Research Article

Influence of Neutral Currents on Electron and Gamma Polarizations in the Process $e + N \rightarrow e' + N + \gamma$

N. V. Samsonenko, 1 Adamou Ousmane Manga, 2 Almoustapha Aboubacar, 2 and Aboubacar Moussa 3

1 Department of Theoretical Physics, Faculty of Sciences, Russian Friendship University, 3 Ordjonikidze, Moscow 117923, Russia
2 Department of Physics, Faculty of Sciences and Techniques, Abdou Moumouni University, 10662 Niamey, Niger
3 Department of Mathematics and Computer Science, Faculty of Sciences and Techniques, Abdou Moumouni University, 10662 Niamey, Niger

Correspondence should be addressed to Adamou Ousmane Manga; manga_adamou@yahoo.com

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The differential cross section of electron inelastic scattering by nuclei followed by $\gamma$ radiation is calculated using the multipole decomposition of the hadronic currents and by taking into account the longitudinal polarization of the initial electron and the circular polarization of the $\gamma$ radiation. We performed the analysis of the angular and energy dependence of the degree of electron and photon polarization which can yield information on values of weak neutral currents parameters.

1. Introduction

For many years lepton scattering by nucleon and nuclei has been playing a key role in the determination of the nuclear electromagnetic form factors and in testing the standard model of electroweak interactions [1–4]. During the last decade there has been a considerable effort to measure parity-violating asymmetries in electron scattering [5–7]. These asymmetries can improve the precision of the determination of the mixing angle $\theta_W$ [8–11]. Electron scattering has also been used for the determination of the vector strange-quark matrix element [11–16].

In this work we study the influence of weak neutral currents parameters on the spin asymmetry coefficient of electron and photon polarization in the electron inelastic scattering by nuclei followed by gamma radiation. This process is illustrated by the following equation:

$$e^- + (A, Z) \rightarrow e'^- + (A, Z)^* + (A, Z) + \gamma_{RL}$$  \hspace{1cm} (1)

2. Differential Cross Section

In the first order perturbation theory the square of matrix element of the process (1) can be expressed as [17]

$$\sum_{M_n, M_f} |M_{fi}|^2 = \frac{1}{2J_i + 1} \times \sum_{M_n, M_f} |\langle J_n, M_n | H_\gamma (\Omega, \sigma) | J_f, M_f \rangle|^2 \times \langle J_f, M_f | H_{ee'} (\tilde{q}, s') | J_i, M_i \rangle|^2,$$

where $M_{ni}$ and $M_{fi}$ are the matrix elements of $|J_i, M_i \rangle \rightarrow |J_n, M_n \rangle$ and $|J_n, M_n \rangle \rightarrow |J_f, M_f \rangle$ transitions. $J_i, J_n, and J_f are, respectively, the spins of the initial, intermediate (excited), and the final states of the nucleus. Let us consider

$$H_\gamma (\Omega, \sigma) = -\frac{2ne}{\sqrt{\omega_1} \sum_{j, i | j > i} (-i)^j |J_1| (\sigma T_{j\sigma}^m - T_{j\sigma}^e),$$

(3)
the interaction Hamiltonian for the emission of gamma rays with polarization $\sigma$ in the direction $\Omega$. Here $|J_1| = (2|J_1| + 1)^{1/2}$. The magnetic and electric multipole operators $\tilde{T}_{i,\sigma}^m$ and $\tilde{T}_{i,\sigma}^e$ are defined in a system of coordinates $X'Y'Z'$ where $z$-axis is oriented along the momentum $\vec{k}$ of the photon (Figure 3).

