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Research Article

P-V Criticality of Conformal Anomaly Corrected AdS Black Holes

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The effects of conformal anomaly on the thermodynamics of black holes are investigated in this paper from the perspective of P-V criticality of AdS black holes. Treating the cosmological constant as thermodynamic pressure, we extend the recent research to the extended phase space. Firstly, we study the P-V criticality of the uncharged AdS black holes with conformal anomaly and find that conformal anomaly does not influence whether there exists Van der Waals like critical behavior. Secondly, we investigate the P-V criticality of the charged cases and find that conformal anomaly influences not only the critical physical quantities but also the ratio $P_c r_c/T_c$. The ratio is no longer a constant as before but a function of conformal anomaly parameter $\tilde{\alpha}$. We also show that the conformal parameter should satisfy a certain range to guarantee the existence of critical point that has physical meaning. Our results show the effects of conformal anomaly.

1. Introduction

Conformal anomaly is such an important concept that it has lots of applications in quantum field theory, cosmology, and black hole physics. So it would be of great interest to investigate its effects on the thermodynamics of black holes. Many efforts have been devoted to this issue. Cai et al. [1] investigated the thermodynamics of static and spherically symmetric black holes with conformal anomaly. In addition to Bekenstein-Hawking area entropy, an extra logarithmic correction was found. The entropy was also studied by Li who took an approach of quantum tunneling [2]. Considering the back reaction through the conformal anomaly, phase transitions of Schwarzschild black hole were investigated [3] and an additional phase transition was discovered. Ehrenfest equations were also studied [4]. Moreover, we studied the phase structures of black holes with conformal anomaly [5] and found that they have much richer phase structures than black holes without conformal anomaly. Recently, Cai [6] obtained analytical AdS black hole solutions with conformal anomaly. Hawking-Page transition was investigated and novel features due to conformal anomaly were found in black holes with a negative constant curvature horizon. Although many efforts have been made, all of them were carried out in the nonextended phase space. In this paper, we would like to extend the research of [6] to the extended phase space, hoping to observe more evidences of the effects of conformal anomaly on the thermodynamics of black holes.

The extended phase space refers to the phase space which includes the pressure and volume as thermodynamic variables. Thermodynamics in the extended phase space has gained more and more attention recently [7–39]. The main idea is to treat the cosmological constant as thermodynamic pressure and the conjugate quantity as volume. The motivation comes from the consideration of variation of cosmological constant to make the first law of black hole thermodynamics consistent with the Smarr relation. The critical behavior of black holes in the extended phase space, especially P-V criticality, has been extensively investigated [13–37]. For nice review, see [28, 39].

The outline of this paper is as follows. In Section 2, thermodynamics of AdS black holes with conformal anomaly will be briefly studied in the extended phase space. Then P-V criticality of the uncharged AdS black holes with conformal

anomaly will be investigated in Section 3 while the charged cases will be discussed in Section 4. To observe the effects of conformal anomaly, comparisons will be made between AdS black holes with conformal anomaly and those without it. In the end, conclusions will be drawn in Section 5.

2. Thermodynamics of Conformal Anomaly Corrected AdS Black Hole in the Extended Phase Space

Trace anomaly of the stress-energy tensor of conformal field theory can be caused by one loop quantum correction in four-dimensional spacetime. The trace anomaly consists of two parts. One is type B anomaly while the other is type A anomaly. Its expression reads

$$g^{\mu\nu}\left\langle T_{\mu\nu}\right\rangle = \beta I_4 - \alpha E_4, \tag{1}$$

where $I_4 = C_{\mu\nu\lambda\sigma}C^{\mu\nu\lambda\sigma}$, $E_4 = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\lambda\sigma}R^{\mu\nu\lambda\sigma}$, and β and α are two positive constants related to the content of conformal field theory.

