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Research Article

The Scattering and Bound States of the Schrödinger Particle in Generalized Asymmetric Manning-Rosen Type Potential

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We solve exactly one-dimensional Schrödinger equation for the generalized asymmetric Manning-Rosen (GAMAR) type potential containing the different types of physical potential that have many application fields in the nonrelativistic quantum mechanics and obtain the solutions in terms of the Gauss hypergeometric functions. Then we determine the solutions for scattering and bound states. By using these states we calculate the reflection and transmission coefficients for scattering states and achieve a correlation that gives the energy eigenvalues for the bound states. In addition to these, we show how the transmission and reflection coefficients depend on the parameters which describe shape of the GAMAR type potential and compare our results with the results obtained in earlier studies.

1. Introduction

One of the most important issues discussed in physics is to understand the structures of nucleus, atoms, molecules, and the material objects. Therefore it is important to create models, which contain the potential concept, describing the interactions between the two objects. Some of the potential models have been identified in order to describe the interactions in the nuclei and nuclei-particle and structures of the diatomic and polyatomic molecules. Some of these potentials are called as follows: the Kratzer, Morse, Eckart, Rosen-Morse, Manning-Rosen, Pöschl-Teller, Hulthen, Woods-Saxon, Scarf, Schiöberg, Deng-Fan, and Cusp potentials [1–13]. The other important and current topic is to obtain the exact solutions, which describe the scattering and bound states, of the Schrödinger equation that is known as the fundamental equation of nonrelativistic quantum mechanics in the existence of an external potential [14–40]. In order to get complete information about a quantum mechanical system under consideration, one needs to study the scattering and bound states. In [17], the authors have presented the exact solutions which describe s-wave scattering states for the Schrödinger equation with the Manning-Rosen potential via the standard method. Ikhdair and Sever have suggested a new approximated scheme to centrifugal term to achieve the $l \neq 0$ solutions of the Schrödinger equation for the Manning-Rosen potential by using the Nikiforov-Uvarov method and they have obtained the corresponding normalized wave functions in terms of the Jacobi polynomial. They have also calculated the bound state energies of various states for HCl, LiH, and CO diatomic molecules [26]. In the presence of the Hulthen potential, approximate analytical solutions of the radial Schrödinger equation with $l \neq 0$ have been presented by using the Exact Quantization Rule in [27]. Arda et al. have solved the one-dimensional Schrödinger equation for the asymmetric Hulthen potential [28]. In this study they have obtained the scattering and bound states solutions in terms of the hypergeometric functions. In [30], the writers have acquired the energy eigenvalues of the bound states and the corresponding eigenfunctions of the generalized Woods-Saxon potential. Arda et al. have achieved the scattering solutions of the one-dimensional Schrödinger equation with the position-dependent mass in the existence of the Woods-Saxon potential [32]. For the modified Pöschl-Teller potential, the approximation solutions of the Schrödinger equation in one dimension have been obtained with an approximation of the centrifugal term by Agboola [33]. In this work the author has obtained some expectation values using the

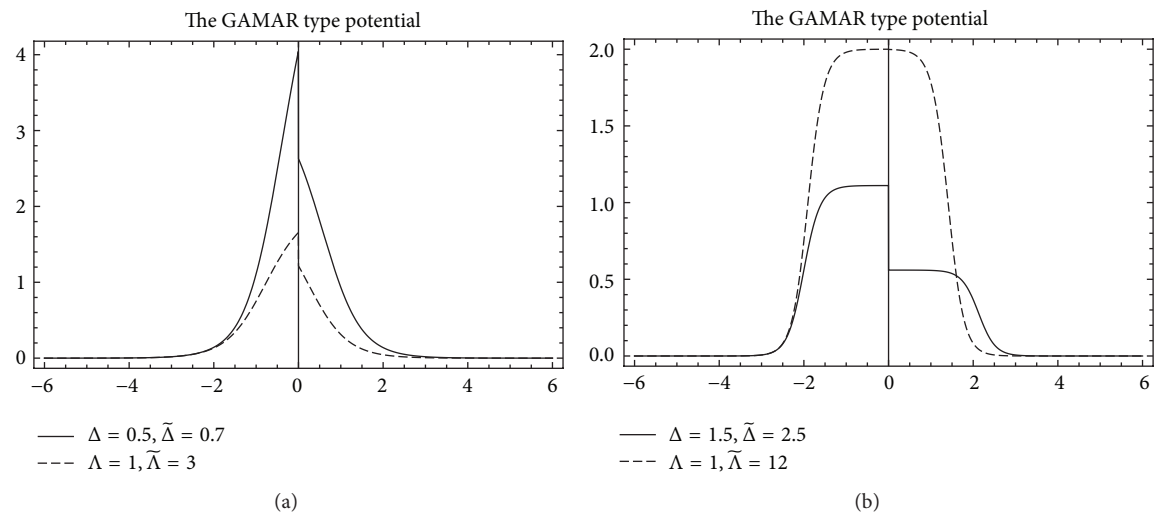

Figure 1: The shapes of the GAMAR type potential for different values of the potential parameters. (a) shows $\alpha = \beta = 2$, $x_0 = \tilde{x}_0 = 1$, and $A = B = C = D = 1$ (lower values of the $\alpha x_0, \beta\tilde{x}_0$). (b) shows $\alpha = \beta = 5$, $x_0 = \tilde{x}_0 = 2$, and $A = B = C = D = 1$ (large values of the $\alpha x_0, \beta\tilde{x}_0$).

Hellmann-Feynman method. Qiang et al. have carried out the approximately scattering states solutions of the l-wave Schrödinger equation for the second Pöschl-Teller-like potential by taking a new approximation scheme to the centrifugal term [34]. Tezcan and Sever have obtained the exact solution of the Schrödinger equation for the Rosen-Morse and Scarf potentials with position-dependent mass by using the general point of the canonical transformation [36]. Analytical solutions of the Schrödinger equation with the Makarow and ring-shaped Hartmann potentials for any n and l (states) quantum numbers have been presented by using the asymptotic iteration method in [40].

