Research Article

Comparing Multicomponent Erlang Distribution and Lévy Distribution of Particle Transverse Momentums

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The transverse momentum spectrums of final-state products produced in nucleus-nucleus and proton-proton collisions at different center-of-mass energies are analyzed by using a multicomponent Erlang distribution and the Lévy distribution. The results calculated by the two models are found in most cases to be in agreement with experimental data from the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC). The multicomponent Erlang distribution that resulted from a multisource thermal model seems to give a better description as compared with the Lévy distribution. The temperature parameters of interacting system corresponding to different types of final-state products are obtained. Light particles correspond to a low temperature emission, and heavy particles correspond to a high temperature emission. Extracted temperature from central collisions is higher than that from peripheral collisions.

1. Introduction

The Relativistic Heavy Ion Collider (RHIC) in USA and the Large Hadron Collider (LHC) in Switzerland have been built to study properties of matters formed in high-energy collisions. These collisions are helpful in understanding particles' statistical behavior, production process, interaction mechanism, and related phenomenon in high-density and high-temperature states. Such high-energy collisions offer us opportunities to carry out investigations not only on the Higgs and dark matter [1–3], but also on particle statistical behavior at ultrahigh energy.

Transverse momentum spectrums of final-state products are very important in high-energy collisions. Many models have been introduced to describe the transverse momentum spectrums of different final-state products [4]. From the spectrums, one can extract temperature parameter of interacting system. It is expected that temperature parameters extracted from different particle spectrums are different due to different emission stages and regions in collisions. Although we can compare nuclear temperature with classical temperature, they have different physical meanings.

Temperature parameter in high-energy collisions is very important. Generally speaking, temperatures of interacting system at initial, intermediate, and final states are different [5]. Since these temperatures cannot be measured directly, it may, therefore, be interesting to find out an indirect method for obtaining the temperature of the interesting system. Traditionally, temperature can be extracted from measurements of spectrum slopes or double isotopic ratios at lower energies [5, 6]. In some cases, we cannot obtain absolute values of concerned temperature parameters, but relative values corresponding to different particle spectrums.

Multicomponent Erlang distribution derived from multisource thermal model [7, 8] has been applied to collisions in relatively low energy region comparing to RHIC and LHC energies. Energy spectrum of nuclear fragments, multiplicity distribution of charged particles, neutron number distribution of isotope in nuclear fragments, transverse momentum (mass) spectrum of relativistic particles, and so forth were described by the multicomponent Erlang distribution. The Lévy distribution has been also applied to transverse momentum spectrums in high-energy collisions [9–11]. We can study transverse momentum spectrums by

using the multicomponent Erlang distribution [7, 8] or the Lévy distribution [9–11] to extract temperature parameters.

In this paper, the transverse momentum spectrums of different final-state products produced in nucleus-nucleus and proton-proton collisions at RHIC and LHC energies are studied with the two distributions mentioned above. Temperature parameters are then obtained from fitting experimental data of the STAR, CMS, and ALICE Collaborations.

2. Formalism

The multicomponent Erlang distribution can be derived from the multisource thermal model [7, 8]. In the model, many emission sources of particles are assumed to form in high energy collisions. According to different interaction mechanisms, geometrical relations, selected conditions, or other factors, the emission sources are divided into l groups. Source number in the j th group is assumed to be m_j . Each source contributes final-state distribution to be an exponential function. We have the transverse momentum p_{tij} spectrum contributed by the i th source in the j th group to be

$$f_{ij}(p_{tij}) = \frac{1}{N} \frac{dN}{dp_{tij}} = \frac{1}{\langle p_{tij} \rangle} \exp\left(-\frac{p_{tij}}{\langle p_{tij} \rangle}\right), \quad (1)$$

where N denotes number of final-state particles and

$$\langle p_{tij} \rangle = \int p_{tij} f_{ij}(p_{tij}) dp_{tij} \quad (2)$$

is mean transverse momentum contributed by the sources in the j th group.

The transverse momentum p_T spectrum contributed by the j th group is the fold of m_j exponential functions; that is,

$$f_j(p_T) = \frac{1}{N} \frac{dN}{dp_T} = \frac{p_T^{m_j-1}}{(m_j-1)! \langle p_{tij} \rangle^{m_j}} \exp\left(-\frac{p_T}{\langle p_{tij} \rangle}\right). \quad (3)$$

This is an Erlang distribution. In final state, the p_T spectrum contributed by the l groups can be written as

$$\begin{aligned} f(p_T) &= \frac{1}{N} \frac{dN}{dp_T} = \sum_{j=1}^l k_j f_j(p_T) \\ &= \sum_{j=1}^l \frac{k_j p_T^{m_j-1}}{(m_j-1)! \langle p_{tij} \rangle^{m_j}} \exp\left(-\frac{p_T}{\langle p_{tij} \rangle}\right), \end{aligned} \quad (4)$$

where k_j is the relative weight contributed by the j th group. It is a multicomponent Erlang distribution.