Solving the Einstein equation

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G\left\langle T_{\mu\nu}\right\rangle, \tag{2}$$

where $\langle T_{\mu\nu}\rangle$ corresponds to the stress-energy tensor related to the trace anomaly, the conformal anomaly corrected AdS black holes' metric can be written as [6]

$$ds^2 = -f(r)\,dt^2 + f(r)^{-1}\,dr^2 + r^2 d\Omega_{2k}^2, \tag{3}$$

where

$$f(r) = k - \frac{r^2}{4\tilde{\alpha}}\left(1 - \sqrt{1 + \frac{8\tilde{\alpha}}{l^2} - \frac{16\tilde{\alpha}GM}{r^3} + \frac{8\tilde{\alpha}Q^2}{r^4}}\right). \tag{4}$$

$d\Omega_{2k}^2$ denotes the line element of two-dimensional Einstein space with constant scalar curvature $2k$, where k can be taken as $0, 1$ or -1 without loss of generality [6]. It was argued that the integration constants M and Q should be interpreted as the mass of black holes and the $U(1)$ conserved charge of the conformal field theory which leads to the anomaly, respectively [6]. When $\tilde{\alpha} \to 0$ [6], the solution reduces to the Reissner-Nordström-AdS (RN-AdS) black holes as

$$f(r) = k + \frac{r^2}{l^2} - \frac{2GM}{r} + \frac{Q^2}{r^2}. \tag{5}$$

One can obtain the event horizon radius r_+ by solving the equation $f(r_+) = 0$ for the largest root. With the event horizon radius, one can derive the expression of the mass of black holes as

$$M = \frac{r_+^4 + kl^2r_+^2 - 2k^2l^2\tilde{\alpha} + l^2Q^2}{2l^2r_+}. \tag{6}$$

Note that G has been set to one.

The Hawking temperature can be obtained as

$$\begin{aligned} T &= \frac{f'(r_+)}{4\pi} \\ &= \frac{r_+}{4\pi\left(r_+^2 - 4k\tilde{\alpha}\right)}\left(k + \frac{3r_+^2}{l^2} - \frac{Q^2}{r_+^2} + \frac{2k^2\tilde{\alpha}}{r_+^2}\right). \end{aligned} \tag{7}$$

Note that the Hawking temperature is obtained by requiring the absence of conical singularity at the horizon in the Euclidean sector of the black hole solution.

The entropy can be derived as

$$S = \int \frac{1}{T}\left(\frac{\partial M}{\partial r_+}\right)dr_+ = \pi r_+^2 - 8\pi k\tilde{\alpha}\ln r_+ + S_0. \tag{8}$$

Thermodynamic pressure and volume in the extended phase space can be defined as

$$P = -\frac{\Lambda}{8\pi} = \frac{3}{8\pi l^2}, \tag{9}$$

$$V = \left(\frac{\partial M}{\partial P}\right)_{S,Q}. \tag{10}$$

Note that the second equation in (9) holds for four-dimensional black holes. For $(n+1)$-dimensional AdS black hole, the cosmological constant satisfies the relation $\Lambda = -n(n-1)/2l^2$, where l represents the AdS radius. Specifically for four-dimensional case, $\Lambda = -3/l^2$.

Utilizing (6) and (9), one can reorganize the expression of the mass as

$$M = \frac{8P\pi r_+^4 + 3kr_+^2 - 6k^2\tilde{\alpha} + 3Q^2}{6r_+}, \tag{11}$$

from which one can easily derive the thermodynamic volume as

$$V = \frac{4\pi r_+^3}{3}. \tag{12}$$

Note that the entropy gains an extra logarithmic term due to the effect of conformal anomaly while the thermodynamic volume is the same as that of RN-AdS black holes. It is quite easy to verify that the first law of black hole thermodynamics in the extended phase space and the Smarr relation can be written as

$$\begin{aligned} dM &= TdS + \Phi dQ + VdP, \\ M &= 2TS + \Phi Q - 2VP, \end{aligned} \tag{13}$$

where

$$\Phi = \left(\frac{\partial M}{\partial Q}\right)_{S,P} = \frac{Q}{r_+}. \tag{14}$$

From (13), one can see clearly again the first law of black hole thermodynamics matches the Smarr relation quite well. That is one reason why one should introduce the extended phase space where the cosmological constant is viewed as thermodynamic pressure. Otherwise, the first law $dM = TdS + \Phi dQ$ does not match the Smarr relation.

