5)} \right)^2 - 4\mathcal{R}_{\text{AB}}^{(5)} \mathcal{R}^{(5)\text{AB}} + \mathcal{R}_{\text{ABCD}}^{(5)} \mathcal{R}^{(5)\text{ABCD}}, \\ S_{\text{brane}} &= \frac{1}{16\pi G^{(4)}} \int d^4x \sqrt{-q} \left[\mathcal{R} - 2\Lambda^{(4)} \right]. \end{aligned} \quad (3)$$

$G^{(5)}$ is the 5D Newton constant in the bulk, and $G^{(4)}$ is the corresponding 4D quantity on the brane. $\mathcal{L}_{\text{GB}}$ is the Gauss-Bonnet term, and α is a constant with dimension of $[\text{length}]^2$. q is the induced metric on the brane. We choose the coordinate of the extra dimension to be y , so that the brane is located at $y = 0$. The DGP crossover distance, which is defined as

$$r_c = \frac{G^{(5)}}{G^{(4)}} = \frac{\kappa_5^2}{2\kappa_4^2}, \quad (4)$$

has the dimension of $[\text{length}]$ and will appear in our forthcoming equations. We note that this scenario is UV/IR complete in some sense, since it contains both the Gauss-Bonnet term as a string-inspired modification of the UV (ultraviolet) sector and the induced gravity as IR (infrared) modification to the general relativity. The cosmological dynamics of this GBIG scenario is described fully by the following Friedmann equation [81–84]:

$$\begin{aligned} &\left[H^2 + \frac{k}{a^2} - \frac{8\pi G^{(4)}(\rho + \lambda)}{3} \right]^2 \\ &= \frac{4}{r_c^2} \left[1 + \frac{8\alpha}{3} \left(H^2 + \frac{k}{a^2} + \frac{U}{2} \right) \right]^2 \left(H^2 + \frac{k}{a^2} - U \right), \end{aligned} \quad (5)$$

where

$$\begin{aligned} U &= -\frac{1}{4\alpha} \pm \frac{1}{4\alpha} \sqrt{1 + 4\alpha \left(\frac{\Lambda^{(5)}}{6} + \frac{2\mathcal{E}_0 G^{(5)}}{a^4} \right)}, \\ \lambda &\equiv \frac{\Lambda^{(4)}}{8\pi G^{(4)}}. \end{aligned} \quad (6)$$

$\mathcal{E}_0$ is referred to hypothetically as the mass of the bulk black hole, and the corresponding term is called the bulk radiation term. Note that, when one adopts the positive sign, the above equation can be reduced to the generalized DGP model as $\alpha \rightarrow 0$, but the branch with negative sign cannot be reduced to the generalized DGP model in this regime. Therefore, we just consider the plus sign of the above equation [81–84]. We note that, depending on the choice of the bulk space, the brane FRW equations are different (see [85] for details). The bulk space in the present model is a 5-dimensional AdS black hole. In which follows, we assume that there is no cosmological constant on the brane or in the bulk; that is, $\Lambda^{(4)} = \Lambda^{(5)} = 0$. Also, we ignore the bulk radiation term since it decays very fast in the early stages of the evolution (note, however, that this term is important when one treats cosmological perturbations on the brane). So the Friedmann equation in this case reduces to the following form:

$$\left[H^2 - \frac{8\pi G^{(4)}}{3} \rho \right]^2 = \frac{4}{r_c^2} \left[1 + \frac{8\alpha}{3} H^2 \right]^2 H^2. \quad (7)$$

It has been shown that it is possible to realize the phantom-like behavior in this setup without introducing any phantom matter on the brane [86–91]. In which follows, we generalize this setup to the case that induced gravity on the brane is modified in the spirit of $f(R)$ gravity, and we explore the cosmological dynamics of this extended braneworld scenario.

2.2. Modified GBIG Gravity. In this subsection, we firstly formulate a GBIG scenario that induced gravity on the brane acquires a modification in the spirit of $f(R)$ gravity. To obtain the generalized Friedmann equation of this model, we proceed as follows. Firstly, the Friedmann equation for pure DGP scenario is given as follows [49–51, 92, 93]:

$$\begin{aligned} &\epsilon \sqrt{H^2 - \frac{\mathcal{E}_0}{a^4} - \frac{\Lambda_5}{6} + \frac{k}{a^2}} \\ &= r_c \left[\left(H^2 + \frac{k}{a^2} \right) - \frac{8\pi G^{(4)}}{3} (\rho + \lambda) \right], \end{aligned} \quad (8)$$

where $\epsilon = \pm 1$ is corresponding to two possible embeddings of the brane in the bulk. Considering a Minkowski bulk with $\Lambda_5 = 0$ and by setting $\mathcal{E}_0 = 0$ with a tensionless brane ($\lambda = 0$), for a flat brane ($k = 0$), we find that

$$H^2 = \frac{8\pi G^{(4)}}{3} \rho \pm \frac{H}{r_c}. \quad (9)$$

The normal branch of the scenario is corresponding to the minus sign on the right-hand side of this equation.

