Research Article

What Can We Learn from (Pseudo)Rapidity Distribution in High Energy Collisions?

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Based on the (pseudo)rapidity distribution of final-state particles produced in proton-proton (pp) collisions at high energy, the probability distributions of momenta, longitudinal momenta, transverse momenta (transverse masses), energies, velocities, longitudinal velocities, transverse velocities, and emission angles of the considered particles are obtained in the framework of a multisource thermal model. The number density distributions of particles in coordinate and momentum spaces and related transverse planes, the particle dispersion plots in longitudinal and transverse coordinate spaces, and the particle dispersion plots in transverse momentum plane at the stage of freeze out in high energy pp collisions are also obtained.

1. Introduction

Due to the complexity of high energy collisions, it is impossible to measure all quantities and characteristics. Instead, some quantities and characteristics can be measured by experiment [1], while others can be obtained from the theoretical explanations based on the experimental results.

In experiments, high energy proton-proton (pp), proton-nucleus (pA), deuteron-nucleus (dA), and nucleus-nucleus (AA) collisions have been performed at available accelerators such as the Super Proton Synchrotron (SPS) [2–4] and colliders such as the Relativistic Heavy Ion Collider (RHIC) [5–8] and the Large Hadron Collider (LHC) [9–12]. Some quantities and characteristics such as the pseudorapidity (or rapidity) distributions, transverse momentum distributions, azimuthal correlations, flow effects, and some other interesting information have been obtained [13].

A lot of models were introduced in the past several decades. Different phenomenological mechanisms including initial interactions, intermediate processes, and final-state statistical laws have been proposed [14–19]. In a workshop held a few years ago at the CERN Theory Institute [20], many models reported their predictions for the heavy ion program at the LHC based on the explanations of the experimental

results at the RHIC. Recently, some of the models are tested by pp , pA , and AA collisions at the LHC. Meanwhile, these models are further tested in pp , pA , dA , and AA collisions at the RHIC and SPS.

In the past years, we suggested a multisource thermal model [21–23] to describe the (pseudo)rapidity distributions, multiplicity distributions, transverse momentum distributions, azimuthal distributions, flow effects, and so forth. Some interesting quantities such as the temperature, speed of sound, and number of sources are obtained. It is noticed that some quantities and characteristics are related to others. Usually, based on a few distributions, other distributions can be obtained due to some modelling assumptions.

Very recently, the NA61/SHINE Collaboration reported the negatively charged pion productions in inelastic pp collisions at 20–158 GeV/c at the SPS [1]. The rapidity spectrums are parameterized by the sum of two Gaussian functions symmetrically displaced with respect to midrapidity. It is interesting for us to give a further test for the multisource thermal model by using the rapidity spectrums in pp collisions at SPS momenta (energies). According to the rapidity spectrums, we hope some other distributions can be obtained by the model.

FIGURE 1: Rapidity and pseudorapidity distributions of negatively charged pions produced in (a) 158, (b) 80, (c) 40, and (d) 20 GeV/c pp collisions. The two-dotted curves represent the rapidity distribution range of the NA61/SHINE experiment [1]. The solid and dashed curves are our modelling results on the rapidity and pseudorapidity distributions.

In this paper, in the framework of the multisource thermal model [21–23], we analyze the rapidity spectrums in pp collisions measured by the NA61/SHINE Collaboration at the SPS [1]. Then, a series of other distributions are obtained due to the description of rapidity spectrums.

2. The Model and Calculation Method

In the multisource thermal model [21–23], we assume that many emission sources are formed in high energy

collisions. In rapidity space in the laboratory or center-of-mass reference frame, these sources distribute at different rapidity y_x and form a target cylinder in rapidity interval $[y_{T\min}, y_{T\max}]$ and a projectile cylinder in rapidity interval $[y_{P\min}, y_{P\max}]$, respectively. Because of the symmetry in pp collision, we have $[y_{T\min} = -y_{P\max}]$ and $[y_{T\max} = -y_{P\min}]$.