The Manning-Rosen potential was first proposed to define the vibrational behavior of diatomic molecules by Manning and Rosen in 1933 [5]. Afterwards, it has been used to describe the interactions between two atoms in a diatomic molecule and also it is very reasonable in describing such interactions close to the surface [41–43]. Some of the potentials can be generalized to describe the interactions consisting of more than one process. Therefore, in our study, we have defined the generalized asymmetric Manning-Rosen (GAMAR) type potential which is the similar type of the Manning-Rosen potential, in the following form [44]:

$$V(x) = \theta(-x)\left[\frac{Ae^{2\alpha(x+x_0)}}{\left(\Lambda + \Delta e^{\alpha(x+x_0)}\right)^2} + \frac{Be^{\alpha(x+x_0)}}{\Lambda + \Delta e^{\alpha(x+x_0)}}\right] + \theta(x)\left[\frac{Ce^{-2\beta(x-\tilde{x}_0)}}{\left(\tilde{\Lambda} + \tilde{\Delta} e^{-\beta(x-\tilde{x}_0)}\right)^2} + \frac{De^{-\beta(x-\tilde{x}_0)}}{\tilde{\Lambda} + \tilde{\Delta} e^{-\beta(x-\tilde{x}_0)}}\right], \quad (1)$$

where $\theta(x)$ is the Heaviside step function. All of the parameters in the potential are real. The shape of the GAMAR type potential varies according to the values of the parameters. If A, B, C, and D are positive, it becomes a potential barrier. When A, B, C, and D are negative values, it takes into a potential well. The GAMAR type potential form is displayed in Figure 1. In special cases, it reduces to potentials such as the Manning-Rosen, generalized Wood-Saxon (GAWS) [45], Woods-Saxon, asymmetric Hulthen (ASH) [46], Hulthen, asymmetric Cusp (ASC) [47], and Cusp, potentials that have many applications in the relativistic and nonrelativistic quantum mechanics. The special cases of the GAMAR type potential are displayed in Table 1.

The content of this study is arranged as follows: in Section 2, the Schrödinger equation with the GAMAR type potential barrier is solved, the solutions are written in terms of the hypergeometric functions, and the asymptotic behaviors of the solutions are obtained. In the same section, by using the continuity conditions of the wave function and its derivative, the transmission and reflection coefficients are calculated. In Section 3, we get a condition that gives the energy eigenvalues for a Schrödinger particle in the GAMAR type potential well. Finally, we discuss the results in Section 4.

2. Scattering States and Finding Coefficients of Reflection and Transmission

The stationary Schrödinger equation in one dimension for a particle with mass m and energy E moving in a external potential is written as the following form (in natural units $\hbar = 1$):

$$\left\{\frac{d^2}{dx^2} + 2m\left[E - V(x)\right]\right\}\psi(x) = 0. \quad (2)$$

To determine scattering states occurring as a result of interaction particles with the GAMAR type barrier potential,

Table 1: The special potentials which are derived from the GAMAR type potential.

The potentials	Varying parameters	The shape of the potential
The Manning-Rosen	$A = B = 0, 0 < x < \infty$ $\Lambda = \widetilde{\Lambda} = 1, x_0 = \widetilde{x}_0 = 0$ $\Delta \to -1, \widetilde{\Delta} \to -1, \alpha = \beta$	$V(x) = \frac{C}{(e^{\beta x} - 1)^2} - \frac{D}{e^{\beta x} - 1}$
The GAWS	$A = C = 0, B = D = V_0$	$V(x) = V_0 \left[\frac{\theta(-x)}{\Delta + \Lambda e^{-\alpha(x+x_0)}} - \frac{\theta(x)}{\widetilde{\Delta} + \widetilde{\Lambda} e^{\beta(x-\widetilde{x}_0)}} \right]$
The Woods-Saxon	$A = C = 0, B = D = V_0$ $\Delta = \widetilde{\Delta} = 1, \Lambda = \widetilde{\Lambda} = 1$ $\alpha = \beta, x_0 = \widetilde{x}_0$	$V(x) = V_0 \left[\frac{\theta(-x)}{1 + e^{-\alpha(x+x_0)}} - \frac{\theta(x)}{1 + e^{\alpha(x-x_0)}} \right]$
The ASH	$A = C = 0, B = D = V_0$ $\Delta \to -\Delta, \Lambda = \widetilde{\Lambda} = 1$ $\widetilde{\Delta} \to -\widetilde{\Delta}, x_0 = \widetilde{x}_0 = 0$	$V(x) = V_0 \left[\frac{\theta(-x)}{e^{-\alpha x} - \Delta} - \frac{\theta(x)}{e^{\beta x} - \widetilde{\Delta}} \right]$
The Hulthen	$A = C = 0, B = D = V_0, \alpha = \beta$ $\Delta \to -\Delta, \Lambda = \widetilde{\Lambda} = 1$ $\widetilde{\Delta} \to -\widetilde{\Delta}, x_0 = \widetilde{x}_0 = 0$	$V(x) = V_0 \left[\frac{\theta(-x)}{e^{-\alpha x} - \Delta} - \frac{\theta(x)}{e^{\alpha x} - \widetilde{\Delta}} \right]$
The ASC	$A = C = 0, B = D = V_0$ $\Delta \to 0, \widetilde{\Delta} \to 0$ $x_0 = \widetilde{x}_0 = 0, \Lambda = \widetilde{\Lambda} = 1$	$V(x) = V_0 \left[\frac{\theta(-x)}{e^{-\alpha x}} - \frac{\theta(x)}{e^{\beta x}} \right]$
The Cusp	$A = C = 0, B = D = V_0$ $\Delta \to 0, \widetilde{\Delta} \to 0, x_0 = \widetilde{x}_0 = 0$ $\Lambda = \widetilde{\Lambda} = 1, \alpha = \beta$	$V(x) = V_0 \left[\frac{\theta(-x)}{e^{-\alpha x}} - \frac{\theta(x)}{e^{\alpha x}} \right]$

we need to solve the Schrödinger equation for both regions $x < 0$ and $x > 0$. Putting (1) in (2) for the left region, we get

$$\left\{ \frac{d^2}{dx^2} + 2m \left[E - \frac{A e^{2\alpha(x+x_0)}}{(\Lambda + \Delta e^{\alpha(x+x_0)})^2} - \frac{B e^{\alpha(x+x_0)}}{\Lambda + \Delta e^{\alpha(x+x_0)}} \right] \right\} \psi_L(x) = 0. \tag{3}$$