3. P-V Criticality of Uncharged AdS Black Holes with Conformal Anomaly

In this section, we focus on the uncharged case $Q = 0$. Substituting (9) and $Q = 0$ into (7), we obtain the Hawking temperature for the uncharge case as

$$T = \frac{kr_+^2 + 8P\pi r_+^4 + 2k^2\tilde{\alpha}}{4\pi r_+\left(r_+^2 - 4k\tilde{\alpha}\right)}, \tag{15}$$

from which we can derive the equation of state as

$$P = \frac{T}{2r_+} - \frac{k}{8\pi r_+^2} - \frac{2kT\tilde{\alpha}}{r_+^3} - \frac{2k^2\tilde{\alpha}}{8\pi r_+^4}. \tag{16}$$

The critical point can be defined as follows:

$$\left.\frac{\partial P}{\partial r_+}\right|_{r=r_c,T=T_c} = 0, \tag{17}$$

$$\left.\frac{\partial^2 P}{\partial r_+^2}\right|_{r=r_c,T=T_c} = 0, \tag{18}$$

where the subscript "c" denotes the physical quantities at the critical point. Utilizing (16) and (17), one can get

$$T_c = \frac{kr_c^2 + 4k^2\tilde{\alpha}}{2\pi r_c\left(r_c^2 - 12k\tilde{\alpha}\right)}. \tag{19}$$

Solving (18) and then substituting (19) into the result, one can obtain

$$k\left(r_c^4 + 24k\tilde{\alpha}r_c^2 - 48k^2\tilde{\alpha}^2\right) = 0. \tag{20}$$

Except $k = 0$, (20) has two positive roots for r_c, namely,

$$r_c = 2\sqrt{\left(-3k \mp 2\sqrt{3}\right)k\tilde{\alpha}}, \tag{21}$$

where "−" corresponds to the case $k = -1$ while "+" corresponds to the case $k = 1$.

Note that, for $k = 0$, the critical Hawking temperature is zero and we would not consider this case for physical consideration.

Substituting (21) back into (19), one can derive that

$$T_c = -\frac{\sqrt{\left(3 \mp 2\sqrt{3}\right)k\tilde{\alpha}}}{12\pi\tilde{\alpha}}, \tag{22}$$

where "−" corresponds to the case $k = -1$ while "+" corresponds to the case $k = 1$. It is obvious that the Hawking temperature at these two critical points is negative. So these two critical points do not make any sense physically and there would be no Van der Waals like critical behavior for the uncharged case. This is in accord with Schwarzschild AdS black holes, implying that the conformal anomaly does not influence whether there exists Van der Waals like critical behavior.

4. P-V Criticality of Charged AdS Black Holes with Conformal Anomaly

Utilizing (7) and (9), one can obtain the equation of state of charged AdS black holes with conformal anomaly as

$$P = \frac{T}{2r_+} - \frac{k}{8\pi r_+^2} - \frac{2kT\tilde{\alpha}}{r_+^3} + \frac{Q^2 - 2k^2\tilde{\alpha}}{8\pi r_+^4}. \tag{23}$$

Substituting (23) into (17), one can get

$$T_c = \frac{kr_c^2 + 4k^2\tilde{\alpha} - 2Q^2}{2\pi r_c\left(r_c^2 - 12k\tilde{\alpha}\right)}. \tag{24}$$

Utilizing (18), (23), and (24), one can obtain

$$kr_c^4 + \left(24k^2\tilde{\alpha} - 6Q^2\right)r_c^2 - 48k^3\tilde{\alpha}^2 + 24kQ^2\tilde{\alpha} = 0. \tag{25}$$

Solving the above equation, one can obtain the positive roots as

$$r_c = \sqrt{\frac{3Q^2}{k} - 12k\tilde{\alpha} \mp \frac{\sqrt{3}\sqrt{3Q^4 - 32k^2Q^2\tilde{\alpha} + 64k^4\tilde{\alpha}^2}}{k}}, \tag{26}$$

where "−" corresponds to r_{c1} while "+" corresponds to r_{c2}. Note that we also omit the case $k = 0$ when we solve (25) because the Hawking temperature is negative.

