Research Article

Quintessence and Holographic Dark Energy in $f(T)$ Gravity

M. Zubair

Department of Mathematics, COMSATS Institute of Information Technology, Lahore 54000, Pakistan

Correspondence should be addressed to M. Zubair; mzubairkk@gmail.com

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We regard $f(T)$ theory as an efficient tool to explain the current cosmic acceleration and associate its evolution with the known dark energy models. The numerical scheme is applied to reconstruct $f(T)$ theory from dark energy model with constant equation of state parameter and holographic dark energy model. We set the model parameters ω_0 and c as describing the different evolution eras and show the distinctive behavior of each case realized in $f(T)$ theory. We also present the future evolution of reconstructed $f(T)$ and find that it is consistent with the recent observations.

1. Introduction

The development of cosmology and gravitation can be seen as one of the scientific triumphs of the twentieth century. Since 1998, when observations of supernova type Ia [1, 2] pointed accelerated cosmic expansion, various observational measurements [3, 4] have affirmed such evolutionary change in the history of the universe. In spite of tremendous efforts, late cosmic acceleration is certainly a major challenge for cosmologists. In the current scenario, the unknown form of energy component, usually named as dark energy (DE), is said to be responsible for such mechanism. DE is recognized by its distinctive nature from ordinary matter sources having negative pressure which may lead to cosmic expansion counterstriking the gravitational pull.

DE has appeared as enigmatic cosmic ingredient and interpretation of its gravitational effects is a dynamic research field. There are two representative directions to address the issue of cosmic acceleration. Introducing an exotic cosmic fluid in the framework of the Einstein gravity is one direction to deal with such issue. The most likely theoretical campaigner of DE is the cosmological constant Λ characterized by constant EoS [5]. A number of alternative models have been proposed in this perspective to explain the role of DE in the present cosmic acceleration [6–11]. The issue of cosmic acceleration can also be counted on the basis of modified theories of gravity like $f(R)$ gravity [12], $f(T)$ gravity (T is

the torsion scalar) [13], $f(R, \mathcal{T})$ gravity ($\mathcal{T}$ is the trace of energy-momentum tensor $T_{\alpha\beta}$) [14–17], $f(R, \mathcal{T}, \mathcal{Q})$ gravity (where $\mathcal{Q} = R_{\alpha\beta}T^{\alpha\beta}$) [18–20], Gauss-Bonnet gravity [21], and scalar-tensor theories [22].

Recently, $f(T)$ gravity a generalization of the action of teleparallel gravity theory has gained special attention. This theory assumes Weitzenböck connection instead of the Levi-Civita connection which has no curvature but nonzero torsion. $f(T)$ theory has gained more importance to explain the accelerated cosmic expansion without introducing DE components [23, 24]. Wu and Yu [25] discussed the dynamical behavior of $f(T)$ theory for a concrete power law model that has a stable de Sitter phase along with an unstable radiation dominated phase and an unstable matter dominated one. In recent literature, various aspects have been discussed such as local Lorentz invariance [26], static spherically symmetric solutions [27], wormhole solutions [28], Noether symmetries [29, 30], cosmographic analysis [31], thermodynamics [32], and phase space analysis [33]. Another significant feature in modified theories of gravity is the choice of Lagrangian such as $f(R)$ or $f(T)$ function. Wu and Yu [34] proposed two new models in $f(T)$ theory to explain the phantom crossing of effective equation of state parameter. They also constrain the model parameters according to recent observations.