Introducing a new variable $\chi = -(\Delta/\Lambda) e^{\alpha(x+x_0)}$ in (3), one acquires the following equation:

$$\chi(1-\chi) \frac{d^2 \psi_L(\chi)}{d\chi^2} + (1-\chi) \frac{d\psi_L(\chi)}{d\chi} + \frac{1}{\chi(1-\chi)} \left(a_1 - a_2 \chi + a_3 \chi^2 \right) \psi_L(\chi) = 0, \tag{4}$$

where

$$a_1 = \frac{2mE}{\alpha^2}, \qquad a_2 = \frac{2mE}{\alpha^2} \left(2E - \frac{B}{\Delta} \right), \qquad a_3 = \frac{2mE}{\alpha^2} \left(2E - \frac{A}{\Delta^2} - \frac{B}{\Delta} \right), \tag{5}$$

and ψ_L is the solution for the left region. In order to get a solution of (4), we take the trial wave function as $\psi_L(\chi) = \chi^{\lambda_1}(1-\chi)^{\delta_1}$ and in that case (4) converts the Gaussian differential equation [48]:

$$\chi(1-\chi) \frac{d^2 f(\chi)}{d\chi^2} + \left[(1+\lambda_1) - (2\lambda_1 + 2\delta_1 + 1) \chi \right] \frac{df(\chi)}{d\chi} - (\lambda_1 + \delta_1 + \gamma_1)(\lambda_1 + \delta_1 - \gamma_1) f(\chi) = 0, \tag{6}$$

where

$$\lambda_1 = \frac{ik}{\alpha}, \quad k = \sqrt{2mE}, \qquad \delta_1 = \frac{1}{2} \left(1 + \sqrt{1 + \frac{8mA}{\alpha^2 \Delta^2}} \right), \qquad \gamma_1 = i \sqrt{\frac{2m}{\alpha^2} \left(E - \frac{A}{\Delta^2} - \frac{B}{\Delta} \right)}. \tag{7}$$

The general solution of (6) is given in terms of the Gauss hypergeometric functions as the following form:

$$f(\chi) = A_1 {}_2F_1(\lambda_1 + \delta_1 - \gamma_1, \lambda_1 + \delta_1 + \gamma_1, 1 + 2\lambda_1; \chi) + A_2 \chi^{-2\lambda_1} {}_2F_1(-\lambda_1 + \delta_1 - \gamma_1, -\lambda_1 + \delta_1 + \gamma_1, 1 - 2\lambda_1; \chi). \tag{8}$$

Therefore, we obtain the complete solution for the left-region ($x < 0$) as

$$\psi_L(\chi) = A_1\chi^{\lambda_1}(1-\chi)^{\delta_1}{}_2F_1(\lambda_1+\delta_1-\gamma_1, \lambda_1+\delta_1+\gamma_1, 1+2\lambda_1;\chi) + A_2\chi^{-\lambda_1}(1-\chi)^{\delta_1}{}_2F_1(-\lambda_1+\delta_1-\gamma_1, -\lambda_1+\delta_1+\gamma_1, 1-2\lambda_1;\chi). \tag{9}$$

Now we search the solution for the right-region ($x > 0$) of the GAMAR type potential. At that rate, (2) turns out to be

$$\left\{\frac{d^2}{dx^2} + 2m\left[E - \frac{Ce^{-2\beta(x-\tilde{x}_0)}}{\left(\tilde{\Lambda}+\tilde{\Delta}e^{-\beta(x-\tilde{x}_0)}\right)^2} - \frac{De^{-\beta(x-\tilde{x}_0)}}{\tilde{\Lambda}+\tilde{\Delta}e^{-\beta(x-\tilde{x}_0)}}\right]\right\} \times \psi_R(x), \tag{10}$$

where $\psi_R(x)$ is the right-region solution. If we use a new variable $\eta = -(\tilde{\Delta}/\tilde{\Lambda})e^{-\beta(x-\tilde{x}_0)}$ in (10) we achieve the following equation:

$$\eta(1-\eta)\frac{d^2\psi_R(\eta)}{d\eta^2} + (1-\eta)\frac{d\psi_R(\eta)}{d\eta} + \frac{1}{\eta(1-\eta)}\left(b_1 - b_2\eta + b_3\eta^2\right)\psi_R(\eta) = 0, \tag{11}$$

where

$$b_1 = \frac{2mE}{\beta^2}, \qquad b_2 = \frac{2mE}{\beta^2}\left(2E - \frac{D}{\tilde{\Delta}}\right), \qquad b_3 = \frac{2mE}{\beta^2}\left(2E - \frac{C}{\tilde{\Delta}^2} - \frac{D}{\tilde{\Delta}}\right). \tag{12}$$

Similarly to the solution that has been suggested for the left-region ($x < 0$) of the potential, taking the trial wave function $\psi_R(\eta) = \eta^{\lambda_2}(1-\eta)^{\delta_2}h(\eta)$, (11) transforms the Gaussian differential equation. In this way, after a little algebra, the general solution for the right-region is obtained as follows:

$$\psi_R(\eta) = A_3\eta^{\lambda_2}(1-\eta)^{\delta_2}{}_2F_1(\lambda_2+\delta_2-\gamma_2, \lambda_2+\delta_2+\gamma_2, 1+2\lambda_2;\eta) + A_4\eta^{-\lambda_2}(1-\eta)^{\delta_2}{}_2F_1(-\lambda_2+\delta_2-\gamma_2, -\lambda_2+\delta_2+\gamma_2, 1-2\lambda_2;\eta), \tag{13}$$

where

$$\lambda_2 = \frac{ik}{\beta}, \quad k = \sqrt{2mE},$$
$$\delta_2 = \frac{1}{2}\left(1 + \sqrt{1 + \frac{8mC}{\beta^2\tilde{\Delta}^2}}\right), \qquad \gamma_2 = i\sqrt{\frac{2m}{\beta^2}\left(E - \frac{A}{\tilde{\Delta}^2} - \frac{B}{\tilde{\Delta}}\right)}. \tag{14}$$

We have to acquire the asymptotic forms of the solutions taking part in (9) and (13) to calculate the reflection and transmission coefficients. To do it, we need to use the convenient boundary conditions $x \to -\infty$ and $x \to +\infty$. For the left-region ($x < 0$), as $x \to -\infty$, $\chi \to 0$, and $(1-\chi)^{\delta_1} \to 1$, we obtain from (9)

$$\psi_L(x) \sim A_1\left(-\frac{\Delta}{\Lambda}\right)^{i(k/\alpha)}e^{ik(x+x_0)} + A_2\left(-\frac{\Delta}{\Lambda}\right)^{-i(k/\alpha)}e^{-ik(x+x_0)}. \tag{15}$$

