Research Article

Magnetic String with a Nonlinear $U(1)$ Source

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Considering the Einstein gravity in the presence of Born-Infeld type electromagnetic fields, we introduce a class of 4-dimensional static horizonless solutions which produce longitudinal magnetic fields. Although these solutions do not have any curvature singularity and horizon, there exists a conic singularity. We investigate the effects of nonlinear electromagnetic fields on the properties of the solutions and find that the asymptotic behavior of the solutions is adS . Next, we generalize the static metric to the case of rotating solutions and find that the value of the electric charge depends on the rotation parameter. Furthermore, conserved quantities will be calculated through the use of the counterterm method. Finally, we extend four-dimensional magnetic solutions to higher dimensional solutions. We present higher dimensional rotating magnetic branes with maximum rotation parameters and obtain their conserved quantities.

1. Introduction

One of the interesting topological defects is cosmic string which may be originated during the early universe phase transitions [1] (see Kibble mechanism for more details [2]). Furthermore, considering the inflationary models [3, 4], it has been proposed that cosmic strings can form at the end of inflation. Moreover, one of the predictions of supersymmetric hybrid inflation [5] (and also grand unified models of inflation [6]) is the cosmic string. Interesting properties and interaction of the superconducting cosmic string with astrophysical magnetic fields have been found in [7–9]. Besides, magnetic strings have been studied in Brans-Dicke theory as well as dilaton gravity [10–13]. From cosmological point of view, one can find the properties of the magnetic (cosmic) string in various literatures [14–16].

In addition to cosmic strings, other kinds of strings may be considered in QCD and also gravity. Properties of the QCD static strings have been investigated extensively in [17–20] and it has been shown that QCD magnetic string can contribute to hadron dynamics [21]. Applications of magnetic string in quantum theories have been presented in [22–24]. Magnetic strings in antiferromagnetic crystals have been investigated in [25]. Application of the (chromo)magnetic

string model to some experimental data on the inclusive pion asymmetries has been studied in [26, 27]. Some arguments about the magnetic strings in the Yang-Mills plasma have been found in [28].

On gravitational aspect, the horizonless solutions and spacetime with conical singularity have been investigated in gravitating electromagnetic field background (see [7, 8, 29–49] and references therein). Interesting properties of the magnetic string in branes, M-theory, and string theory have been investigated [50, 51]. Calculations of the vacuum energy of two different fields in the background of a magnetic string have been analyzed in [52, 53].

One of the generalizations of the Einstein-Maxwell field equations is gravitating nonlinear electrodynamics (NLED), whose most popular theory is Born-Infeld [54–59]. In addition to Lorentz and $U(1)$ gauge invariances, we know that the Lagrangian of the Maxwell electrodynamics contains only quadratic forms of gauge potential and its first derivative. One can consider both invariances and leave out the third condition to obtain NLED [60]. From historical point of view, NLED were introduced to eliminate infinite quantities in theoretical analysis of charged point-like particles [54–59]. Recently, we have more motivations for considering NLED theories, for example, various limitations of the linear

electrodynamics [61, 62], clarification of the self-interaction of virtual electron-positron pairs [63–65], explanation of electrodynamics on D-branes [66–68], and description of radiation propagation inside specific materials [69–72]. In addition, from astrophysical viewpoint, we know that the effects of NLED become indeed quite important in super-strongly magnetized compact objects, such as pulsars, and particular neutron stars (some examples include the so-called magnetars and strange quark magnetars) [73–75]. Moreover, NLED modifies in a fundamental basis the concept of gravitational redshift and its dependency on any background magnetic field as compared to the well-established method introduced by standard general relativity. Furthermore, it has been recently shown that NLED objects can remove both of the big bang and black hole singularities [76–78].

Amongst the nonlinear generalization of Maxwell electrodynamics, the so-called BI type NLED, whose first nonlinear correction is quadratic function of Maxwell invariant, is completely special. It has been shown that BI type NLED may be arisen as a low energy limit of heterotic string theory [66, 68, 79–83], which led to an increased interest for BI type NLED theories. In addition, BI type theories have some interesting properties; for example, these theories enjoy the birefringence phenomena, free of the shock waves [84, 85] and electric-magnetic duality [86]. Furthermore, considering the relation between AdS/CFT correspondence and superconductivity phenomenon, it was shown that the BI type theories make a crucial effect on the condensation, the critical temperature, and energy gap of the superconductors [87].