In the rest frame of a given source, we can use different formalizations to describe the production of particles. For example, we can use the relativistic ideal gas model to give

FIGURE 2: Distributions of (a) p , (b) p_z , (c) p_T , and (d) $m_T - m_0$ of negatively charged pions produced in pp collisions at the SPS. The solid, dotted, dashed, and dotted-dashed curves correspond to the results at 158, 80, 40, and 20 GeV/c, respectively. The circles in Figure 2(b) represent the contribution of the target cylinder to p_z distribution at 158 GeV/c.

a description for the source. Then, we have the momentum p' distribution of particles to be

$$f_{p'}(p') = \frac{1}{n} \frac{dn}{dp'} = Cp'^2 \exp\left(-\frac{\sqrt{p'^2 + m_0^2}}{T}\right), \quad (1)$$

where $C = (1/m_0^2 T)(1/K_2(m_0/T))$ is the normalization constant, $K_2(m_0/T)$ is the modified Bessel function of order 2, m_0 is the rest mass of the considered particle, n is the particle number, and T is the temperature parameter of the source.

The particles are assumed to emit isotropically in the rest frame of the source. Then, the distributions of emission angle θ' and the azimuth ϕ' can be given by $f_{\theta'}(\theta') = (1/2) \sin \theta'$ and $f_{\phi'}(\phi') = 1/(2\pi)$, respectively.

Let p'_z , p'_T , p'_x , p'_y , m'_T , E' , y' , and η' denote, respectively, the longitudinal momentum, transverse momentum, x -component of momentum, y -component of momentum, transverse mass, energy, rapidity, and pseudorapidity of a considered particle in the rest frame of the considered source. In the Monte Carlo calculation, let $r_{1,2,3}$ denote, respectively,

FIGURE 3: The same as Figure 2, but showing the distributions of (a) E , (b) β , (c) β_z , and (d) β_T . The circles in Figure 3(c) represent the contribution of the target cylinder to β_z distribution at 158 GeV/c.

random numbers distributed evenly in $[0, 1]$. The value of p' satisfies

$$\int_0^{p'} f_{p'}(p') dp' < r_1 < \int_0^{p'+dp'} f_{p'}(p') dp'. \quad (2)$$

For other quantities, we have $\theta' = \arctan[2\sqrt{r_2(1-r_2)} / (1-2r_2)]$ if the right side of the equation is greater than 0; otherwise $\theta' = \arctan[2\sqrt{r_2(1-r_2)} / (1-2r_2)] + \pi$. Further, we have some quantities such as $\phi' = 2\pi r_3$, $p'_z = p' \cos \theta'$, $p'_T = p' \sin \theta'$, $p'_x = p'_T \cos \phi'$, $p'_y = p'_T \sin \phi'$, $m'_T = \sqrt{p'^2_T + m_0^2}$,

$$E' = \sqrt{p'^2 + m_0^2} = \sqrt{p'^2_z + p'^2_T + m_0^2} = \sqrt{p'^2_z + m'^2_T}, \text{ and } y' \equiv (1/2) \ln[(E' + p'_z)/(E' - p'_z)].$$

In the laboratory or center-of-mass reference frame, let $r_{5,6}$ denote, respectively, random numbers distributed evenly in $[0, 1]$; we have the related quantities $y_x = (y_{T \max} - y_{T \min})r_5 + y_{T \min}$ if $r_6 < 0.5$, or $y_x = (y_{P \max} - y_{P \min})r_5 + y_{P \min}$ if $r_6 > 0.5$. Further, we have $y = y_x + y'$, $p_T = p'_T$, $p_x = p'_x$, $p_y = p'_y$, $m_T = m'_T$, $p_z = m_T \sinh y$, $p = \sqrt{p'^2_T + p'^2_z}$, $E = m_T \cosh y$, and $\theta = \arctan(p_T/p_z)$ if $p_z > 0$, or $\theta = \arctan(p_T/p_z) + \pi$ if $p_z < 0$. The pseudorapidity $\eta \equiv -\ln \tan(\theta/2)$. Then, the probability distributions of the

FIGURE 4: Distributions of (a) θ and (b) η of negatively charged pions produced in pp collisions at the SPS. The solid, dotted, dashed, and dotted-dashed curves correspond to the results at 158, 80, 40, and 20 GeV/c, respectively. The closed circles show the results of the isotropic emission. The open circles represent the contribution of the target cylinder at 158 GeV/c.