For the right-region ($x > 0$), as $x \to +\infty$, $\eta \to 0$ and $(1-\eta)^{\delta_2} \to 1$, we get from (13)

$$\psi_R(x) \sim A_4\left(-\frac{\tilde{\Delta}}{\tilde{\Lambda}}\right)^{i(k/\tilde{\beta})}e^{ik(x-\tilde{x}_0)}. \tag{16}$$

The one-dimensional current density for the Schrödinger equation is defined as follows:

$$j = \frac{1}{2mi}\left[\psi(x)\nabla_x\psi^*(x) - \psi^*(x)\nabla_x\psi(x)\right]. \tag{17}$$

Putting the asymptotic behaviors of the obtained solutions for the both regions that are given in (15) and (16) into (17), we achieve the reflection (R) and transmission (T) coefficients, respectively:

$$R = \frac{j_{\text{ref.}}}{j_{\text{inc.}}} = e^{4\pi k/\alpha}\left|\frac{A_2}{A_1}\right|^2, \qquad T = \frac{j_{\text{trans.}}}{j_{\text{inc.}}} = e^{2\pi k(1/\alpha+1/\beta)}\left|\frac{A_4}{A_1}\right|^2, \tag{18}$$

where $j_{\text{ref.}}$, $j_{\text{inc.}}$, and $j_{\text{trans.}}$ are called reflection, incident, and transmission currents, respectively. To obtain these coefficients clearly, we should use the continuity conditions of the wave function and its derivative given as

$$\psi_L(x=0) = \psi_R(x=0), \qquad \left(\frac{d\psi_L}{dx}\right)_{x=0} = \left(\frac{d\psi_R}{dx}\right)_{x=0}. \tag{19}$$

By using the above equations, after cumbersome algebra, we come to the following results:

$$\begin{aligned}\frac{A_2}{A_1} &= \left(\left[(K_{10}+K_{11})H_3 + K_{12}H_6\right]K_1H_1 - \left[(K_4+K_5)H_1 + K_6H_4\right]K_3H_3\right) \times \left(\left[(K_7+K_8)H_2 + K_9H_8\right]K_3H_3 - \left[(K_{10}+K_{11})H_3 + K_{12}H_6\right]K_2H_2\right)^{-1},\\ \frac{A_4}{A_1} &= \left(\left[(K_7+K_8)H_2 + K_9H_5\right]K_1H_1 - \left[(K_4+K_5)H_1 + K_6H_4\right]K_2H_2\right) \times \left(\left[(K_7+K_8)H_2 + K_9H_8\right]K_3H_3 - \left[(K_{10}+K_{11})H_3 + K_{12}H_6\right]K_2H_2\right),\end{aligned} \tag{20}$$

Table 2: The abbreviations defined in the calculations for the scattering states.

$$\sigma = -\frac{\Delta}{\Lambda}, \tilde{\sigma} = -\frac{\tilde{\Delta}}{\tilde{\Lambda}}$$
$$K_1 = \sigma^{\lambda_1} e^{\alpha\lambda_1 x_0}\left(1 - \sigma e^{\alpha x_0}\right)^{\delta_1}$$
$$K_2 = \sigma^{-\lambda_1} e^{-\alpha\lambda_1 x_0}\left(1 - \sigma e^{\alpha x_0}\right)^{\delta_1}$$
$$K_3 = \tilde{\sigma}^{\lambda_2} e^{-\beta\lambda_2 \tilde{x}_0}\left(1 - \tilde{\sigma} e^{\beta \tilde{x}_0}\right)^{\delta_2}$$
$$K_4 = \sigma^{\lambda_1}(\alpha\lambda_1) e^{\alpha\lambda_1 x_0}\left(1 - \sigma e^{\alpha x_0}\right)^{\delta_1}$$
$$K_5 = \sigma^{\lambda_1+1}(-\alpha\delta_1) e^{\alpha(\lambda_1+1)x_0}\left(1 - \sigma e^{\alpha x_0}\right)^{\delta_1-1}$$
$$K_6 = \sigma^{\lambda_1+1}(\alpha) e^{\alpha(\lambda_1+)x_0}\left[\frac{(\lambda_1+\delta_1-\gamma_1)(\lambda_1+\delta_1+\gamma_1)}{1+2\lambda_1}\right]\left(1 - \sigma e^{\alpha x_0}\right)^{\delta_1}$$
$$K_7 = \sigma^{-\lambda_1}(-\alpha\lambda_1) e^{-\alpha\lambda_1 x_0}\left(1 - \sigma e^{\alpha x_0}\right)^{\delta_1}$$
$$K_8 = \sigma^{-\lambda_1+1}(-\alpha\delta_1) e^{-\alpha(\lambda_1+1)x_0}\left(1 - \sigma e^{\alpha x_0}\right)^{\delta_1-1}$$
$$K_9 = \sigma^{-\lambda_1+1}(\alpha) e^{-\alpha(\lambda_1+1)x_0}\left[\frac{(-\lambda_1+\delta_1-\gamma_1)(-\lambda_1+\delta_1+\gamma_1)}{1-2\lambda_1}\right]\left(1 - \sigma e^{\alpha x_0}\right)^{\delta_1}$$
$$K_{10} = \tilde{\sigma}^{\lambda_2}(\beta\lambda_2) e^{-\beta\lambda_2 \tilde{x}_0}\left(1 - \tilde{\sigma} e^{\beta \tilde{x}_0}\right)^{\delta_2}$$
$$K_{11} = \tilde{\sigma}^{\lambda_2+1}(\beta\delta_2) e^{-\beta(\lambda_2-1)\tilde{x}_0}\left(1 - \tilde{\sigma} e^{\beta \tilde{x}_0}\right)^{\delta_2-1}$$
$$K_{12} = \tilde{\sigma}^{\lambda_2+1}(-\beta) e^{-\beta(\lambda_2-1)\tilde{x}_0}\left[\frac{(-\lambda_2+\delta_2-\gamma_2)(-\lambda_2+\delta_2+\gamma_2)}{1-2\lambda_2}\right]\left(1 - \tilde{\sigma} e^{\beta \tilde{x}_0}\right)^{\delta_2}$$
$$H_1 = {}_2F_1\left(\lambda_1+\delta_1-\gamma_1, \lambda_1+\delta_1+\gamma_1, 1+2\lambda_1; \sigma e^{\alpha x_0}\right)$$
$$H_2 = {}_2F_1\left(-\lambda_1+\delta_1-\gamma_1, -\lambda_1+\delta_1+\gamma_1, 1-2\lambda_1; \sigma e^{\alpha x_0}\right)$$
$$H_3 = {}_2F_1\left(-\lambda_2+\delta_2-\gamma_2, -\lambda_2+\delta_2+\gamma_2, 1-2\lambda_2; \tilde{\sigma} e^{\alpha \tilde{x}_0}\right)$$
$$H_4 = {}_2F_1\left(\lambda_1+\delta_1-\gamma_1+1, \lambda_1+\delta_1+\gamma_1+1, 2+2\lambda_1; \sigma e^{\alpha x_0}\right)$$
$$H_5 = {}_2F_1\left(-\lambda_1+\delta_1-\gamma_1+1, -\lambda_1+\delta_1+\gamma_1+1, 2-2\lambda_1; \sigma e^{\alpha x_0}\right)$$
$$H_6 = {}_2F_1\left(-\lambda_2+\delta_2-\gamma_2+1, -\lambda_2+\delta_2+\gamma_2+1, 2-2\lambda_2; \tilde{\sigma} e^{\alpha \tilde{x}_0}\right)$$