Considering relative contribution of the j th group, we have the mean transverse momentum of final-state particles to be

$$\langle p_T \rangle = \sum_{j=1}^l k_j m_j \langle p_{tij} \rangle. \quad (5)$$

Generally, $\langle p_T \rangle$ reflects the mean excitation degree of the emission sources and can be used to describe the source temperature parameter T_E . As in the ideal gas model in which p_T obeys Rayleigh distribution, we have

$$T_E \approx \frac{2}{\pi} \frac{\langle p_T \rangle^2}{m_0 \bar{\gamma}}, \quad (6)$$

where m_0 denotes rest mass and $\bar{\gamma}$ is mean Lorentz factor of considered particles. Further,

$$m_0 \bar{\gamma} = \bar{E} \approx \sqrt{\langle p \rangle^2 + m_0^2} = \sqrt{1.5 \langle p_T \rangle^2 + m_0^2}, \quad (7)$$

where $\bar{E}$ and $\langle p \rangle$ are mean energy and mean momentum of considered particles, respectively. On other hand, as the inverse slope parameter, $\langle p_{tij} \rangle$ can be used to describe excitation degree of the emission sources. We define

$$T_{ES} \equiv \sum_{j=1}^l k_j \langle p_{tij} \rangle \quad (8)$$

as a new temperature parameter.

The Lévy distributions appear in many branches of physics, mathematics, biology, economy, computer science, and other areas, where the distribution forms may be different in different branches and the scale of fluctuations may be characterized by long tails and an asymptotic power-law-like behavior. The Lévy distributions are a generalization of the Gaussian distribution. They are similar to the Gaussian distribution and remain stable under the convolution. In fact the Lévy distributions are quite general distributions which contain Gaussian and Cauchy distributions as special cases [12].

Let q be the nonextensive parameter. As a probability distribution, the Lévy distribution is commonly the following power-like distribution [9]:

$$G_q(x) = C_q \left[1 - (1-q) \frac{x}{\langle x \rangle (3-2q)} \right]^{1/(1-q)} \quad (9)$$

which is just a one-parameter generation of the Boltzmann-Gibbs exponential formula with $1 \leq q < 1.5$, where C_q is the normalization constant and x is in the range from 0 to infinity. For the transverse momentum distributions in high-energy collisions, we use directly the function form of Lévy distribution [10]:

$$\begin{aligned} \frac{1}{2\pi p_T} \frac{d^2 N}{dy dp_T} &= \frac{dN}{dy} \frac{(n-1)(n-2)}{2\pi n T_L [n T_L + m_0 (n-2)]} \\ &\times \left(1 + \frac{\sqrt{p_T^2 + m_0^2} - m_0}{n T_L} \right)^{-n}, \end{aligned} \quad (10)$$

where T_L is the slope parameter and n represents the scale of possible fluctuation in T_L . The parameter T_L can be regarded as the temperature parameter in the Lévy distribution.

FIGURE 1: Transverse momentum spectrums of final-state particles produced in Cu-Cu and Au-Au collisions at $\sqrt{s_{NN}} = 0.2$ TeV. The symbols represent experimental data of the STAR Collaboration [11]. The solid and dashed curves represent results calculated by the multicomponent Erlang distribution and the Lévy distribution, respectively, (a), (b), (c), (d), (e), and (f) correspond to different final-state particles and collisions.

3. Comparisons with Experimental Data

The transverse momentum spectrums of final-state particles produced in Cu-Cu and Au-Au collisions at RHIC energy ($\sqrt{s_{NN}} = 0.2$ TeV) are shown in Figure 1. The symbols represent experimental data of the STAR Collaboration [11]. The solid and dashed curves represent results calculated by

the multicomponent Erlang distribution with $l = 1$ or 2 and the Lévy distribution, respectively. The results for different centralities (0–10%, 20–30%, and 40–60% in Cu + Cu, as well as 0–5%, 20–40%, and 60–80% in Au + Au) and also for different particles (K_s^0 , Λ , Ξ , and $\Omega + \bar{\Omega}$ in Cu + Cu, as well as K_s^0 and Λ in Au + Au) in central rapidity range ($|y| < 0.5$) are displayed in different panels. For the sake of

TABLE 1: Parameter values for the two kinds of curves in Figure 1. The values of χ^2/dof and extracted temperatures are given. The errors for m_1 , m_2 , and n can be neglected, and the relative errors for other parameters are less than 10%.

