Research Article

Electron Capture Cross Sections for Stellar Nucleosynthesis

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In the first stage of this work, we perform detailed calculations for the cross sections of the electron capture on nuclei under laboratory conditions. Towards this aim we exploit the advantages of a refined version of the proton-neutron quasiparticle random-phase approximation (pn-QRPA) and carry out state-by-state evaluations of the rates of exclusive processes that lead to any of the accessible transitions within the chosen model space. In the second stage of our present study, we translate the abovementioned e^- -capture cross sections to the stellar environment ones by inserting the temperature dependence through a Maxwell-Boltzmann distribution describing the stellar electron gas. As a concrete nuclear target we use the ^{66}Zn isotope, which belongs to the iron group nuclei and plays prominent role in stellar nucleosynthesis at core collapse supernovae environment.

1. Introduction

Weak interaction processes occurring in the presence of nuclei under stellar conditions play crucial role in the late stages of the evolution of massive stars and in the presupernova stellar collapse [1–6]. As it is known, the core of a massive star, at the end of its hydrostatic burning, is stabilized by electron degeneracy pressure as long as its mass does not exceed an appropriate mass (the Chandrasekhar mass limit, M_{Ch}) [6–10]. When the core mass exceeds M_{Ch} , electron degeneracy pressure cannot longer stabilize the center of the star and the collapse starts. In the early stage of collapse electrons are captured by nuclei in the iron group region [6, 10].

During the presupernova evolution of core collapse supernova, the Fermi energy (or equivalently the chemical potential) of the degenerate electron gas is sufficiently large to overcome the threshold energy E_{thr} (E_{thr} is given by negative Q values of the reactions involved in the interior of the stars) [11] and the nuclear matter in the stellar core is neutronized. This high Fermi energy of the degenerate electron gas leads to enormous e^- -capture on nuclei and reduces the electron to baryon ratio Y_e [12, 13]. In this way, the electron pressure is reduced and the energy as well as the entropy drop. One of the important characteristics of the early pre-explosion evolution

is the fact that electron capture on nuclei (specifically on nuclei of the pf shell) plays a key role [14, 15].

In the early stage of collapse (for densities lower than a few $10^{10} \text{ g cm}^{-3}$), the electron chemical potential is of the same order of magnitude as the nuclear Q value, and the e^- -capture cross sections are sensitive to the details of GT strength distributions in daughter nuclei. For this reason, some authors restrict the calculations only to the GT strength and evaluate e^- -capture rates on the basis of the GT transitions (at these densities, electrons are captured mostly on nuclei with mass number $A \leq 60$) [9–12, 15, 16]. Various methods, used for calculating e^- -capture on nuclei during the collapse phase, have shown that this process produces neutrinos with rather low energies in contrast to the inelastic neutrino-nucleus reactions occurring in supernova [17–20]. These neutrinos escape the star carrying away energy and entropy from the core which is an effective cooling mechanism of the exploding massive star [21]. For higher densities and temperatures, e^- capture occurs on heavier nuclei $A \geq 65$ [8, 10, 13–15]. As a consequence, the nuclear composition is shifted to more neutron-rich and heavier nuclei (including those with $N > 40$) which dominate the matter composition for densities larger than about $10^{10} \text{ g cm}^{-3}$ [1, 15, 21, 22].

The first calculations of stellar electron capture rates for iron group nuclei have been performed by employing the

independent particle model (IPM) [2–4]. Recently, similar studies have been addressed by using continuum RPA (CRPA) [25], large scale shell model [26, 27], RPA [11], and so forth [28]. In the present work e^- -capture cross sections are obtained within a refined version of the quasiparticle random phase approximation (QRPA) which is reliable for constructing all the accessible final (excited) states of the daughter nuclei in the iron group region of the periodic table [29–38]. For the description of the required correlated nuclear ground states we determine single-particle occupation numbers calculated within the BCS theory [29, 31, 32]. Our nuclear method is tested through the reproducibility of experimental muon capture rates relying on detailed calculations of exclusive, partial, and total muon capture rates [23, 24, 39–42]. The agreement with experimental data provided us with high confidence level of our method and we continued with the calculations of electron capture cross sections in supernova conditions (where the densities and temperatures are high) using the pn-QRPA method. In this paper, we performed calculations for ^{66}Zn isotope (it belongs to the iron group nuclei) that plays prominent role in core collapse supernovae stellar nucleosynthesis [18, 19, 43].