where the explicit forms of the abbreviations are given in Table 2.

3. Bound State Solutions and Condition for Energy Eigenvalues

The aim of this section is to obtain a relation for the energy eigenvalues. If the shape of the potential is a potential well, the bound states occurred. The GAMAR type potential converted the potential well if the parameters are selected such as any of the following options:

(i) $A \to -A$, $B \to -B$, $C \to -C$, and $D \to -D$,

(ii) $A > 0$, $|B| \geq A$, $B \to -B$, and $D \to -D$,

(iii) $B > 0$, $|A| \gg B$, $A \to -A$, and $C \to -C$,

(iv) $\frac{B}{\tilde{\Lambda}} > 0$, $\frac{B}{\tilde{\Lambda}} > A$, $\Delta \to -\Delta$, $\tilde{\Delta} \to -\tilde{\Delta}$, $\Lambda \to -\Lambda$, and $\tilde{\Lambda} \to -\tilde{\Lambda}$.

Considering the first option, (2) for $x < 0$ yields

$$\left\{\frac{d^2}{dx^2} + 2m\left[E + \frac{Ae^{2\alpha(x+x_0)}}{\left(\Lambda + \Delta e^{\alpha(x+x_0)}\right)^2} + \frac{Be^{\alpha(x+x_0)}}{\Lambda + \Delta e^{\alpha(x+x_0)}}\right]\right\} \times \psi_L(x) = 0. \tag{21}$$

Taking the new variable $\chi = -(\Delta/\Lambda)e^{\alpha(x+x_0)}$ and setting the trial wave function $\psi_L(\chi) = \chi^{\lambda_3}(1-\chi)^{\delta_3}$ in (21), it reduces to the Gaussian differential equation. Hence, the general solution for $x < 0$ becomes

$$\psi_L(\chi) = A_5\chi^{\lambda_3}(1-\chi)^{\delta_3}{}_2F_1\left(\lambda_3+\delta_3-\gamma_3, \lambda_3+\delta_3+\gamma_3, 1+2\lambda_3; \chi\right) + A_6\chi^{-\lambda_3}(1-\chi)^{\delta_3}{}_2F_1\left(-\lambda_3+\delta_3-\gamma_3, -\lambda_3+\delta_3+\gamma_3, 1-2\lambda_3; \chi\right), \tag{22}$$

where

$$\lambda_3 = \frac{k}{\alpha}, \quad k = \sqrt{2mE},$$
$$\delta_3 = \frac{1}{2}\left(1 + \sqrt{1 - \frac{8mA}{\alpha^2\Delta^2}}\right), \qquad \gamma_3 = i\sqrt{\frac{2m}{\alpha^2}\left(E + \frac{A}{\Delta^2} + \frac{B}{\Delta}\right)}. \tag{23}$$

For $x > 0$, (2) turns into

$$\left\{\frac{d^2}{dx^2} + 2m\left[E + \frac{Ce^{-2\beta(x-\tilde{x}_0)}}{\left(\tilde{\Lambda} + \tilde{\Delta}e^{-\beta(x-\tilde{x}_0)}\right)^2} + \frac{De^{-\beta(x-\tilde{x}_0)}}{\tilde{\Lambda} + \tilde{\Delta}e^{-\beta(x-\tilde{x}_0)}}\right]\right\} \times \psi_R(x). \tag{24}$$

Table 3: The abbreviations defined in the calculations for the bound states.

$\sigma = -\frac{\Delta}{\Lambda}, \tilde{\sigma} = -\frac{\tilde{\Delta}}{\tilde{\Lambda}}$
$K_{13} = \sigma^{\lambda_3} e^{\alpha\lambda_3 x_0}\left(1-\sigma e^{\alpha x_0}\right)^{\delta_3}$
$K_{14} = \tilde{\sigma}^{\lambda_4} e^{\beta\lambda_4 \tilde{x}_0}\left(1-\tilde{\sigma} e^{\beta \tilde{x}_0}\right)^{\delta_4}$
$K_{15} = \sigma^{\lambda_3}(\alpha\lambda_3) e^{\alpha\lambda_3 x_0}\left(1-\sigma e^{\alpha x_0}\right)^{\delta_3}$
$K_{16} = \sigma^{\lambda_3+1}(-\alpha\delta_3) e^{\alpha(\lambda_3+1)x_0}\left(1-\sigma e^{\alpha x_0}\right)^{\delta_3-1}$
$K_{17} = \sigma^{\lambda_3+1}(\alpha) e^{\alpha(\lambda_3+1)x_0}\left(1-\sigma e^{\alpha x_0}\right)^{\delta_3}\left[\frac{(\lambda_3+\delta_3-\gamma_3)(\lambda_3+\delta_3+\gamma_3)}{1+2\lambda_3}\right]$
$K_{18} = \tilde{\sigma}^{\lambda_4}(-\beta\lambda_4) e^{\beta\lambda_4 \tilde{x}_0}\left(1-\tilde{\sigma} e^{\beta \tilde{x}_0}\right)^{\delta_4}$
$K_{19} = \tilde{\sigma}^{\lambda_4+1}(-\beta\delta_4) e^{\beta(\lambda_4+1)\tilde{x}_0}\left(1-\tilde{\sigma} e^{\beta \tilde{x}_0}\right)^{\delta_4-1}$
$K_{20} = \tilde{\sigma}^{\lambda_4+1}(-\beta) e^{\beta(\lambda_4+1)\tilde{x}_0}\left(1-\tilde{\sigma} e^{\beta \tilde{x}_0}\right)^{\delta_4}\left[\frac{(\lambda_4+\delta_4-\gamma_4)(\lambda_4+\delta_4+\gamma_4)}{1+2\lambda_4}\right]$
$H_7 = {}_2F_1\left(\lambda_3+\delta_3-\gamma_3, \lambda_3+\delta_3+\gamma_3, 1+2\lambda_3; \sigma e^{\alpha x_0}\right)$
$H_8 = {}_2F_1\left(\lambda_4+\delta_4-\gamma_4, \lambda_4+\delta_4+\gamma_4, 1+2\lambda_4; \tilde{\sigma} e^{\alpha \tilde{x}_0}\right)$
$H_9 = {}_2F_1\left(\lambda_3+\delta_3-\gamma_3+1, \lambda_3+\delta_3+\gamma_3+1, 2+2\lambda_3; \sigma e^{\alpha x_0}\right)$
$H_{10} = {}_2F_1\left(\lambda_4+\delta_4-\gamma_4+1, \lambda_4+\delta_4+\gamma_4+1, 2+2\lambda_4; \tilde{\sigma} e^{\alpha \tilde{x}_0}\right)$