Figures	Centralities	m_1	$\langle p_{ti1} \rangle$ (GeV/c)	k_1	m_2	$\langle p_{ti2} \rangle$ (GeV/c)	T_E (GeV)	χ^2/dof	n	T_L (GeV)	χ^2/dof
Figure 1(a)	0–10%	2	0.310	0.990	2	0.680	0.274	1.162	13	0.200	2.823
	20–30%	2	0.320	0.987	2	0.680	0.286	1.004	12	0.190	3.991
	40–60%	2	0.310	0.982	2	0.680	0.277	1.160	11	0.180	1.260
Figure 1(b)	0–10%	4	0.275	0.950	2	0.550	0.440	0.928	23	0.260	7.272
	20–30%	4	0.260	0.920	2	0.540	0.408	0.209	21	0.250	4.588
	40–60%	4	0.240	0.900	2	0.530	0.367	0.988	18	0.240	4.000
Figure 1(c)	0–10%	4	0.290	0.950	4	0.420	0.456	0.322	31	0.310	0.890
	20–30%	4	0.290	0.950	4	0.450	0.460	0.341	26	0.310	1.160
	40–60%	4	0.290	0.950	4	0.430	0.458	0.594	33	0.310	0.877
Figure 1(d)	0–10%	3	0.420	1.000	—	—	0.444	1.130	30	0.360	0.940
	20–30%	3	0.410	1.000	—	—	0.428	0.932	25	0.330	0.354
	40–60%	3	0.410	1.000	—	—	0.428	0.474	25	0.340	0.147
Figure 1(e)	0–5%	2	0.320	0.990	2	0.640	0.284	1.260	16	0.230	2.840
	20–40%	2	0.320	0.984	2	0.650	0.287	0.797	14	0.230	4.481
	60–80%	2	0.310	0.970	2	0.610	0.280	0.812	14	0.220	2.540
Figure 1(f)	0–5%	5	0.240	0.900	2	0.510	0.486	0.939	42	0.300	9.609
	20–40%	5	0.230	0.800	2	0.490	0.450	1.537	35	0.310	4.180
	60–80%	5	0.200	0.620	2	0.455	0.365	0.834	22	0.270	1.670

convenience, the spectrums are for various centrality bins, with each being scaled by the amount indicated in the legend. The parameter values used in the calculations are shown in Table 1 along with values of χ^2 per degree of freedom (χ^2/dof) and extracted temperatures. One can see that the concerned experimental data are described approximately by the two distributions. Light particles correspond to a lower temperature comparing with the heavy particles. The multicomponent Erlang distribution seems to give a better description than the Lévy distribution. We can use the new distribution, the multicomponent Erlang distribution, to describe the transverse momentum spectrums.

In Figure 2, we give the transverse momentum spectrums of leading and subleading jets produced in Pb-Pb and p-p collisions at the LHC energy ($\sqrt{s_{NN}}$ or $\sqrt{s} = 2.76 \text{ TeV}$), where the selections of leading and subleading jets can be found in experimental material [13]. The symbols represent experimental data of the CMS Collaboration [13]. The solid and dashed curves represent results calculated by the multicomponent Erlang distribution and the Lévy distribution, respectively. Figures 2(a), 2(b), and 2(c) correspond to different selected conditions shown in the panels, where $\int L dt$, ϕ , *anti- k_T* , *R*, and *PFlow* denote the integral luminosity, azimuth, sequential recombination algorithm for high- p_T particle, resolution parameter, and particle flow, respectively. The parameter values used in the calculations are shown in Table 2 with values of χ^2/dof and extracted temperatures.

It is again observed that the two distributions describe approximately the concerned experimental data.

In the Lévy distribution, we need to know the rest mass of final-state product. However, the rest mass of jet is uncertain. In fact, we regarded m_0 as a parameter in Figure 2. To see dependence of jet p_T spectrum on m_0 in the Lévy distribution, we redraw the Lévy distribution curves for different m_0 values in Figure 3, where the same experimental data [13] as those cited in Figure 2 are used. Different values of m_0 correspond to different results shown in the figure by different types of curves. All the parameter values with values of χ^2/dof are given in Table 3. We see that the temperature extracted from a given jet spectrum decreases with increase of the jet mass and is greater than that extracted from particle spectrums. It should be noticed that the jet mass is the total mass of particles in the jet. For a jet with a given total transverse momentum, a larger mass corresponds to more particle number. Then, the transverse momentum per particle will be smaller, which renders a lower temperature.