Our strategy in this work is, at first, to perform extensive calculations of the transition rates for all the abovementioned nuclear processes, assuming laboratory conditions, and then to translate these rates to the corresponding quantities within stellar environment through the use of an appropriate convolution procedure [9, 14, 15, 21, 26]. To this purpose, we assume that leptons under such conditions follow Maxwell-Boltzmann energy distribution [9, 26].

2. Construction of Nuclear Ground and Excited States

Electrons of energy E_e are captured by nuclei interacting weakly with them via W^- boson exchange as follows:

$$(A, Z) + e^- \longrightarrow (A, Z - 1)^* + \nu_e. \quad (1)$$

The outgoing ν_e neutrino carries energy E_ν , while the daughter nucleus $(A, Z - 1)$ absorbs a part of the incident electron energy given by the difference between the initial E_i and the final E_f nuclear energies as $E_\nu = E_f - E_i$.

The nuclear calculations for the cross sections of reaction (1) start by writing down the weak interaction Hamiltonian $\widehat{\mathcal{H}}_w$ which is given as a product of the leptonic, j_μ^{lept} , and the hadronic, $\widehat{\mathcal{F}}^\mu$, currents (current-current interaction Hamiltonian) as follows:

$$\widehat{\mathcal{H}}_w = \frac{G}{\sqrt{2}} j_\mu^{\text{lept}} \widehat{\mathcal{F}}^\mu, \quad (2)$$

where $G = G_F \cos \theta_c$ with G_F and θ_c being the well-known weak interaction coupling constant and the Cabibbo angle, respectively [31, 32, 44].

From the nuclear theory point of view, the main task is to calculate the cross sections of reaction (1) which are based

on the evaluation of the nuclear transition matrix elements between the initial $|i\rangle$ and a final $|f\rangle$ nuclear states of the form

$$\langle f | \widehat{H}_w | i \rangle = \frac{G}{\sqrt{2}} \ell^\mu \int d^3x e^{-i\mathbf{q}\cdot\mathbf{x}} \langle f | \widehat{\mathcal{F}}_\mu | i \rangle. \quad (3)$$

The quantity $\ell^\mu e^{-i\mathbf{q}\cdot\mathbf{x}}$ stands for the leptonic matrix element written in coordinate space with $\mathbf{q}$ being the 3-momentum transfer. For the calculation of these transition matrix elements one may take advantage of the Donnelly-Walecka multipole decomposition which leads to a set of eight independent irreducible tensor multipole operators containing polar-vector and axial-vector components [44] (see Appendix A).

In the present work, in (3) the ground state of the parent nucleus $|i\rangle$ is computed by solving the relevant BCS equations which give us the quasiparticle energies and the amplitudes V and U that determine the probability for each single particle level to be occupied or unoccupied, respectively [31]. Towards this aim, at first, we consider a Coulomb corrected Woods-Saxon potential with a spin orbit part as a mean field for the description of the strong nuclear field [45, 46]. For the latter potential we adopt the parameterization of IOWA group [47]. Then, we use as pairing interaction the monopole part of the Bonn C-D one meson exchange potential. The renormalization of this interaction, to fit in the ^{66}Zn isotope, is achieved through the two pairing parameters $g_{\text{pair}}^{p,n}$, $p(n)$ for proton (neutron) pairs, and the values of which are tabulated in Table 1.