By using the transform $\eta = -(\tilde{\Delta}/\tilde{\Lambda})e^{-\beta(x-\tilde{x}_0)}$ and organizing the trial wave function $\psi_R(\eta) = \eta^{\lambda_4}(1-\eta)^{\delta_4}h(\eta)$ in (24), the general solution for the right-region of the potential ($x > 0$) is obtained in the following form:

$$\begin{aligned}\psi_R(\eta) = {} & A_7\eta^{\lambda_4}(1-\eta)^{\delta_4}{}_2F_1\left(\lambda_4+\delta_4-\gamma_4,\right.\\ & \left.\lambda_4+\delta_4+\gamma_4, 1+2\lambda_4; \eta\right)\\ & + A_8\eta^{-\lambda_4}(1-\eta)^{\delta_4}{}_2F_1\left(-\lambda_4+\delta_4-\gamma_4,\right.\\ & \left.-\lambda_4+\delta_4+\gamma_4, 1-2\lambda_4; \eta\right),\end{aligned} \tag{25}$$

where

$$\lambda_4 = \frac{k}{\beta}, \quad k = \sqrt{2mE},$$
$$\delta_4 = \frac{1}{2}\left(1+\sqrt{1-\frac{8mC}{\beta^2\tilde{\Delta}^2}}\right), \qquad \gamma_4 = i\sqrt{\frac{2m}{\beta^2}\left(E+\frac{A}{\tilde{\Delta}^2}+\frac{B}{\tilde{\Delta}}\right)}. \tag{26}$$

In order to obtain a regular wave function from (22) and (25), we should set $A_6 = A_8 = 0$ and we obtain

$$\begin{aligned}\psi_L(\chi) = {} & A_5\chi^{\lambda_3}(1-\chi)^{\delta_3}{}_2F_1\left(\lambda_3+\delta_3-\gamma_3,\right.\\ & \left.\lambda_3+\delta_3+\gamma_3, 1+2\lambda_3; \chi\right),\\ \psi_R(\eta) = {} & A_7\eta^{\lambda_4}(1-\eta)^{\delta_4}{}_2F_1\left(\lambda_4+\delta_4-\gamma_4,\right.\\ & \left.\lambda_4+\delta_4+\gamma_4, 1+2\lambda_4; \eta\right).\end{aligned} \tag{27}$$

Then, using boundary conditions which are given in (19), we get the energy eigenvalues equation for the bound states as follows:

$$\begin{aligned}&\left\{\left[\left(K_{18}+K_{19}\right)K_{13}-\left(K_{15}+K_{16}\right)K_{14}\right]H_8+K_{13}K_{20}H_{10}\right\}H_7\\ &\quad - K_{14}K_{17}H_8H_9 = 0,\end{aligned} \tag{28}$$

where the abbreviations are given in Table 3.

4. Discussion and Conclusion

In this study, we solve the one-dimensional Schrödinger equation, which is the fundamental equation of the quantum mechanic, with the GAMAR type potential and we achieve the wave functions describing the bound and scattering states in terms of hypergeometric functions within the framework of the change of variable method. The reflection and transmission coefficients are obtained by using the asymptotic behavior of the scattering states wave functions as well as to continuity conditions. We also derive a relation for the bound state which is described by the energy eigenvalues and defined as the energy eigenvalue condition. Further, we show how the transmission and reflection coefficients depend on the GAMAR type potential by using Mathematica Software package.

As shown in Figure 2, the unitary condition ($T + R = 1$) is satisfied certainly for the different values of the potential parameters: the transmission coefficient approaches unity asymptotically for increasing the energy value and the reflection coefficient goes to zero asymptotically for decreasing values of the energy. The left and right illustrations of Figure 3 show effect of parameters $\Delta, \tilde{\Delta}$ on the transmission coefficient according to the energy and also potential strength, respectively. According to left illustration, the transmission

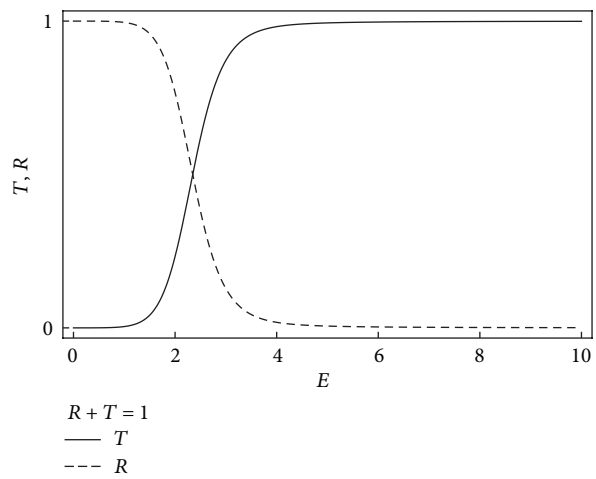

Figure 2: The unitarity condition. The transmission T and reflection R coefficient for the GAMAR type potential barrier varying with E for $\alpha = \beta = 2$, $x_0 = \tilde{x}_0 = 1$, $\Lambda = \tilde{\Lambda} = 1$, $\Delta = \tilde{\Delta} = 2$, $A = 4$, $C = 5$, $B = D = 3$, and $m = 1$.