In Figure 4, another data sample on p_T spectrums of leading and subleading jets produced in Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76 \text{ TeV}$ is analyzed. The symbols represent experimental data of the CMS Collaboration [13]. The solid and dashed curves represent results calculated by the multicomponent Erlang distribution and the Lévy distribution, respectively. The values of all the parameters along with the values of χ^2/dof are given in Table 2. We see that except for a few points the two distributions describe approximately

FIGURE 2: Transverse momentum spectrums of leading and subleading jets produced in Pb-Pb and p-p collisions at $\sqrt{s_{NN}}$ or $\sqrt{s} = 2.76$ TeV. The symbols represent experimental data of the CMS Collaboration [13]. The solid and dashed curves represent results calculated by the multicomponent Erlang distribution and the Lévy distribution, respectively, (a), (b), and (c) correspond to different selected conditions.

the experimental data. Different spectrums corresponding to different A_J (dijet imbalance parameter) ranges can be described by the same distribution which reflects a common law in the spectrums.

The p_T spectrums of charged jets produced in Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV is given in Figure 5. The symbols represent experimental data of the ALICE Collaboration [14]. The solid and dashed curves represent results calculated by the multicomponent Erlang distribution and the Lévy

distribution, respectively. All the parameter values along with values of χ^2/dof and extracted temperatures are given in Table 2. One can see that both the distributions describe approximately the experimental data, and the former one gives a better description than the latter one.

The p_T spectrums of charged particles (which can be approximately regarded as $\pi^\pm$) produced in $\sqrt{s_{NN}} = 2.76$ TeV Pb-Pb collisions in different centrality bins with different multiplications are shown in Figure 6(a). Meanwhile,

TABLE 2: Parameter values for the two kinds of curves in Figures 2, 4, and 5. The values of χ^2/dof and extracted temperatures are given. The abbreviations LJ and SJ represent leading and subleading jets, respectively. The errors for m_1 , m_2 , and n can be neglected, and the relative errors for other parameters are less than 10%.

Figures	Types	m_1	$\langle p_{t1} \rangle$ (GeV/c)	k_1	m_2	$\langle p_{t2} \rangle$ (GeV/c)	T_E (GeV)	χ^2/dof	n	m_0 (GeV/c ²)	T_L (GeV)	χ^2/dof
Figure 2(a)	LJ	5	14.600	0.9800	6	22.000	25.925	0.865	14	100	13.000	1.022
	SJ	4	16.000	0.9000	5	18.500	13.163	1.648	17	200	10.500	2.442
Figure 2(b)	LJ	5	12.600	0.9900	6	26.500	20.499	1.207	10	100	9.500	2.322
	SJ	7	10.500	0.9300	6	23.000	17.479	0.939	8	200	8.500	3.841
Figure 2(c)	LJ	5	12.200	0.9600	6	17.800	19.915	1.006	13	100	10.500	1.327
	SJ	8	10.000	0.9500	9	15.500	19.535	1.449	12	200	10.000	4.304
Figure 4(a)	LJ	5	11.200	0.9000	6	17.800	19.016	0.754	9	100	11.000	1.019
	SJ	8	9.500	0.9200	9	14.200	18.352	22.441	12	200	10.000	32.055
Figure 4(b)	LJ	5	11.500	0.9200	6	16.800	18.956	2.904	10	100	10.300	2.154
	SJ	6	8.450	0.9400	7	13.000	15.055	2.024	12	100	9.000	3.266
Figure 4(c)	LJ	5	12.000	0.9000	6	17.000	20.624	1.446	13	100	12.000	1.788
	SJ	4	9.500	0.9700	5	15.500	8.812	33.377	10	100	6.000	50.246
Figure 4(d)	LJ	5	11.500	0.8900	6	16.900	11.067	0.907	10	210	7.800	1.320
	SJ	4	6.900	0.9900	5	12.500	4.704	0.439	8	100	3.000	0.996
Figure 5	0–10%	2	5.210	0.9993	1	30.000	4.270	0.449	18	10	3.800	6.910
	10–30%	2	4.800	0.9992	1	25.000	3.808	0.245	15	10	3.200	7.534
	30–50%	2	3.500	0.9986	1	17.000	1.004	1.216	6	30	0.800	6.223
	50–80%	1	3.000	0.9980	1	12.200	0.180	2.070	4	32	0.190	1.630

TABLE 3: Parameter values for different curves of the Lévy distributions in Figure 3. The values of χ^2/dof and extracted temperatures are given. The little marks LJ and SJ represent leading and subleading jets, respectively. The relative errors for the parameters are less than 10%.