As it is well known, the pairing parameters, $g_{\text{pair}}^{p,n}$, are determined through the reproduction of the energy gaps, $\Delta_{p,n}^{\text{exp}}$, from neighboring nuclei as follows (3-point formula):

$$\begin{aligned} \Delta_n^{\text{exp}} &= -\frac{1}{4} [S_n(A-1, Z) - 2S_n(A, Z) + S_n(A+1, Z)], \\ \Delta_p^{\text{exp}} &= -\frac{1}{4} [S_p(A-1, Z-1) \\ &\quad - 2S_p(A, Z) + S_p(A+1, Z+1)], \end{aligned} \quad (4)$$

where S_p and S_n are the experimental separation energies for protons and neutrons, respectively, of the target nucleus (A, Z) and the neighboring nuclei $(A \pm 1, Z \pm 1)$ and $(A \pm 1, Z)$. For the readers convenience in Table 2 we show the values of experimental separation energies for the target ^{66}Zn and the neighboring nuclei ^{65}Cu , ^{67}Ga , ^{65}Zn , and ^{67}Zn .

Subsequently, the excited states $|f\rangle$ of the studied daughter nucleus ^{66}Cu are constructed by solving the pn-QRPA equations [29–38], which are written as follows in matrix form [29]:

$$\begin{pmatrix} \mathcal{A} & \mathcal{B} \\ -\mathcal{B} & -\mathcal{A} \end{pmatrix} \begin{pmatrix} X^\nu \\ Y^\nu \end{pmatrix} = \Omega_{J^\pi}^\nu \begin{pmatrix} X^\nu \\ Y^\nu \end{pmatrix}, \quad (5)$$

$\Omega_{J^\pi}^\nu$ denotes the excitation energy of the QRPA state $|J^\pi_\nu\rangle$ with spin J and parity π .

The solution of (5) is an eigenvalue problem which provides the amplitudes for forward and backward scattering X and Y , respectively, and the QRPA excitation energies $\Omega_{J^\pi}^\nu$

TABLE 1: Parameters for the renormalization of the interaction of proton pairs, g_{pair}^p , and neutron pairs, g_{pair}^n . They have been fixed in such a way that the corresponding experimental gaps, Δ_p^{exp} and Δ_n^{exp} of ^{66}Zn isotope, are quite accurately reproduced.

Nucleus	g_{pair}^n	g_{pair}^p	Δ_n^{exp} (MeV)	Δ_n^{theor} (MeV)	Δ_p^{exp} (MeV)	Δ_p^{theor} (MeV)
^{66}Zn	1.0059	0.9271	1.7715	1.7716	1.2815	1.2814

TABLE 2: The experimental separation energies in MeV for protons and neutrons of the target (A, Z) and neighboring ($A \pm 1, Z \pm 1$) and ($A \pm 1, Z$) nuclei.

Nucleus	$S_n(A-1, Z)$	$S_n(A, Z)$	$S_n(A+1, Z)$	$S_p(A-1, Z-1)$	$S_p(A, Z)$	$S_p(A+1, Z+1)$
^{66}Zn	7.979	11.059	7.052	7.454	8.924	5.269

[31–34]. In our method the solution of the QRPA equations is carried out separately for each multipole set of states $|J^\pi\rangle$.

For the renormalization of the residual 2-body interaction (Bonn C-D potential), the strength parameters, for the particle-particle (g_{pp}) and particle-hole (g_{ph}) interaction entering the QRPA matrices $\mathcal{A}$ and $\mathcal{B}$, are determined (separately for each multipolarity) from the reproducibility of the low-lying experimental energy spectrum of the final nucleus. The values of these parameters in the case of the spectrum of ^{66}Cu are listed in Table 3.

At this point, it is worth mentioning that, for measuring the excitation energies of the daughter nucleus ^{66}Cu from the ground state of the initial one ^{66}Zn , a shifting of the entire set of QRPA eigenvalues is necessary. Such a shifting is required whenever in the pn-QRPA a BCS ground state is used, a treatment adopted by other groups previously [9, 40, 48, 49]. The shifting, for the spectrum of the daughter nucleus ^{66}Cu , is done in such a way that the first calculated value of each multipole state of ^{66}Cu (i.e. $1_1^+, 2_1^+, \dots$, etc.) approaches as close as possible the corresponding lowest experimental multipole excitation. Table 4 shows the shifting applied to our QRPA spectrum for each multipolarity of the parent nucleus ^{66}Zn . We note that a similar treatment is required in pn-QRPA calculations performed for double-beta decay studies where the excitations derived for the intermediate odd-odd nucleus (intermediate states) through p - n and n - p reactions from the neighboring nuclei, left or right nuclear isotope, do not match to each other [48, 49]. The resulting low-energy spectrum, after using the parameters of Tables 1 and 3 and the shifting shown in Table 4, agrees well with the experimental one (see Figure 1).