By using (3) and (4) and applying the Wigner-Eckart theorem the matrix element of the Hamiltonian $H_f$ can take the following form:

$$\langle J_f M_f | H_f (\Omega, \sigma) | J_n M_n \rangle$$

$$= \frac{2\pi e}{\sqrt{\omega \Omega}} \times \sum_{\ell=1}^{\infty} \langle J_f M_f | I_{i,\ell}^M | J_n M_n \rangle \langle J_f | \sigma \tilde{T}_{i,\sigma}^m - \tilde{T}_{i,\sigma}^e \rangle | J_n \rangle$$

$$\times \left( \frac{J_f}{-M_f} \frac{J_1}{M_1} \frac{J_n}{M_n} \right)$$

So the matrix element for the nuclear transition from the initial state $|J_f M_f \rangle$ to the intermediate state $|J_n M_n \rangle$ is given by

$$M_{nf} = \langle J_n M_n | \tilde{H}_{ec} | J_f M_f \rangle = C_e \epsilon^{*}_{\ell} \ell_{\mu}^e + C_W \epsilon^{*}_{\ell} \ell_{\mu}^w$$

where

$$\epsilon^{*}_{\ell} \ell_{\mu}^e = \bar{u}(p') \gamma_{\mu} (a_\nu + a_A \gamma_5) u(p)$$

$$\tilde{\epsilon}_{\mu} (\ell) = \langle J_f M_f | \int d\vec{x} \exp(-i\vec{q} \cdot \vec{x}) \tilde{\epsilon}^{*}_{\mu} (\vec{x}) | J_n M_n \rangle$$

are the leptonic and hadronic currents. $C_e = 4\alpha q^2_{\mu}$ and $C_W = -G_F/\sqrt{2}$ are related, respectively, to the electromagnetic and weak interactions. $s_{\mu} = (p' - p)_{\mu}$ is the transferred 4-momentum, $\alpha$ is the fine structure constant, and $G_F$ is the Fermi coupling constant for weak interaction. The constants $a_\nu$ and $a_A$ take specific values depending on the process. In the Weinberg-Salam model, for electron or muon scattering processes we have according to [19]

$$a_\nu = -1 + 4\sin^2\theta_W \quad a_A = -1.$$

The density of hadronic current $\tilde{\epsilon}_{\mu} (\ell)$ is composed by vector and axial-vector currents with isoscalar ($\tau = 0$) and isovector ($\tau = 1$) components [18]:

$$\tilde{\epsilon}_{\mu} = \tilde{\epsilon}^{(i)}_{\mu} (\ell)_{rM} + \tilde{\epsilon}^{(a)}_{\mu} (\ell)_{rM},$$

where $\tilde{\epsilon}^{(i)}_{\mu}$ and $\tilde{\epsilon}^{(a)}_{\mu}$ are the linking constants whose values depend on the process considered. For the electromagnetic interactions $M_\tau = 0$, $\tilde{\epsilon}^{(i)}_{\mu} = 1$, and $\tilde{\epsilon}^{(a)}_{\mu} = 0$ ($\tau = 0, 1$) and the hadronic current is given by

$$J_{\mu}^r = (J_{\mu})_{00} + (J_{\mu})_{10}.$$

In the case of processes with weak neutral currents, $M_\tau = 0$ ($\tau = 0, 1$) so the hadronic current is given by

$$J_{\mu}^r = \tilde{\epsilon}^{(0)}_{\mu} (J_{\mu})_{00} + \tilde{\epsilon}^{(0)}_{\mu} (J_{\mu})_{10} + \tilde{\epsilon}^{(1)}_{\mu} (J_{\mu})_{00} + \tilde{\epsilon}^{(1)}_{\mu} (J_{\mu})_{10}.$$

In the Weinberg-Salam model, for scattering processes according to [19] we have

$$\tilde{\epsilon}^{(0)}_{\mu} = -2\sin^2\theta_W \quad \tilde{\epsilon}^{(0)}_{\mu} = 0,$n $$\tilde{\epsilon}^{(1)}_{\mu} = 1 - 2\sin^2\theta_W \quad \tilde{\epsilon}^{(1)}_{\mu} = 1,$$

where $\theta_W$ is the Weinberg angle.