The second term on the right is the source of the phantom-like behavior on the normal branch: the key feature of this phase is that the brane is extrinsically curved in such a way that shortcuts through the bulk allow gravity to screen the effects of the brane energy-momentum contents at Hubble parameters $H \sim r_c^{-1}$, and this is not the case for the self-accelerating phase [54–60].

In the next step, we incorporate possible modification of the induced gravity by inclusion of a $f(R)$ term on the brane. This extension can be considered as a manifestation of the scalar-tensor gravity on the brane. In this case, we find the following generalized Friedmann equation (see, e.g., [61–68, 92, 93]):

$$\begin{aligned} & \epsilon \sqrt{H^2 - \frac{MG^{(5)}}{a^4} - \frac{\Lambda_5}{6} + \frac{k}{a^2}} \\ &= r_c \left[\left(H^2 + \frac{k}{a^2} \right) f'(R) - \frac{8\pi G^{(4)}}{3} \right. \\ & \quad \left. \times \left[\rho + \lambda + \left(\frac{1}{2} [Rf'(R) - f'(R)] - 3H\dot{R}f''(R) \right) \right] \right], \end{aligned} \quad (10)$$

where a prime marks derivative with respect to R . In the third step, we need the GBIG Friedmann equation in the absence of any modification of the induced gravity on the brane, that is, without $f(R)$ term on the brane. This has been obtained in the previous subsection; see (5). Now, we have all prerequisites to obtain the Friedmann equation of our GBIG-modified gravity scenario. The comparison between previous equations gives this Friedmann equation of cosmological dynamics as follows (we note that this equation can be derived using the generalized junction conditions on the brane straightforwardly; see [81–84]):

$$H^2 = \frac{\kappa_4^2}{3f'(R)}\rho + \frac{\kappa_4^2}{3}\rho_{\text{curv}} \pm \frac{1}{r_c f'(R)} \left(1 + \frac{8}{3}\alpha H^2 \right) H, \quad (11)$$

where we have defined hypothetically the following energy density corresponding to curvature effect:

$$\rho_{\text{curv}} = \frac{1}{f'(R)} \left(\frac{1}{2} [Rf'(R) - f(R)] - 3H\dot{R}f''(R) \right). \quad (12)$$

Note that, to obtain this relation, $\mathcal{E}_0$, Λ_5 , λ , and k are set equal to zero. From now on, we restrict our attention to the normal branch of the scenario, that is, the minus sign in (11) because there are no ghost instabilities in this branch only if the DGP character of the model is considered.

Note, however, that although we refer to the normal (ghost-free) branch of the DGP model (in the sense that for $f(R) = R$ the obtained solutions reduce to this branch) as an indication of the ghost-free property of the considered solutions, it is not a priori guaranteed that, on the obtained de Sitter backgrounds which generalize the normal DGP branch, the ghost does not reappear. In fact, the ghost on the self-accelerated branch of the DGP model is entirely the problem of the de Sitter background. Within the crossover

scale r_c which is the horizon for the self-accelerated branch, the theory reduces to a scalar-tensor model, with the scalar sector (brane bending mode) being described by a simple Galileon self-interaction [94] as

$$\mathcal{L}_\pi = \pi \square \pi - \frac{(\partial\pi)^2 \square \pi}{\Lambda^3}, \quad (13)$$

which, in spite of the presence of higher derivatives, propagates a single healthy degree of freedom. The ghost on the self-accelerated branch arises merely due to the fact that π gets a nontrivial profile and the kinetic term for its perturbation flips the sign on the background. A similar argument can be applied in the present work to overcome the ghost instabilities in this extended braneworld setup.