We must also mention that, usually, in nuclear structure calculations we test a nuclear method in two phases: first through the construction of the excitation spectrum as discussed before and second through the calculations of electron scattering cross sections or muon capture rates. Following the above steps, we test the reproducibility of the relevant experimental data for many nuclear models employed in nuclear applications (nuclear structure and nuclear reactions) and in nuclear astrophysics [1, 9].

3. Results and Discussion

In this work we perform detailed cross section calculations for the electron capture on ^{66}Zn isotope on the basis of

FIGURE 1: Comparison of the theoretical excitation spectrum (resulting from the solution of the QRPA eigenvalue problem) with the low-lying (up to about 3 MeV) experimental one for ^{66}Cu nucleus. As it can be seen, the agreement is very good below 1 MeV but for higher excitation energies it becomes moderate.

the pn-QRPA method. The required nuclear matrix elements between the initial $|J_i\rangle$ and the final $|J_f\rangle$ states are determined by solving the BCS equations for the ground state [29, 31, 32] and the pn-QRPA equations for the excited states [31–34] (see Section 2). For the calculations of the original cross sections, a quenched value of g_A (see Appendix B) is considered which subsequently modifies all relevant multipole matrix elements [23, 24, 50, 51].

At this point of the present work and in order to increase the confidence level of our method, we perform total muon capture rates calculations [23, 24, 39–42]. The comparison

TABLE 3: Strength parameters for the particle-particle (g_{pp}) and particle-hole (g_{ph}) interaction for various multipolarities (for the rest of multipolarities, $J^\pi \leq 5^\pm$, the bare 2-body interaction has been used), in the case of the spectrum of ^{66}Cu nucleus.

J^π	Positive parity states					Negative parity states			
	0^+	1^+	2^+	3^+	4^+	1^-	2^-	3^-	4^-
g_{pp}	0.827	0.547	0.686	0.854	1.300	0.994	0.200	0.486	0.622
g_{ph}	0.336	0.200	1.079	0.235	0.200	1.200	0.200	1.200	1.200

TABLE 4: The shift (in MeV) applied on the spectrum (seperately of each multipole set of states) of ^{66}Cu isotope, daughter nucleus of the electron capture on ^{66}Zn .

Positive parity states		Negative parity states	
0^+	0.90	0^-	5.00
1^+	2.50	1^-	6.80
2^+	2.55	2^-	3.85
3^+	2.50	3^-	2.60
4^+	1.75	4^-	3.55
5^+	0.55	5^-	3.00

with experimental and other theoretical results is shown in Section 3.1. Afterwards, we study in detail the electron capture process as follows. (i) Initially we consider laboratory conditions; that is, the initial (parent) nucleus is considered in the ground state and no temperature dependence is assumed (see Section 3.2.1). (ii) Second, we consider stellar conditions; that is, the parent nucleus is assumed to be in any initial excited state and due to the e^- -capture process it goes to any final excited state of the daughter nucleus. At these conditions it is necessary to take into account the temperature dependence of the cross sections (see Section 3.2.2) [10].

3.1. Calculations of Muon Capture Rates for ^{66}Zn . Despite the fact that the muon capture on nuclei does not play a crucial role in stellar-nucleosynthesis, it is, however, important to start our study from this process since the nuclear matrix elements required for an accurate description of the μ -capture are the same for all semileptonic charge-changing weak interaction processes. In addition, the excitation spectrum of the daughter ($A, Z - 1$) nucleus, as we saw before, is in good agreement with the experimental data.