TABLE 1: Values of parameters corresponding to the solid curves in Figure 1.

Figure	T (GeV)	$y_{P\max}$	$y_{P\min}$
Figure 1(a)	0.150 ± 0.012	2.00 ± 0.02	-0.21 ± 0.01
Figure 1(b)	0.145 ± 0.012	1.71 ± 0.02	-0.21 ± 0.01
Figure 1(c)	0.141 ± 0.011	1.43 ± 0.01	-0.21 ± 0.01
Figure 1(d)	0.138 ± 0.011	1.20 ± 0.01	-0.21 ± 0.01

considered quantities and their correlations can be obtained. The space density distributions of p and p_T are $\rho(p) = [1/(4\pi p^2 n)](dn/dp)$ and $\rho(p_T) = [1/(2\pi p_T n)](dn/dp_T)$, respectively. Besides, some exchangeable quantities in the above expressions are $p_z = p'_z \cosh y_x + E' \sinh y_x$, $E = p'_z \sinh y_x + E' \cosh y_x$, and $y \equiv (1/2) \ln[(E + p_z)/(E - p_z)]$.

Further, the velocity and corresponding longitudinal and transverse components of the particle can be given by $\beta = p/E$, $\beta_z = p_z/E$, and $\beta_T = p_T/E$, respectively. If we assume that the time interval from initial collision to the stage of freeze-out is t_0 , then we have the space coordinates $x = t_0 \beta_T \cos \phi$, $y = t_0 \beta_T \sin \phi$, $z = r_z = t_0 \beta_z$, $r_T = \sqrt{x^2 + y^2} = t_0 \beta_T$, and $r = t_0 \beta$. Thus, the probability distributions of the considered quantities and their correlations can be obtained. The space density distributions of r and r_T are $\rho(r) = [1/(4\pi r^2 n)](dn/dr)$ and $\rho(r_T) = [1/(2\pi r_T n)](dn/dr_T)$, respectively.

3. Comparison and Extraction

The NA61/SHINE Collaboration has measured the rapidity distributions and other features of negatively charged pions

produced in 20–158 GeV/c pp collisions at the SPS [1]. It is shown that the rapidity spectrums are parameterized by the sum of two Gaussian functions symmetrically displaced with respect to midrapidity:

$$\frac{dn}{dy} = \frac{n}{2\sigma_0 \sqrt{2\pi}} \cdot \left[\exp\left(-\frac{(y - y_0)^2}{2\sigma_0^2}\right) + \exp\left(-\frac{(y + y_0)^2}{2\sigma_0^2}\right) \right], \quad (3)$$

where n , σ_0 , and y_0 are the normalization constant, distribution width for one Gaussian, and peak position parameter, respectively. The values with errors of the three parameters at different momenta are obtained by the NA61/SHINE Collaboration [1].

We can compare directly our modelling results with the experimental distribution function of the NA61/SHINE Collaboration [1]. In Figure 1, the rapidity and pseudorapidity distributions of negatively charged pions produced in (a) 158, (b) 80, (c) 40, and (d) 20 GeV/c pp collisions are displayed. The two-dotted curves represent the rapidity distribution range of the NA61/SHINE experiment [1]. The solid curves which fall into the experimental distribution range are our modelling results obtained by using the multisource thermal model. The modelling results in fact give almost whole superpositions to the two-Gaussian functions which are not presented in the panels. Correspondingly, the modelling results on the pseudorapidity distributions are given by the dashed curves. In the calculation, we have used the Monte Carlo technique and χ^2 -testing method. The values

FIGURE 5: Space density distributions of (a) r , (b) r_T , (c) p , and (d) p_T of negatively charged pions produced in pp collisions at the SPS. The solid, dotted, dashed, and dotted-dashed curves correspond to the results at 158, 80, 40, and 20 GeV/c, respectively.

