Review Article

Balance Function in High-Energy Collisions

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Received 2 January 2015; Revised 13 March 2015; Accepted 16 March 2015

Academic Editor: Ming Liu

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Aspects and implications of the balance functions (BF) in high-energy physics are reviewed. The various calculations and measurements depending on different quantities, for example, system size, collisions centrality, and beam energy, are discussed. First, the different definitions including advantages and even short-comings are highlighted. It is found that BF, which are mainly presented in terms of relative rapidity, and relative azimuthal and invariant relative momentum, are sensitive to the interaction centrality but not to the beam energy and can be used in estimating the hadronization time and the hadron-quark phase transition. Furthermore, the quark chemistry can be determined. The chemical evolution of the new-state-of-matter, the quark-gluon plasma, and its temporal-spatial evolution, femtoscopy of two-particle correlations, are accessible. The production time of positive-negative pair of charges can be determined from the widths of BF. Due to the reduction in the diffusion time, narrowed widths refer to delayed hadronization. It is concluded that BF are powerful tools characterizing hadron-quark phase transition and estimating some essential properties.

1. Introduction

The quark-gluon plasma (QGP), a state of matter created at 0.1–1 μ s after the Big Bang, is believed to be discovered in the relativistic heavy-ion collider (RHIC) at BNL, ten years ago [1–5]. The heavy-ion program at the large hadron collider (LHC) at CERN was designed to explore, among others, the properties of QGP. In such sophisticated experimental facilities, the nucleus-nucleus collisions at ultrarelativistic energies are devoted to characterize the dynamical processes by which matter at extreme temperatures is produced and the fundamental properties that this matter exhibits. Over the last four decades, various high-energy experiments using nucleon and nucleus beams have been evaluated. Based on Bjorken model, the latter are likely able to produce a new-state-of-matter with partonic degrees of freedom, where quarks and gluons deconfine forming a state similar to the plasma state in atomic physics, thus called quark-gluon plasma (QGP). In early Universe, QGP is believed to entirely fill the cosmological background geometry. Furthermore, the extreme conditions available inside the cores of compact

stars are likely able to compress the hadronic matter. Such extreme compression has the same effect as that of extreme temperature. Both are necessary to derive the confined hadrons into deconfined partons. The temporal and spatial evolution of hot matter till the creation of hadrons is sketched in Figure 1.

The discovery of QGP imposes extreme experimental and theoretical challenges and is a good example about physical problems which should wait even decades for their proper explanation [6]. One of the main QGP signatures is the J/ψ suppression that was proposed in 1986 by Matsui and Satz [7]. During three decades, the theoretical interpretation is still under debate [8–10]. Other challenges can be summarized as follows.

- (i) Mechanism of the elliptic flow: there is an unconfirmed point-of-view about the scaling of constituent quarks, which is still not perfect because the results are not dealing directly with the constituents quarks [11].

FIGURE 1: The space-time evolution of heavy-ion collision. The figure is taken from [28].

- (ii) Lattice QCD results predicted two orders for phase transition(s). It is argued that a first-order phase transition is likely in system consisting of two flavors while a second-order one is likely in the three-flavor system. Furthermore, a smooth cross-over was seen in the QCD simulations. Linking such theoretical predictions with the experimental results would be possible through varying the critical temperature. For instance, at low temperature, the matter is confined, that is, hadronic phase, while at high temperature, QGP phase is likely [12].
- (iii) The strangeness enhancement at alternating gradient synchrotron (AGS) is found larger than that at super proton synchrotron (SPS), which obviously seems to weaken the concept of strangeness enhancement as a signal of QGP [13]. Nevertheless, the search for enhancement at RHIC and LHC energies should be continued.
- (iv) The estimation of the time span till equilibration refers to very small value (~ 10 fm/c). Thus, the evolution of the equilibrated states cannot be evident [14]. Thus, it would not be possible to assure that the hadronic phase was originated in a partonic state (prior to hadronization) [14]. The situation becomes more drastic at RHIC and LHC energies. The critical and freeze-out temperatures become almost indistinguishable [14].

The balance functions (BF) were proposed by Bass et al. [15] as a measure for the correlation of the positive and negative charged particles produced during the relativistic heavy-ion collisions. Their width can be related to the hadronization time. The charge correlation functions which are devoted to study the jets hadronization [16] are used to derive BF. So far they have been estimated in pp collisions at intersecting storage rings (ISR) [17–19], $e^+ + e^-$ annihilation at PETRA at DESY [20–24], Au+Au, in STAR experiment at BNL RHIC [25], and Pb+Pb in NA49 experiment at CERN SPS [26, 27]. Due to charge conservation, oppositely charged particles are produced in pairs. But the produced pairs are separated in the rapidity region due to their different momenta. This implies

that BF can be extracted from the fact that the pairs of opposite charges are created in the local space. This idea defines how to proceed with the measurement of balance between produced pairs.

The different heavy-ion experiments can be differentiated according to the collision energy or nucleon-nucleon (NN) center-of-mass energy $\sqrt{s_{NN}}$ [46], the system size, and type of reactants whether being elementary, NN, or nucleus-nucleus (AA) collisions

$$y = \frac{1}{2} \ln \left(\frac{E + p_L}{E - p_L} \right) = \ln \left(\frac{E + p_L}{m_\perp} \right), \quad (1)$$

where p_L is the longitudinal momentum and $m_\perp = \sqrt{m^2 + p_T^2}$ is the transverse mass. The Lorentz boosts are the transformations with respect to one of three dimensions taking as the frame of reference. At ultrarelativistic energies, it is convenient to deal with the pseudorapidity, η , which is defined in analogy to y , (1):

$$\eta = -\ln \left[\tan \left(\frac{\theta}{2} \right) \right], \quad (2)$$

where θ is the angle of emitted particles relative to the beam axis.

The present work is organized as follows. Section 1 presents a general overview about the history of QGP. Section 2 is devoted to the various definitions of BF. The experimental measurements will be discussed in Section 3. Section 4 discusses some effective models used to calculate BF in high-energy physics. Finally, Section 5 presents the discussion and conclusions.

2. Definitions

In relativistic heavy-ion collisions, it is assumed that many produced particles of different charges expand in temporal and spatial dimensions [39]. Due to charge conservation, both positive and negative charges have to be produced in the same space-time during the evolution of the medium. The correlation between the opposite charges is characterized through BF, which apparently measure the balance between both types of charges [47]. In early studies, Bass et al. [15] have proposed that BF are signatures differentiating between early- and late-stage of the hadronization. The balance functions are proposed to work as a “clock” determining whether the quark production occurred at early times, $t < 1$ fm/c, or at late-stage [15]. For charges created in the early stage, balancing charges are separated by the order of one unit of rapidity, while those formed in a late stage are far from the correlation. Delayed hadronization means that the QGP stays for a long time. This implies that the QGP might be formed at a certain time before the evolution of the hot matter. In principal, BF were proposed to investigate the hadronization from jets production in proton-proton collisions [17, 18]. In a series of papers [17, 18, 48], BF were associated with charge correlations.

Furthermore, the conditional probability is the probability that an event will occur under some conditions, while

another event is predicted to occur or to have occurred [49]. According to the conditional probability, a particle with charge a produced within a rapidity interval y_a should be accompanied by another particle with charge b separated from a by a specified rapidity difference Δy or $\delta y = y_b - y_a$. The balance functions are defined as the linear combination of these conditional probabilities [49]. In terms of different quantities such as azimuthal angle ϕ , rapidity difference Δy , pseudorapidity difference $\Delta\eta$, and invariant momentum q_{inv} , BF can be expressed, (3) [28, 30].

(i) The balance functions are defined as [15]

$$B(P_1 | P_2) \equiv \frac{1}{2} \rho(b, P_2 | a, P_1) - \rho(b, P_2 | b, P_1) + \rho(a, P_2 | b, P_1) - \rho(a, P_2 | a, P_1), \quad (3)$$

where $\rho(b, P_2 | a, P_1)$ is the conditional probability of finding particle of type b in a bin at momentum P_2 accompanied with another particle in a bin a with momentum P_1 . a and b are two types/variables like positive and negative charges. For all charged hadrons, BF should be normalized in order to highlight the charge conservation condition.

In terms of rapidity distributions, the balance functions can be defined as [39]

$$B(P_1 | P_2) \equiv \frac{1}{2} \frac{N_{-+}(P_1, P_2) - N_{++}(P_1, P_2)}{N_+(P_1)} + \frac{N_{+-}(P_1, P_2) - N_{--}(P_1, P_2)}{N_-(P_2)}, \quad (4)$$

where $N_{-+}(P_1, P_2)$ denotes the number of charged particle pair (momenta of the observed positive and negative charges). In a similar way, the number of positive (negative) pair charges for the different distributions reads N_{+-} , N_{--} and N_{++} .