In this paper, we investigate the horizonless magnetic strings in the presence of two kinds of the BI type NLED [88, 89]. One of the elemental motivations for analyzing the horizonless string solutions is that they may be interpreted as cosmic strings.

2. Basic Field Equations

Our goal in this work is to construct a class of four-dimensional solutions to the Einstein equations with negative cosmological constant in the presence of nonlinear electromagnetic source, $L(\mathcal{F})$, which describes a magnetic string. The Euler-Lagrange equations of motion for the metric $g_{\mu\nu}$ and the gauge potential A_μ may be written as [90]

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}(R - 2\Lambda) = T_{\mu\nu}, \quad (1)$$

$$\partial_\mu (\sqrt{-g} L_{\mathcal{F}} F^{\mu\nu}) = 0, \quad (2)$$

where $\Lambda = -3/l^2$, $L_{\mathcal{F}} = dL(\mathcal{F})/d\mathcal{F}$, $\mathcal{F} = F_{\mu\nu}F^{\mu\nu}$ denotes the Maxwell invariant, and the energy-momentum tensor is given by

$$T_{\mu\nu} = \left(\frac{1}{2}g_{\mu\nu}L(\mathcal{F}) - 2L_{\mathcal{F}}F_{\mu\lambda}F_{\nu}^{\lambda} \right). \quad (3)$$

It is notable that these field equations can be obtained from variation of the following action:

$$I_G = -\frac{1}{16\pi} \int_{\mathcal{M}} d^4x \sqrt{-g} [R - 2\Lambda + L(\mathcal{F})] - \frac{1}{8\pi} \int_{\partial\mathcal{M}} d^3x \sqrt{-\gamma} \Theta(\gamma), \quad (4)$$

where the bulk action (first term) is supplemented with a Gibbons-Hawking surface term (second term) whose variation will cancel the extra normal derivative term in deriving the equation of motion. The quantities Θ and γ denote the trace of the extrinsic curvature and the induced metric for the boundary $\partial\mathcal{M}$, respectively.

In this work, we take into account the recently proposed BI type models of NLED [88, 89]. They have been nominated the *Exponential form of Nonlinear Electromagnetic Field* (ENEF) and the *Logarithmic form of Nonlinear Electromagnetic Field* (LNEF), in which their Lagrangians are

$$L(\mathcal{F}) = \begin{cases} \beta^2 \left(\exp\left(-\frac{\mathcal{F}}{\beta^2}\right) - 1 \right), & \text{ENEF,} \\ -8\beta^2 \ln\left(1 + \frac{\mathcal{F}}{8\beta^2}\right), & \text{LNEF.} \end{cases} \quad (5)$$

Here, we want to obtain magnetic solutions. It is well known that the electric field comes from the time component of the vector potential (A_t), while the magnetic field is associated with the angular component (A_ϕ). Hence one expects that a magnetic solution may be written in a metric gauge in which the components g_{tt} and $g_{\phi\phi}$ interchange their roles relatively to that present in the Schwarzschild gauge used to describe electric solution. Therefore, we start with a class of the four-dimensional metrics which produces longitudinal magnetic fields along the z direction [48]:

$$ds^2 = -\frac{\rho^2}{l^2} dt^2 + \frac{d\rho^2}{f(\rho)} + l^2 f(\rho) d\varphi^2 + \frac{\rho^2}{l^2} dz^2, \quad (6)$$

where $f(\rho)$ is an arbitrary function of coordinate ρ . It is notable that this metric may be obtained from the horizon flat Schwarzschild-like metric:

$$ds^2 = -f(\rho) dt^2 + \frac{d\rho^2}{f(\rho)} + \rho^2 d\varphi^2 + \frac{\rho^2}{l^2} dz^2, \quad (7)$$

with the following *local* transformation:

$$t \longrightarrow il\varphi, \quad \varphi \longrightarrow \frac{it}{l}. \quad (8)$$

Since the mentioned transformation is not a global mapping and metric (7) can be locally mapped to metric (6), one can find that both (6) and (7) do not describe a unique spacetime. Using the nonlinear Maxwell equation (2) with the metric (6), one can obtain

$$\begin{aligned} \left[1 - \left(\frac{2F_{\phi\rho}}{\beta l} \right)^2 \right] F'_{\phi\rho} + \frac{2F_{\phi\rho}}{\rho} &= 0, & \text{ENEF,} \\ \left[1 - \left(\frac{F_{\phi\rho}}{2\beta l} \right)^2 \right] F'_{\phi\rho} + \left[4 + \left(\frac{F_{\phi\rho}}{\beta l} \right)^2 \right] \frac{F_{\phi\rho}}{2\rho} &= 0, & \text{LNEF} \end{aligned} \quad (9)$$