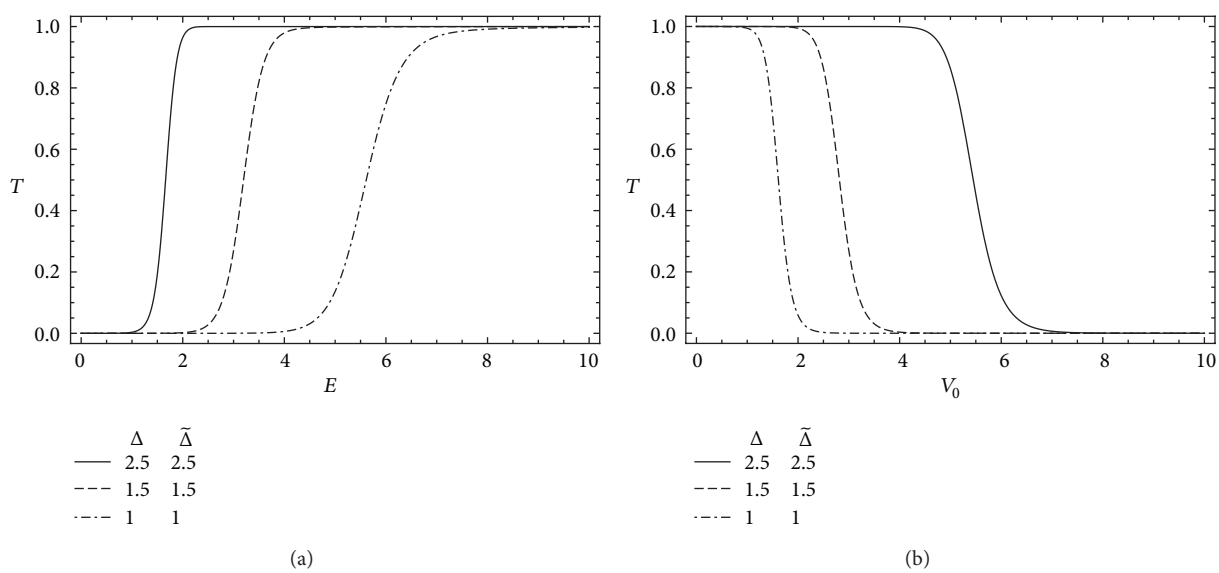

Figure 3: The effect of the GAMAR type potential parameters $(\Delta, \tilde{\Delta})$ on the transmission coefficient for $\alpha x_0 = 2$, $\beta \tilde{x}_0 = 4$ (lower values of the αx_0, $\beta \tilde{x}_0$). (a) shows T for varying E with $\alpha = \beta = 2$, $x_0 = 1$, $\tilde{x}_0 = 2$, $\Lambda = \tilde{\Lambda} = 1$, $A = B = C = D = 3$, and $m = 1$. (b) displays T for varying V_0 with $\alpha = \beta = 2$, $x_0 = 1$, $\tilde{x}_0 = 2$, $\Lambda = \tilde{\Lambda} = 1$, $A = B = C = D = V_0$, $E = 3$, and $m = 1$.

probability of the Schrödinger particle from the GAMAR type potential barrier takes place at a lower energy value for increasing values of the parameters $\Delta, \tilde{\Delta}$. In the right illustration, the transmission probability of the particle exactly gets the value of one if the potential strength (V_0) is zero and it gets the value of zero at the higher values of the potential strength for decreasing values of the parameters $\Delta, \tilde{\Delta}$. It is seen in Figure 4, if we increase the numerical values of the αx_0 and $\beta \tilde{x}_0$ which are stayed as a multiplier in the denominator of the potential function, the transmission resonances occur. One can clearly see from left illustration of Figure 4 that the transmission coefficient goes to one; the condition provides $E \sim [(A + B\Delta)/\Delta^2]$, where $\Delta = \tilde{\Delta}$. In the right illustration, transmission coefficient approaches zero; the condition must be $V_0 \sim [\Delta^2/(\Delta + 1)]$, where $A = B = C = D = V_0$. Figures 5 and 6 display the transmission probability of Schrödinger particles from the potentials which can be derived from the GAMAR type potential. As shown in Figure 5, the transmission coefficient of Schrödinger particles goes to unity for the GAWS potential at lower values of energy, while it goes to unity for the ASH potential at higher values of energy. Similarly, in Figure 6, the transmission coefficient goes to unity for the Woods-Saxon potential at lower values of energy, while it goes to unity for the Hulthen potential at higher values of energy. From Figure 5, we see that our results are consistent with the results