Cosmological reconstruction has appeared as one of the promising aspects in modified theories. In this perspective, different schemes have been proposed for known cosmic

evolutions to find the corresponding particular Lagrangian [35–44]. Nojiri et al. [35] developed a general reconstruction scheme for $f(R)$ gravity and formulated different epochs in FRW cosmology including matter dominated phase, transition from deceleration to acceleration, accelerating epoch, and Λ CDM phase. In this study, one interesting way is to consider the known cosmic evolution and use the field equations to find particular form of Lagrangian that can reproduce the given evolution background. Nojiri et al. [36] executed such reconstruction scheme in order to find some realistic models in $f(R)$ theory which was applied in $f(R, G)$ and modified GaussBonnet theories [37]. Capozziello et al. [39] proposed an efficient scheme to reconstruct $f(R)$ gravity from the known Hubble parameter in terms of redshift. In this procedure, one needs to know the relation of $H(z)$ for a given DE model so that $f(R)$ Lagrangian can be reconstructed implying the same dynamics. This scheme is also applied for a class of holographic dark energy (HDE) models [40–42]. The finite-time future singularities may appear for the universe dominated by dark energy and these are classified in four types [43]. Such singularities have been discussed in $f(R)$ and other modified theories [44, 45].

Bamba et al. [45] reconstructed power-law, exponential and logarithmic $f(T)$ cosmological models with realizing early inflation, Λ CDM, little rip, and pseudo rip cosmology. The issue of finite-time future singularities is also discussed in $f(T)$ gravity and it is shown that power-law type correction terms can be useful to get rid of such singularities. Setare and Mohammadipour [46] developed dynamical system to reconstruct $f(T)$ function for a background Λ CDM cosmology. They investigated the stability condition with respect to the homogeneous scalar perturbations in different cosmological eras. Cosmological reconstruction corresponding to holographic DE (HDE) model has also been studied in the literature. Daouda et al. [47] developed the reconstruction of $f(T)$ gravity according to HDE by imposing the initial condition on constant parameter. The holographic $f(T)$ gravity model with power-law entropy correction is also studied in [48], where model parameters are fitted using the latest observational data including type Ia supernovae, baryon acoustic oscillations, cosmic microwave background, and Hubble parameter data. It is shown that in the power-law entropy-corrected holographic $f(T)$ gravity model, the universe begins a matter dominated phase and approaches a de Sitter regime at late times. The equation of state parameter has appeared as a significant tool and various people discussed the properties of $f(T)$ models regarding the crossing of phantom divide line. In [49], Wu and Yu proposed two $f(T)$ models realizing the phantom crossing and discussed the observational parameters. Bamba et al. [50] presented the evolution of effective equation of state for the $f(T)$ models involving exponential and logarithmic functions. They also discussed the $f(T)$ model with combination of exponential and logarithmic functions and it was found that this model can cross the phantom divide line. The cosmological observations favor such feature of $f(T)$ models.

In this paper we are interested in reconstructing some $f(T)$ models for known cosmic evolution which has been

discussed in the literature. Following [39], we apply the numerical reconstruction scheme to develop the $f(T)$ models corresponding to quintessence and HDE models. The future evolution of $f(T)$ function is also presented for different choices of essential parameters. The paper has the following format: in the next section, we overview the $f(T)$ theory and present the modified field equations. We reconstruct the $f(T)$ theory as an effective description to known DE models and present their corresponding evolution. Section 3 summarizes our findings.

2. Reconstruction of $f(T)$ Theory

The action of $f(T)$ gravity with matter Lagrangian $\mathcal{L}_m$ reads

$$\mathcal{S} = \int dx^4 e \left[\frac{1}{2\kappa^2} f(T) + \mathcal{L}_m \right], \quad (1)$$

where $e = \text{diag}(e_\mu^i) \sqrt{-g}$, $\kappa^2 = 8\pi G = 1$, and T is the torsion scalar defined by the relation

$$T = T_{\mu\nu}^{\rho} S_{\rho}^{\mu\nu}, \quad (2)$$

where

$$\begin{aligned} T_{\mu\nu}^{\rho} &= e_i^{\rho} (\partial_{\mu} e_{\nu}^i - \partial_{\nu} e_{\mu}^i), \\ S_{\rho}^{\mu\nu} &= \frac{1}{2} (K_{\rho}^{\mu\nu} + \delta_{\rho}^{\mu} T_{\alpha}^{\alpha\nu} - \delta_{\rho}^{\nu} T_{\alpha}^{\alpha\mu}), \end{aligned} \quad (3)$$