FIGURE 1: $T_{\tilde{t}\tilde{t}}$ versus ρ for $l = 1$, $q = 1$, and $\beta = 1$ (solid line), $\beta = 1.3$ (bold line), and $\beta = 3$ (dashed line). “ENEF branch (a) and LNEF branch (b).”

with the following solutions:

$$F_{\phi\rho} = \begin{cases} \frac{2ql^2}{\rho^2} \exp\left(-\frac{1}{2}L_W\right), & \text{ENEF,} \\ \frac{\beta^2\rho^2}{q} (1 - \Gamma), & \text{LNEF,} \end{cases} \quad (10)$$

where the prime denotes differentiation with respect to ρ , the parameter q is an integration constant, $L_W = \text{Lambert } W(-16l^2q^2/\beta^2\rho^4)$ which satisfies $\text{Lambert } W(x) \exp[\text{Lambert } W(x)] = x$ [91, 92], and $\Gamma = \sqrt{1 - (2lq/\beta\rho^2)^2}$. It is worthwhile to note that in order to have a real electromagnetic field, we should consider $\rho > \rho_0$, where

$$\rho_0 = \sqrt{\frac{2lq}{\beta}} \times \begin{cases} \sqrt{2} \exp\left(\frac{1}{4}\right), & \text{ENEF,} \\ 1, & \text{LNEF.} \end{cases} \quad (11)$$

Here, we use the orthonormal contravariant (hatted) basis vectors to study the effect of nonlinearity on the energy density. Considering the mentioned diagonal metric in this basis, one should apply $\mathbf{e}_{\tilde{t}} = (l/\rho)(\partial/\partial t)$ and therefore the $\tilde{t}\tilde{t}$ component of the stress-energy tensor is

$$T_{\tilde{t}\tilde{t}} = \begin{cases} \frac{\beta^2}{2} \left[1 - \exp\left(-\frac{2F_{\phi\rho}^2}{l^2\beta^2}\right) \right], & \text{ENEF,} \\ \beta^2 \ln\left(1 + \frac{F_{\phi\rho}^2}{4l^2\beta^2}\right), & \text{LNEF.} \end{cases} \quad (12)$$

We plot $T_{\tilde{t}\tilde{t}}$ versus $\rho > \rho_0$ in Figure 1 and find that, for a fixed value of ρ , as nonlinearity parameter increases, the energy density of the spacetime decreases and therefore, in order to

reduce the concentration volume of the energy density, we should increase the nonlinearity parameter.

Now, we should obtain the metric function $f(\rho)$. One can take into account (6) and (10) in the gravitational field equation (1) to obtain its nonzero components as

$$\begin{aligned} (\Psi(\rho) + \rho\beta^2) \exp\left(-\frac{1}{2}L_W\right) - \rho\beta^2 [1 + H^2(\rho)] &= 0, & \text{ENEF,} \\ \Psi(\rho) + 8\rho\beta^2 \ln\left[1 + \left(\frac{\beta\rho^2(1-\Gamma)}{2ql}\right)^2\right] - J(\rho) &= 0, & \text{LNEF,} \end{aligned} \quad (13)$$

where $\Psi(\rho) = 2f'(\rho) + g(\rho) - (6\rho/l^2)$ and

$$g(\rho) = \begin{cases} \rho f''(\rho), & tt(zz) \text{ component,} \\ \frac{2f(\rho)}{\rho}, & \rho\rho(\phi\phi) \text{ component,} \end{cases}$$

$$H(\rho) = \begin{cases} 0, & tt(zz) \text{ component,} \\ \frac{4ql}{\beta\rho^2} \exp\left(-\frac{1}{2}L_W\right), & \rho\rho(\phi\phi) \text{ component,} \end{cases}$$

$$J(\rho) = \begin{cases} 0, & tt(zz) \text{ component} \\ \frac{4\rho^5\beta^4(1-\Gamma)^2}{l^2q^2[1 + (\beta\rho^2(1-\Gamma)/2ql)^2]}, & \rho\rho(\phi\phi) \text{ component.} \end{cases} \quad (14)$$