The calculations of the muon capture rates are performed in three steps. In the first step we carry out realistic state-by-state calculations of exclusive ordinary muon capture (OMC) rates in ^{66}Zn isotope for all multipolarities with $J^\pi \leq 5^\pm$ (higher multipolarities contribute negligibly). The appropriate expression for the exclusive muon capture rates is written as follows:

$$\begin{aligned} \Lambda_{gs \rightarrow J_f^\pi} &\equiv \Lambda_{J_f^\pi} \\ &= 2G^2 \langle \Phi_{1s} \rangle^2 R_f q_f^2 \\ &\times \left[\left| \langle J_f^\pi \| (\widehat{\mathcal{M}}_J - \widehat{\mathcal{D}}_J) \| 0_{gs}^+ \rangle \right|^2 \right. \\ &\quad \left. + \left| \langle J_f^\pi \| (\widehat{\mathcal{T}}_J^{\text{el}} - \widehat{\mathcal{T}}_J^{\text{magn}}) \| 0_{gs}^+ \rangle \right|^2 \right], \end{aligned} \quad (6)$$

where Φ_{1s} represents the muon wave function in the 1s muonic orbit. The operators in (6) are referred to as Coulomb $\widehat{\mathcal{M}}_J$, longitudinal $\widehat{\mathcal{D}}_J$, transverse electric $\widehat{\mathcal{T}}_J^{\text{el}}$, and transverse magnetic $\widehat{\mathcal{T}}_J^{\text{mag}}$ multipole operators (see Appendix A). The factor R_f in (6) takes into consideration the nuclear recoil which is written as $R_f = (1 + q_f/M_{\text{targ}})^{-1}$, with M_{targ} being the mass of the target (parent) nucleus.

Due to the fact that there are no available data in the literature for exclusive muon capture rates, the test of our method is realized by comparing partial and total muon capture rates with experimental data and other theoretical results [23, 24]. Towards this purpose, our second step includes calculations of the partial μ^- -capture rates for various low-spin multipolarities, Λ_{J^π} (for $J^\pi \leq 4^\pm$), in the studied nucleus. These partial rates are found by summing over the contributions of all the individual multipole states of the studied multipolarity as follows:

$$\begin{aligned} \Lambda_{J^\pi} &= \sum_f \Lambda_{gs \rightarrow J_f^\pi} \\ &= 2G^2 \langle \Phi_{1s} \rangle^2 \left[\sum_f q_f^2 R_f \left| \langle J_f^\pi \| (\widehat{\mathcal{M}}_J - \widehat{\mathcal{D}}_J) \| 0_{gs}^+ \rangle \right|^2 \right. \\ &\quad \left. + \sum_f q_f^2 R_f \left| \langle J_f^\pi \| (\widehat{\mathcal{T}}_J^{\text{el}} - \widehat{\mathcal{T}}_J^{\text{magn}}) \| 0_{gs}^+ \rangle \right|^2 \right] \end{aligned} \quad (7)$$

(f runs over all states of the multipolarity $|J^\pi\rangle$). We also estimate the percentage (portion) of their contribution into the total μ -capture rate for the most important multipolarities. In Table 5 we tabulate the individual portions of the low-spin multipole transitions ($J^\pi = 4^\pm$) into the total muon capture rate. As it can be seen, the contribution of the 1^- multipole transitions is the most important multipolarity exhausting about 44% of the total muon capture rate. Such an important contribution was found in ^{16}O and ^{48}Ca isotopes studied in [41].

In the last step of testing our method, we evaluate total muon capture rates for the ^{66}Zn isotope. These rates are obtained by summing over all partial multipole transition rates (up to $J^\pi = 4^\pm$) as follows:

$$\Lambda_{\text{tot}} = \sum_{J^\pi} \Lambda_{J^\pi} = \sum_{J^\pi} \sum_f \Lambda_{J_f^\pi}. \quad (8)$$

For the sake of comparison, the abovementioned μ -capture calculations have been carried out using the

TABLE 5: The percentage of each multipolarity into the total muon capture rate evaluated with our pn-QRPA method.