and $K_{\rho}^{\mu\nu}$ is the contorsion tensor

$$K_{\rho}^{\mu\nu} = -\frac{1}{2} (T_{\rho}^{\mu\nu} - T_{\rho}^{\nu\mu} - T_{\rho}^{\mu\mu}) \quad (4)$$

which equals the difference between Weitzenböck and Levi-Civita connections. The variation of the action with respect to the vierbein vector field e_i^{μ} presents

$$\begin{aligned} &e^{-1} \partial_{\mu} (e S_i^{\mu\nu}) f'(T) - e_i^{\lambda} T_{\mu\lambda}^{\rho} S_{\rho}^{\nu\mu} f'(T) \\ &+ S_i^{\nu\mu} \partial_{\mu} (T) f''(T) + \frac{1}{4} e_i^{\nu} f(T) \\ &= \frac{\kappa^2}{2} e_i^{\rho} T_{\rho}^{(m)\nu}, \end{aligned} \quad (5)$$

where $T_{\mu\nu}^{(m)}$ is the matter energy momentum tensor and the prime denotes the differentiation with respect to T .

We take the homogeneous and isotropic flat FRW metric defined as

$$ds^2 = dt^2 - a^2(t) (dx^2 + dy^2 + dz^2), \quad (6)$$

where $a(t)$ represents the scale factor and corresponding tetrad components are $e_{\mu}^i = (1, a(t), a(t), a(t))$. Using relations (2)–(4), we obtain the torsion scalar $T = -6H^2$, where H is the Hubble parameter. We consider the matter

content described by an energy momentum tensor of perfect fluid.

The substitution of $i = 0 = \nu$ and $i = 1 = \nu$ in (5) results in the following Friedmann equations:

$$\begin{aligned} -Tf'(T) + \frac{1}{2}f(T) &= \kappa^2 \rho_m, \\ 2H\dot{T}f''(T) + (2\dot{H} - T)f'(T) + \frac{1}{2}f(T) &= -\kappa^2 p_m, \end{aligned} \quad (7)$$

where ρ_m and p_m are the energy density and pressure of matter components. One can rewrite these equations as the usual form of effective Einstein equations

$$\begin{aligned} 3H^2 &= \frac{\kappa^2}{f'(T)} (\rho_m + \rho_T), \\ \dot{H} &= -\frac{\kappa^2}{2f'(T)} (\rho_m + p_m + \rho_T + p_T), \end{aligned} \quad (8)$$

where H is the Hubble parameter and

$$\begin{aligned} \rho_T &= \frac{1}{2\kappa^2} (Tf'(T) - f(T)), \\ p_T &= \frac{1}{2\kappa^2} (4H\dot{T}f''(T) - Tf'(T) + f(T)). \end{aligned} \quad (9)$$

In this discussion, we consider the pressureless matter and ρ_m is determined from the conservation equation $\dot{\rho}_m + 3H\rho_m = 0$. We combine (8) and continuity equation to a single equation:

$$\dot{H} = \frac{-1}{2f(T)} [3H_0^2 \Omega_{m0} (1+z)^3 + 2H\dot{T}f''(T)]. \quad (10)$$

Using the relation $d/dt = -(1+z)Hd/dz$, (10) can be expressed as

$$\mathcal{E}_1(z) \frac{d^2 f}{dz^2} + \mathcal{E}_2(z) \frac{df}{dz} = 3H_0^2 \Omega_{m0} (1+z)^3, \quad (11)$$

where $\mathcal{E}_i$ ($i = 1, 2$) is defined in terms of $H(z)$ and its derivatives as

$$\begin{aligned} \mathcal{E}_1(z) &= 2H^2 (1+z) \left(\frac{dT}{dz} \right)^{-1}, \\ \mathcal{E}_2(z) &= 2H (1+z) \left(\frac{dH}{dz} \right) \left(\frac{dT}{dz} \right)^{-1} \\ &\quad - 2H^2 (1+z) \left(\frac{dT}{dz} \right)^{-2} \left(\frac{d^2 T}{dz^2} \right). \end{aligned} \quad (12)$$