After some calculations one can show that these equations have the following solutions:

$$f(\rho) = \frac{\rho^2}{l^2} + \frac{2ml^3}{\rho} - \frac{\chi\beta^2\rho^2}{6} - \frac{2\beta ql}{\rho} \Upsilon(\rho), \quad (15)$$

where m is the integration constant which is related to mass parameter, χ is equal to 1 and -16 for ENEF and LNEF branches, respectively, and

$$\Upsilon(\rho) = \begin{cases} \int (-L_W)^{-1/2} [1 - L_W] d\rho, & \text{ENEF,} \\ \frac{2\beta}{ql} \int \left[\rho^2 \ln \left(\frac{\beta^2 (1 - \Gamma) \rho^4}{2l^2 q^2} \right) + \frac{4q^2 l^2}{\beta^2 \rho^2 (1 - \Gamma)} \right] d\rho, & \text{LNEF,} \end{cases} \quad (16)$$

where one may calculate these integrations. We should note that the obtained solutions are the same as asymptotically anti-de Sitter magnetic solution of Einstein-Maxwell gravity [49], asymptotically (large values of radial coordinate). In addition, one may expect to recover the solution of [49] for $\beta \rightarrow \infty$.

Taking into account the metric (6), it is clearly desirable to have an examination on the geometric structure of the solutions. The first step is investigation of the spacetime curvature. It is easy to show that the Kretschmann scalar is

$$R_{\mu\nu\lambda\kappa} R^{\mu\nu\lambda\kappa} = \left(\frac{d^2 f(\rho)}{d\rho^2} \right)^2 + 4 \left(\frac{1}{\rho} \frac{df(\rho)}{d\rho} \right)^2 + 4 \left(\frac{f(\rho)}{\rho^2} \right)^2. \quad (17)$$

Numerical calculations show that the Kretschmann is finite for nonzero ρ . Furthermore, we can show that

$$\lim_{\rho \rightarrow 0} R_{\mu\nu\lambda\kappa} R^{\mu\nu\lambda\kappa} = \frac{48M^2 l^6}{\rho^6} + \frac{A(M, q, \beta, l)}{\rho^5} + O\left(\frac{1}{\rho^4}\right), \quad (18)$$

$$\lim_{\rho \rightarrow \infty} R_{\mu\nu\lambda\kappa} R^{\mu\nu\lambda\kappa} = \frac{8\Lambda^2}{3} + \frac{48M^2 l^6}{\rho^6} - \frac{384M l^5 q^2}{\rho^7} + \frac{B(M, q, \beta, l)}{\rho^8} + O\left(\frac{1}{\rho^{11}}\right), \quad (19)$$

where A and B are different functions of metric parameters M , q , l , and β . Equation (19) confirms that the asymptotic behavior of the solutions is adS. In addition, one may take into account (18) to think about the existence of a curvature singularity located at $\rho = 0$ and therefore conclude that there are magnetically charged black hole solutions. Since $\rho > \rho_0$, one concludes that, for charged solutions with finite β , the spacetime never achieves $\rho = 0$. In addition, we should obtain the zeroes of the function $f(\rho) = (g_{\rho\rho})^{-1}$. Considering the largest positive real root of $f(\rho) = 0$ by r_0 (suppose $r_0 > \rho_0$; for $r_0 < \rho_0$, the metric function $f(\rho)$ is positive definite which we are not interested in), one can find that the function $f(\rho)$ is negative for $\rho < r_0$. We should note that $g_{\rho\rho}$ and $g_{\phi\phi}$ are related by $f(\rho) = g_{\rho\rho}^{-1} = l^{-2} g_{\phi\phi}$, and therefore negativity of $g_{\rho\rho}$ (which occurs for $\rho < r_0$) leads to negativity of $g_{\phi\phi}$ and hence the signature of the metric changes from $+2$ to -2 . This indicates that we could not extend the spacetime from

$\rho > r_0$ to $\rho < r_0$. In order to get rid of this incorrect extension, one may introduce a new radial coordinate r in the following form:

$$\begin{aligned} r^2 &= \rho^2 - r_0^2, \\ d\rho^2 &= \frac{r^2}{r^2 + r_0^2} dr^2. \end{aligned} \quad (20)$$