In an equivalent expression, BF can be given as [50]

$$B(\Delta_2 | \Delta_1) = \frac{D(+, \Delta_2 | -, \Delta_1) - D(+, \Delta_2 | +, \Delta_1)}{N_+(\Delta_1)} + \frac{D(-, \Delta_2 | +, \Delta_1) - D(-, \Delta_2 | -, \Delta_1)}{N_-(\Delta_1)}, \quad (5)$$

where N and D refer to the single and double (pair) particle functions. In literature, the distribution of double and single particle is expressed in different forms $D(+, \Delta_2 | -, \Delta_1)/N_+(\Delta_1)$ or $\rho(b, \Delta_2 | a, \Delta_1) = N(b, \Delta_2 | a, \Delta_1)/N(a, \Delta_1)$ in which a and b are the positive and negative charges [30].

(a) Rapidity dependence [50]:

$$N_i(\Delta) = \int_{\Delta} \frac{dn_i}{dy} dy, \quad (6)$$

$$D(i, \Delta_2 | j, \Delta_1) = \int_{\Delta_2} dy_2 \int_{\Delta_1} dy_1 \frac{d^2 n_{ij}}{dy_1 dy_2},$$

where dn/dy is the particle density. Some remarks on the STAR measurements, for instance, for the charge balance functions, are now in order. The rapidity acceptance ranges between $y_1 \geq -\Delta$ and $y_2 \leq \Delta$ and the pseudorapidity differences are kept constant while the pairs of produced particles are detected. In this regard, notations like $y_1 - y_2 \equiv \Delta$, $(y_1 + y_2)/2 \equiv z$ were introduced [50].

(b) Momentum dependence [30]:

$$\rho(b, \Delta_2 | a, \Delta_1) = \frac{N(b, \Delta_2 | a, \Delta_1)}{N(a, \Delta_1)}, \quad (7)$$

where

$$N(b, \Delta_2 | a, \Delta_1) = \int_{\Delta_1} d^3 p_1 \int_{\Delta_2} d^3 p_2 f_{ab}(p_1, p_2), \quad (8)$$

$$N(a, \Delta_1) = \int_{\Delta_1} d^3 p_1 f_a(p_1),$$

where $f_a(p_1)$ or $f_b(p_1)$ are the single particle distribution function and $f_a(p_1, p_2)$ is the two-particle (joint) momentum distribution.

The joint momentum distributions, $f_{ab}(p_1, p_2)$, can be classified into quark-antiquark, quark-quark, or antiquarks created pairs. These distributions are *the product of the corresponding single particle momentum distribution* [30],

$$f_{q_1 q_2}(p_1, p_2) = N_{q_1} N_{q_2} n_{q_1}(p_1) n_{q_2}(p_2) (1 - \delta_{q_1 q_2}) + N_{q_1} (N_{q_2} - 1) n_{q_2}(p_2) \delta_{q_1 q_2}, \quad (9)$$

$$f_{\bar{q}_1 \bar{q}_2}(p_1, p_2) = N_{\bar{q}_1} N_{\bar{q}_2} n_{\bar{q}_1}(p_1) n_{\bar{q}_2}(p_2) (1 - \delta_{q_1 q_2}) + N_{\bar{q}_1} (N_{\bar{q}_2} - 1) n_{\bar{q}_2}(p_2) \delta_{q_1 q_2},$$

where q_1 and q_2 are the quarks flavors. The subscripts, a and b , refer to the quark-pair, antiquark-pair, or quark-antiquark pair. The distribution of the quark-antiquark is given as

$$f_{q_1 \bar{q}_2}(p_1, p_2) = N_{q_1} N_{\bar{q}_2} n_{q_1}(p_1) n_{\bar{q}_2}(p_2) + N_{\bar{q}_1} [n_{q\bar{q}}^{\text{pair}}(p_1, p_2) - n_{q_1}(p_1) n_{\bar{q}_2}(p_2)] \delta_{q_1 q_2}.$$

The single particle distribution for bosons and fermions reads [46]

$$f_{F/B}(E; \beta, \lambda, \gamma) = \frac{1}{\gamma^{-1} \lambda^{-1} e^{\beta E} \pm 1}, \quad (11)$$

where the dispersion relation reads $E = \sqrt{p^2 + m^2}$, $\beta = 1/T$, the fugacity $\lambda = e^{\mu/T}$, and γ is a Lagrange multiplier related to the conservation of the number of members of the ensemble. In the same matter, the single particle distribution for antiquarks can be expressed in terms of $n_{q\bar{q}}^{\text{pair}}$

$$n_{\bar{q}}(p_2) = \int d^3 p_1 n_{q\bar{q}}^{\text{pair}}(p_1, p_2). \quad (12)$$

With this regard, the following frames should be defined:

- (1) laboratory frame is the inertial reference frame with the coordinates t, x, y , and z ;
- (2) comoving frame: at a time t_0 , this is the inertial frame in which the accelerated observer is instantaneously at rest at $t = t_0$. Thus the term “comoving frame” refers to a different frame at each t_0 .

It is argued that the physical quantities which are significant and meaningful are the ones corresponding to the laboratory frame. This means that the quantities are conserved only with respect to laboratory frame, because the comoving frame is an accelerated reference frame [51]. In comoving frame, the single particle momentum distribution for quarks or antiquarks in Boltzmann limit is given as [30]

$$n_q^*(p^*) = n_{\text{th}}(p^*) = \frac{e^{-E^*/T}}{4\pi m^2 T K_2(m/T)}, \quad (13)$$

where astride refers to the quantities in the comoving frame.

- (ii) Uniform binning: for charge a the multiplicity can be determined from $n_{ai}(\delta y)$, where y is the rapidity axis of the bin with the acceptance Δy . The bin size is δy and the bin number is $M(\Delta y, \delta y)$. The total multiplicity reads [49]

$$\sum_{i=1}^{M(\Delta y, \delta y)} n_{ai}(\delta y) \equiv N_a(\Delta y). \quad (14)$$

The bin counts represent integrals of the form

$$\bar{n}_{ai}(\delta y) \approx \int_{y_i - \delta y/2}^{y_i + \delta y/2} \rho_1 n_a(y) dy, \quad (15)$$

where $n_a(y)$ is the number density of a single-particle distribution determined from the histogram of

the ensemble averages and $\bar{n}_{ai}(\delta y)$. Thus, BF are defined as [49]

$$\begin{aligned} B_K(\Delta y, \delta y) &\equiv \frac{1}{2} \sum_{a,b=-}^+ - ab \frac{1}{\sum_{i=1}^M \bar{n}_{ai}} \sum_{i=1}^{M(\Delta y, \delta y)-k} \overline{n_{ai} \cdot (n_{b(i+k)} - \delta_{ab} \delta_{k0})}, \\ B_K(\Delta y, \delta y) &\equiv \frac{1}{2} \sum_{a,b=-}^+ - ab \frac{1}{\sum_{i=1}^M \bar{n}_{ai}} \sum_{i=1-k}^{M(\Delta y, \delta y)} \overline{n_{ai} \cdot (n_{b(i+k)} - \delta_{ab} \delta_{k0})}, \end{aligned} \quad (16)$$

where $k \in [0, M-1]$ and delta functions indicate the cancellation of self-pair distributions.

- (iii) Conditional probabilities: the single- and two-point probabilities can be given in terms of the joint multiplicity

$$P_i(a) = \frac{\overline{n_{ai}(n_b - \delta_{ab})}}{N_a(N_b - \delta_{ab})}, \quad P_{ij}(ab) = \frac{\overline{n_{ai}(n_{bj} - \delta_{ab} \delta_{ij})}}{N_a(N_b - \delta_{ab})}. \quad (17)$$

In statistics and probability theory, the Bayes theorem shows the importance of the mathematical manipulation of the conditional probabilities. The Bayesian probability is one of different interpretations of probability and belongs to evidential probabilities. In an ensemble, the Bayes theorem gives

$$P_{j:1}(ab, \Delta y, \delta y) \equiv \frac{P_{ij}(ab)}{P_i(a)} = \frac{n_{ai}(n_{bj} - \delta_{ab} \delta_{ij})}{n_{ai}(N_b - \delta_{ab})}. \quad (18)$$

This is the conditional probability that predicted that a particle with charge b occupies the i th bin while the j th bin is occupied by another particle with charge a as determined by the joint distribution $n_{ai}(n_{bj} - \delta_{ab} \delta_{ij})$.