One can see that an expression of $H(z)$ is needed to reconstruct $f(T)$. Consequently, the respective $f(T)$ theories can be determined for a known $H(z)$ predicted by a given DE model. To figure out the evolution of $f(T)$, we set the boundary conditions [31]

$$\begin{aligned} \left(\frac{df}{dz} \right)_{z=0} &= \left(\frac{dT}{dz} \right)_{z=0}, \\ f(z=0) &= f(T_0) = 6H_0^2 (\Omega_{m0} - 2). \end{aligned} \quad (13)$$

FIGURE 1: Evolution trajectories of reconstructed $f(T)$ for quintessence model in $lf - lT$ planes with $\Omega_{m0} = 0.315$, $H_0 = 67.3$, and $0 \leq z \leq 10$.

In the following we consider two well-known models of DE to numerically reconstruct the corresponding $f(T)$ function.

2.1. Quintessence Model. In the first place, we consider the familiar quintessence model of DE with constant equation of state parameter ω_ϕ and $\Omega_\phi = 1 - \Omega_m$. The corresponding Hubble parameter is defined as [39]

$$H(z) = H_0 \sqrt{\Omega_m (1+z)^3 + \Omega_\phi (1+z)^{3(1+\omega_\phi)}}. \quad (14)$$

In fact it is the generalization of Λ CDM model with EoS $\omega_\phi = -1$. One can figure out other DE models depending on ω_ϕ such as quintessence $\omega_\phi > -1$ and phantom $\omega_\phi < -1$, violating the null energy condition. ω_ϕ has been constrained through distinct observations: the Planck and WMAP9 observations set the constraints for ω_ϕ as $\omega_\phi = -1.13^{+0.13}_{-0.10}$ [51] and $-1.71 < \omega_\phi < -0.34$ [52], respectively. In this study, we set the present day values of Ω_m and H as $\Omega_{m0} = 0.315$ and $H_0 = 67.3$ [51]. We choose three different values of ω_ϕ as $\omega_\phi = -0.5, -1, -1.5$ representing quintessence, Λ CDM, and phantom regimes of the universe. Equations (11) and (14) can be solved numerically using conditions (13) as well as $\Omega_{\phi0} = 1 - \Omega_{m0}$. The reconstructed function $f(T)$ is presented on $lf - lT$ plane in Figure 1, where $lf = \ln(-f)$ and $lT = \ln(-T)$. For the sake of comparison, we also show the evolution trajectories on $lf - z$ plane in Figure 2. The evolution trajectories show distinct behavior for $\omega_\phi = -0.5$ and coincide for $\omega_\phi = -1, -1.5$. Figures 1 and 2 indicate the role of ω_ϕ to determine the evolution of $f(T)$ in remote past. To further explain the effect of ω_ϕ , we consider the future evolution of $|T|$. Figure 3 shows the future variation of $|T|$. As is expected for $\omega_\phi = -1.5$, the DE dominates over the matter and $|T|$ would take infinitely large values. For $\omega_\phi = -1$, we have the cosmological constant regime which is also assured from the corresponding curve in Figure 3.

FIGURE 2: Evolution trajectories of reconstructed $f(T)$ for quintessence model in $lf - z$ plane with $\Omega_{m0} = 0.315$, $H_0 = 67.3$, and $0 \leq z \leq 10$.

FIGURE 3: Evolution trajectories of T with $\Omega_{m0} = 0.315$, $H_0 = 67.3$, and $-1 \leq z \leq 1$.