Figures	n_{LJ}	m_0 (GeV/c ²)	T_{LJ} (GeV)	χ^2/dof	n_{SJ}	m_0 (GeV/c ²)	T_{SJ} (GeV)	χ^2/dof
Figure 3(a)	7	200	6.0	0.813	17	200	10.5	2.442
	9.3	150	9.0	0.911	40	150	14.2	3.170
	14	100	13.0	1.022	55	100	18.0	2.444
	14.5	50	14.0	0.902	300	50	25.0	3.914
	18	1	14.0	0.919	500	1	30.0	7.714
Figure 3(b)	9	200	7.0	2.883	8	200	8.5	3.841
	10	100	9.5	2.322	9	100	12.0	4.684
	14	10	12.0	2.135	12	10	15.0	8.050
Figure 3(c)	9	200	6.5	1.279	12	200	10.0	4.304
	13	100	10.5	1.327	22	100	15.0	5.891
	14.5	10	11.0	1.282	42	10	20.0	9.446

the p_T spectrums of π^- , K_s^0 , K^- , and $\bar{p}$ produced in central (0–5%) Pb-Pb collisions at the same energy are shown in Figure 6(b). The symbols represent experimental data of the ALICE Collaboration [14, 15] measured in the pseudorapidity range of $|\eta| < 0.8$. The solid and dashed curves represent results calculated by the multicomponent Erlang distribution and the Lévy distribution, respectively. Corresponding to Figures 6(a) and 6(b), the parameter values with values of χ^2/dof and extracted temperatures are given in Tables 4 and 5, respectively. One can see that the multicomponent Erlang distribution describes well the p_T spectrums in all the cases.

The Lévy distribution describes well the spectrums in some cases, and in other cases it describes approximately the mean trends of the spectrums.

Figures 7(a), 7(b), and 7(c) show, respectively, p_T spectrums of final-state particles $\pi^+ + \pi^-$, π^0 , and $\bar{p}$ produced in $\sqrt{s_{NN}} = 2.76$ TeV Pb-Pb collisions in different centrality bins with different multiplications. Selected condition for $\bar{p}$ is rapidity being in the range of $|y| < 0.5$. For the sake of comparison, the results for $\pi^+ + \pi^-$ and π^0 produced in 2.76 TeV p-p collisions are also given in Figures 7(a) and 7(b), respectively. The symbols represent experimental data

FIGURE 3: Dependence of jet p_T spectrum on m_0 in the Lévy distribution. The same experimental data [13] as those cited in Figure 2 are used. Different values of m_0 correspond to different results shown in the figure by different types of curves. The unit of m_0 is GeV/c^2 .

of the ALICE Collaboration [16, 17]. The solid and dashed curves represent results calculated by the multicomponent Erlang distribution and the Lévy distribution, respectively. All the parameter values with values of χ^2/dof and extracted temperatures are given in Tables 5 (for Figures 7(b) and 7(c))

and 6 (for Figure 7(a)), respectively. One can see that the multicomponent Erlang distribution describes well the p_T spectrums in all the cases. The Lévy distribution describes well the spectrums in some cases, and in other cases it describes approximately mean trends of the spectrums.

FIGURE 4: The same as that for Figure 2, but showing another data sample in which the dijet imbalance parameter A_J is regarded as the selected condition. (a), (b), (c), and (d) correspond to different A_J ranges.

The transverse momentum spectrums of Ξ and Ω as well as inclusive electrons produced in inelastic p-p collision at 7 TeV are given in Figures 8(a) and 8(b), respectively. Experimental data measured by the ALICE Collaboration [15, 18] are shown by the symbols. Results calculated by using the multicomponent Erlang distributions and the Lévy distributions are shown by the solid and dashed curves, respectively. The parameter values used in the calculation are listed in Table 4. We see that both distributions describe approximately the experimental data.

The transverse momentum spectrums of π^+ , K^+ , and p ; π^- , K^- , and $\bar{p}$; K_s^0 , Λ , $\bar{\Lambda}$, ϕ , and $\Xi^- + \bar{\Xi}^+$ produced in p-p collisions at 0.9 TeV are displayed in Figures 9(a), 9(b),

and 9(c), respectively. The symbols represent experimental data of the ALICE Collaboration [19, 20]. The solid and dashed curves represent results calculated by using the multicomponent Erlang distribution and the Lévy distribution, respectively. The related parameter values are given in Table 5. One can see that both the two distributions describe approximately the experimental data.