The square of the matrix element (7) for electron electroweak scattering by nuclei takes the following form:

$$|M_{nl}|^2 = C_e^2 \left( \epsilon^{*}_{\mu} \epsilon^{\nu}_{\mu} (J_{\mu}^{*} J_{\mu}^{*}) + 2\rho \text{Re} \left( \epsilon^{*}_{\mu} \epsilon^{\nu}_{\mu} (J_{\mu}^{*} J_{\mu}^{*}) \right) \right),$$

where $\rho = C_W/C_e$.

In (13) we neglect the term proportional to $\rho^2$. In the ultrarelativistic longitudinally polarized electrons case we obtain the relation

$$\epsilon^{*}_{\mu} \epsilon^{\nu}_{\mu} = (a_\nu - a_A s_\epsilon) \epsilon^{*}_{\mu} \epsilon^{\nu}_{\mu},$$

where the tensor $\epsilon^{*}_{\mu} \epsilon^{\nu}_{\mu}$ is given by

$$\epsilon^{*}_{\mu} \epsilon^{\nu}_{\mu} = \frac{\delta}{2E} \left\{ \delta_{\nu} (pp') - p_\nu p_\nu - (p' - p)^2 - s_\epsilon \epsilon_{\nu} p_\rho b_{\rho} \right\}.$$

Here $E$ and $p$ ($E'$ and $p'$) are the energy and the 4-momentum of the initial (final) electron; $s_\epsilon = \pm 1$ is the helicity of the initial electron. The factor $\delta$ takes the following values:

$$\delta = \begin{cases} -1 & \text{for } \mu = 1, 2, 3, \\ +1 & \text{for } \mu = 4. \end{cases}$$

The differential cross section, calculated by multipole decomposition of matrix elements [19, 20] and by taking into
account the circular polarization of the emitted photon, is given by the following formula:

\[
\frac{da}{dΩ\nu dΩ_γ} = \frac{υ_n \rightarrow f}{υ_n \rightarrow f} \left( \cos θ_γ \right) R^0_L \times \left( 1 + \frac{1}{R^0(q)_{ni}} \right)
\]

\[
\times \sum_{L=2}^{4} \frac{s_γ}{R^0(q)_{ni}} \sum_{L=1}^{3} \sum_{L=1}^{3} f_{L}^{(n-f+γ)} \left( P_L \left( \cos θ_γ \right) R^0_L \right)
\]

\[
+ \frac{s_γ}{R^0(q)_{ni}} \sum_{L=1}^{3} \sum_{L=1}^{3} f_{L}^{(n-f+γ)} \left( P_L \left( \cos θ_γ \right) R^0_L \right)
\]

\[
\times \left\{ \begin{array}{cl}
P_L \left( \cos θ_γ \right) \cos φ_α \frac{R^0_L}{R^0_L} \\
+ P_L \left( \cos θ_γ \right) \cos φ_α \frac{R^0_L}{R^0_L}
\end{array} \right\}
\]

\[
+ \frac{s_γ}{R^0(q)_{ni}} \sum_{L=1}^{3} \sum_{L=1}^{3} f_{L}^{(n-f+γ)} \left( P_L \left( \cos θ_γ \right) \cos φ_α \frac{R^0_L}{R^0_L} \right)
\]

\[
\times \left\{ \begin{array}{cl}
P_L \left( \cos θ_γ \right) \cos φ_α \frac{R^0_L}{R^0_L} \\
+ P_L \left( \cos θ_γ \right) \cos φ_α \frac{R^0_L}{R^0_L}
\end{array} \right\}
\]

\[
\times \left\{ \begin{array}{cl}
P_L \left( \cos θ_γ \right) \cos φ_α \frac{R^0_L}{R^0_L} \\
+ P_L \left( \cos θ_γ \right) \cos φ_α \frac{R^0_L}{R^0_L}
\end{array} \right\}
\]

\[
\text{for } L \text{ even}
\]

\[
\text{for } L \text{ odd}
\]