of parameters T , $y_{P_{\max}} (= -y_{T_{\min}})$, and $y_{P_{\min}} (= -y_{T_{\max}})$ for Figures 1(a), 1(b), 1(c), and 1(d) are listed in Table 1. One can see that the model describes the experimental rapidity distributions of negatively charged pions produced in pp collisions at the SPS. The pseudorapidity distribution has a lower peak and wider range than those of the rapidity distribution. The temperature increases and the rapidity shift $y_{P_{\max}}$ increases with increase of the incident momentum, and the rapidity shift $y_{P_{\min}}$ does not change with the momentum.

By using the above parameter values, we can obtain some interesting features for other quantities. Figure 2 presents the

distributions of (a) p , (b) p_z , (c) p_T , and (d) $m_T - m_0$ of negatively charged pions produced in pp collisions at the SPS. The solid, dotted, dashed, and dotted-dashed curves correspond to the results at 158, 80, 40, and 20 GeV/c, respectively. The circles in Figure 2(b) represent the contribution of the target cylinder to p_z distribution at 158 GeV/c, while the contribution of the projectile cylinder is symmetrical. For the p , p_T , and $m_T - m_0$ distributions, both the contributions of the target and projectile cylinders are the same and equal to a half of the distributions. Similarly, Figure 3 presents the distributions of (a) E , (b) β , (c) β_z , and (d) β_T of the

FIGURE 6: Dispersion plots in $x(y)$ - z planes for negatively charged pions produced in (a) 158, (b) 80, (c), 40, and (d) 20 GeV/c pp collisions. The circles and squares correspond, respectively, to the contributions of the target and projectile cylinders in 500 events.

negatively charged pions. The circles in Figure 3(c) represent the contribution of the target cylinder to β_z distribution at 158 GeV/c, while the contribution of the projectile cylinder is symmetrical. For the E , β , and β_T distributions, both the contributions of the target and projectile cylinders are the same and equal to a half of the distributions.

The space angular distributions of negatively charged pions produced in pp collisions at the SPS are displayed in Figure 4(a). The solid, dotted, dashed, and dotted-dashed curves represent the results at 158, 80, 40, and 20 GeV/c, respectively. The open circles are the contribution of the target cylinder at 158 GeV/c, while the contribution of the

projectile cylinder is symmetrical. For comparison, the result of an isotropic emission is given by the closed circles. To compare the pseudorapidity distributions at different incident momenta, the dashed curves in Figures 1(a)–1(d) are plotted together in Figure 4(b) by the solid, dotted, dashed, and dotted-dashed curves, respectively. The contribution of the target cylinder at 158 GeV/c is shown by the open circles, while the contribution of the projectile cylinder is symmetrical. For comparison, the result of an isotropic emission is given by the closed circles. One can see that the final-state particles are far from the isotropic emission.

FIGURE 7: The same as Figure 6, but showing the dispersion plots in y - x planes.

The space density distributions of (a) r , (b) r_T , (c) p , and (d) p_T are presented in Figure 5, where the value of t_0 is taken to be 2 fm/c for convenience. The solid, dotted, dashed, and dotted-dashed curves correspond to the results at 158, 80, 40, and 20 GeV/c, respectively. One can see that the maximum $\rho(r)$ appears at the maximum r and the minimum $\rho(r)$ appears in the region close to 0. The maximum values of $\rho(r_T)$, $\rho(p)$, and $\rho(p_T)$ appear at 0 and the minimum values appear at the maximum coordinate or momentum (transverse momentum) values.

The dispersion plots in $x(y)$ - z , y - x , and $p_x(p_y)$ - p_z planes for negatively charged pions produced in (a) 158, (b) 80, (c), 40, and (d) 20 GeV/c pp collisions are presented in Figures 6, 7, and 8, respectively. The circles and squares correspond,

respectively, to the contributions of the target and projectile cylinders in 500 events. We can see that the density in large $|z|$ and small $|x(y)|$ region is greater than those in other regions. The density in small $|p_x(p_y)|$ and $|p_z|$ region is greater than those in other regions. The contribution points of the two cylinders are mixed in small longitudinal momentum region. The particle number densities in coordinate space and momentum space increase obviously with increase of the incident momentum.