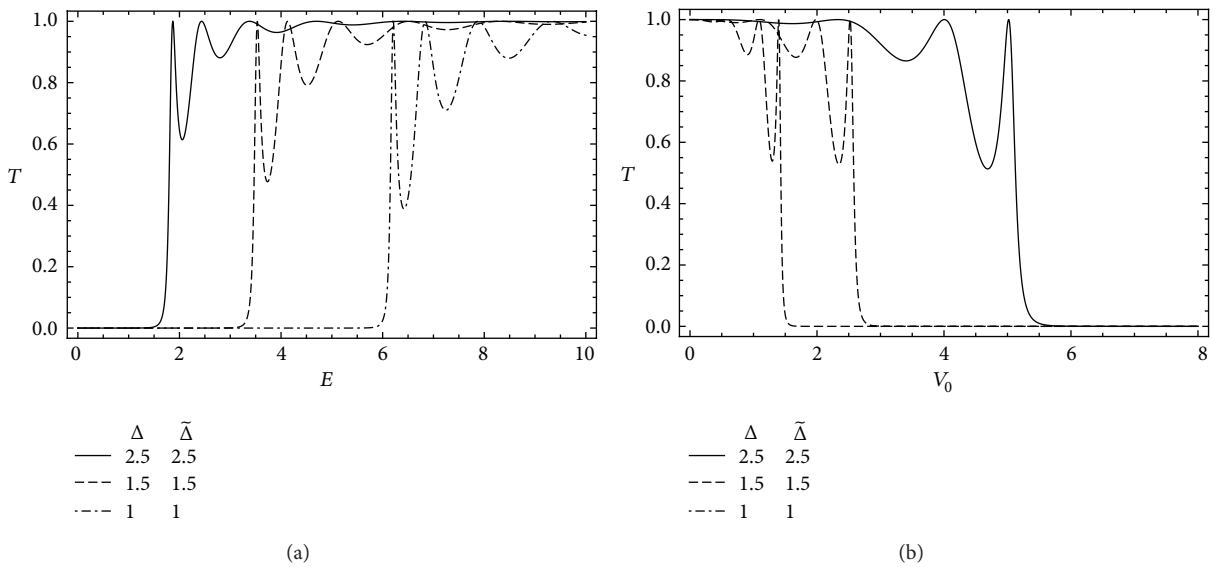

FIGURE 4: The effect of the GAMAR type potential parameters $(\Delta, \widetilde{\Delta})$ on the transmission coefficient for $\alpha x_0 = 20$, $\beta \widetilde{x}_0 = 30$ (large values of the αx_0, $\beta \widetilde{x}_0$). (a) shows T for varying E with $\alpha = \beta = 10$, $x_0 = 2$, $\widetilde{x}_0 = 3$, $\Lambda = \widetilde{\Lambda} = 1$, $A = B = C = D = 3$, and $m = 1$. (b) shows T for varying V_0 with $\alpha = \beta = 10$, $x_0 = 2$, $\widetilde{x}_0 = 3$, $\Lambda = \widetilde{\Lambda} = 1$, $A = B = C = D = V_0$, $E = 3$, and $m = 1$.

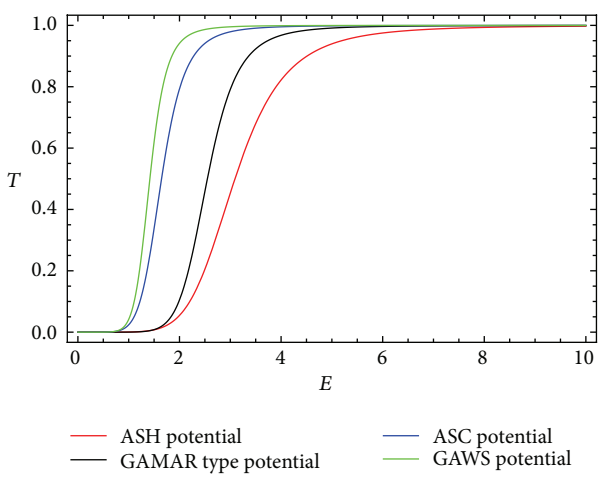

FIGURE 5: The effect of the asymmetric potentials on the transmission coefficient: the ASH potential for $\alpha = 0.5$, $\beta = 0.8$, $x_0 = \widetilde{x}_0 = 0$, $\Lambda = \widetilde{\Lambda} = 1$, $\Delta = \widetilde{\Delta} = -0.5$, $A = C = 0$, $B = D = 2$, and $m = 1$, the GAMAR potential for $\alpha = 0.5$, $\beta = 0.8$, $x_0 = \widetilde{x}_0 = 0.5$, $\Lambda = \widetilde{\Lambda} = 1$, $\Delta = \widetilde{\Delta} = 0.5$, $A = B = C = D = 2$, and $m = 1$, the ASC potential for $\alpha = 0.5$, $\beta = 0.8$, $x_0 = \widetilde{x}_0 = 0$, $\Lambda = \widetilde{\Lambda} = 1$, $\Delta \to 0$, $\widetilde{\Delta} \to 0$, $A = C = 0$, $B = D = 2$, and $m = 1$, and the GAWS potential for $\alpha = 0.5$, $\beta = 0.8$, $x_0 = \widetilde{x}_0 = 0.5$, $\Lambda = \widetilde{\Lambda} = 1$, $\Delta = \widetilde{\Delta} = 0.5$, $A = C = 0$, $B = D = 2$, and $m = 1$.

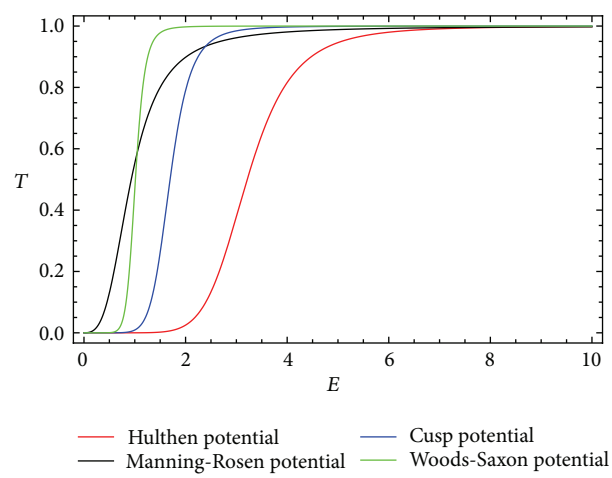

FIGURE 6: The effect of the potentials on the transmission coefficient: the Hulthen potential for $\alpha = \beta = 0.5$, $x_0 = \widetilde{x}_0 = 0$, $\Lambda = \widetilde{\Lambda} = 1$, $\Delta = \widetilde{\Delta} = -0.5$, $A = C = 0$, $B = D = 2$, and $m = 1$, the Manning-Rosen potential for $\alpha = \beta = 0.5$, $x_0 = \widetilde{x}_0 = 0$, $\Lambda = \widetilde{\Lambda} = 1$, $\Delta \to -1$, $\widetilde{\Delta} \to -1$, $A = B = 0$, $C = D = 2$, and $m = 1$, the Cusp potential for $\alpha = \beta = 0.5$, $x_0 = \widetilde{x}_0 = 0$, $\Lambda = \widetilde{\Lambda} = 1$, $\Delta \to 0$, $\widetilde{\Delta} \to 0$, $A = C = 0$, $B = D = 2$, and $m = 1$, and the Woods-Saxon potential for $\alpha = \beta = 0.5$, $x_0 = \widetilde{x}_0 = 0.5$, $\Lambda = \widetilde{\Lambda} = 1$, $\Delta = \widetilde{\Delta} = 0.5$, $A = C = 0$, $B = D = 2$, and $m = 1$.

of the work of Arda et al. [28] for the ASH potential and it is also compatible with [49] for the GAWS potential.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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