where $1_{n \rightarrow f}^{\text{rel}}$ and $1_{\text{total} \rightarrow f}$ indicate, respectively, the relative and total disintegration widths of the excited nucleus state and $P_L^{\nu_0}(\cos θ_γ)$ is Legendre polynomial. The angles $θ_γ$ and $φ_α$ determine the photon direction, $s_γ$ is photon helicity, and $σ_{ee'}$ is the differential cross section of the electron scattering by unpolarized nuclei given by

\[
σ_{ee'} = 4πσ_M R^0(q)_{ni} f_{rec}^{-1}
\]

where $σ_M$ and $f_{rec}$ are, respectively, the Mott cross section and the nuclear correction factor given by

\[
σ_M = \left( \frac{Zα \cos(θ/2)}{2E \sin^2(θ/2)} \right)^2, \quad f_{rec} = 1 + \frac{2E}{M} \sin^2 \frac{θ}{2}
\]

where $M$ is the mass of the nucleus and $Z$ is the atomic number.

The functions $R^0_L$ and $R^0_L$ are given by the following formulae:

\[
R^0_L = X_1 \left( v_i W^{i' L}_n + v_i W^{i' L}_o \right) + X_2 v_i W^{i' L}_n,
\]

\[
R^1_L = X_2 v_i W^{i' L}_n + X_3 v_i W^{i' L}_o + X_4 v_i W^{i' L}_n,
\]

\[
R^2_L = X_4 v_i W^{i' L}_n + X_2 v_i W^{i' L}_o + X_3 v_i W^{i' L}_n,
\]

\[
R^3_L = X_4 v_i W^{i' L}_n + X_2 v_i W^{i' L}_o + X_3 v_i W^{i' L}_n,
\]

\[
X_1 = 1 + 2ρ β_{ℓ' L} (a_ℓ - a_s s_e),
\]

\[
X_2 = 2ρ β_{ℓ' L} (a_ℓ - a_s s_e),
\]

\[
X_3 = 2ρ β_{L' L} (a_ℓ - a_s s_e),
\]

\[
X_4 = 1 + \rho \left( β_{ℓ' L} + β_{L' L} \right) (a_ℓ - a_s s_e),
\]

Here $v_i (i = ℓ, t, . . . , a, c')$ are the leptonic functions and $W^{i' L}_n$ are the hadronic functions. The expressions of the hadronic and the leptonic functions by taking into account the longitudinal polarization of electrons are given in the Appendix. The quantum number $L$ is defined by the relation $0 ≤ L ≤ 2J_n$. The functions $f^{(n-f+γ)}_L$ are defined as follows:

\[
f^{(n-f+γ)}_L = \left\{ \begin{array}{cl}
W^i_L (q_γ) & \text{for } L \text{ even} \\
W^j_L (q_γ) & \text{for } L \text{ odd}
\end{array} \right\}
\]

where

\[
W^i_L (q_γ) = (-1)^{j' + J_o} \times \sum_{j', J} \sqrt{(2L + 1) (2J_n + 1) (2J_n' + 1)}
\]

\[
\times \left( \begin{array}{c}
J' L \\
1 - 1 0
\end{array} \right) \left( \begin{array}{c}
J'_n J_n L \\
I_n I_n I_f
\end{array} \right)
\]

\[
\times \left\{ P_{J' + J}_n \left( F_{E|I'|n} F_{E|I|n} + F_{M|I'|n} F_{M|I|n} \right) \right. \\
\left. + P_{j' + J}_n \left( F_{E|J'|n} F_{E|J|n} - F_{M|J'|n} F_{M|J|n} \right) \right\}.
\]

Hence $q_γ = E_γ$ is the gamma transition energy.
Quantum numbers $J_1$ and $J'_1$ are given by

$$|J_n - J_f| \leq J_1 \leq J_n + J_f, \quad |J_n - J_f| \leq J'_1 \leq J_n + J_f,$$

where

$$W_L^J (q_y) = W_L^J (q_y) \text{ for } L \text{ odd,}$$

$$P_{i_1}^{i_1} = \frac{1}{2} (-1)^{(1/2)(i_1-J_1)} \left( 1 + (-1)^{i_1} y_i \right),$$

$$P_{i_1}^{i_1} = \frac{1}{2} (-1)^{(1/2)(i_1-J_1)} \left( 1 - (-1)^{i_1} y_i \right).$$