For $\omega_9 = -0.5$, $|T|$ vanishes in the future. We also plot the future evolution of reconstructed function $f(T)$ which more effectively reflects the difference in choice of parameter ω_9 . For $\omega_9 = -0.5$, $|T|$ decreases and $f(T)$ attains a constant value as shown in Figure 4(a). In case of $\omega_9 = -1$, Figure 4(b) shows the linear dependence of f on T up to the present day value and in future evolution it approaches definite value preferring the Λ CDM model. For the phantom evolution with $\omega_9 = -1.5$, the corresponding evolution of function $f(T)$ is shown in Figure 4(c). It can be seen that $|T|$ decreases initially and after the recent epoch it starts increasing leading to the corresponding function $f(T)$.

2.2. Holographic Dark Energy Model. Holographic DE (HDE) appeared as one of the most eminent candidates to address the issue of cosmic acceleration. The density of the HDE has

been proposed by incorporating the mathematical form of the holographic principle as [53, 54]

$$\rho_9 = \frac{3c^2 M_p^2}{L^2}, \quad (15)$$

where $3c^2$ is a numerical constant, $M_p = 1/\sqrt{8\pi G}$ is the reduced Planck mass, and L is the infrared cutoff. Li [53, 54] proposed future event horizon as an IR cutoff which is found to be well coherent with recent observations [55, 56]. Many works have been executed to justify the cosmic acceleration using the evolution of EoS parameter for this horizon [57–59]. The IR cutoff L (event horizon) is defined as

$$L = R_{\hat{E}} = a(t) \int_t^\infty \frac{d\hat{t}}{a(\hat{t})} = a(t) \int_a^\infty \frac{da'}{Ha'^2}. \quad (16)$$

The first Friedmann equation is given by

$$3M_p^2 H^2 = \rho_M + \rho_9. \quad (17)$$

By introducing the fractional energy densities of matter and DE $\Omega_M = \rho_M/\rho_{cri}$, $\Omega_9 = \rho_9/\rho_{cri}$, $\rho_{cri} = 3M_p^2 H^2$, one obtains the Hubble parameter as

$$H(x) = H_0 \left(\frac{\Omega_{M0} e^{-3x}}{1 - \Omega_9} \right)^{1/2}. \quad (18)$$

Employing the definition of HDE (15) and event horizon (16), we obtain the dynamical equation of fractional density of DE as

$$\Omega_9' = \frac{-1}{1+z} \Omega_9 (1 - \Omega_9) \left(1 + \frac{2}{c} \sqrt{\Omega_9} \right), \quad (19)$$

where the prime denotes derivative with respect to z . Using the energy conservation equation of DE, we obtain

$$\omega_9 = -\frac{1}{3} - \frac{2}{3c} \sqrt{\Omega_9}, \quad (20)$$

which can result in three significant eras of the universe. It can be seen that when $\Omega_9 \rightarrow 1$ in the future (i.e., the HDE dominates the contents of the universe), for $c > 1$, we have $\omega_9 > -1$ which depicts quintessence era such that the universe escapes from entering the de Sitter and big rip phases. For $c = 1$, it represents the de Sitter universe and if $c < 1$, it may end up with phantom phase and behaves as quintom era because EoS parameter intersects the cosmological constant boundary (the phantom divide) throughout evolution. The value of c plays a vital role in deciding the evolutionary features of HDE and ultimate fate of the universe. The HDE has been constrained from observations of SNeIa, CMB, and galaxy clusters and the best fit favors $c < 1$, although $c > 1$ is also compatible with the data in one-sigma error range [55, 56].

In case of HDE, we do not have explicit form of $H(z)$ whereas, using (18) and (19), $H(z)$ and its derivatives can be expressed by Ω_9 . Hence, the coefficients C_i 's are expressed in the form of Ω_9 and its derivative. The system of (11) and

FIGURE 4: Future evolution of $f(T)$ for (a) $\omega_g = -0.5$, (b) $\omega_g = -1$, and (c) $\omega_g = -1.5$ with $-1 \leq z \leq 1$. The dot denotes the present day value of the function $f(T)$.

FIGURE 5: Evolution trajectories of reconstructed $f(T)$ for HDE model in $lf - lT$ plane with $\Omega_{m0} = 0.315$, $H_0 = 67.3$, and $0 \leq z \leq 10$.