Regarding balance functions, the conditional probability is defined as

$$P_{k:\Delta y}(ab) \equiv \frac{\bar{N}_k(ab, \Delta y, \delta y)}{N_a(\Delta y)(N_b(\Delta y) - \delta_{ab})},$$

$$\begin{aligned} B_k(\Delta y, \delta y) &\equiv \frac{1}{2} \sum_{a,b=-}^+ - \frac{ab P_{k:\Delta y}(ab) \bar{N}_a(\Delta y)(N_b(\Delta y) - \delta_{ab})}{\bar{N}_a(\Delta y)}, \end{aligned}$$

$$\begin{aligned} B_k(\Delta y, \delta y) &\equiv \frac{1}{2} \sum_a P_{k:\Delta y}(aa) - \frac{\sum_{a,b} P_{k:\Delta y}(ab) \bar{N}_a(\Delta y) N_b(\Delta y)}{\bar{N}_a(\Delta y)}, \end{aligned} \quad (19)$$

where $\sum_k P_{k:\Delta y}(ab, \Delta y, \delta y) \equiv 1$.

FIGURE 2: Angular correlations as measured by STAR for Au+Au and Cu+Cu collisions. The shaded areas stand for systematic uncertainties in the analysis relative to the elliptic flow. The figure is taken from [29].

2.1. Angular Correlation. For odd-parity observables in STAR experiment at RHIC, large fluctuations have been observed [52, 53]. These fluctuations are supposed to arise from the color flux tubes, which carry both kinds of color charges, that is, color-electric and color-magnetic flux. The color flux tubes generate electric field with random signs [29]. The electric field fluctuates as $1/(\sqrt{N_{\text{fluxtubes}}})$, where $N_{\text{fluxtubes}}$ is the number of tubes. The correlation between positive and negative charges are conjectured to include large fluctuations from odd-parity. Obviously, both types of charges should be produced at same space-time coordinates. In other words, both charges should have the same rapidity and azimuthal angle in the collective flow. Such correlations can be described by BF. The correlations can be expressed as $\langle \cos(\Delta\phi_{\text{balance}}) \rangle$ [29]

$$\gamma_{+-} = F_Q \left(\left(\sum_i \cos 2\phi_i \langle \cos(\Delta\phi_{\text{balance}}) \rangle(\phi_i) - \sin 2\phi_i \langle \sin(\Delta\phi_{\text{balance}}) \rangle(\phi_i) \right) \cdot (M_+)^{-1} \right), \quad (20)$$

where $\Delta\phi_{\text{balance}} = \phi_j - \phi_i$ and F_Q is the fraction of charge. Momentum conservation means $\sum_i p_x^i = 0$, $\sum_i p_y^i = 0$. The correlations are shown in Figure 2 in dependence on the collision centrality.

When the momentum $p_x^i = p_t^i \cos(\phi_i)$, the correlation can be written as [29]

$$\gamma = -F_p \frac{\sum_i (\cos^2 \phi_i - \sin^2 \phi_i)}{M_{\text{tot}}^2}. \quad (21)$$

Here, F_p is fraction of the momentum balance and $M_{\text{tot}} = M_+ + M_- + M_0$ sums over positive, negative, and neutral charges. The fluctuations are essential in estimating the electric field in the initial conditions, which is found 10% of the magnetic field. Thus, the charge and momentum conservation should be attributed to the correlation with one unit of rapidity, while the fluctuations for the initial conditions are found with several units of rapidity.

2.2. Advantages of Balance Functions. In light of the various definitions of BF, Section 2, different advantages can be listed out.

(i) *Charge-Density Balance.* Instead of determining the net-charge density, it is advantageous to study the associated charge density balance [17].

(ii) *Associated Charge-Density Distributions.* The charge-density balance allows us to select out the associated charge density distributions and the correlated fractions [17]. The associated charge-density balance has a further advantage. This is less sensitive to the acceptance corrections than the associated charge density, itself. Taking the trigger of a large transverse momentum event as the selected particle(s), the dependence of the associated charge-density balance Δq on the rapidity of other particles was presented in [17].

(iii) *Relative Distance.* The balance functions are able to measure the relative distance between the positive and negative charges produced in heavy-ion collisions. In the same way, they can be applied to the baryon and antibaryons and so forth.

(iv) *Charge Fluctuation.* The charge fluctuations which occur in heavy-ion collisions are related to the charge-balance functions. So that, it is very important to study the evolution of state of matter created during the collision. This can be done by calculating the charge correlations in dependence on the rapidity.

(v) *Width of Balance Functions.* The production time of the positive-negative pair of charges can be determined by studying widths of BF in terms of the rapidity [38]. It is argued that narrowed balance functions are considered as probes of delayed hadronization, due to the reduction in the diffusion time. This implies long-lived stage before hadronization. In other words, this might refer to delayed hadronization [54].

(vi) *Rapidity Correlation.* One of the most important features of the balance functions is the boost invariance variable such as rapidity. The rapidity correlations describe what so-called the conditional probability. This estimates the probability of the charge produced in a rapidity bin associated to the opposite charge in the other rapidity bin. Rapidity and

FIGURE 3: The p_T -integrated $B(\Delta y | y_w)$ of final hadron system at different rapidity positions with same (a) and different (b) window sizes, as well as the $B_s(\Delta y)$ (c). Correlation coefficient ρ is taken to be 0.3. The graph is taken from [30].

pseudorapidity were given in (1) and (2), respectively. Both act as measure for the speed.

(vii) *Probing Hadron- and QGP-Formation.* One of the signatures for the QGP formation is the sudden drop in the balance function width [55]. On the other hand, having an access to the occurrence of quark-pairs can be utilized as a signature for the hadron formation or hadron diffusion.

2.3. Short-Comings of Balance Functions. The balance functions can have some short-comings.

(i) *Binning Geometry and Bayes Theorem.* The conditional probability is not a *true* probability. Using it leads to contradiction between the binning geometry and Bayes theorem [49].

(ii) *Nonstandard Normalization.* The normalization of BF is not standard one [49].

(iii) *Length Scale Inconsistency.* It is argued that in nucleus-nucleus collisions the production of pair separation length at the formation stage is zero [15]. This is not compatible with the fragmentation scenario [49]. In the thermal and diffusion process of elementary particle collisions, the hadron diffusion is negligible, while the correlation length that would be charge-dependent is larger [49].

3. Experimental Measurements

The experimental features of NA22 [56] and STAR experiments [57] were essential to enable both of them analyzing the characteristics of BF [15, 28], which can be used as effective probes for the phase transition in heavy-ion collisions and $e^+ + e^-$ collisions at ISR and PETRA energies [58]. Many measurements for the dependence of BF on the collision centrality [35], the system size [25–27], and the transverse momentum [57] have been conducted. All properties mentioned above which can be categorized under what so-called

the longitudinal boost invariance are very useful in studying BF. The boost invariance means that the single particle density will be independent of the rapidity. Therefore, it is essential to study BF in terms of rapidity in order to investigate the boost invariance. The widths of balance functions get narrower by increasing the window size y_w [30]. This relation can be formulated from the following relation:

$$B(\Delta y | y_w) = B(\Delta y | \infty) (1 - \Delta y). \quad (22)$$

3.1. Various Measurements. One can categorize the experimental measurements [54] according to the type of the reaction and the dependence of the quantities of common interest.

- (i) The type of the reaction whether nuclei, hadron, or hadron-nuclei interaction: the hadron-hadron collisions like positive pion and kaon $\pi^+ p, k^+ p$ at $\sqrt{s_{NN}} = 22$ GeV in NA22 experiment were introduced in [56]. This experiment can compromise the full momentum and 4π azimuthal acceptance, so that one can very well determine the properties of BF.
- (ii) The dependence on the rapidity (pseudorapidity) and the window size: the window size can be arbitrary but it should be restricted by the rapidity range. Figure 3 shows BF in terms of the rapidity positions and at different window sizes [30].
- (iii) Multiplicity dependence: it is found that as the system size becomes large (in central collisions), most of QGP signatures can be observed [28]. Due to the difficulty of the experimental determination of the collision centrality, we are left with the Monte-Carlo simulations to play this role. Therefore, the multiplicity of observed particles can be correlated to the collision centrality [28]. The balance functions are integrated for all events (multiplicities) in the pp collisions and plotted in Figure 4, which shows

FIGURE 4: The balance functions are given in terms of q_{inv} for charged pion pairs in panel (a) and charged kaon pairs in panel (b) from pp collisions at $\sqrt{s_{\text{NN}}} = 200$ GeV integrated over all multiplicities. The graph is taken from [28].

the dependence on the q_{inv} integrated over all multiplicities at $\sqrt{s_{\text{NN}}} = 200$ GeV [28].