Considering this suitable coordinate transformation, the electromagnetic field can be written as

$$F_{\phi r} = \begin{cases} \frac{2ql^2}{r^2 + r_0^2} \exp\left(-\frac{1}{2} L'_W\right), & \text{ENEF,} \\ \frac{\beta^2 (r^2 + r_0^2)}{q} (1 - \Gamma'), & \text{LNEF,} \end{cases} \quad (21)$$

where $L'_W = \text{Lambert } W(-16l^2 q^2 / \beta^2 (r^2 + r_0^2)^2)$ and $\Gamma' = \sqrt{1 - (2lq/\beta(r^2 + r_0^2))^2}$. Moreover, the metric (6) in the new coordinate is

$$\begin{aligned} ds^2 &= -\frac{r^2 + r_0^2}{l^2} dt^2 + l^2 f(r) d\phi^2 + \frac{r^2}{(r^2 + r_0^2)} \frac{dr^2}{f(r)} \\ &\quad + \frac{r^2 + r_0^2}{l^2} dz^2, \end{aligned} \quad (22)$$

with $0 \leq r < \infty$, and $f(r)$ is now given as

$$\begin{aligned} f(r) &= \frac{r^2 + r_0^2}{l^2} + \frac{2ml^3}{\sqrt{r^2 + r_0^2}} - \frac{\chi\beta^2 (r^2 + r_0^2)}{6} \\ &\quad - \frac{2\beta ql}{\sqrt{r^2 + r_0^2}} \Upsilon(r), \end{aligned} \quad (23)$$

where

$$\begin{aligned} \Upsilon(r) &= \begin{cases} \int \frac{[1 - L'_W] r}{\sqrt{-(r^2 + r_0^2)} L'_W} dr, & \text{ENEF,} \\ \frac{2\beta}{ql} \int \left(\frac{\ln\left(\left(\beta^2 (1 - \Gamma') (r^2 + r_0^2)^2\right)/2l^2 q^2\right)}{(r^2 + r_0^2)^{-1/2}} + \frac{4q^2 l^2}{\beta^2 (1 - \Gamma') (r^2 + r_0^2)^{3/2}} \right) r dr & \text{LNEF.} \end{cases} \end{aligned} \quad (24)$$

Numerical calculations show that not only Kretschmann scalar but also other curvature invariants are finite in the range $0 \leq r < \infty$ ($r_0 \leq \rho < \infty$) and therefore the mentioned spacetime has no curvature singularity and no horizon. It is notable that the above-mentioned magnetic solutions differ from the electric solutions and the properties

FIGURE 2: The deficit angle versus β for $r_0 = 1$, $l = 1$, and $q = 1$. “ENEF branch (a) and LNEF branch (b).”

of electric and magnetic solutions are distinct. For example, the electric solutions lead to black objects interpretation, while the magnetic solutions do not.

In spite of the fact that the obtained magnetic solutions have no essential singularity, one can show that $\lim_{r \rightarrow 0} (1/r) \sqrt{g_{\phi\phi}/g_{rr}} \neq 1$ and so, when r goes to zero, the limit of the ratio “circumference/radius” is not 2π . This indicates that there is a conic singularity located at $r = 0$. In order to remove the conic singularity, one can identify the angular coordinate ϕ with the period

$$\text{Period}_\phi = 2\pi \left(\lim_{r \rightarrow 0} \frac{1}{r} \sqrt{\frac{g_{\phi\phi}}{g_{rr}}} \right)^{-1} = 2\pi (1 - 4\mu), \quad (25)$$

where the conical singularity has a deficit angle $\delta\phi = 8\pi\mu$. Expanding the metric function for $r \rightarrow 0$, one can show that

$$\begin{aligned} f(r)|_{r=0} &= \left. \frac{df(r)}{dr} \right|_{r=0} = 0, \\ f''(0) &= \left. \frac{d^2 f(r)}{dr^2} \right|_{r=0} \neq 0, \end{aligned} \quad (26)$$

and therefore μ is given by

$$\mu = \frac{1}{4} \left(1 - \frac{2}{lr_0 f''(0)} \right). \quad (27)$$

It is easy to show that the near origin metric can be written as

$$\begin{aligned} ds^2 &= -\frac{r_0^2}{l^2} dt^2 + \frac{r^2 l^2}{2} \frac{d^2 f(r)}{dr^2} \Big|_{r=0} d\phi^2 \\ &+ \frac{2}{r_0^2} \frac{dr^2}{(d^2 f(r)/dr^2)|_{r=0}} + \frac{r_0^2}{l^2} dz^2. \end{aligned} \quad (28)$$

Following the Vilenkin procedure [93], one can identify the near origin metric (28) with a cosmic string and interpret μ as the mass per unit length of the string [93].