FIGURE 6: Future evolution of T in HDE with $\Omega_{m0} = 0.315$, $H_0 = 67.3$, and $-1 \leq z \leq 1$.

(19) can be solved numerically with boundary condition (13) and $\Omega_g = 1 - \Omega_m$. Here, we set $c = 0.6, 1, 1.4, 1.8$, $\Omega_{m0} = 0.315$ and plot the reconstructed function $f(T)$ on $lf - lT$ plane in Figure 5, where $lf = \ln(-f)$ and $lT = \ln(-T)$. It is clear that different values of c do not affect the shape of the curves and their evolution is quite similar in these plots. One can compare these results with reconstructed function

in $f(R)$ gravity and the evolution curves are consistent with that in [39–42]. To further explore the effect of parameter c on reconstructed functions $f(T)$, we consider the future evolution scenario. The future evolution of T is presented in Figure 6. For $c = 0.6$, $|T|$ would take infinitely large values

FIGURE 7: Future evolution of $f(T)$ for HDE with $\Omega_{m0} = 0.315$, $H_0 = 67.3$, and $-1 \leq z \leq 1$. The dot denotes the present day value of the function $f(T)$.

showing the typical phantom evolution. The variation in $|T|$ is comparatively small for $c = 1$ and the DE model is more likely cosmological constant. For $c = 1.4, 1.8$, $|T|$ vanishes in future evolution. The difference in parameter c is more effectively reflected from the reconstructed functions $f(T)$ corresponding to HDE as shown in Figure 7. In case of $c = 1$, the dependence of function on T is smooth which results in a constant value representing the cosmological constant regime. For $c = 0.6$, the curve shows that initially $|T|$ decreases before reaching the turnaround point ($z = 0$), which changes its direction and it would increase leading to the phantom DE. In this scenario, $|T|$ grows linearly, and $f(T)$ decreases first and then approaches a constant value as shown in Figure 7(b). For $c = 1.4$, the reconstructed function is significantly different and vanishes in later times.

3. Conclusions

$f(T)$ theory is the generalization of teleparallel theory which is an analogy of the Einstein gravity and it would be quite interesting to investigate the aspects of modified Einstein theories such as $f(R)$ theory in this modified theory. In this paper, we have applied numerical scheme to reconstruct

$f(T)$ theory from the well-known DE models. In fact such approach was initially introduced in $f(R)$ theory [39] and different DE models have been studied in this perspective [39–42]. We considered the FRW universe to formulate the dynamical field equations which can be combined to linear second order differential equation. Consequently, this equation is expressed in terms of redshift, Hubble parameter and its derivative. For a given $H(z)$ from the recent observations, this equation can be solved numerically depending on the boundary conditions. We set the boundary conditions from physical considerations based on the cosmographic analysis in $f(T)$ theory [31].

In this scheme, one needs a precise knowledge of $H(z)$ to fit the $f(T)$ theory according to given evolution background. In the first place, we present the evolution trajectories of reconstructed function $f(T)$ on $lf - lT$ plane for $\omega_9 = -0.5, -1, -1.5$. We also show the reconstructed function on $lf - z$ plane. The future evolution curves of T and f with respect to z are also shown for known eras of the universe.

The deceleration parameter and EoS parameter for $f(T)$ gravity are defined as

$$q = \frac{1}{2} (1 + 3\omega_T \Omega_T), \quad (21)$$

FIGURE 8: Evolution of NEC for reconstructed $f(T)$ in quintessence model for $\Omega_{m0} = 0.315$ and $H_0 = 67.3$.