In Figure 10, the transverse momentum spectrum of charged particles (which can be approximately regarded as $\pi^\pm$) produced in nonsingle diffractive (NSD) p-p collisions at 0.9 TeV is presented. The symbols represent experimental data measured in the pseudorapidity range of $|\eta| < 0.8$ by the ALICE Collaboration [19]. The solid and dashed curves

FIGURE 5: The p_T spectrums of charged jets produced in Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV. The symbols represent experimental data of the ALICE Collaboration [14]. The solid and dashed curves represent results calculated by the multicomponent Erlang distribution and the Lévy distribution, respectively.

FIGURE 6: The p_T spectrums of (a) charged and (b) identified particles produced in Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV. The symbols represent experimental data of the ALICE Collaboration [14, 15]. The solid and dashed curves represent results calculated by the multicomponent Erlang distribution and the Lévy distribution, respectively.

represent results of the multicomponent Erlang distribution and the Lévy distribution, respectively. The related parameter values are given in Table 4. One can see that both the two distributions describe approximately the experimental data.

To see dependences of temperature T (T_E and T_L) on centrality and $\sqrt{s_{NN}}$, in Figures 11 and 12, we plot different values of T_E and T_L taken from Tables 1–6. The related impacting types, $\sqrt{s_{NN}}$, centralities, and final-state products

are shown in the figures. Figures 11(a), 11(b), 11(c) and 11(d) as well as 11(e) and 11(f) correspond to dependence on centrality for particle productions at 0.2 and 2.76 TeV and jet production at 2.76 TeV, respectively. Figure 12 corresponds to dependence on $\sqrt{s_{NN}}$ for particle productions at RHIC and LHC energies. One can see that the extracted temperature for light particles is less than that for heavy particles. Central collisions or high $\sqrt{s_{NN}}$ correspond to a relative high temperature. The multicomponent Erlang distribution

FIGURE 7: The p_T spectrums of (a) $\pi^+ + \pi^-$, (b) π^0 , and (c) $\bar{p}$ produced in $\sqrt{s_{NN}} = 2.76$ TeV Pb-Pb collisions in different centrality bins. For the sake of comparison, the results for $\pi^+ + \pi^-$ and π^0 produced in 2.76 TeV p-p collisions are also given. The symbols represent experimental data of the ALICE Collaboration [16, 17]. The solid and dashed curves represent results calculated by the multicomponent Erlang distribution and the Lévy distribution, respectively.

extracts a relatively high temperature comparing to the Lévy distribution. Besides, from the parameter tables (Tables 1, 2, and 4–6) and (8), one can easily obtain values of T_{ES} which show similar behaviors as those of T_E .

4. Conclusions and Discussions

The transverse momentum spectrums of final-state products produced in high-energy collisions are analysed by using the multicomponent Erlang distribution and the Lévy distribution. In most cases, both the distributions are approximately

in agreement with experimental data at RHIC and LHC energies. The multicomponent Erlang distribution seems to give a better description as compared to the Lévy distribution. Although the Lévy distribution is well known to give the transverse momentum spectrums, the multicomponent Erlang distribution gives a new method to describe the transverse momentum spectrums.

The temperature parameters of interacting system corresponding to different types of final-state products are extracted from transverse momentum spectrums. Light particles correspond to a low temperature emission, and heavy

FIGURE 8: The p_T spectra of (a) Ξ and Ω as well as (b) inclusive electrons produced in inelastic p-p collision at 7 TeV. Experimental data measured by the ALICE Collaboration [15, 18] are shown by the symbols. Results calculated by using the multicomponent Erlang distributions and the Lévy distributions are shown by the solid and dashed curves, respectively.

FIGURE 9: The p_T spectra of (a) π^+ , K^+ , and p ; (b) π^- , K^- , and $\bar{p}$; and (c) K_s^0 , Λ , $\bar{\Lambda}$, ϕ , and $\Xi^- + \Xi^+$ produced in p-p collisions at 0.9 TeV. The symbols represent experimental data of the ALICE Collaboration [19, 20]. The solid and dashed curves represent results calculated by using the multicomponent Erlang distribution and the Lévy distribution, respectively.

particles correspond to a high temperature emission. For a jet with a given transverse momentum, larger mass corresponds to larger particle number and lesser transverse momentum per particle, which renders a lower temperature. Central collisions or high $\sqrt{s_{NN}}$ correspond to a relative

high temperature. The multicomponent Erlang distribution extracts a relatively high temperature comparing with the Lévy distribution.