(23)

3. Study of the Transitions $J_i = 0 \rightarrow J_n = 1 \rightarrow J_f = 0$

As an example of transitions let us consider the following process:

$$e^- +^{12}C(0^+ 0) \rightarrow e^- +^{12}C^* (1^+ 1) \rightarrow^{12}C(0^+ 0) + \gamma_{RL}$$

(24)

So the quantum numbers $L, J,$ and $J'$ are defined as

$$0 \leq L \leq 2J_n \Rightarrow 0 \leq L \leq 2;$$

$$|J_i - J_n| \leq J \leq J_i + J_n \Rightarrow J = 1;$$

$$|J_i - J_n| \leq J' \leq J_i + J_n \Rightarrow J' = 1.$$

(25)

The differential cross section of the process (24) which takes into account the longitudinal polarization of the electron and the circular polarization of the photon is given by the formula

$$\frac{d\sigma}{d\Omega_e d\Omega_\gamma} = \Sigma_0 \left[ R_0^0 + \sqrt{2 \psi} F_{M1} (q_y) \sigma_{ee'} + f_1^{(n+f+\gamma)} + \sqrt{2 \psi} F_{C1} (q_y) \sigma_{ee'} + f_1^{(n+f+\gamma)} + \sqrt{2 \psi} F_{L1} (q_y) \sigma_{ee'} + f_1^{(n+f+\gamma)} + \sqrt{2 \psi} F_{E1} (q_y) \sigma_{ee'} + f_1^{(n+f+\gamma)} \right],$$

(26)

where

$$\Sigma_0 = \frac{1}{\sqrt{5}} F_{M1}^2 (q_y) \sigma_{ee'}, \quad f_1^{(n+f+\gamma)} = \frac{-\sqrt{3}}{\sqrt{2}}$$

$$f_2^{(n+f+\gamma)} = \frac{1}{\sqrt{2}}$$

$$R_0^0 (q_y) = \frac{1}{3 \sqrt{3}} F_{M1}$$

$$\times \left[ v_1 F_{M1} (1+2\rho \beta_V (a_v - a_s e)) + 2 v_1 F_{E1} (a_v - a_s e) \right],$$

$$R_1^1 (q_y) = \frac{1}{3 \sqrt{3}} v_{ac} F_{M1} \rho A (a_v - a_s e) \left( F_{C1}^5 - \frac{q_0}{q} F_{L1}^5 \right),$$

$$R_2^2 (q_y) = \frac{1}{3 \sqrt{2}} F_{M1}$$

$$\times \left[ v_1 F_{M1} (1+2\rho \beta_V (a_v - a_s e)) + 2 v_1 F_{E1} \rho A (a_v - a_s e) \right],$$

$$\tilde{R}_1^1 (q_y) = -\frac{1}{3 \sqrt{3}} v_{ac} F_{M1} \rho A (a_v - a_s e) \left( F_{C1}^5 - \frac{q_0}{q} F_{L1}^5 \right).$$

(27)

The matrix elements $F_{M1}, F_{C1}, F_{L1}$, and $F_{E1}$, calculated in the shell model with a harmonic oscillator potential, are given by [21]:

$$F_{M1} = \frac{\psi}{6 \sqrt{2 \pi} M} e^{-\gamma} (F_1 - \mu (2 - y));$$

$$F_{C1} = -\frac{\psi}{3 \sqrt{2 \pi} M} e^{-\gamma} \left( \frac{3}{2} F_A - (1 - y) \left( q_0 F_P + 2MF_\gamma \right) \right);$$

$$F_{L1} = -\frac{\sqrt{2} \psi}{3 \sqrt{2 \pi} M} e^{-\gamma} (1 - y) \left( F_A - \frac{q^2}{2M} F_P \right);$$

$$F_{E1} = -\frac{\psi}{3 \sqrt{2 \pi} M} e^{-\gamma} F_A (2 - y),$$

(28)

where $\psi = -0.003$ [19], $\mu = F_1 + 2M_F F_2$, $q_0 = -\Delta E$ is the energy of the transition, and $y = (bq/2)^2$, where $b$ is the parameter of the harmonic oscillator.