Positive parity transitions		Negative parity transitions	
J^π	Portions (%)	J^π	Portions (%)
0^+	8.22	0^-	7.94
1^+	21.29	1^-	44.21
2^+	2.85	2^-	13.32
3^+	1.58	3^-	0.34
4^+	0.01	4^-	0.23

quenched value $g_A = 1.135$ [23, 24]. The results are listed in Table 6, where we also include the experimental total rates and the theoretical ones of [23, 24]. Moreover, in Table 6 we show the individual contribution into the total muon capture rate of the polar-vector (Λ_{tot}^V), the axial-vector (Λ_{tot}^A), and the overlap ($\Lambda_{\text{tot}}^{\text{VA}}$) parts. As it can be seen, our results obtained with the quenched g_A coupling constant are in very good agreement with the experimental total muon capture rates (the deviations from the corresponding experimental rates are smaller than 7%). This agreement provides us with high confidence level for our method.

3.2. Electron Capture Cross Section. After acquiring a high confidence level for our nuclear method, we proceed with the main goal of the present study which concerns the calculations of the electron capture cross sections. As mentioned before, this includes original (see Section 3.2.1) and stellar electron capture investigations (Section 3.2.2).

3.2.1. Original Electron Capture Cross Section on ^{66}Zn Isotope. The original cross sections for the electron capture process in the ^{66}Zn isotope are obtained by using the pn-QRPA method considering all the accessible transitions of the final nucleus ^{66}Cu . In the Donnelly-Walecka formalism the expression for the differential cross section in electron capture by nuclei reads [10]

$$\begin{aligned}
 \frac{d\sigma_{ec}}{d\Omega} = & \frac{G_F^2 \cos^2 \theta_c}{2\pi} \frac{F(Z, E_e)}{(2J_i + 1)} \\
 & \times \left\{ \sum_{J \geq 1} \mathcal{W}(E_e, E_\nu) \right. \\
 & \times \left\{ [1 - \alpha \cos \Phi + b \sin^2 \Phi] \right. \\
 & \times [|\langle J_f \| \widehat{\mathcal{T}}_J^{\text{mag}} \| J_i \rangle|^2 + |\langle J_f \| \widehat{\mathcal{T}}_J^{\text{el}} \| J_i \rangle|^2] \\
 & - \left[\frac{(\varepsilon_i + \varepsilon_f)}{q} (1 - \alpha \cos \Phi) - d \right] \\
 & \left. \left. \times 2 \text{Re} \langle J_f \| \widehat{\mathcal{T}}_J^{\text{mag}} \| J_i \rangle \langle J_f \| \widehat{\mathcal{T}}_J^{\text{el}} \| J_i \rangle^* \right\} \right\}
 \end{aligned}$$

$$\begin{aligned}
 & + \sum_{J \geq 0} \mathcal{W}(E_e, E_\nu) \\
 & \times \left\{ (1 + \alpha \cos \Phi) |\langle J_f \| \widehat{\mathcal{M}}_J \| J_i \rangle|^2 \right. \\
 & + (1 + \alpha \cos \Phi - 2b \sin^2 \Phi) |\langle J_f \| \widehat{\mathcal{P}}_J \| J_i \rangle|^2 \\
 & - \left[\frac{\omega}{q} (1 + \alpha \cos \Phi) + d \right] 2 \text{Re} \langle J_f \| \widehat{\mathcal{P}}_J \| J_i \rangle \\
 & \left. \left. \times \langle J_f \| \widehat{\mathcal{M}}_J \| J_i \rangle^* \right\} \right\}, \tag{9}
 \end{aligned}$$

where $F(Z, E_e)$ is the well-known Fermi function [18]. The factor $W(E_e, E_\nu) = E_\nu^2 / (1 + E_\nu / M_T)$ accounts for the nuclear recoil [8], M_T is the mass of the target nucleus, and parameters α , b , and d are given, for example, in [31]. The nuclear transition matrix elements between the initial state $|J_i\rangle$ and the final state $|J_f\rangle$ correspond to the Coulomb $\widehat{\mathcal{M}}_{JM}$, longitudinal $\widehat{\mathcal{P}}_{JM}$, transverse electric $\widehat{\mathcal{T}}_{JM}^{\text{el}}$, and transverse magnetic $\widehat{\mathcal{T}}_{JM}^{\text{mag}}$ multipole operators (discussed in Appendix A).