Here, we are in a position to investigate the effect of nonlinearity parameter on the deficit angle $\delta\phi$. At first, we should note that $\delta\phi$ is a smooth real function for $\beta > \beta_{\text{ext}}$, where

$$\beta_{\text{ext}} = \frac{2lq}{r_0^2} \times \begin{cases} 2 \exp\left(\frac{1}{2}\right), & \text{ENEF,} \\ 1, & \text{LNEF.} \end{cases} \quad (29)$$

Second, it is interesting to note that the minimum and maximum values of the deficit angle are

$$\begin{aligned} \delta\phi|_{\text{Min}} &= \lim_{\beta \rightarrow \infty} \delta\phi, \\ \delta\phi|_{\text{Max}} &= \lim_{\beta \rightarrow \beta_{\text{ext}}} \delta\phi, \end{aligned} \quad (30)$$

which means that increasing the nonlinearity parameter β leads to decreasing the deficit angle (see Figure 2 for more clarifications).

3. Spinning Magnetic String

In this section, we apply a local rotation boost to the static metric (22) to obtain rotating spacetime solutions. In 4-dimensional spacetime the rotation group is $SO(3)$, and so one can find that there is only one independent rotation parameter. In order to apply rotation, one may use the following local transformation in the $t - \phi$ plane:

$$t \mapsto \sqrt{1 + \frac{a^2}{l^2}} t - a\phi, \quad \phi \mapsto \sqrt{1 + \frac{a^2}{l^2}} \phi - \frac{a}{l^2} t, \quad (31)$$

where a is a rotation parameter. Taking into account the static metric (20) and applying (31), we can obtain

$$ds^2 = -\frac{r^2 + r_0^2}{l^2} \left(\sqrt{1 + \frac{a^2}{l^2}} dt - ad\phi \right)^2 + \frac{r^2 dr^2}{(r^2 + r_0^2) f(r)} + l^2 f(r) \left(\frac{a}{l^2} dt - \sqrt{1 + \frac{a^2}{l^2}} d\phi \right)^2 + \frac{r^2 + r_0^2}{l^2} dz^2, \quad (32)$$

where the metric function $f(r)$ is the same as that in (23). According to the mentioned transformation, one can find that, in spite of the static case, F_{rt} does not vanish for rotating solutions. Straightforward calculations show that the nonvanishing components of the electromagnetic fields are

$$F_{rt} = -\frac{a}{l^2 \sqrt{1 + a^2/l^2}} F_{r\phi} = \frac{a}{\sqrt{1 + a^2/l^2}} \times \begin{cases} \frac{2q}{r^2 + r_0^2} \exp\left(-\frac{1}{2} L'_W\right), & \text{ENEF,} \\ \frac{\beta^2 (r^2 + r_0^2)}{ql^2} (1 - \Gamma'), & \text{LNEF.} \end{cases} \quad (33)$$

Considering (22), (31), and (32), one may think that there is a one-to-one correspondence between static and rotating spacetimes and so they are the same. But this statement is not correct. It is worthwhile to mention that the coordinate ϕ is periodic and therefore (31) is not a proper coordinate transformation on the entire manifold. In other words, the metrics (22) and (32) can be locally mapped into each other but not globally, and so (31) generates a new metric (for some details about this local transformation see, e.g., [94]).

In order to finalize this section, we should discuss the conserved quantities of the magnetic string. Using the counterterm method [95–98] and following the procedure of magnetic solutions papers [7, 8, 29–49], one can find that the mass and angular momentum per unit length of the string can be written as

$$M = \frac{\pi}{2} \left(1 + \frac{3a^2}{l^2} \right) m, \quad (34)$$

$$J = \frac{3\pi ma}{2} \sqrt{1 + \frac{a^2}{l^2}}. \quad (35)$$

Equation (35) shows that considering $a = 0$ leads to vanishing angular momentum and it confirms that a is the rotational parameter of the spacetime. In addition, it is interesting to calculate the electric charge of the solutions. Using Gauss's law and calculating the flux of the electric field at infinity, we find that the electric charge per unit length Q can be given by

$$Q = \pi qa. \quad (36)$$

We should note that the electric charge may be originated from the electric field. Since, for rotating solutions, besides the magnetic field along the ϕ coordinate, there is also a radial electric field F_{tr} (see (33)), one may expect to obtain an electric charge which is related to the rotating parameter.