- (iv) Beam energy dependence: Figure 5 shows the dependence of BF on the center-of-mass energy $\sqrt{s_{\text{NN}}}$ ranging from 7.7 to 200 GeV [28]. The figure shows the relation between BF and pseudorapidity for the most central collisions 0–5%. It is to be noticed that BF behave as well at different energies. The data from STAR is narrower than the shuffled results.
- (v) Correlation: the balance functions of charge correlations and fluctuations depend on the charges square [56, 59, 60]:

$$\langle (\delta Q^2) \rangle = \langle Q^2 \rangle - \langle Q \rangle^2 = q^2 (\langle N^2 \rangle - \langle N \rangle^2), \quad (23)$$

where $Q = n_+ - n_-$ and $N_{\text{ch}} = n_+ + n_-$. For hadron gas $q = \pm 1$ while $q = \pm 1/3, \pm 2/3$ for QGP.

Furthermore,

$$\langle N_{\text{ch}} \rangle \langle \delta R^2 \rangle = 4 \frac{\langle (\delta Q^2) \rangle}{\langle N_{\text{ch}} \rangle}, \quad (24)$$

where

$$R = \frac{\langle N_+ N_- \rangle - \langle N_- \rangle \langle N_+ \rangle}{\langle N_- \rangle \langle N_+ \rangle}. \quad (25)$$

Then the D -measure for fluctuation can be written as

$$D(Q) = 4 \frac{\langle (\delta Q)^2 \rangle}{N_{\text{ch}}}. \quad (26)$$

The correlations of all charges are conjectured to combine with BF,

$$\frac{D(Q)}{4} = 1 - \int_0^{y_w} B(\Delta y | y_w) d\Delta y + \frac{\langle Q \rangle}{N_{\text{ch}}}. \quad (27)$$

- (vi) Centrality dependence: BF have been studied at different collision centralities and noticed that they coincide but the width changes due to the different positions of the rapidity ranges $-0.5 < y < 0.5$, $0 < y < 1$, $1 < y < 2$, and $1.5 < y < 2.5$ [28]. Shuffled data and mixed collisions are analyzed, as well. For mixed collisions, the balance functions are zero at all the nine centrality bins, Figure 6.
- (vii) Transverse momentum dependence: BF can also be studied in terms of the difference of momenta (invariant) of the produced particles, that is, q_{inv} . In a Gaussian-like form,

$$B(q_{\text{inv}}) = a q_{\text{inv}}^2 e^{-q_{\text{inv}}^2/2\sigma^2}. \quad (28)$$

This was implemented for charged kaons $K^\pm$ from Au+Au collisions at $\sqrt{s_{\text{NN}}} = 200$ GeV in different centrality bins. The mixed events were abstracted from these balance functions. The solid curves are the one calculated from (28). In [28], the authors stated that the peaks observed in each curve are due to the decay of $\phi \rightarrow k^+ + k^-$; Figure 7 shows these relations.

3.2. Confronting to STAR Experiments. Measuring BF dates back to 2003, where the STAR experiment announced its first measurements [25].

3.2.1. System Size and Centrality Dependence. The balance functions were measured in various system sizes, for example Au+Au at $\sqrt{s_{\text{NN}}} = 200$ GeV in the STAR experiment [54] and Pb+Pb collisions at $\sqrt{s_{\text{NN}}} = 17.2$ GeV in the ALICE experiment at LHC [37]. Also the width of BF was measured in Pb+Pb, C+C, and Si+Si collisions at $\sqrt{s_{\text{NN}}} = 8.8, 17.2$ GeV at SPS [16]. It was observed that BF behave as well in both the central and peripheral collisions but the widths change. This behavior was investigated at different pseudorapidity windows [54]. The width of BF is considered as a timometer for the hadronization. It was observed that the narrowing of

FIGURE 5: The balance functions in terms of $\Delta\eta$ for all charged particles. Central events (0–5%) are shown here at $\sqrt{s_{NN}}$ ranging from 7.7 to 200 GeV. The graph is taken from [28].

BF in central collisions is more than in peripheral collisions [37] and this agrees well with the theoretical results [37] for late hadronization or long-lived QGP. In Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV, it was concluded that increasing the centrality and the transverse momentum decreases the width of BF [54] due to the radial flow [54]. The dependence of balance functions $\langle\Delta\eta\rangle$ on the mean number of wounded nucleons was studied [27]. A strong centrality dependence was found in pp collisions and width of $\langle\Delta\eta\rangle$ decreases with increasing centrality of Pb+Pb collisions [27].

3.2.2. Chemical Evolution of QGP. In heavy-ion collisions, it is conjectured that the creation of quarks occurs in specific space-time, while the antiquarks may occupy the same coordinates [33]. This would mean that the charge balance functions can identify the location of the balancing for the produced hadron [55]. Then, the rapidity distribution of the balancing charges can be observed for any pair flavors [55].

Therefore, the charge correlation function can be analysed even in the QGP medium [55]. Obviously, BF can be related to the correlation function [55]. In order to determine BF for different particle species (hadrons), the longitudinal position in the Bjorken coordinates, in which the charge density is depending, should be analyzed [33]. The correlations from charge conservation should be affected by the time of creation of charge-anticharge pairs [47]. By analysing correlations from STAR experiment for different particle species, Pratt [55] distinguished the two separate waves of charge creation expected in high-energy collisions, one at early times when the QGP should be formed and a second at hadronization. Further, the density of up, down, and strange quarks was extracted in QGP and found in agreement with predictions for a chemically thermalized plasma (at a level of 20%).

In relativistic heavy-ion collisions, thousands of hadrons are created. For every quark flavor detected in the final state

FIGURE 6: The balance functions in terms of $\Delta\phi$ for all charged particle pairs from Au+Au collisions at $\sqrt{s_{\text{NN}}} = 39$ GeV. The graph is taken from [28].

like u , d , and s quarks, there should be antiquarks $\bar{u}$, $\bar{d}$, and $\bar{s}$, too. Such quark correlations are defined as [33]

$$\chi_{ab} = \frac{\langle Q_a Q_b \rangle}{V}, \quad (29)$$

where Q_a is the net-charge of u , d , and s quarks within the volume V . For a parton gas

$$\chi_{ab}^{\text{QGP}} = \Delta_{ab} (n_a + n_{\bar{a}}), \quad (30)$$

where n_a , $n_{\bar{a}}$ are densities for u and d quarks and their antiquarks, respectively. For a noninteracting hadron gas, the correlation is defined as

$$\chi_{ab}^{\text{HG}} = \sum_{\alpha} n_{\alpha} q_{\alpha,a} q_{\alpha,b}, \quad (31)$$

where $q_{\alpha,a}$ is the charge of type a and α is the particle type. The correlations for different species were calculated by lattice gauge theory [31, 32], Figure 8.

The correlation of hadrons is given as [33]

$$G_{\alpha\beta}(\eta) = 4 \sum_{abcd} \langle n_{\alpha} \rangle q_{\alpha,a} \chi_{ab}^{(\text{had})(-1)}(0) g_{bc}^{(\text{had})} \cdot (\eta) \chi_{cd}^{(\text{had})(-1)}(\eta) q_{\beta,d} \langle n_{\beta} \rangle. \quad (32)$$

The balance functions should be related to that correlation

$$B_{\alpha\beta}(\Delta\eta) = \frac{G_{\alpha\beta}(\Delta\eta)}{n_{\beta} + n_{\bar{\beta}}}, \quad (33)$$

where β is the hadron species and n_{β} is the number per rapidity of that species. Therefore, BF for identified pair of species can be calculated [33].

3.2.3. Dependence on Beam Energy and Reaction Plane. Information on the creation of hot and dense matter can be extracted by studying the correlations and fluctuations [34].

FIGURE 7: The balance functions in terms of q_{inv} for charged kaon pairs from Au+Au collisions at $\sqrt{s_{\text{NN}}} = 200$ GeV in different centrality bins. Solid lines correspond to (28). The graph is taken from [28].