System size dependence of the hadronic spectrums is well described by the two modeling distributions in the present

TABLE 4: Parameter values for the two kinds of curves in Figures 6(a), 8, and 10. The values of χ^2/dof and extracted temperatures are given. The errors for $m_{1,2,3,4}$ and n can be neglected, and the relative errors for other parameters are less than 10%.

	Figures						
	Figure 6(a)	Figure 6(a)	Figure 6(a)	Figure 8(a)	Figure 8(a)	Figure 8(b)	Figure 10
Types	0–5%	20–40%	40–80%	Ξ	Ω	Inclusive electron	Charged particle
m_1	1	1	1	3	3	1	1
$\langle p_{ti1} \rangle$ (GeV/c)	0.500	0.500	0.500	0.390	0.390	0.108	0.370
k_1	0.799184	0.928772	0.918786	0.91000	0.70000	0.88822	0.93906
m_2	1	1	1	4	4	1	1
$\langle p_{ti2} \rangle$ (GeV/c)	0.600	0.800	0.820	0.625	0.530	0.380	0.800
k_2	0.199796	0.069908	0.079894	0.09000	0.30000	0.10978	0.05994
m_3	1	1	1	—	—	5	1
$\langle p_{ti3} \rangle$ (GeV/c)	2.300	2.300	2.300	—	—	0.480	1.800
k_3	0.001000	0.001300	0.001300	—	—	0.00200	0.00100
m_4	1	1	1	—	—	—	—
$\langle p_{ti4} \rangle$ (GeV/c)	6.700	6.700	6.700	—	—	—	—
T_E (GeV)	0.265	0.266	0.268	0.515	0.551	0.074	0.198
χ^2/dof	0.303	0.299	0.512	0.535	0.023	0.070	0.563
n	7.5	7	7	9	10	5	8
T_L (GeV)	0.200	0.190	0.160	0.320	0.390	0.040	0.150
χ^2/dof	66.600	46.825	4.944	0.288	0.099	1.184	0.503

FIGURE 10: The p_T spectrum of charged particles produced in NSD p-p collisions at 0.9 TeV. The symbols represent experimental data measured by the ALICE Collaboration [19]. The solid and dashed curves represent results of the multicomponent Erlang distribution and the Lévy distribution, respectively.

work. We see some correlations between the parameter values and system size. Particularly, the extracted temperature increases with increase of the system size from p-p collision

to Cu-Cu and Au-Au (Pb-Pb) collisions at the same $\sqrt{s_{NN}}$. This renders that the excitation degree of the interacting system increases with increase of the system size. Comparing with light nuclear collisions, a participant nucleon in heavy nuclear collisions takes part in more binary collisions, and more energy per nucleon deposits in heavy nuclear collisions.

It is well known that most of the hadrons in low transverse momentum region are produced in the process dominated by soft interaction, whereas the hadrons with high transverse momenta are produced in the process dominated by hard parton-parton scattering. According to the discussions in the present work, the first group of sources in the multicomponent Erlang distribution corresponds generally to the soft interaction, and the second or third group of sources corresponds to the hard scattering. The Lévy distribution does not distinguish the transverse momentum regions of soft interaction and hard scattering.

Although there are more or less differences in both the modeling distributions for the observed transverse momentum spectrums, the multicomponent Erlang distribution and the Lévy distribution describing approximately the transverse momentum spectrums in different systems render that there are some common laws or universality in multihadron production [21, 22], even in general probability distributions. For example, the multicomponent Erlang distribution is also used to describe the probability distributions of some plant seed masses and sizes [23], and the Lévy distribution has more

TABLE 5: Parameter values for the two kinds of curves in Figures 6(b), 7(b), 7(c), and 9. The values of χ^2/dof and extracted temperatures are given. The errors for m_1 , m_2 , and n can be neglected, and the relative errors for other parameters are less than 10%.