In which follows, we assume that the energy density ρ on the brane is due to cold dark matter (CDM) with $\rho_m = \rho_{0m}(1+z)^3$. We can rewrite the Friedmann equation in terms of observational parameters such as the redshift z and dimensionless energy densities Ω_i as follows:

$$\begin{aligned} E^2 &= \frac{\Omega_m}{f'(R)}(1+z)^3 + \Omega_{\text{curv}}(1+z)^{3(1+w_{\text{curv}})} \\ &\quad - 2\frac{\sqrt{\Omega_{r_c}}}{f'(R)} [1 + \Omega_\alpha E^2(z)] E(z), \end{aligned} \quad (14)$$

where

$$\begin{aligned} E(z) &\equiv \frac{H}{H_0}, \\ \Omega_m &\equiv \frac{\kappa_4^2}{3H_0^2}\rho_{0m}, \quad \Omega_\alpha \equiv \frac{8}{3}\alpha H_0^2, \\ \Omega_{r_c} &\equiv \frac{1}{4r_c^2 H_0^2}, \quad \Omega_{\text{curv}} \equiv \frac{\kappa_4^2}{3H_0^2}\rho_{0\text{curv}}, \\ w_{\text{curv}} &= \frac{p_{\text{curv}}}{\rho_{\text{curv}}}. \end{aligned} \quad (15)$$

p_{curv} , the hypothetical pressure of the curvature effect, can be obtained by the following equation of continuity:

$$\begin{aligned} \dot{\rho}_{\text{curv}} + 3H \left(\rho_{\text{curv}} + p_{\text{curv}} + \frac{\dot{R}f''(R)}{r_c[f'(R)]^2} \right) \\ = \frac{3H_0^2\Omega_m\dot{R}f''(R)}{[f'(R)]^2} a^{-3}. \end{aligned} \quad (16)$$

One can obtain a constraint on the cosmological parameters of the model at $z = 0$ as follows:

$$\Omega_m = 1 - \Omega_{\text{curv}} + 2\sqrt{\Omega_{r_c}}(1 + \Omega_\alpha). \quad (17)$$

Note that we have used the normalization $f'(R)|_{z=0} = 1$ in this relation which is observationally a viable assumption.

3. Cosmological Dynamics in the Modified GBIG Scenario

Now, we study cosmological dynamics in this setup. To solve the Friedmann equation for the normal branch of this scenario, it is convenient (following the papers by Bohamdi-Lopez in [86–90]) to introduce the dimensionless variables as follows:

$$\begin{aligned}\bar{H} &\equiv \frac{8}{3} \frac{\alpha}{r_c f'(R)} H = 2\Omega_\alpha \sqrt{\Omega_r E(z)}, \\ \bar{\rho} &\equiv \frac{32}{27} \frac{\kappa_5^2 \alpha^2}{[r_c f'(R)]^3} (\rho_m + f'(R) \rho_{\text{curv}}) \\ &= 4\Omega_\alpha^2 \Omega_r [\Omega_m (1+z)^3 + \Omega_{\text{curv}} (1+z)^{3(1+w_{\text{curv}})}], \\ b &\equiv \frac{8}{3} \frac{\alpha}{[r_c f'(R)]^2} = 4\Omega_\alpha \Omega_r.\end{aligned}\quad (18)$$

An effective crossover distance which appeared on the right-hand side of these relations can be defined as follows,

$$r \equiv r_c f'(R), \quad (19)$$

and, by definition, $\Omega_r \equiv 1/4r^2 H_0^2$. Then, the Friedmann equation can be rewritten as

$$\bar{H}^3 + \bar{H}^2 + b\bar{H} - \bar{\rho} = 0. \quad (20)$$

The number of real roots of this equation is determined by the sign of the discriminant function $\mathcal{N}$ defined as

$$\mathcal{N} = Q^3 + S^2, \quad (21)$$

where Q and S are defined as

$$\begin{aligned}Q &= \frac{1}{3} \left(b - \frac{1}{3} \right), \\ S &= \frac{1}{6} b + \frac{1}{2} \bar{\rho} - \frac{1}{27},\end{aligned}\quad (22)$$

respectively. We can rewrite $\mathcal{N}$ as

$$\mathcal{N} = \frac{1}{4} (\bar{\rho} - \bar{\rho}_1) (\bar{\rho} - \bar{\rho}_2), \quad (23)$$

where

$$\begin{aligned}\bar{\rho}_1 &= -\frac{1}{3} \left\{ b - \frac{2}{9} \left[1 + \sqrt{(1-3b)^3} \right] \right\}, \\ \bar{\rho}_2 &= -\frac{1}{3} \left\{ b - \frac{2}{9} \left[1 - \sqrt{(1-3b)^3} \right] \right\}.\end{aligned}\quad (24)$$

In which follows, we consider just the real and positive roots of the Friedmann equation (20). For $0 < b < 1/4$, $\bar{\rho}_1 > 0$ and $\bar{\rho}_2 < 0$. Then, the number of real roots of the cubic Friedmann equation depends on the minimum energy density of the brane and the situation of ρ_1 relative to this minimum. Since, in our setup, curvature effect plays the role of the dark energy component on the brane, we can consider two different regimes to determine the minimum value of $\bar{\rho}$ as follows.