When $k = 1$, $\tilde{\alpha} = 0$, (26) reduces to $r_c = \sqrt{6}Q$, recovering the result of RN-AdS black holes in former literature [13]. When $\tilde{\alpha} \neq 0$, one can easily draw the conclusion from (26) that the location of critical point relies on the conformal anomaly parameter $\tilde{\alpha}$. To observe the effect of conformal anomaly, we fix $k = 1$, $Q = 1$ and show the behavior of r_c for different $\tilde{\alpha}$ in Figure 1(a). The black holes with conformal anomaly have much richer phase structure. They may have none, one or two critical points due to different $\tilde{\alpha}$. However, one should check the Hawking temperature to make sure whether the critical points are in the physical region. The positive Hawking temperature means

$$T_c\big|_{k=1,Q=1} = \frac{r_c^2 + 4\tilde{\alpha} - 2}{2\pi r_c\left(r_c^2 - 12\tilde{\alpha}\right)} > 0, \tag{27}$$

which can be solved as

$$\begin{gathered} r_c > A \\ \text{or } 0 < r_c < B, \end{gathered} \tag{28}$$

where $A = \max\{2\sqrt{3}\tilde{\alpha}, \sqrt{2 - 4\tilde{\alpha}}\}$ and $B = \min\{2\sqrt{3}\tilde{\alpha}, \sqrt{2 - 4\tilde{\alpha}}\}$. To witness the constraint of the Hawking temperature intuitively, we plot Figure 1(b). From (28) and Figure 1(b), we find that only one critical point exists under the constraining condition of Hawking temperature. And the conformal anomaly parameter $\tilde{\alpha}$ should satisfy $0 < \tilde{\alpha} < 1/8$.

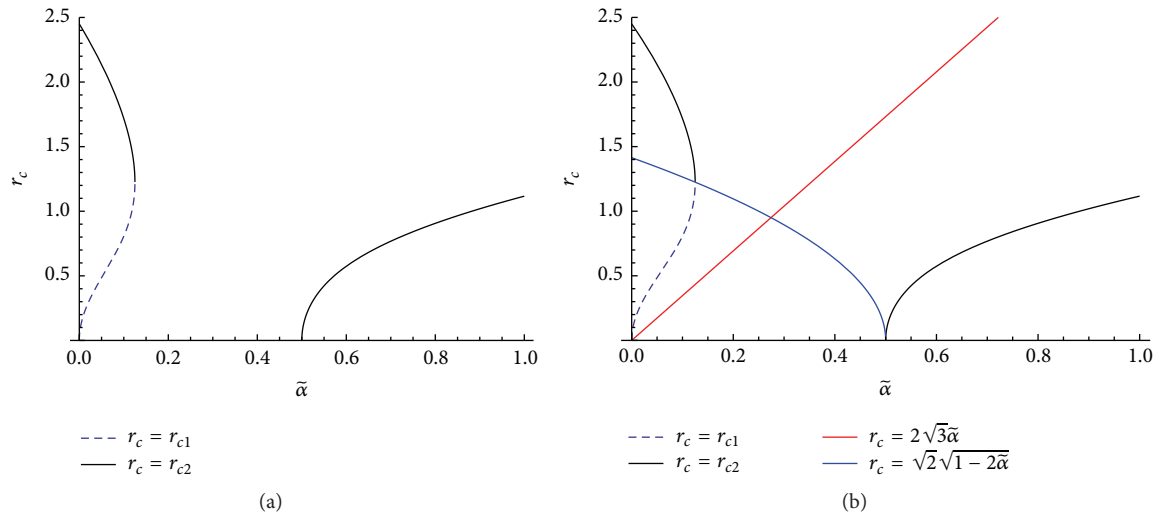

Figure 1: (a) r_c versus $\tilde{\alpha}$ for $k = 1, Q = 1$. (b) Curve constrained by positive Hawking temperature.