Writing explicitly the electron and photon polarization, the differential cross section (26) takes the following form:

$$\frac{d\sigma}{d\Omega_e d\Omega_\gamma} = \frac{1}{3 \sqrt{3}} F_{M1} \Sigma_0 \left( \Sigma + s_s \Delta + s_s \Delta' + s_s s_s \Delta'' \right).$$

(29)
where

$$
\Sigma = \left( 1 + \frac{1}{2} P_1 (\cos \theta) \right) \times \left( \nu_{F_{1M}} \left( 1 + 2 \rho \beta_{1V} a_V - 2 \nu_{F_{1E}} \rho \beta_{1A} a_A \right) 
- \frac{1}{\sqrt{2}} P_2 \left( \cos \phi \right) \cos \theta \right) \times \left( F_{C1}^5 - \frac{q_0 F_{L1}^5}{q} \right) 
- \frac{1}{2} P_2 \left( \cos \theta \right) \cos 2 \phi \nu_{F_{1M}} \left( 1 + 2 \rho \beta_{1V} a_V \right),
$$

$$
\Delta = \rho \left\{ \left( 2 + P_2 \left( \cos \theta \right) \right) \left( \nu_{F_{1E}} \beta_{1V} a_V - \nu_{F_{1M}} \beta_{1V} a_A \right) 
+ \frac{1}{\sqrt{2}} P_2 \left( \cos \theta \right) \cos \phi \right\} \times \left( F_{C1}^5 - \frac{q_0 F_{L1}^5}{q} \right) 
+ \frac{1}{2} P_2 \left( \cos \theta \right) \cos 2 \phi \nu_{F_{1M}} \beta_{1V} a_A \right\},
$$

$$
\Delta' = \rho \left\{ \left( 2 + P_2 \left( \cos \theta \right) \right) \left( \nu_{F_{1E}} \beta_{1V} a_V - \nu_{F_{1M}} \beta_{1V} a_A \right) 
+ \sqrt{3} P_1 \left( \cos \theta \right) \cos \phi \right\} \times \nu_{ac} \beta_{1A} a_V \left( F_{C1}^5 - \frac{q_0 F_{L1}^5}{q} \right),
$$

$$
\Delta'' = \rho \left\{ \left( 2 + P_2 \left( \cos \theta \right) \right) \left( \nu_{F_{1E}} \beta_{1V} a_V - \nu_{F_{1M}} \beta_{1V} a_A \right) 
- \sqrt{3} P_1 \left( \cos \theta \right) \cos \phi \nu_{ac} \beta_{1A} a_V \left( F_{C1}^5 - \frac{q_0 F_{L1}^5}{q} \right) \right\},
$$

Let us consider now two coefficients of asymmetry which can be experimentally determined, that is, the electron polarization ratio and the photon circular polarization degree:

$$
A_{R.L} = \frac{\frac{d\sigma}{d\sigma} (s_\gamma = +1) - \frac{d\sigma}{d\sigma} (s_\gamma = -1)}{\frac{d\sigma}{d\sigma} (s_\gamma = +1) + \frac{d\sigma}{d\sigma} (s_\gamma = -1)}.
$$

$$
B_{R.L} = \frac{\frac{d\sigma}{d\sigma} (s_\gamma = +1) - \frac{d\sigma}{d\sigma} (s_\gamma = -1)}{\frac{d\sigma}{d\sigma} (s_\gamma = +1) + \frac{d\sigma}{d\sigma} (s_\gamma = -1)}. \tag{32}
$$