- (1) $w_{\text{curv}} > -1$: in this case, curvature fluid plays the role of quintessence component; then, the minimum value happens asymptotically at $z = -1$, and we will obtain $\bar{\rho}_{\min} = 0$. In this situation, we can define three possible regimes: a high-energy regime with $\bar{\rho}_1 < \bar{\rho}$, a limiting regime with $\bar{\rho}_1 = \bar{\rho}$, and a low-energy regime with $\bar{\rho}_1 > \bar{\rho}$. In each of these cases, depending on the sign of N , there are different solutions [86–91].
- (2) $w_{\text{curv}} < -1$: in this case, the curvature fluid plays the role of a phantom component (we will investigate its phantom-like behavior in the next section), and the minimum value of $\bar{\rho}$ happens at $z = 0.18$. So we find the $\bar{\rho}_{\min}$ as follows:

$$\bar{\rho}_{\min} = 4\Omega_\alpha^2 \Omega_r \left[0.43 + \Omega_{\text{curv}} (1 + 0.18)^{3(1+w_{\text{curv}})} \right], \quad (25)$$

where we have set $\Omega_m = 0.26$. We note that w_{curv} is not constant, and, as we will show, it evolves from quintessence to phantom phase. We note also that the value of redshift when $\bar{\rho}_{\min}$ occurs (i.e., $z = 0.18$) has no dependence on the values of w_{curv} . Here, we treat only the case $\bar{\rho}_1 < \bar{\rho}_{\min}$ with details. When this condition is satisfied, the function $\mathcal{N}$ is positive, and there is a unique solution for expansion of the brane described by

$$\bar{H}_1 = \frac{1}{3} \left[2\sqrt{1-3b} \cosh\left(\frac{\eta}{3}\right) \right], \quad (26)$$

where η is defined as

$$\cosh(\eta) = \frac{S}{\sqrt{-Q^3}}. \quad (27)$$

We note that this condition provides a constraint on the dimensionless parameters Ω_i as follows:

$$\begin{aligned}-\frac{1}{3b} \left[b - \frac{2}{9} \left[1 + \sqrt{(1-3b)^3} \right] \right] \\ < \Omega_\alpha \left[0.43 + \Omega_{\text{curv}} (1 + 0.18)^{3(1+w_{\text{curv}})} \right].\end{aligned}\quad (28)$$

Figure 1 shows the phase space of the above relation. In this figure, we have defined $\Psi \equiv 0.43 + \Omega_{\text{curv}} (1 + 0.18)^{3(1+w_{\text{curv}})}$. The relation (28) is fulfilled for upper region (region II) of this figure. In this case, there are three possible regimes as was mentioned above. A point that should be emphasized here is the fact that, in the presence of modified induced gravity on the brane, the solution of the generalized Friedmann equation (11) is actually rather involved due to simultaneous presence of H , $\dot{H}$, and $\ddot{H}$. A thorough analysis of this problem is out of the scope of this study, but there are some attempts (such as cosmography) in this direction to construct an operational framework to treat this problem; see, for instance [41]. Here, we have tried to find a solution of (11) by using the discriminant function $\mathcal{N}$, the result of which is given by (26). However, we note that a complete analysis is needed, for instance, in the framework of cosmography of brane $f(R)$ gravity [95].

To investigate cosmology described by solution (26), we rewrite the original Friedmann equation in the following

FIGURE 1: The upper region (region II) of this figure is corresponding to the set $(\Omega_r, \Omega_\alpha, \Omega_{\text{curv}}, w_{\text{curv}})$ that fulfils inequality (28).