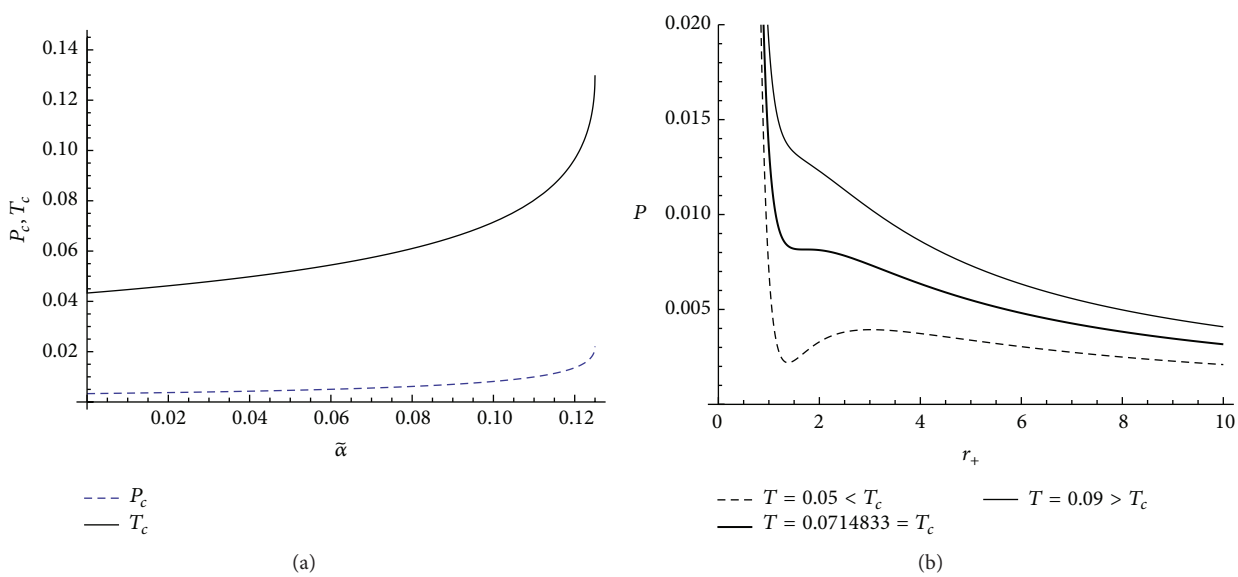

Figure 2: (a) P_c, T_c versus $\tilde{\alpha}$ for $k = 1, Q = 1$. (b) P versus r_+ for $k = 1, Q = 1, \tilde{\alpha} = 0.1$.

From (23), (26), (27), and Figure 1(b), one can derive the explicit expressions for all the critical quantities as

$$\begin{aligned} r_c|_{k=1,Q=1} &= \sqrt{3 - 12\tilde{\alpha} + \sqrt{9 - 96\tilde{\alpha} + 192\tilde{\alpha}^2}}, \\ T_c|_{k=1,Q=1} &= \frac{\sqrt{3 - 12\tilde{\alpha} + \sqrt{9 - 96\tilde{\alpha} + 192\tilde{\alpha}^2}}}{192\pi\tilde{\alpha}^2}\left(3 \right. \\ &\quad \left. - 16\tilde{\alpha} - \sqrt{9 - 96\tilde{\alpha} + 192\tilde{\alpha}^2}\right), \\ P_c|_{k=1,Q=1} &= \frac{6 - 18\tilde{\alpha} + \sqrt{9 - 96\tilde{\alpha} + 192\tilde{\alpha}^2}}{24\pi\left(3 - 12\tilde{\alpha} + \sqrt{9 - 96\tilde{\alpha} + 192\tilde{\alpha}^2}\right)^2}. \end{aligned} \tag{29}$$