4. Conclusions

From the above discussions, we obtain the following conclusions.

FIGURE 8: The same as Figure 6, but showing the dispersion plots in $p_x(p_y)$ - p_z planes.

(a) The multisource thermal model is used to describe the rapidity distributions of negatively charged pions produced in pp collisions at SPS energies (momenta). A target cylinder and a projectile cylinder are assumed to form in the collisions. For each source, a relativistic ideal gas model is used. The calculated results are in agreement with the rapidity distributions measured by the NA61/SHINE Collaboration [1]. In our calculation, the main parameters are the temperature, the maximum rapidity shift, and the minimum rapidity shift. The temperature and the maximum rapidity shift increase with increase of the incident momentum, and the minimum rapidity shift does not change with the momentum.

(b) According to the parameter values obtained from the rapidity distributions, the other distributions are obtained in the framework of the multisource thermal model. The other distributions include the probability distributions of momenta, longitudinal momenta, transverse momenta (transverse masses), energies, velocities, longitudinal velocities, transverse velocities, space angles, and pseudorapidities, as well as the number density distributions of particles in coordinate and momentum spaces. The final-state particles are far from the isotropic emission. The maximum $\rho(r)$ appears at the maximum r and the minimum $\rho(r)$ appears in the region close to 0. The maximum values of $\rho(r_T)$, $\rho(p)$,

and $\rho(p_T)$ appear at 0 and the minimum values appear at the maximum coordinate or momentum (transverse momentum) values.

(c) According to the parameter values obtained from the rapidity distributions, the particle dispersion plots in $x(y)$ - z , y - x , and $p_x(p_y)$ - p_z planes are also obtained in the framework of the model. The density in large $|z|$ and small $|x(y)|$ region is greater than those in other regions, and the density in small $|p_x(p_y)|$ and $|p_z|$ region is greater than those in other regions. The contributions of the target cylinder and the projectile cylinder are mixed in small longitudinal momentum region.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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References