Assuming that the photon is emitted in the direction of the incident electron we have

$$
\cos \theta = \frac{E' \cos \theta - E}{q}, \quad \varphi = 0, \quad \sin \theta = \frac{E' \sin \theta}{q}, \tag{33}
$$

and after averaging on the photon polarization, the asymmetry coefficient for the electron defined by (31) is given by

$$
A_{R.L} = \frac{\Delta}{\Sigma}. \tag{34}
$$

Averaging the differential cross section (29) on the electron polarization, the photon polarization degree defined by (32) takes the form

$$
B_{R.L} = \frac{\Delta'}{\Sigma}. \tag{35}
$$

Figures 1(a) and 1(b) show the dependence on the diffusion angle $\theta$ of the asymmetry coefficient for electron and photon.
polarization degrees for three values of the energy for incident electron. Calculation is done by using $\sin^2 \theta_W = 0.23125$ from [8] and formulas (12) relative to the coupling constants of standard model.

Figure 1(a) shows for $E = 300$ MeV and $E = 500$ MeV variation of asymmetry coefficient. This variation is greater for electron scattering around value of angle $\theta = 45^\circ$. However, the photon degree of polarization undergoes greater variation for large values of scattering angle.

Figures 2(a)–2(f) represent the angular dependence of asymmetry coefficient for electron with energy values $E = 300$ and $E = 500$ MeV and for different values of parameters for electron scattering around value of angle $\theta = 45^\circ$. However, the photon degree of polarization undergoes greater variation for large values of scattering angle.
Electron asymmetry coefficient is very sensitive to changes of these parameters, but it could be sensitive to the change of $\sin^2 \theta_W^\pm$ which depend on these parameters in the standard model.

4. Conclusion
The differential cross section of the inelastic electron scattering by nuclei is calculated and the expressions of the asymmetries coefficients are obtained; their angular-energy dependence is analyzed and some results are carried out. The experimental study of the electron scattering processes accompanied by gamma radiation can play an important role in research on the parameters of the weak neutral currents.

Appendix
By taking into account the longitudinal polarization of electrons we obtain for the leptonic functions the following expressions:

$$v_e = \left( \frac{q_\mu}{q^2} \right)^2, \quad \nu_t = \frac{1}{2} \frac{q_\mu^2}{q^2} + \tan^2 \frac{\theta}{2}, \quad \nu_{\tilde{t}} = -\frac{1}{2} \frac{q_\mu^2}{q^2} \tan \frac{\theta}{2},$$

$$v_{\ell e} = \frac{1}{\sqrt{2}} \frac{q_\mu^2}{q^2} \left( \frac{q_\mu^5}{q^2} + \tan^2 \frac{\theta}{2} \right)^{1/2}, \quad \nu_{\tilde{\ell} e} = \frac{1}{\sqrt{2}} \frac{s_e q_\mu^5}{q^2} \tan \frac{\theta}{2},$$

$$v_{\ell e} = \frac{q_\mu^2}{q^2}, \quad \nu_{\tilde{\ell} e} = -s_e \frac{1}{\sqrt{2}} \left( \frac{q_\mu^5}{q^2} + \tan^2 \frac{\theta}{2} \right)^{1/2} \tan \frac{\theta}{2} = s_e \tilde{\nu}_{\ell e},$$

$$v_{\ell e} = s_e \tan \frac{\theta}{2} = s_e \tilde{\nu}_{\ell e}, \quad v_{\ell e} = \frac{1}{2} \frac{q_\mu^2}{q^2} \left( \frac{q_\mu^5}{q^2} + \tan^2 \frac{\theta}{2} \right)^{1/2}.$$

(A.1)