4. Magnetic Brane Solutions

Here, we start with a class of the $(n+1)$ -dimensional metrics to obtain magnetic brane solutions with the following ansatz:

$$ds^2 = -\frac{\rho^2}{l^2} dt^2 + \frac{d\rho^2}{f(\rho)} + l^2 f(\rho) d\varphi^2 + \frac{\rho^2}{l^2} dX_1^2, \quad (37)$$

where $dX_1^2 = \sum_{i=1}^{n-2} (dx^i)^2$ is the Euclidean metric on the $(n-2)$ -dimensional submanifold. Using the nonlinear Maxwell equation (2) with the metric (37), we find that the nonzero components of Maxwell field are

$$F_{\phi\rho} = -F_{\rho\phi} = \begin{cases} \frac{2ql^{n-1}}{\rho^{n-1}} \exp\left(-\frac{1}{2} L_W\right), & \text{ENEF,} \\ \frac{\beta^2 \rho^{n-1}}{q} (1 - \Gamma), & \text{LNEF,} \end{cases} \quad (38)$$

where $L_W = \text{Lambert } W(-16l^{2(n-2)} q^2 / \beta^2 \rho^{2(n-1)})$ and $\Gamma = \sqrt{1 - (2lq/\beta\rho^{n-1})^2}$. It is notable that considering the real electromagnetic field leads to $\rho > \rho_0$, where

$$\rho_0^{2n-2} = \left(\frac{2lq}{\beta} \right)^2 \times \begin{cases} 4l^{2(n-3)} \exp(1), & \text{ENEF,} \\ 1, & \text{LNEF.} \end{cases} \quad (39)$$

Now, we are in a position to obtain the metric function $f(\rho)$. Considering (37) with (38), we find that the solution of the gravitational field equation (1) is

$$f(\rho) = \frac{\rho^2}{l^2} + \frac{2ml^3}{\rho^{n-2}} - \frac{\chi\beta^2 \rho^2}{n(n-1)} - \frac{4\beta ql^{n-2}}{(n-1)\rho^{n-2}} Y(\rho), \quad (40)$$

where

$$Y(\rho) = \begin{cases} \int (-L_W)^{-1/2} [1 - L_W] d\rho, & \text{ENEF,} \\ \frac{2\beta}{ql^{n-2}} \int \left[\rho^{n-1} \ln\left(\frac{\beta^2 (1-\Gamma) \rho^{2(n-1)}}{2l^2 q^2}\right) + \frac{4q^2 l^2}{\beta^2 \rho^{n-1} (1-\Gamma)} d\rho \right] & \text{LNEF.} \end{cases} \quad (41)$$

It is easy to show that the Kretschmann scalar diverges when $\rho \rightarrow 0$ and is finite for $\rho \neq 0$. Following the same method, we find that one could not extend the spacetime from $\rho > r_0$ to $\rho < r_0$ in which r_0 is the largest positive real root of $f(\rho) = 0$. Therefore, we can use radial coordinate transformation (20) to obtain a real well-defined spacetime for $0 \leq r < \infty$. Here, we leave details for reasons of economy.

Final step is generalization of static magnetic branes to spinning ones. We know that the rotation group in $n+1$ dimensions is $SO(n)$ and hence the maximum number of independent rotation parameters is integer part of $n/2$.

Generalization of static solutions to the spinning case with $k \leq [n/2]$ rotation parameters leads to the following metric:

$$\begin{aligned}
 ds^2 = & -\frac{r^2 + r_0^2}{l^2} \left(\Xi dt - \sum_{i=1}^k a_i d\phi^i \right)^2 \\
 & + f(r) \left(\sqrt{\Xi^2 - 1} dt - \frac{\Xi}{\sqrt{\Xi^2 - 1}} \sum_{i=1}^k a_i d\phi^i \right)^2 \\
 & + \frac{r^2 dr^2}{(r^2 + r_0^2) f(r)} + \frac{r^2 + r_0^2}{l^2 (\Xi^2 - 1)} \sum_{i < j}^k (a_i d\phi_j - a_j d\phi_i)^2 \\
 & + \frac{r^2 + r_0^2}{l^2} dX_2^2,
 \end{aligned} \quad (42)$$

where $\Xi = \sqrt{1 + \sum_i^k a_i^2/l^2}$; dX_2^2 is the Euclidean metric on the $(n-k-1)$ -dimensional submanifold with volume V_{n-k-1} . The nonvanishing components of electromagnetic field tensor and the metric function are, respectively,