The balance functions can directly measure the correlations between negative and positive charge pairs [34]. They are sensitive to the changes in the formation or diffusion processes of the balancing charges [34]. If the hadronization process delays, the particle and antiparticle are correlated due to the conservation of the charge [34]. In addition to that, the reaction plane would play a vital role as BF depend on the azimuthal angle,

$$B(\phi, \Delta\phi) = \frac{1}{2} \frac{\Delta_{+-}(\phi, \Delta\phi) - \Delta_{++}\phi, \Delta\phi}{N_+(\phi)} + \frac{\Delta_{-+}(\phi, \Delta\phi) - \Delta_{--}\phi, \Delta\phi}{N_-(\phi)}, \quad (34)$$

where $N_{+(-)}(\phi)$ is the total number of +ve and (-ve) particles. $\Delta_{+-}(\phi, \Delta\phi)$ is total number of positive particles with azimuthal angle ϕ with respect to the reaction plane and the

negative particles with $\Delta\phi$ with respect to the positive one [34]. The width of BF is given as

$$\langle \Delta\eta \rangle = \frac{\sum_i B(\Delta\eta_i) \Delta\eta_i}{\sum_i B(\Delta\eta_i)}. \quad (35)$$

Figure 9 shows the widths of BF in terms of the pseudorapidity, $\Delta\eta$, and azimuthal angle, $\Delta\phi$, in dependence on the participant particles and the center-of-mass energy, respectively. The calculations are compared with the STAR data for the most central events (0–5%) of Au+Au collisions at $\sqrt{s_{\text{NN}}} = 200, 62.4, 39, 11.5$, and 7.7 GeV. It can be concluded that the narrower width indicates an early hadronization time, while a wider one indicates the diffusion after the freeze-out [34]. Also, it is noticed that the dependence of identified kaons on the centrality is weak in contrast to the pions [34] indicating that the kaons are likely produced in very early stage of the collision.

FIGURE 8: The charge fluctuations as functions of temperature in the lattice gauge theory [31, 32]. Results at temperatures <160 MeV are likely belonging to hadronic state. At higher temperatures, the system is characterized by partonic degrees of freedom. The graph is taken from [33].

3.3. Confronting to ALICE Experiment

3.3.1. Energy Dependence. When comparing the results given in [35, 37] with each other, one finds that in [37] the width of the balance functions is studied in terms of the pseudorapidity $\langle\Delta\eta\rangle$ and $\langle\Delta\phi\rangle$. For a better comparison with STAR results, ALICE measurements were corrected for acceptance and detector effects. So that, terms $B_{+-}(\Delta\eta | \eta_{\max})$ should be corrected

$$B_{+-}(\Delta\eta | \eta_{\max}) = B_{+-}(\Delta\eta | \infty) \left(1 - \frac{\Delta\eta}{\eta_{\max}}\right). \quad (36)$$

It is obvious that the BF width is narrower at LHC than at RHIC energies, Figure 10.

On the other hand, Figure 11 represents $\langle\Delta\eta\rangle$ and $\langle\Delta\phi\rangle$ as function of the average number of participant particles, from peripheral to central collisions. The dependence on the number of participants is appropriate choice for scaling to the centrality classes.

4. Effective Model Calculations

4.1. Coalescence Model. One of the strongest signatures for QGP [61] is the suppression of charmonium system J/ψ as measured in Pb+Pb collisions [62]. The quark coalescence from deconfined quarks to produce charmed hadrons can be best described by the algebraic coalescence model for rehadronization of charmed quark matter (ALCOR). The number of produced hadrons is given by the number of quarks or antiquarks, which mainly are the compositions of those hadrons multiplied by the coalescence coefficient C_q and the nonlinear normalization coefficient b_q , in which

the latter indicates the conservation of the quark number during the quark coalescence [63]. The ALCOR model begins with the valence quarks and antiquarks that create the final hadron-state in thermal equilibrium [64]. In the ALCOR model, meson and baryon coalescence coefficients are represented by $C_M(i, j)$ and $C_B(i, j, k)$, respectively, where i, j , and k refer to the quark species numbers. Also a normalization factor and spin degeneracy factor $D^h = 2S_h + 1$ can be introduced in this model, where S_h is the hadron spin. Thus, the number of a certain type of meson that has flavors i and j is given as [64]

$$N_M^{(h)} = D^h C_M(i, j) b_{q_i} N_{q_i} b_{\bar{q}_j} N_{\bar{q}_j}, \quad (37)$$

where N_{q_i} and $N_{\bar{q}_j}$ are the number of quarks and antiquarks [65] and b_{q_i} and $b_{\bar{q}_j}$ are the corresponding parameters, respectively. The number of a certain baryon with flavors i, j , and k is given by

$$N_B^{(h)} = D^h C_B(i, j, k) b(i) b(j) b(k) N_q(i) N_q(j) N_q(k), \quad (38)$$

$$N_{\bar{B}}^{(h)} = D^h C_{\bar{B}}(i, j, k) \bar{b}(i) \bar{b}(j) \bar{b}(k) N_{\bar{q}}(i) N_{\bar{q}}(j) N_{\bar{q}}(k), \quad (39)$$

where $N_{q(i)}$ and $N_{\bar{q}(i)}$ are the number of quarks and antiquarks of type i , for instance. One can reformulate (39) as sum over i, j , and k for each hadron from 1 to n_f flavors. So that, in ALCOR model one can calculate the hadron multiplicity and compare between the model and the experimental results [65, 66].

Changing linear to nonlinear rehadronization coalescence model is doable. The linear coalescence model is based on the counting of quarks and the determination of probabilities in the heavy-ion collisions. It was assumed [67] that the number of produced particles is directly proportional to the product of constituent quarks in the reaction volume [68],

$$\begin{aligned} p &= a_p q^3, \\ \Lambda | \Sigma &= a_\Lambda q^2 s, \\ \Xi &= a_\Xi q s^2, \\ \Omega &= a_\Omega s^3. \end{aligned} \quad (40)$$

The antiparticles are straightforwardly constructed [68],

$$\begin{aligned} \bar{p} &= a_{\bar{p}} \bar{q}^3, \\ \bar{\Lambda} | \bar{\Sigma} &= a_{\bar{\Lambda}} \bar{q}^2 \bar{s}, \\ \bar{\Xi} &= a_{\bar{\Xi}} \bar{q} \bar{s}^2, \\ \bar{\Omega} &= a_{\bar{\Omega}} \bar{s}^3. \end{aligned} \quad (41)$$

The coalescence model can be used to predict the small width of the baryon-antibaryon BF [50]. It is observed that

FIGURE 9: The widths of balance functions are given as functions of $\Delta\eta$, $\Delta\phi$ for all charged particle pairs from Au+Au collisions. Left-hand panel shows centrality dependence, while the right-hand panel shows beam energy dependence of most central events (0–5%). The graph is taken from [34].

in the central heavy-ion collision at RHIC energies [25], the hadron constituents of quarks which are described by coalescence model [63] can explain the small pseudorapidity width of BF. Furthermore, the coalescence concept would explain cluster from pairs of charges,

$$u\bar{u} + d\bar{d} \longrightarrow u\bar{d} + d\bar{u}. \quad (42)$$

For the above processes, the momentum distribution for the two particles can be written as [50]

$$\begin{aligned} \rho(p, \bar{p}) = & \int dP_1 dP_2 \rho_c(P_1) \rho_c(P_2) \\ & \cdot \int dP_{u_1} d\bar{P}_{u_1} dp_{d_2} d\bar{p}_{d_2} f(P_1, p_{u_1}) \\ & \cdot f(P_1, \bar{p}_{u_1}) f(P_2, p_{d_2}) \end{aligned}$$

$$\begin{aligned} & \cdot f(P_2, \bar{p}_{d_2}) \delta \left[p_+ - \frac{(p_{u_1} + \bar{p}_{d_2})}{2} \right] \\ & \cdot \delta \left[p_- - \frac{(\bar{p}_{u_1} + p_{d_2})}{2} \right] G_m(p_{u_1} - \bar{p}_{d_2}) \\ & \cdot G_m(p_{d_1} - \bar{p}_{u_2}), \end{aligned} \quad (43)$$

where P_1 and P_2 are the momenta of the two clusters. The momenta of quarks and antiquarks are p and $\bar{p}$, respectively. $\rho_c(P)$ is the distribution of clusters and $f(P, p)$ and $f(P, \bar{p})$ are the cluster dissociation probabilities of finding a quark or antiquark of momentum p and/or $\bar{p}$ in the cluster, respectively. $G(P - \bar{p})$ is the coalescence probability, in which the quark-antiquark pair coalesce to create a hadron.

FIGURE 10: From ALICE and STAR experiments, the centrality dependence of the balance function width $\langle \Delta\eta \rangle$ (a) and $\langle \Delta\phi \rangle$ (b). The STAR results [35] have been corrected for the finite acceptance as suggested in [36]. The figure is taken from [37].

FIGURE 11: The centrality dependence of the relative decrease of BF width in relative pseudorapidity (a) and relative azimuthal angle (b). The ALICE results are compared with the results for the highest SPS [26, 27] and RHIC [35] energies. The figure is taken from [37].