The hadronic functions are given by the following formulae:

$$W_L^e (q) = \sum_{j,l} A_{0j}^{(L)} P_{j+l}^e F_C j F_C j,$$

$$W_L^e (q) = -\sum_{j,l} A_{1j}^{(L)} \left\{ P_{j+l}^e (F_{Ej} F_{El} + F_{Mj} F_{Ml}) \right\},$$

$$W_L^d (q) = -\sum_{j,l} A_{1j}^{(L)} \left\{ P_{j+l}^e (F_{Ej} F_{El} - F_{Mj} F_{Ml}) \right\},$$

$$W_L^t (q) = 2 \sqrt{2} \sum_{j,l} A_{0j}^{(L)} F_{Cj} \left\{ P_{j+l}^e F_{Mj} + P_{j+l}^e F_{Ml} \right\},$$

$$W_L^{ae} (q) = -\sum_{j,l} A_{1j}^{(L)} \left\{ P_{j+l}^e (F_{Ej} F_{Mj} + F_{Mj} F_{El}) \right\},$$

$$W_L^{ae} (q) = -\sum_{j,l} A_{1j}^{(L)} \left\{ P_{j+l}^e (F_{Ej} F_{Mj} - F_{Mj} F_{El}) \right\},$$

$$W_L^{ae} (q) = 2 \sqrt{2} \sum_{j,l} A_{0j}^{(L)} F_{Cj} \left\{ P_{j+l}^e F_{Mj} - P_{j+l}^e F_{Ml} \right\},$$

$$W_L^{ae} (q) = -\sum_{j,l} A_{1j}^{(L)} \left\{ P_{j+l}^e (F_{Ej} F_{Mj} - F_{Mj} F_{El}) \right\},$$

$$W_L^{ae} (q) = -\sum_{j,l} A_{1j}^{(L)} \left\{ P_{j+l}^e (F_{Ej} F_{Mj} + F_{Mj} F_{El}) \right\}.$$  

(A.2)

The coefficients $A_{m,m}^{(L)}$ and $P_{j+l}^\pm$ are given by the following formulae:

$$A_{m,m}^{(L)} = (-1)^{J_1+J_2} \left[ J' \right] \left[ J \right] \left[ L \right] \left( \frac{L + M}{L - M} \right)^{1/2} \times \left( \begin{array}{c} J' \ J' \ J \ J \ \ 1 \ M \\ m' \ m \ m \ M \ \ J_n \ J_n \ J_l \ J_l \end{array} \right) ,$$

A.3

$$P_{j+l}^{(J)} = \frac{1}{2} (-1)^{(J' J_2 - J_1)} \left( 1 + (-1)^J \right),$$

$$P_{j+l}^{(J)} = \frac{1}{2} (-1)^{(J' J_2 - J_1)} \left( 1 - (-1)^J \right).$$

(A.3)
The functions $F_{CJ}, F_{LJ}, F_{MJ}$, and $F_{EJ}$ ($F_{CJ}^5, F_{LJ}^5, F_{MJ}^5$, and $F_{EJ}^5$) are the matrix elements of the multipole vector (axial-vector) Coulomb, longitudinal, transverse magnetic, and transverse electric operators defined as follows:

$$F_{CJ} = \langle J_f | \hat{M}_c^{\text{coul}} | I_i \rangle; \quad F_{LJ} = \langle J_f | \hat{L}_c | I_i \rangle;$$

$$F_{MJ} = \langle J_f | \hat{L}_m | I_i \rangle; \quad F_{EJ} = \langle J_f | \hat{L}_e | I_i \rangle;$$

$$F_{CJ}^5 = \langle J_f | \hat{M}_c^{5\text{coul}} | I_i \rangle; \quad F_{LJ}^5 = \langle J_f | \hat{L}_c^5 | I_i \rangle;$$

$$F_{MJ}^5 = \langle J_f | \hat{L}_m^5 | I_i \rangle; \quad F_{EJ}^5 = \langle J_f | \hat{L}_e^5 | I_i \rangle.$$

\[ \text{(A.4)} \]

**Conflict of Interests**

The authors declare that there is no conflict of interests regarding the publication of this paper.

**References**