Figures	Types	m_1	$\langle p_{ti1} \rangle$ (GeV/c)	k_1	m_2	$\langle p_{ti2} \rangle$ (GeV/c)	T_E (GeV)	χ^2/dof	n	T_L (GeV)	χ^2/dof
Figure 6(b)	π^-	2	0.195	0.785	3	0.320	0.260	0.045	10	0.185	0.435
	K^-, K_s^0	8	0.230	0.060	3	0.270	0.411	0.155	68	0.350	0.335
	$\bar{p}$	11	0.131	0.130	3	0.463	0.636	0.193	22	0.450	12.390
Figure 7(b)	0–20%	1	0.450	0.990	1	1.250	0.231	0.406	9	0.180	1.171
	20–40%	1	0.400	0.980	1	1.000	0.207	0.283	9	0.175	0.599
	40–60%	1	0.400	0.975	1	1.100	0.210	0.122	8	0.170	0.362
	60–80%	1	0.420	0.975	1	1.220	0.222	0.152	7	0.145	0.361
	p-p	1	0.390	0.970	1	1.150	0.207	0.258	7	0.140	0.097
Figure 7(c)	0–5%	6	0.272	0.500	3	0.370	0.622	0.199	300	0.450	21.836
	5–10%	6	0.262	0.400	3	0.410	0.620	0.212	300	0.450	21.136
	10–20%	6	0.269	0.380	3	0.390	0.604	0.113	300	0.450	16.885
	20–30%	6	0.267	0.330	3	0.395	0.595	0.065	300	0.470	6.772
	30–40%	6	0.260	0.220	3	0.395	0.564	0.161	500	0.470	2.645
	40–50%	6	0.260	0.230	3	0.365	0.527	0.187	200	0.450	1.074
	50–60%	6	0.280	0.110	3	0.355	0.488	0.145	100	0.420	0.509
	60–70%	6	0.320	0.070	3	0.330	0.444	0.627	22	0.346	1.117
Figure 9(a)	π^+	2	0.159	0.850	3	0.280	0.198	0.834	8	0.129	0.270
	K^+	3	0.180	0.870	7	0.210	0.293	0.310	7	0.180	0.999
	p	8	0.230	0.060	3	0.240	0.293	0.304	8	0.195	0.486
Figure 9(b)	π^-	2	0.170	0.895	3	0.305	0.200	0.302	8	0.129	0.176
	K^-	3	0.180	0.130	7	0.192	0.284	0.684	8	0.185	1.025
	$\bar{p}$	8	0.210	0.050	3	0.230	0.267	0.347	9	0.190	0.606
Figure 9(c)	K_s^0	2	0.250	0.860	3	0.410	0.259	0.136	6.8	0.168	0.426
	Λ	2	0.290	0.780	3	0.400	0.230	0.484	9	0.220	0.953
	$\bar{\Lambda}$	2	0.300	0.800	3	0.400	0.232	0.053	8	0.198	0.126
	ϕ	2	0.448	1.000	—	—	0.341	0.396	9	0.270	0.224
	$\Xi^- + \bar{\Xi}^+$	2	0.430	1.000	—	—	0.279	0.147	8	0.230	0.076

TABLE 6: Parameter values for the two kinds of curves in Figure 7(a). The values of χ^2/dof and extracted temperatures are given. The errors for $m_{1,2,3}$ and n can be neglected, and the relative errors for other parameters are less than 10%.

	Types						
	0–5%	5–10%	10–20%	20–40%	40–60%	60–80%	p-p
m_1	1	1	1	1	1	1	1
$\langle p_{ti1} \rangle$ (GeV/c)	0.550	0.500	0.600	0.650	0.650	0.6400	0.660
k_1	0.994980	0.995480	0.990941	0.984941	0.981941	0.979951	0.974942
m_2	1	1	1	1	1	1	1
$\langle p_{ti2} \rangle$ (GeV/c)	1.85	1.700	1.800	1.800	1.750	1.720	1.680
k_2	0.005000	0.004500	0.008999	0.014999	0.017999	0.019999	0.024998
m_3	5	5	5	5	5	5	5
$\langle p_{ti3} \rangle$ (GeV/c)	2.800	2.200	2.000	2.200	2.300	2.200	2.300
T_E (GeV)	0.283	0.256	0.312	0.342	0.343	0.339	0.352
χ^2/dof	0.166	0.208	0.139	0.320	0.105	0.074	0.077
n	6	6	6	6	6	7	7
T_L (GeV)	0.00001	0.00010	0.00080	0.00700	0.03000	0.15000	0.14000
χ^2/dof	2.656	1.544	1.373	0.814	0.480	0.531	1.220

FIGURE 11: Dependences of temperatures T_E and T_L on centrality. (a), (b), (c), and (d) as well as (e) and (f) correspond to dependence on centrality for particle productions at 0.2 and 2.76 TeV and jet production at 2.76 TeV, respectively.

FIGURE 12: Dependences of temperatures T_E and T_L on $\sqrt{s_{NN}}$.

other applications [12, 24]. We are interested in searching new applications of the two distributions.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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