Similarity, the distribution of baryon and antibaryon “three particles” distribution can be written as

$$\begin{aligned} \rho(p, \bar{p}) &= \int dP_1 dP_2 dP_3 \rho_c(P_1) \rho_c(P_2) \rho_c(P_3) \\ &\cdot \int d\bar{P}_1 d\bar{P}_2 d\bar{P}_3 \\ &\cdot f(P_1, p_1) f(P_2, p_2) f(P_3, p_3) \\ &\cdot f(P_1, \bar{p}_1) f(P_2, \bar{p}_2) f(P_3, \bar{p}_3) \end{aligned}$$

$$\begin{aligned} &\cdot \delta \left[p - \frac{(p_1 + p_2 + p_3)}{3} \right] \\ &\cdot \delta \left[\bar{p} - \frac{(\bar{p}_1 + \bar{p}_2 + \bar{p}_3)}{3} \right] \\ &\cdot G_B(p_1 - p_2, p_2 - p_3, p_3 - p_1) \\ &\cdot G_{\bar{B}}(\bar{p}_1 - \bar{p}_2, \bar{p}_2 - \bar{p}_3, \bar{p}_3 - \bar{p}_1), \end{aligned} \quad (44)$$

which is valid for each quark and antiquark [64]. This sums over the different number of flavors so that the number of quarks and antiquarks of type i is given by $N_q(i)$ and $N_{\bar{q}}(i)$, respectively,

$$\begin{aligned}
 N_q(i) &= \sum_i \sum_{j=1}^{N_f} \sum_{k=1}^{N_f} (1 + \delta_{i,j} + \delta_{i,k}) D^h C_B(i, j, k) \\
 &\quad \times b(i) b(j) b(k) N_q(i) N_q(j) N_q(k) \\
 &\quad + \sum_h \sum_{j=1}^{N_f} D^h C_M(i, j) b(i) \bar{b}(j) N_q(i) N_{\bar{q}}(j), \\
 N_{\bar{q}}(i) &= \sum_i \sum_{j=1}^{N_f} \sum_{k=1}^{N_f} (1 + \delta_{i,j} + \delta_{i,k}) D^h C_{\bar{B}}(i, j, k) \\
 &\quad \times \bar{b}(i) \bar{b}(j) \bar{b}(k) N_{\bar{q}}(i) N_{\bar{q}}(j) N_{\bar{q}}(k) \\
 &\quad + \sum_h \sum_{j=1}^{N_f} D^h C_M(i, j) \bar{b}(i) b(j) N_{\bar{q}}(i) N_q(j).
 \end{aligned} \tag{45}$$

The calculation of BF in the coalescence model has the ability to explain the small pseudorapidity width of BF observed for central heavy-ion collisions [63], where the parameter $\alpha = c^2/h^2$. For uncorrelated decay, $c/h \approx 0$.

4.2. Thermal Resonances. As discussed in previous sections, the STAR analysis of balance functions is based on multiplicities [25]

$$\begin{aligned}
 B(\Delta, Y) &= \frac{1}{2} \frac{\langle N_{+-}(\Delta) \rangle - \langle N_{++}(\Delta) \rangle}{N_+} \\
 &\quad + \frac{N_{-+}(\Delta) - \langle N_{--}(\Delta) \rangle}{N_-},
 \end{aligned} \tag{46}$$

where $N_{+-}(\Delta)$ counts the opposite-charge pairs having rapidity Y relative to $|y_2 - y_1| = \Delta$, at $Y \sim Y^{\max}$, and BF of all charged hadrons are normalized to unity. The separation of balancing charges at kinetic freeze-out is studied [69]. To characterize the possible contributions, we highlight that the $\pi^+\pi^-$ BF have two types of contributions corresponding to two different mechanisms of their creation. The resonances may come up with an additional contribution. The decay channels of neutral hadronic resonances likely lead to $\pi^+ - \pi^-$ pairs. Also, a nonresonance contribution is related to other correlations among the charged particles. The two opposite-charge particles are produced at the same space-time coordinates with thermal velocities. A neutral resonance ends up as a $\pi^+ - \pi^-$ pair, where, as in the nonresonance mechanism of charge balancing, a charged pion can be balanced with another charged hadron, not necessarily a pion [38]. In light of this, the $\pi^+\pi^-$ balance functions can be constructed as

$$B(\Delta, Y) = B_R(\Delta, Y) + B_{NR}(\Delta, Y). \tag{47}$$

The resonance contribution $B_R(\Delta, Y)$ is obtained from the expressions describing the phase-space of the pions emitted in a decay [38]. The calculation in the neutral clusters model [63] does not depend on the correlations between the clusters, themselves. But they are determined by the single-particle distribution, or by two-particle distribution in which the pair of particles can be formed from one cluster and others from different clusters [63]. Replacing the neutral clusters by the neutral resonances in order to obtain the two-particle rapidity distribution of the $\pi^+ - \pi^-$ pairs stemming from the decay of a neutral resonance, then the two-particle pion momentum distribution in two-body $\pi^+ - \pi^-$ resonance decay can be expressed by Dirac δ function

$$\rho_{R \rightarrow \pi^+ \pi^-} = \frac{b_{\pi\pi}}{N_2} \delta^{(4)}(p - p_1 - p_2), \tag{48}$$

where p , p_1 , and p_2 are total momentum, momentum of positive pion, and momentum of negative pion, respectively, and the $b_{\pi\pi}$ is the branching ratio. The normalization factor N_2 is given by [38]

$$N_2 = \int \frac{d^3 p_1}{E_1} \frac{d^3 p_2}{E_2} \delta^{(4)}(p - p_1 - p_2). \tag{49}$$

The correlation between nonresonance pions is not specified by the model introduced in [38]. It is assumed that the creation of an opposite pair occurs in the fireball cylinder; that is, the two charges have the same longitudinal and transverse collective velocity [38]. The results are shown in Figure 12. The calculations for four different centrality windows are compared to the STAR data [25].

4.3. Statistical and Dynamical Model. At top RHIC energies, an energy density can be as high as $\approx 10 \text{ GeV/fm}^3$. Apparently, this would cover a volume of several hundred fm^3 in the Au+Au collisions [25]. Therefore, quark and gluon degrees of freedom provide a description of the microscopic motion for several fm/c, until the matter expands and cools down till the hadronic degrees of freedom become appropriate [39]. The conversion from partonic to hadronic degrees of freedom accompanied by increasing production of quark antiquark pairs on the entropy stored in gluons and quarks is converted to hadrons, each of which has at least two quark. The change in the degrees of freedom accompanying the hadron-quark phase transition was revised in [70–75]. There newly created charges are more correlated to their anticharges than pairs created early [39]

$$\begin{aligned}
 B(P_2 | P_1) &\equiv \frac{1}{2} \frac{N_{+-}(P_1, P_2) - N_{++}(P_1, P_2)}{N_+(P_1)} \\
 &\quad + \frac{N_{-+}(P_1, P_2) - N_{--}(P_1, P_2)}{N_-(P_1)},
 \end{aligned} \tag{50}$$

where P_1 and P_2 are “the extra particle of the opposite charge with momentum P_2 given the observation of the first particle with momentum P_1 ” as stated in [39] and $+/-$ indices refer to particles or antiparticles, respectively. The balance functions

FIGURE 12: The balance functions for pions in the thermal model calculated for four different centralities are compared to data [25], $\delta \equiv \Delta$. The graph is taken from [38].

are designed as measure for the probability of observing an extra particle with opposite charge and momentum P_2 gives the observation of the first particle with momentum P_1 . P_1 refers to a particle observed anywhere in the detector, and P_2 refers to either the relative rapidity Δy or the relative momentum Q_{inv} . The STAR measurements were performed for all charged particles as functions of relative pseudorapidity and for identical pions as functions of relative rapidity [25]. The behavior of the balance function is compared between the STAR data [25] and the one calculated from the microscopic hadronic simulations, RQMD (relativistic quantum molecular dynamic) [76]. Figure 13 has shown the $\pi^+\pi^-$ balance functions from RQMD for p+p and Au+Au collisions compared to the STAR data [25].

4.4. Thermal Blast-Wave Model. The dynamical evolution of the system created in heavy-ion collisions can also be studied in the blast-wave model [77], which describes the kinetic freeze-out properties, in which the particles are thermalized at the kinetic freeze-out temperature [28]. The creation of particles in a very hot and dense matter has the features of explosion [78]. The explosion wave called blast wave due to sequential collisions. The hot and dense medium would be anisotropic so that the velocity of the particles is also

anisotropic [78]. Finally, the net-flow of velocity β can be estimated [78]. The model has eight parameters, R_x , R_y , T , ρ_0 , ρ_2 , a_s , τ_0 , and $\Delta\tau$, where R_x , R_y , and T are the radii of the transverse shape and the temperature, respectively. a_s is the surface diffuseness parameters. ρ_0 and ρ_2 are the radial and anisotropy flow parameters, respectively. The schematic diagram, Figure 14, shows the elliptic flow with R_x and R_y [40].