form in order to create a general relativistic description of our model:

$$H^2 = \frac{(\kappa_4^2)_{\text{eff}}}{3} \rho_m + \rho_{\text{curv}} - \frac{H}{r_c f'(R)} \left(1 + \frac{8}{3} \alpha H^2 \right), \quad (29)$$

where ρ_{curv} is defined in (12). Comparing this relation with the Friedmann equation in GR

$$H^2 = \frac{\kappa_4^2}{3} (\rho_m + \rho_{\text{eff}}), \quad (30)$$

we obtain an effective energy density

$$\frac{\kappa_4^2}{3} \rho_{\text{eff}} = \rho_{\text{curv}} - \frac{H}{r_c f'(R)} \left(1 + \frac{8}{3} \alpha H^2 \right), \quad (31)$$

which can be rewritten as follows:

$$\rho_{\text{eff}} = \Omega_{\text{curv}} (1+z)^{3(1+w_{\text{curv}})} - \frac{2\sqrt{\Omega_{r_c}}}{f'(R)} \left(1 + \Omega_\alpha E^2 \right) E(z). \quad (32)$$

The dependence of ρ_{eff} on the redshift depends itself on the regimes introduced above and the form of $f(R)$ function. Figure 2 shows the variation of ρ_{eff} versus the redshift for

$$f(R) = R - (n-1) \zeta^2 \left(\frac{R}{\zeta^2} \right)^n \quad (33)$$

with $n = 0.25$. This value of n lies well in the range of observationally acceptable values of n from solar-system tests [32–45]. (We note, however, that the key issue with regards to passing solar-system tests is not the value of n but the value of $f'(R)$ today. In fact, experimental data tell us that $f'(R) - 1 < 10^{-6}$, when $f'(R)$ is parameterized to be exactly 1 in the far past.)

FIGURE 2: Variation of effective energy density versus the redshift for a specific $f(R)$ model described as (33) with $n = 0.25$.

The effective energy density shows a phantom-like behavior; that is, it increases with cosmic time. This is a necessary condition to have phantom-like behavior, but it is not sufficient: we should check status of the deceleration parameter and also equation of state parameter. In a general relativistic description of our model, one can rewrite the energy conservation equation as follows:

$$\dot{\rho}_{\text{eff}} + 3H(1 + \omega_{\text{eff}}) \rho_{\text{eff}} = 0 \quad (34)$$

which leads to the following relation for $1 + \omega_{\text{eff}}$:

$$1 + \omega_{\text{eff}} = -\frac{\dot{\rho}_{\text{eff}}}{3H\rho_{\text{eff}}}. \quad (35)$$

Figure 3 shows variation of the effective equation of state parameter versus the redshift for a specific $f(R)$ model described as (33) with $n = 0.25$. The effective equation of state parameter transits to the phantom phase $1 + \omega_{\text{eff}} < 0$, but there is no smooth crossing of the phantom divide line in this setup. We note that adopting other general ansatz, such as the Hu-Sawicki model [96]

$$f(R) = R - R_c \frac{\alpha_0 (R/R_c)^n}{1 + \beta_0 (R/R_c)^n} \quad (36)$$

(where both α and R_c are free positive parameters), does not change this result in our framework. The deceleration parameter defined as

$$q \equiv -1 - \frac{\dot{H}}{H^2} \quad (37)$$

FIGURE 3: Variation of effective equation of state parameter versus the redshift for a specific $f(R)$ model described as (33) with $n = 0.25$.

FIGURE 4: Variation of the deceleration parameter q versus z for $n = 0.25$. The transition from deceleration to the acceleration phase occurs at $z \approx 2$.

takes the following form in our setup:

$$q = -1 - \frac{\Omega_m(1+z)^3 \left((3/2) - (\dot{f}'(R)/f'(R)) (H_0/E(z)) \right)}{f'(R) E^2(z) + \sqrt{\Omega_{r_c}} (1 + 3\Omega_\alpha E^2(z)) E(z)} - \left(\sqrt{\Omega_{r_c}} \frac{\dot{f}'(R)}{f'(R)} (1 + \Omega_\alpha E^2(z)) \frac{1}{H_0} - \frac{3}{2} \Omega_{\text{curv}} f'(R) (1+z)^{3(1+\omega_c)} (1 + \omega_c) \right) \times \left(f'(R) E^2(z) + \sqrt{\Omega_{r_c}} (1 + 3\Omega_\alpha E^2(z)) E(z) \right)^{-1}. \quad (38)$$

In this relation, H and $f(R)$ are defined as (26) and (33). Figure 4 shows variation of q versus the redshift for $n = 0.25$. The universe enters the accelerated phase of expansion at $z \approx 2$. Another important issue to be investigated in this setup is the big-rip singularity. To avoid super acceleration on the brane, it is necessary to show that Hubble rate decreases as the brane expands and that there is no big-rip singularity in the future. Figure 5 shows variation of $\dot{H}$ versus z . We see that in this model $\dot{H} < 0$ always, and, therefore, there is no super-acceleration or future big-rip singularity in this setup. All of the previous considerations show that this model accounts for realization of the phantom-like behavior without introducing a phantom field neither on the brane nor in the bulk. Nevertheless, we have to check the status of the null energy condition in this setup. Figure 6 shows the variation of $(\rho + p)_{\text{tot}}$ versus the redshift. We see that this condition is fulfilled at least in some subspaces of the phantom-like region of the model parameter space.