- [1] N. Abgrall, A. Aduszkiewicz, Y. Ali et al., "Measurement of negatively charged pion spectra in inelastic p + p interactions at 20, 31, 40, 80 and 158 GeV/c," submitted, <http://arXiv:1310.2417v2>.
- [2] M. I. Adamovich, M. M. Agarwall, Y. A. Alexandrov et al., "Angular distributions of light projectile fragments in deep inelastic Pb + Em interactions at 160A GeV," *The European Physical Journal A*, vol. 6, no. 4, pp. 421–425, 1999.
- [3] M. M. Aggarwal, Z. Ahammed, A. L. S. Angelis et al., "Event-by-event charged-neutral fluctuations in Pb + Pb collisions at 158A GeV," *Physics Letters B*, vol. 701, no. 3, pp. 300–305, 2011.
- [4] M. M. Aggarwal, A. L. S. Angelis, V. Antonenko et al., "Multiplicity and pseudorapidity distribution of photons in S + Au reaction at 200A GeV," *Physical Review C*, vol. 58, no. 2, pp. 1146–1154, 1998.
- [5] I. G. Bearden, D. Beavis, C. Besliu et al., "Charged particle densities from Au + Au collisions at $\sqrt{s_{NN}} = 130$ GeV," *Physics Letters B*, vol. 523, no. 3-4, pp. 227–233, 2001.
- [6] K. Adcox, S. S. Adler, S. Afanasiev et al., "Formation of dense partonic matter in relativistic nucleus-nucleus collisions at RHIC: experimental evaluation by the PHENIX Collaboration," *Nuclear Physics A*, vol. 757, no. 1-2, pp. 184–283, 2005.
- [7] B. B. Back, M. D. Baker, D. S. Barton et al., "Charged-particle pseudorapidity density distributions from Au + Au collisions at $\sqrt{s_{NN}} = 130$ GeV," *Physical Review Letters*, vol. 87, no. 10, Article ID 102303, 4 pages, 2001.
- [8] K. H. Ackermann, N. Adams, C. Adler et al., "Elliptic flow in Au + Au Collisions at $\sqrt{s_{NN}} = 130$ GeV," *Physical Review Letters*, vol. 86, no. 3, pp. 402–407, 2001.
- [9] K. Aamodt, A. Abrahantes Quintana, D. Adamová et al., "Rapidity and transverse momentum dependence of inclusive J/ψ production in pp collisions at $\sqrt{s} = 7$ TeV," *Physics Letters B*, vol. 704, no. 5, pp. 442–455, 2011, Erratum vol. 718, no. 2, pp. 692–698, 2012.
- [10] G. Aad, T. Abajyan, B. Abbott et al., "Measurement of Upsilon production in 7 TeV pp collisions at ATLAS," *Physical Review D*, vol. 87, no. 5, Article ID 052004, 31 pages, 2013.
- [11] V. Khachatryan, A. M. Sirunyan, A. Tumasyan, C. M. S. Collaboration et al., "Upsilon production cross section in pp collisions at $\sqrt{s} = 7$ TeV," *Physical Review D*, vol. 83, no. 11, Article ID 112004, 27 pages, 2011.
- [12] R. Aaij, B. Adeva, M. Adinolfi et al., "Measurement of J/ψ production in pp collisions at $\sqrt{s} = 7$ TeV," *The European Physical Journal C*, vol. 71, no. 5, Article ID 1645, 17 pages, 2011.
- [13] R. Singh, L. Kumar, P. K. Netrakanti, and B. Mohanty, "Selected experimental results from heavy-ion collisions at LHC," *Advances in High Energy Physics*, vol. 2013, Article ID 761474, 22 pages, 2013.
- [14] Y.-P. Liu, C. M. Ko, and T. Song, "Hot medium effects on J/ψ production in p + Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV," *Physics Letters B*, vol. 728, pp. 437–442, 2014.
- [15] A. K. Chaudhuri, "Viscous hydrodynamic model for relativistic heavy ion collisions," *Advances in High Energy Physics*, vol. 2013, Article ID 693180, 25 pages, 2013.
- [16] S. V. Akkelin and M. Yu. Sinyukov, "Entanglement of scales as a possible mechanism for decoherence and 6 thermalization in relativistic heavy ion collisions," *Physical Review C*, vol. 89, no. 3, Article ID 034910, 9 pages, 2014.
- [17] E. Marji, A. Canul, Q. MacPherson et al., "Nonperturbative renormalization of the chiral nucleon-nucleon interaction up to next-to-next-to-leading order," *Physical Review C*, vol. 88, no. 5, Article ID 054002, 10 pages, 2013.
- [18] A. Tawfik, "On the higher moments of particle multiplicity, chemical freeze-out, and QCD critical endpoint," *Advances in High Energy Physics*, vol. 2013, Article ID 574871, 22 pages, 2013.
- [19] C. B. Chiu and R. C. Hwa, "Ridge and transverse correlation without long-range longitudinal correlation," *Advances in High Energy Physics*, vol. 2013, Article ID 728365, 11 pages, 2013.
- [20] S. Abreu, N. Borghini, S. Jeon et al., "Heavy-ion collisions at the LHC—last call for predictions," *Journal of Physics G*, vol. 35, no. 5, Article ID 054001, 185 pages, 2008.
- [21] F.-H. Liu, "Particle production in Au-Au collisions at RHIC energies," *Physics Letters B*, vol. 583, no. 1-2, pp. 68–72, 2004.
- [22] F.-H. Liu, "Dependence of charged particle pseudorapidity distributions on centrality and energy in p(d)A collisions at high energies," *Physical Review C*, vol. 78, no. 1, Article ID 014902, 6 pages, 2008.
- [23] F.-H. Liu, "Unified description of multiplicity distributions of final-state particles produced in collisions at high energies," *Nuclear Physics A*, vol. 810, no. 1–4, pp. 159–172, 2008.