In principal, the thermal models can divide the balancing charges into resonant and nonresonant contributions [38]. The resonant contribution is dominated by the decays of the hadron resonances to create $\pi^+\pi^-$ in the most final state [38], while nonresonant contribution is dominated by other process or correlations between charges. Accordingly, BF can be expressed as [38]

$$B(\Delta y, y_w) = B_{\text{resonant}}(\Delta y, y_w) + B_{\text{non-resonant}}(\Delta y, y_w), \quad (51)$$

where $\Delta y = y_1 - y_2$ and y_w is the window size ranging from 1 to 4. The resonant contribution can be estimated from the cluster model [63]. While the nonresonant contribution can not be determined specifically. Božek et al. [38] proposed a form in which the charge-anticharge pair is created in a fireball cylinder [38]. BF calculated due to resonance and

FIGURE 13: $\pi^+\pi^-$ balance functions for RQMD are shown for both p+p and Au+Au collisions assuming a perfect detector. In contrast to the experimental results of [25], the balance functions are slightly broader for central Au+Au collisions. The graph is taken from [39].

FIGURE 14: The elliptical subshell of the source. Here, $R_y R_x$ are radii of the ellipse. The arrows represent the direction and magnitude of the elliptic flow. The graph is taken from [40].

nonresonance contributions [38] replace the neutral cluster [63] by neutral resonances. Then, the two-particle rapidity distribution for pair, for instance, pion pair, is obtained

$$\frac{dN_R^{+-}}{dy_1 dy_2} = \int dy dp_\perp^2 \cdot \int dp_1^\perp dp_2^\perp C_\pi \left(\frac{dN_R}{dy dp_\perp^2} \right) \rho_{R \rightarrow \pi^+ \pi^-}(p, p_1, p_2). \quad (52)$$

The nonresonant rapidity distribution is given as

$$\begin{aligned} \frac{dN_{NR}^{+-}}{dy_1 dy_2} &= A \int dp_1^\perp dp_2^\perp C_\pi \\ &\times \int d\Sigma(x) p_1 \cdot u(x) f_{NR}^\pi(p_1 \cdot u(x)) p_2 \cdot u(x) \\ &\cdot f_{NR}^\pi(p_2 \cdot u(x)). \end{aligned} \quad (53)$$

FIGURE 15: Resonance and nonresonance balance functions plotted as a function of the rapidity difference. δ refers to Δy , $\Delta y \equiv \delta y$. The graph is taken from [38].

From (52) and (53), the resonance and nonresonance BF for pion pairs can be calculated,

$$\begin{aligned} B_R(\Delta y) &= \frac{1}{N_\pi} \sum_R \int dy_1 dy_2 C_\pi \left(\frac{dN_R^{+-}}{dy_1 dy_2} \right) \delta(|y_2 - y_1| - \delta y), \\ B_{NR}(\Delta y) &= \frac{1}{N_\pi} \sum_{NR} \int dy_1 dy_2 C_\pi \left(\frac{dN_{NR}^{+-}}{dy_1 dy_2} \right) \delta(|y_2 - y_1| - \delta y), \end{aligned} \quad (54)$$

in which $N_\pi = (N_{\pi^+} + N_{\pi^-})/2$. The resonance and nonresonance balance functions are given in Figure 15.

In heavy-ion collisions, the quarks and gluons are under collective expansion; that is, geometric asymmetry of plane of the interaction can be studied as anisotropic flow, while the second coefficient is called the elliptic flow [40]. These contributions are Fourier expansion of the differential distribution

$$E \frac{d^3 N}{d^3 p} = \frac{1}{2\pi} \frac{d^2 N}{p_t dp_t dy} \left[1 + 2 \sum_{n=1}^{\infty} v_n \cos(n\phi - \Psi_{PR}) \right]. \quad (55)$$

The Fourier decomposition is given as [79]

$$1 + 2v_1 \cos(\phi - \Psi_{PR}) + 2v_2 \cos(2(\phi - \Psi_{PR})), \quad (56)$$

where v_1 is the directed flow, v_2 is the elliptic flow, and Ψ_{PR} is the real reaction plane [79]. The elliptic flow is essential probe to studying the evolution of the strongly interacting system and the flow fluctuations and balancing between created charges [80–82].

An extended blast wave model was introduced in order to investigate the effect of flow, in which a combination of elliptic flow with the transverse mass spectra and the two-charge correlation was introduced [79]. This blast wave model describes a specific particle elliptic flow that emitted through a finite thin shell. In order to determine the size of pions produced in the reaction, the model has to be extended

FIGURE 16: The balance functions for $\phi = 0^\circ$ (in-plane), $\phi = 45^\circ$, and $\phi = 90^\circ$ (out-of-plane) particles pairs. The 40–50% centrality bins are shown. The points are from the data (not corrected for event plane resolution), while solid lines represent the blast-wave model calculations. The graph is taken from [34].

through a filled cylinder. The significant idea of the extended blast-wave model is to describe the system in the freeze-out conditions in terms of the elliptic flow and temperature [83]. Some new parameters concerning the geometry of the system were introduced, as well [84, 85]. The new parameterization interprets the transverse mass spectra as mentioned above. The probabilities of emitting particles in the space-time X with momentum P can be written as [83]

$$F(X, P) = F(r, \phi_s, t, p_T, \phi_p, m) \\ = \Theta \left(1 - \frac{(r \cos(\phi_s))^2}{(R_x)^2} - \frac{(r \sin(\phi_s))^2}{(R_y)^2} \right) \quad (57) \\ \cdot K_1[(r, \phi_s, p_T)] e^{\beta(r, \phi_s, p_T) \cos(\phi_s, \phi_p)} e^{-t^2/\tau^2},$$

where Θ is the step function modelling the confinement of the system in the filled ellipse. The spatial and azimuthal momentum are ϕ_s and ϕ_p , respectively. The earlier gives the radii of the system in-plane while the latter gives the out-of-plane. Figure 16 shows BF calculated in the blast-wave model compared with STAR data at different azimuthal angles [34], while Figure 17 shows the blast-wave model calculations compared with midcentral, peripheral, and central collisions from STAR data [25].

For completeness, we add that the evolution of the system till the final state would be more convenient to be studied by the Hanbury Brown-Twiss (HBT) interferometry [86–88]. In that case, measured single- and two-particle correlations are essential inputs [89, 90]. The probability for a joint observation of the two quanta with momenta k_1 and k_2 and the correlation function are also studied [89].

FIGURE 17: The balance functions from 200A GeV Au+Au collisions measured by STAR are compared to the canonical blast-wave model described in the text. The model should set a lower bound for the width of a balance function provided that the particles are emitted thermally. The remarkable agreement with the data suggests that charge conservation remains highly localized at breakup. The graph is taken from [39].

4.5. Glue Cluster Model. The experimental results, for instance, from STAR [25, 35, 91] and NA49 [26, 27], should be understood that the charges are produced in a late stage of the hadronization process, that is, in freeze-out region [92]. This means that QGP mostly consisted of gluons, as well. The widths of BF in the central and peripheral collisions are different and also they are different from AA and pp collisions. It is argued that the system would need more correlations in the QGP phase exhibiting a clustering behavior. So that the glue clusters can explain the correlations in QGP. In momentum space, the width of BF can be determined by the short-range correlations as proposed by the STAR experiment [35]. It is believed that the small or narrow width of BF indicate how late is the stage of hadronization. Apparently, this was also measured by the STAR experiment and expected from different models like the coalescence model. The clusters decay to gluons and quark-antiquark pair, for instance, to up and antiup quarks. Both quarks should attempt to recombine again forming pions or any other kind of mesons. The cluster decay distribution is given by

$$\rho(\eta) = \frac{1}{2 (\cosh \eta)^2}. \quad (58)$$

The decay width $\langle |\eta| \rangle = \log 2$. Thus, the width of BF can be affected also by the transverse flow. The clusters are isotropic in their rest frame. However after the transverse flow of clusters they become no longer isotropic.

4.6. UrQMD. The ultrarelativistic quantum molecular dynamics (UrQMD) model is a microscopic model used to

FIGURE 18: (a) The balance function width $\langle \Delta\eta \rangle$ for all charged particles from Au+Au collisions at $\sqrt{s_{\text{NN}}} = 200$ GeV compared with the widths of BF calculated using shuffled events. The balance function widths for p+p and d+Au collisions at $\sqrt{s_{\text{NN}}} = 200$ GeV are also shown. Filtered UrQMD and HIJING calculations are shown for the widths of BF from Au+Au collisions. (b) The same as in (a) but for identified charged pions and charged kaons. The width of BF for pions predicted by the blast-wave model [39] is also shown. The figure is taken from [28].

simulate (ultra)relativistic heavy-ion collisions in the energy range from Bevalac to LHC. Main goals are to gain better understanding about the following physical phenomena within a single transport model:

- (i) creation of dense hadronic matter at high temperatures,
- (ii) properties of nuclear matter, delta and resonance matter,
- (iii) creation of mesonic matter and of antimatter,
- (iv) creation and transport of rare particles in hadronic matter,
- (v) creation, modification, and destruction of strangeness in matter,
- (vi) emission of electromagnetic probes.