FIGURE 5: In this model, $\dot{H}$ is always negative, and, therefore, there is no superacceleration or big-rip singularity.

4. A Dynamical System Viewpoint

Up to this point, we have shown that there are effective quantities that create an effective phantom-like behavior on the brane. In this respect, one can define a potential related to the effective phantom scalar field ϕ_{eff} as follows [97]:

$$\frac{V_{\text{eff}}(z)}{3H_0^2} = E^2 - \frac{\Omega_m(1+z)^3}{f'(R)} + \frac{1}{2} \frac{d \left(E^2 - (\Omega_m(1+z)^3 / f'(R)) \right)}{d \ln(1+z)^3},$$

$$\frac{\phi_{\text{eff}}(z)}{\sqrt{3}} = - \int \frac{dz}{(1+z)E} \sqrt{\frac{d \left(E^2 - (\Omega_m(1+z)^3 / f'(R)) \right)}{d \ln(1+z)^3}}. \quad (39)$$

FIGURE 6: The status of the null energy condition in this model.

We note that in principle these equations can lead to $V_{\text{eff}} = V_{\text{eff}}(\phi_{\text{eff}})$, but in practice the inversion cannot be performed analytically. Now, we define the following normalized expansion variables [98–101]:

$$\begin{aligned} x &= \frac{\sqrt{\Omega_m}}{a^{3/2}E}, & y &= \frac{\sqrt{\Omega_{\text{curv}}f'(R)}}{a^{(3/2)(1+w_{\text{curv}})}E}, \\ v &= \frac{\sqrt{\Omega_r}}{E}, & \chi &= \sqrt{\Omega_\alpha}E. \end{aligned} \quad (40)$$

With these definitions, the Friedmann equation (14) takes the following form:

$$x^2 + y^2 - 2v(1 + \chi^2) = f'(R). \quad (41)$$

This constraint means that the phase space can be defined by the relation $x^2 + y^2 - 2v \geq 1$, since $f'(R) - 1 < 10^{-6}$ by solar-system constraints and v is a positive quantity. Introducing the new time variable $\tau = \ln a$ and eliminating χ and E , we obtain the following autonomous system:

$$\begin{aligned} x' &= x \left(q - \frac{1}{2} \right), \\ y' &= \frac{y}{y^2 - 1} \left[\frac{3}{2} (1 + w_{\text{curv}}) (1 + x^2) - (q + 1) \right. \\ &\quad \times \left. \left[1 + x^2 - 2v + \frac{1}{2} (2 + v) (x^2 + y^2 - 1) \right] \right], \\ v' &= v (q + 1). \end{aligned} \quad (42)$$

Here, primes denote differentiation with respect to τ , and $q = -\ddot{a}a/\dot{a}^2$ stands for the deceleration parameter

$$\begin{aligned} q + 1 &= \frac{3}{2} (x^2 + [1 + w_{\text{curv}} (1 - x^2)] y^2) \\ &\quad \times \left(4x^2 + 8y^2 - x^2 y^2 - \frac{3}{2} y^4 + \frac{1}{4} v y^4 - \frac{9}{4} v y^2 \right. \\ &\quad \times \left. - \frac{1}{2} v x^2 - \frac{19}{2} v - 5 \right)^{-1}. \end{aligned} \quad (43)$$

To study cosmological evolution in the dynamical system approach, it is necessary to find fixed (or critical) points of the model that are denoted by (x^*, y^*, v^*) . These points are achieved by fulfillment of the following condition:

$$g^i(x^*, y^*, v^*) = 0, \quad (44)$$

where

$$x'^i = g^i(x^*, y^*, v^*), \quad (45)$$

where in Table 1 x_B and y_B are as follows:

$$\begin{aligned} y_B &= \sqrt{x_B^2 (w_{\text{curv}} - 1) + (1 + w_{\text{curv}})}, \\ x_B &= \sqrt{\frac{-17w_{\text{curv}} + 4 + 6w_{\text{curv}}^2 + \sqrt{297w_{\text{curv}}^2 - 120w_{\text{curv}} - 180w_{\text{curv}}^3 + 16 + 36w_{\text{curv}}^4}}{2(w_{\text{curv}} - 1)}}. \end{aligned} \quad (46)$$

A part of dynamical system analysis of this model is summarized in Table 1.