The behavior of P_c and T_c is depicted in Figure 2(a). From Figures 1(b) and 2(a), we can see clearly that, with the increasing of $\tilde{\alpha}$, both T_c and P_c increase while r_c decreases. $P-r_+$ diagram for a specific case $\tilde{\alpha} = 0.1$ is shown in Figure 2(b). When the temperature is lower than the critical temperature, the isotherm not only has stable small radius branch and stable large radius branch but also has the unstable medium radius branch. However, when the temperature is higher than the critical temperature, the above phenomenon disappears.

We also plot P-T curve in Figure 3 for three different choices of $\tilde{\alpha}$. Namely, $\tilde{\alpha} = 0$, $\tilde{\alpha} = 0.05$, and $\tilde{\alpha} = 0.1$. Similar curves as the P-T curve of liquid-gas phase transition can be observed. When $\tilde{\alpha} = 0$, the curve recovers that of RN-AdS black hole shown in former literature [13]. The effect of conformal anomaly is reflected in different endpoints of the curves. These endpoints correspond to the critical points. Since we have demonstrated analytically that physical quantities vary with the conformal anomaly parameter, the graphical results match the analytical results quite well.

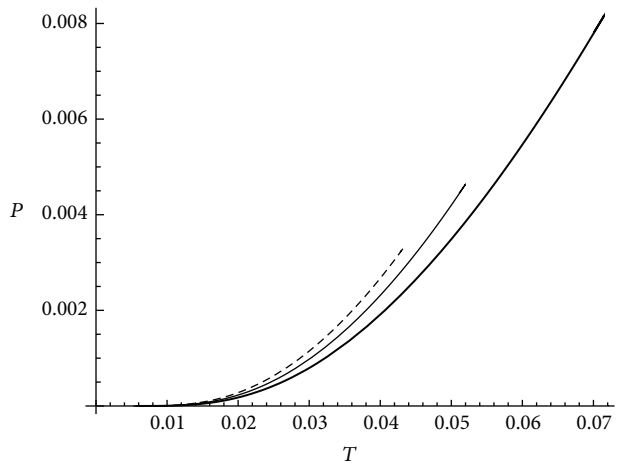

FIGURE 3: P versus T when $k = 1$, $Q = 1$. Dashed curve for $\tilde{\alpha} = 0$, common curve for $\tilde{\alpha} = 0.05$, and thick curve for $\tilde{\alpha} = 0.1$.

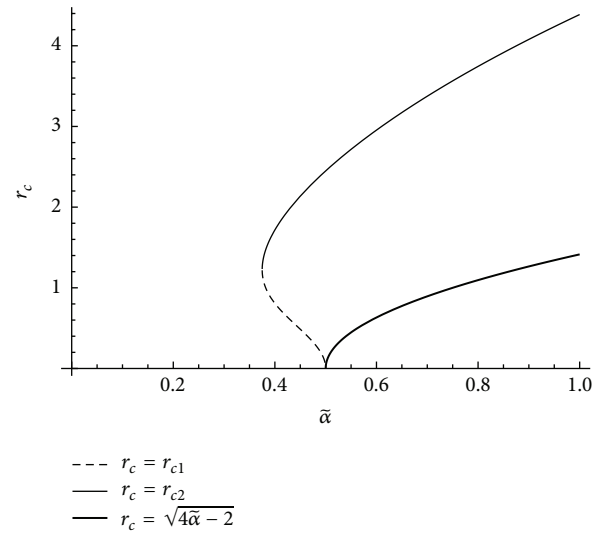

FIGURE 4: r_c versus $\tilde{\alpha}$ for $k = -1$, $Q = 1$.