Figures 18 and 19 show the balance function widths for pions and kaons and also the widths in terms of q_{long} , q_{side} , and q_{out} , respectively. All are compared to the STAR data for Au+Au collision at 200 GeV. Filtered HIJING calculations, Section 4.7, are also shown for the widths of BF from pp and Au+Au collisions.

4.7. HIJING. The heavy ion jet interaction generator (HIJING) was developed by Gyulassy and Wang [93] with

special emphasis on the role of minijets in proton-proton, proton-nucleus, and nucleus-nucleus interactions at collider energies. The perturbative QCD predicts jet production from parton scatterings in high energy hadronic interactions. It is therefore expected that hard or semihard parton scatterings with transverse momentum of a few GeV are expected to dominate high energy heavy ion collisions. The HIJING code has been widely distributed to experimental groups preparing for RHIC and LHC. HIJING is also used to investigate two effects, gluon shadowing, and jet quenching, in heavy ion collisions at RHIC [42]. The study of pA and AA collisions is required to separate between the two effects at RHIC. Therefore, the conclusions from such study will investigate the new physics of the gluon structure of nuclei and the energy loss in QGP. As introduced, the BF width in the rapidity representation can be defined as

$$\langle \Delta y \rangle = \frac{\int_0^{y_w} B(\Delta y y_w) \Delta y d y_w}{\int_0^{y_w} B(\Delta y | y_w) d \Delta y}. \quad (59)$$

HIJING can establish the existence of QGP by the simulation and extracting BF. But, HIJING lacks the collective flow description, so that generation of the balance function widths by HIJING is larger than that measured in experiments. Figure 20 represents the balance function widths from HIJING and the multitransport (AMPT) model with

FIGURE 19: The balance function width σ extracted from $B(q_{inv})$ for identified charged pions and kaons from Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV and pp collisions at $\sqrt{s_{NN}} = 200$ GeV where σ is the width. Filtered HIJING and UrQMD calculations are shown for pions and kaons from Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV. Values are shown for $\sqrt{2mT_{kin}}$ from Au+Au collisions, where m is the mass of a pion or a kaon, and T_{kin} is calculated from identified particle spectra [41]. The width predicted by the blast-wave model [39] is also shown for pions. The graph is taken from [28].

the data from ALICE [37]. Figure 21 [28] compares between BF calculated from HIJING and blast-wave model. The detailed HIJING results are discussed in [47].

4.8. PYTHIA. The PYTHIA is designed to generate high-energy-physics “events,” that is, sets of outgoing particles produced in the interactions between two incoming particles. The objective is to provide as accurate as possible a representation of event properties in a wide range of reactions, within and beyond the Standard Model, with emphasis on those where strong interactions play a role, directly or indirectly,

and therefore multihadronic final states are produced [94]. The PYTHIA 5.72 is an event generator; one can study the proton-proton collision events that are generated at different center of mass (c.m.) energies [45]. This can be shown clearly at different energies in Figure 22 [45]. Then the width of BF can be studied for different multiplicity bins.

The results presented in [28, 54] show that the string fragmentation implemented in PYTHIA describes the production particles and their charge balance functions. They deduced from measured $B(\delta\eta/\eta_w)$ at six different windows; $B(\delta\eta)$ for the six windows η_w coincides with each other. It was shown that the scaled balance functions is corresponding to BF in the whole pseudorapidity range $B(\Delta\eta | \infty)$ [54].

4.9. AMPT Model. A multiphase transport (AMPT) is a Monte Carlo transport model for heavy ion collisions at relativistic energies written in FORTRAN 77. It uses HIJING for generating the initial conditions, Zhang’s Parton Cascade (ZPC) for modelling the partonic scatterings, and a relativistic transport (ART) model for treating hadronic scatterings. The AMPT model consists of four parts [95], the initial conditions which are obtained from HIJING, partonic interactions, the transition from the partonic case to the hadronic matter case, and hadronic interactions. AMPT model uses the coalescence model to coalesce partons to create hadrons.

It was shown in [54] that BF do not depend on the size and position of the windows and are consistent with the results of pp in PYTHIA. The charge balance functions are boost-invariance in both hadron-hadron and nuclear interaction. The boost invariance can scale BF with the window size within the whole range of the rapidity. Therefore, BF are good measures free from the restriction of finite longitudinal acceptance. The dependence on transverse momentum of the longitudinal property of balance functions is a sensitive probe for charge balance in hadronization mechanism.

5. Discussion and Conclusions

The main topics of this review are the study of correlations between opposite-sign charge pairs. Together with the particle-ratio fluctuations these can provide a powerful tool to probe dynamics and properties of QGP beside hadronization and particle production. It has been suggested that the existence of a QCD phase transition would cause an increase and divergence of fluctuations. Thus the fluctuations could be used to study various particle/charge fluctuations near the QCD critical end point (CEP). On the other hand, BF, which measure the correlations between opposite-sign charge pairs, is sensitive to the mechanisms of charge formation and the subsequent relative diffusion of the balancing charges. Their study can provide information about charge creation time as well as the subsequent collective behavior of particles.

In this review, we have attempted to explain most of the important aspects of BF in high-energy physics. The various definitions are introduced and confronted to different experimental measurements and the effective models. The essential points we focused on is BF including the advantages and short-comings. Then, we have discussed the various

FIGURE 20: The centrality dependence of the widths of BF $\langle \Delta\eta \rangle$ and $\langle \Delta\phi \rangle$, for the correlations studied in terms of the relative pseudorapidity and the relative azimuthal angle, respectively. The data points are compared to the predictions from HIJING [42, 43] and AMPT [44]. The graph is taken from [37].

FIGURE 21: The balance functions in terms of Δy for identified charged pion pairs from Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV for nine centrality bins. The graph is taken from [28].

experimental measurements depending on different quantities, for example, the system size, centrality, and the beam energy. The theoretical models describing and calculating BF have been discussed.

FIGURE 22: The width of balance functions in the rapidity region $[-3, 3]$ for different multiplicities in pp collision at $\sqrt{s_{NN}} = 22, 64, 130, 200$ GeV. The graph is taken from [45].

Three main results can be extracted from this review. First, BF have been calculated in terms of rapidity, window size, and pseudorapidity as given in Figure 3. Second, BF in terms of the reaction centrality and the beam energy (center-of-mass energy) are shown in Figures 5 and 6. Third, BF

in terms of the invariant momentum q_{inv} are also studied. BF were measured in various system sizes, for example, Au+Au at $\sqrt{s_{\text{NN}}} = 200$ GeV in the STAR experiment [54] and Pb+Pb collisions at $\sqrt{s_{\text{NN}}} = 17.2$ GeV in the ALICE experiment [37]. Also the width of BF was measured in Pb+Pb, C+C, and Si+Si collisions at $\sqrt{s_{\text{NN}}} = 8.8$ and 17.2 GeV at SPS [16]. The calculations from different effective models have been calculated and compared with the data, Figures 12, 13, and 17. Recent results depending on the system size and centrality for all charged particles have been studied at $\sqrt{s_{\text{NN}}} = 17.3$ GeV for p-p, C-C, Si-Si, and Pb-Pb collisions [26, 27, 35]. The dependence on the rapidity and the beam energies are also studied [35, 96]. While HIJING and UrQMD models fail to reproduce the narrowing in the balance function width observed [35], AMPT does. The net-charge fluctuations are studied at LHC [97] for event-by-event net-charge fluctuations in terms of the pseudorapidity $\Delta\eta$ and azimuthal angle $\Delta\phi$ in Pb-Pb collisions at $\sqrt{s_{\text{NN}}} = 2.76$ TeV. The balance functions confronted to the STAR results show that the quark chemistry can be determined. The results agree within 20% with the expectations [33]. This provides quantitative highlights on the chemical evolution of the QGP, for example, the femtoscopy of two-particle correlations. This study should be extended with new experiment results from STAR, ALICE, CMS, and ATLAS [33].

The main conclusions can be summarized as follows:

- (i) the effective models are well suited to calculate the balance functions,
- (ii) the most important quantities are the rapidity and pseudorapidity,
- (iii) the balance functions are very sensitive to the interaction centrality but not for the beam energy,
- (iv) the balance function width seems to be related to the hadronization time,
- (v) the balance functions can estimate the hadronization time from the jets production in p+p collision,
- (vi) the phase transition from hadron to quark matter and the properties of such matter, the correlations between charge, and anticharge can be studied, directly.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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