The critical points A and B demonstrate the early-time, matter-dominated epoch which leads to a positive deceleration parameter. Points C and D which are phases with vanishing matter character, that is, $\Omega_m = 0$, can explain the positively accelerated phase of the universe expansion for $w_{\text{curv}} \geq -6.88$. Critical curve E also demonstrates a positively accelerated phase for all values of the equation of state parameter of the curvature fluid. Critical curve F , which exists only for the case $w_{\text{curv}} = -1$, is a de Sitter phase

in this model. Existence of a stable de Sitter point and an unstable matter-dominated phase (in addition to radiation-dominated era) in the universe expansion history is required for cosmological viability of any cosmological model. In order to investigate the stability of these points, one can obtain the eigenvalues of these points separately. Based on Table 2, in order for point A to be an unstable point, it is necessary to have $w_{\text{curv}} < 1/4$. Therefore, the point A as a saddle point agrees with what we have shown in Figure 7. Now, the stability of the positively accelerated phases of the model depends on whether the curvature fluid plays the role of a quintessence

TABLE 1: Location and deceleration parameter of the critical points. The location of point B and the deceleration factor of points C and D are dependent on the equation of state parameter of the perfect fluid, w_{curv} . The critical curve F exists just for $w_{\text{curv}} = -1$.

Name	x	y	v	q
A	$\sqrt{15}/3$	0	0	$1/2$
B	x_B	y_B	0	$1/2$
C	0	$1/2$	0	$-(1.17 + 0.17w_{\text{curv}})$
D	0	$7/2$	0	$-(1.13 + 0.13w_{\text{curv}})$
E	0	0	v^*	-1
F	0	y^*	v^*	-1

TABLE 2: Eigenvalues and the stability regions of the critical points. $\zeta(w_{\text{curv}})$, $\xi(w_{\text{curv}})$, and $\zeta(w_{\text{curv}})$ are complicated functions of w_{curv} . The critical curve F for which $w_{\text{curv}} = -1$ is a stable de Sitter phase.

Name	Eigenvalue	Stable region
A	$-9, -1/2, 1 - 4w_{\text{curv}}$	$w_{\text{curv}} > 1/4$
B	$\zeta(w_{\text{curv}}), \xi(w_{\text{curv}}), \zeta(w_{\text{curv}})$	—
C	$-(1 + 0.1w_{\text{curv}}), -(1 + w_{\text{curv}}), -(2 + 0.17w_{\text{curv}})$	$w_{\text{curv}} > -1$
D	$-(1 + 0.1w_{\text{curv}}), -(1 + w_{\text{curv}}), -(2 + 0.13w_{\text{curv}})$	$w_{\text{curv}} > -1$
E	$-3/2, -3/2(1 + w_{\text{curv}}), -2$	$w_{\text{curv}} > -1$
F	$-3/2, 0, -2$	$w_{\text{curv}} = -1$

FIGURE 7: The phase subspace $x - y$ that the curvature fluid plays the role of a cosmological constant. This model is cosmologically viable since there are an unstable matter-dominated point and a stable de Sitter phase.

scalar field or not. Points C , D , and E of Table 2 are stable phases of this model if $w_{\text{curv}} > -1$, whereas if the curvature fluid plays the role of a cosmological constant, the point F will be a stable de Sitter phase. We note that, generally, if a nonlinear system has a critical curve, the Jacobian matrix of the linearized system at a critical point on the curve (line in our 2-dimensional subspace) has a zero eigenvalue with an associated eigenvector tangent to the critical curve at the chosen point. The stability of a specific critical point on

the curve can be determined by the nonzero eigenvalues, because near this critical point there are essentially no dynamics along the critical curve [102]. We have plotted the phase space of this model in subspace $x - y$ with $w_{\text{curv}} = -1$. As we see, point A is a saddle point, and curve F is a stable de Sitter curve.