$$F_{rt} = -\frac{(\Xi^2 - 1)}{\Xi a_i} F_{r\phi^i} = \frac{(\Xi^2 - 1)}{\Xi a_i} \times \begin{cases} \frac{2ql^{n-1}}{(r^2 + r_0^2)^{(n-1)/2}} \exp\left(-\frac{1}{2}L'_W\right), & \text{ENEF,} \\ \frac{\beta^2(r^2 + r_0^2)^{(n-1)/2}}{q} (1 - \Gamma'), & \text{LNEF,} \end{cases} \quad (43)$$

$$\begin{aligned}
 f(r) = & \frac{r^2 + r_0^2}{l^2} + \frac{2ml^3}{(r^2 + r_0^2)^{(n-2)/2}} - \frac{\chi\beta^2(r^2 + r_0^2)}{n(n-1)} \\
 & - \frac{4\beta ql^{n-2}\Upsilon(r)}{(n-1)(r^2 + r_0^2)^{(n-2)/2}},
 \end{aligned} \quad (44)$$

where $L'_W = \text{LambertW}(-16l^{2(n-2)}q^2/\beta^2(r^2 + r_0^2)^{n-1})$, $\Gamma' = \sqrt{1 - 4l^2q^2/\beta^2(r^2 + r_0^2)^{n-1}}$, and the function $\Upsilon(r)$ is

$$\Upsilon(r) = \begin{cases} \int \frac{[1 - L'_W]r}{\sqrt{-(r^2 + r_0^2)}L'_W} dr, & \text{ENEF,} \\ \frac{2\beta}{ql^{n-2}} \int \left(\frac{\ln(\beta^2(1 - \Gamma')(r^2 + r_0^2)^{n-1}/2l^2q^2)}{(r^2 + r_0^2)^{1-n/2}} + \frac{4q^2l^2}{\beta^2(1 - \Gamma')(r^2 + r_0^2)^{n/2}} \right) r dr & \text{LNEF.} \end{cases} \quad (45)$$

Following the known counterterm procedure and Gauss's law, it is easy to calculate the conserved quantities of the magnetic brane solutions. Straightforward calculations show

that the mass, angular momentum, and electric charge per unit volume of the magnetic branes may be written as

$$\begin{aligned}
 M &= \frac{(2\pi)^k}{4} [n(\Xi^2 - 1) + 1] m, \\
 J_i &= \frac{(2\pi)^k}{4} n \Xi m a_i, \\
 Q &= \frac{(2\pi)^k ql}{2} \sqrt{\Xi^2 - 1}.
 \end{aligned} \quad (46)$$

We should note that the electric charge is proportional to the rotation parameter and is zero for the case of static magnetic branes. This is due the fact that radial electric field F_{tr} vanishes for the static solutions.

5. Conclusions

At the first step, we introduced a class of static magnetic string solutions in Einstein gravity in the presence of negative cosmological constant with two types of NLED. In order to have real solutions, we obtained a lower limit for the radial coordinate, ρ . Furthermore, we nominated the largest real root of the metric function as r_0 and, in order to get rid of signature changing, we introduced a new radial coordinate r .

Calculations of geometric quantities showed that although these solutions do not have curvature singularity, there is a conical singularity at $r = 0$ with a deficit angle $\delta\phi = 8\pi\mu$, where one can interpret μ as the mass per unit length of the string. Moreover, we found that, unlike the power Maxwell invariant solutions [45–47], the nonlinearity does not have any effect on the asymptotic behavior of the solutions and, in other words, obtained solutions are asymptotically adS.

In addition, we investigated the effects of nonlinearity parameter on the energy density and deficit angle, separately, and found that when one increases the nonlinearity parameter, the concentration volume of the energy density and the deficit angle reduce.

Using a suitable local transformation, we added an angular momentum to the spacetime and found that for rotating solutions there is an electric field in addition to the magnetic one.

Next, we used the counterterm method and Gauss's law to obtain conserved quantities and electric charge, respectively. It is interesting to note that these quantities depend on the rotation parameter and the static string has no net electric charge.

At the final step, we studied magnetic solutions in higher dimensions. We generalized static magnetic branes to spinning ones and obtained consistent electromagnetic field as well as metric function. Moreover, we obtained conserved quantities of the magnetic branes and found that the electric charge vanishes for the static magnetic branes. In addition, we found that, for $n = 3$, the conserved quantities of the magnetic branes reduce to those of magnetic string, as we expected.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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