FIGURE 9: Evolution of $1 + \omega_T$ reconstructed $f(T)$ in quintessence model for $\Omega_{m0} = 0.315$ and $H_0 = 67.3$.

where

$$\omega_T = -1 + \frac{4H\dot{T}f''(T)}{Tf'(T) - f(T)}, \quad \Omega_T = \frac{\rho_T}{3H^2}. \quad (22)$$

We explore the behavior of NEC for the $f(T)$ functions in quintessence model. Figure 8 shows that NEC is violated and results in $\rho_T + p_T = 0$ representing the Λ CDM model in the future evolution. We present the evolution of EoS parameter $1 + \omega_T$ in Figure 9. The plot shows that these curves correspond to $\omega_T = -1$ in later times of the universe. We also present the evolution of deceleration parameter q in Figure 10. From Figure 9 it is clear that initially ω_T is less than -1 and henceforth approaches $\omega_T = -1$ in future evolution. Hence, the reconstructed functions for this case may favor the little rip scenario [60]. Bamba et al. [45] have discussed

FIGURE 10: Evolution of deceleration parameter q in quintessence model for $\Omega_{m0} = 0.315$ and $H_0 = 67.3$. This plot shows that $q < -1$ in the evolution of quintessence model which favors the accelerated expansion of the universe.

little rip scenario for $f(T)$ gravity and it is shown that future singularities can be avoided for $f(T)$ models.

In the second case, we consider HDE model and developed correspondence with $f(T)$ theory. For this model $H(z)$ is defined in terms of Ω_θ , so one needs to solve the system of evolution equations for both Ω_θ and f . The evolution of f is shown on $lf - lT$ plane for different values of holographic parameter. It is shown that the evolution trajectories coincide and evolutionary parameter c does not affect the $f(T)$ curves. Moreover, the difference in evolution trajectories is evident for later times ($-1 \leq z \leq 1$) as shown in Figures 6 and 7. For the second case, we have considered particular $f(T)$ model with $c = 0.6$ and plot the evolution of NEC and ω_T versus redshift in Figure 11. The NEC is violated for the $f(T)$ model and in future evolution it is trivially satisfied. Figure 11(b) shows that ω_T increases with z and approaches $\omega_T = -1$ as $z \rightarrow -1$.

We have analyzed the stability of $f(T)$ DE models using squared speed of sound. The relation of squared speed of sound can be found using the expressions of ρ_T and p_T as

$$v_s^2 = \frac{\dot{p}_T}{\dot{\rho}_T} = \frac{\dot{T}f'''(T) + (4\dot{H}T + 4H\ddot{T} - T\ddot{T})f''(T)}{T\dot{T}f''(T)}. \quad (23)$$

The squared speed of sound is a significant tool to figure out the stability of matter configuration. The positive values of squared speed of sound v_s^2 represent the potentially stable region and unstable region if $v_s^2 < 0$. In [61–63], stability of reconstructed modified gravity models has been investigated using squared speed of sound. In Figure 12, we show evolution trajectories of squared speed of sound for the $f(T)$ models which favor the Λ CDM regime. The squared speed of sound for quintessence $f(T)$ model with $\omega = -1$ is shown in Figure 12(b). It can be seen that $f(T)$ model is stable for present and future evolution. We also present the evolution of v_s^2 for holographic $f(T)$ model; it is unstable in some regions and favors stable region in future era. It is remarked that quintessence and HDE are developed within the framework of general relativity rather than any other

FIGURE 11: Evolution of NEC and ω_T for the $f(T)$ model in HDE for $\Omega_{m0} = 0.315$, $H_0 = 67.3$, and $c = 0.6$. The left plot shows that NEC is violated and $\rho_T + p_T = 0$ in later times. In the right plot we present the evolution of EoS in $f(T)$ gravity. These plots favor the Λ CDM era of cosmos.

FIGURE 12: Evolution of squared speed of sound versus z for the reconstructed quintessence and holographic $f(T)$ models. Left plot shows the quintessence $f(T)$ model with $\Omega_{m0} = 0.315$, $H_0 = 67.3$, and $\omega = -1$ and right plot shows the evolution of holographic $f(T)$ model for $c = 0.6$.

analogous or modified theory. We have reconstructed $f(T)$ theory by considering torsion as an effective description of these DE models.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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