Now the ratio $P_c r_c / T_c$ can be calculated as

$$\left.\frac{P_c r_c}{T_c}\right|_{k=1,Q=1} = \frac{21 - 48\tilde{\alpha} - \sqrt{9 - 96\tilde{\alpha} + 192\tilde{\alpha}^2}}{96\,(1 - 2\tilde{\alpha})}. \tag{30}$$

When $\tilde{\alpha} = 0$, (30) reduces to $3/16$, recovering the result of RN-AdS black holes [13]. When $\tilde{\alpha} \neq 0$, the ratio is no longer a constant but a function of $\tilde{\alpha}$, showing the effect of conformal anomaly.

The case $k = -1$ can be discussed similarly. The positive Hawking temperature means

$$T_c\big|_{k=-1,Q=1} = \frac{-r_c^2 + 4\tilde{\alpha} - 2}{2\pi r_c\left(r_c^2 + 12\tilde{\alpha}\right)} > 0, \tag{31}$$

which can be solved as

$$0 < r_c < \sqrt{4\tilde{\alpha} - 2}. \tag{32}$$

And the curve of r_c versus $\tilde{\alpha}$ is depicted in Figure 4, which shows that no critical points exist under the constraint of the positive Hawking temperature.

5. Conclusions

To observe the effects of conformal anomaly on the thermodynamics of black holes, we focus on the P-V criticality of AdS black holes with conformal anomaly. Specifically, we probe whether conformal anomaly influences the existence of Van der Waals like critical behavior and the critical physical quantities. Treating the cosmological constant as thermodynamic pressure, we extend the former research to the extended phase space. The entropy gains an extra logarithmic term due to the effect of conformal anomaly while the thermodynamic volume is the same as that of RN-AdS black holes.

Firstly, we study the P-V criticality of the uncharged AdS black holes with conformal anomaly. There are two positive roots of r_c. However, the Hawking temperature at these two critical points is negative. So these two critical points do not make any sense physically and there would be no Van der Waals like critical behavior for the uncharged case. This is in accord with Schwarzschild AdS black holes, implying that the conformal anomaly does not influence whether there exists Van der Waals like critical behavior. Secondly, we investigate the P-V criticality of the charged cases and find that conformal anomaly influences not only the critical quantities but also the ratio $P_c r_c / T_c$. When $k = 1$, with the increasing of conformal anomaly parameter $\tilde{\alpha}$, both T_c and P_c increase while r_c decreases. The ratio $P_c r_c / T_c$ is no longer a constant as before but a function of $\tilde{\alpha}$. The above results show the effects of conformal anomaly. When $k = -1$, there exist no critical points that make any sense physically.

In the context of AdS black holes, P-V criticality suggests the existence of first order small/large black hole phase transition below the critical temperature and second order phase transition at the critical point. It resembles the liquid/gas phase transition. By studying the case of nonzero alpha, we show more characteristics other than the common characteristics shared by AdS black holes. Not only the critical physical quantities but also the ratio $P_c r_c / T_c$ is influenced due to the effect of conformal anomaly, suggesting that conformal anomaly may affect the small/large black hole phase transition. On the other hand, the curves show that the critical volume/radius exists only for some range of alpha. In this paper, we have derived the explicit expression of this range. When the conformal anomaly parameter is above this range, one can not find root of critical point that has physical meaning. The physics is not difficult to explain. When $\tilde{\alpha} \to 0$, the black hole solution reduces to the RN-AdS black hole. So when the conformal anomaly parameter is small enough, one may expect its critical behavior is not affected too much and similar to RN-AdS black hole. However, when the conformal anomaly parameter is above certain range, the effect of conformal anomaly begins to play an important role, leading to different behavior from RN-AdS black hole.

On the other hand, the phenomenological implications of our findings should be further probed in the future so as to predict what is expected to be observed if one can handle-create-measure AdS black holes.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

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