5. Confrontation with Recent Observational Data

In this section, we use the combined data from Planck + WMAP + high L+ lensing + BAO [103] to confront our model with recent observation. In this way, we obtain some constraints on the model parameters, especially, the Gauss-Bonnet curvature contribution. For this purpose, we consider the relation between Ω_{curv} and Ω_m in the background of the mentioned observational data. We suppose that Ω_{curv} plays the role of dark energy in this setup. Figure 8 shows the result of our numerical study. In this model, with $\Omega_{r_c} \sim 10^{-4}$ (see [104], e.g.), Ω_α is constrained as follows:

$$\begin{aligned}
 w_{\text{curv}} = -0.5, & \quad 0.008 < \Omega_\alpha < 0.011, \\
 w_{\text{curv}} = -0.92, & \quad 0.01 < \Omega_\alpha < 0.055, \\
 w_{\text{curv}} = -1.05, & \quad 0.012 < \Omega_\alpha < 0.073.
 \end{aligned} \tag{47}$$

On the other hand, if we consider the Ω_{eff} defined as $\Omega_{\text{eff}} = (\kappa_4^2/3H_0^2)\rho_{\text{eff}}$ as our main parameter, the result will be as shown in Figure 9. In this case, we have the following constraint on Ω_α :

$$\begin{aligned}
 \Omega_{\text{curv}} = 0.7, & \quad 0.0043 < \Omega_\alpha < 0.036, \\
 \Omega_{\text{curv}} = 0.5, & \quad 0.0048 < \Omega_\alpha < 0.0081.
 \end{aligned} \tag{48}$$

FIGURE 8: Ω_{curv} versus Ω_m in the background of Planck + WMAP + high L+ lensing + BAO joint data.

FIGURE 9: Ω_{eff} versus Ω_m in the background of Planck + WMAP + high L+ lensing + BAO joint data.

6. Summary and Conclusion

In this paper, we have constructed a DGP-inspired braneworld scenario where induced gravity on the brane is modified in the spirit of $f(R)$ gravity, and higher-order

curvature effects are taken into account by incorporation of the Gauss-Bonnet term in the bulk action. It is well known that the normal branch of the DGP braneworld scenario, which is not self-accelerating, has the potential to realize phantom-like behavior without introducing any phantom fields neither on the brane nor in the bulk. Our motivation here to study this extension of the DGP setup is to explore possible influences of the curvature corrections, especially, the modified induced gravity, on the cosmological dynamics of the normal branch of the DGP setup. In this regard, cosmological dynamics of this scenario as an alternative for dark energy proposal is studied, and the effects of the curvature corrections on the phantom-like dynamics of the model are investigated. The complete analysis of the generalized Friedmann equation needs a cosmographic viewpoint to $f(R)$ gravity, but here we have tried to find a special solution of this generalized equation via the discriminant function method. In our framework, effective energy density attributed to the curvature plays the role of effective dark energy density. In other words, we defined a *curvature fluid* with varying equation of state parameter that incorporates in the definition of effective dark energy density. The equation of state parameter of this curvature fluid is evolving, and the effective dark energy equation of state parameter has transition from quintessence to the phantom phase in a nonsmooth manner. We have considered a cosmologically viable (Hu-Sawicki) ansatz for $f(R)$ gravity on the brane to have more practical results. We have shown that this model mimics the phantom-like behavior on the normal branch of the scenario in some subspaces of the model parameter space without introduction of any phantom matter neither in the bulk nor on the brane. At the same time, the null energy condition is respected in the phantom-like phase of the model parameter space. There is no super-acceleration or big-rip singularity in this setup. Incorporation of the curvature effects both in the bulk (via the Gauss-Bonnet term) and on the brane (via modified induced gravity) results in the facility that *curvature fluid* plays the role of dark energy component. On the other hand, this extension allows the model to mimic the phantom-like prescription in relatively wider range of redshifts in comparison to the case that induced gravity is not modified. This effective phantom-like behavior permits us to study cosmological dynamics of this setup from a dynamical system viewpoint. This analysis has been performed with details, and its consequences are explained. The detailed dynamical system analysis of this setup is more involved relative to the case that there are no curvature effects. We have shown that, with suitable condition on equation of state parameter of curvature fluid, there are an unstable matter era and a stable de Sitter phase in this scenario leading to the conclusion that this model is cosmologically viable. We have constrained our model based on the recent observational data from joint Planck + WMAP + high L+ lensing + BAO data sets. In this way, some constraints on Gauss-Bonnet coupling contribution are presented. We note that no big-rip singularity is present in this model since the Gauss-Bonnet contribution to this model is essentially a stringy, quantum gravity effect that prevents the big-rip singularity (see, e.g., [105] for details).

Acknowledgment

The work of K. Nozari has been supported financially by the Center for Excellence in Astronomy and Astrophysics (CEAAI-RIAAM), Maragha, Iran.

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