From the energy conservation in the reaction (1), the energy of the outgoing neutrino E_ν is written as follows:

$$E_\nu = E_e - Q + E_i - E_f, \tag{10}$$

which includes the difference between the initial E_i and the final E_f nuclear states. The Q value of the process is determined from the experimental masses of the parent (M_i) and the daughter (M_f) nuclei as $Q = M_f - M_i$ [9].

It is worth mentioning that for low momentum transfer, various authors use the approximation $q \rightarrow 0$ for all multipole operators of (9). Then, the transitions of the Gamow-Teller operator ($\text{GT}^+ = \sum_i \tau_i^+ \sigma_i$) provide the dominant contribution to the total cross section [9].

While performing detailed calculations for the original electron capture cross sections on ^{66}Zn isotope we assumed that (i) the initial state of the parent nucleus ^{66}Zn is the ground state $|0^+\rangle$ and (ii) the nuclear system is under laboratory conditions (no temperature dependence of the cross sections is needed). The cross sections as a function of the incident electron energy E_e are calculated with the use of realistic two-body interactions as mentioned before. The obtained total original electron capture cross sections for ^{66}Zn target nucleus are illustrated in Figure 2 where the individual contributions of various multipole channels ($J^\pi \leq 5^\pm$) are also shown. The electron capture cross sections in Figure 2 exhibit a sharp increase by several orders of magnitude within the first few MeV above energy-threshold, and this reflects the GT^+ strength distribution. For electron energy $E_e \geq 10$ MeV the calculated cross sections show a moderate increase. From experimental and astrophysical point of view, the important range of the incident electron energy E_e is up to 30 MeV. At these energies the 1^+ multipolarity has the largest contribution to the total electron capture cross sections [9, 10]. In the present work we have extended the

TABLE 6: Individual contribution of polar-vector, axial-vector, and overlap parts into the total muon-capture rate. The total muon capture rates, obtained by using the pn-QRPA with the quenched value of $g_A = 1.135$ for the medium-weight nucleus ^{66}Zn , are compared with the available experimental data and with the theoretical rates of [23, 24].

Nucleus	Total muon capture rates $\Lambda_{\text{tot}} (\times 10^6 \text{ s}^{-1})$				Experiment	Other theoretical methods	
	Λ_{tot}^V	Λ_{tot}^A	$\Lambda_{\text{tot}}^{VA}$	Λ_{tot}		$\Lambda_{\text{tot}}^{\text{theor}} [23]$	$\Lambda_{\text{tot}}^{\text{theor}} [24]$
^{66}Zn	1.651	4.487	-0.204	5.934	5.809	4.976	5.809

FIGURE 2: Original total cross sections of electron capture on the ^{66}Zn (parent) nucleus calculated with pn-QRPA method as a function of the incident electron energy E_e . The individual contributions of various multipole channels (for $J^\pi \leq 5^+$) are also demonstrated.

range of E_e up to 50 MeV since at higher energies (around 40 MeV) the contribution of other multipolarities like 1^- , 0^+ , and 0^- becomes noticeable and cannot be omitted (see Figure 2).

From the study of the original electron capture cross sections we conclude that the total cross sections can be well approximated with the Gamow-Teller transitions only in the region of low energies [9–12, 15, 16]. For higher incident electron energies the inclusion of the contributions originating from other multipolarities leads to better agreement [10].

3.2.2. Stellar Electron Capture on ^{66}Zn Isotope. As it is well known, electron capture process plays a crucial role in late stages of evolution of a massive star, in presupernova and in supernova phases [1–6]. In presupernova collapse, that is, at densities $\rho \leq 10^{10} \text{ g cm}^{-3}$ and temperatures $300 \text{ keV} \leq T \leq 800 \text{ keV}$, electrons are captured by nuclei with $A \leq 60$ [9–12, 15, 16]. During the collapse phase, at higher densities $\rho \geq 10^{10} \text{ g cm}^{-3}$ and temperatures $T \approx 1.0 \text{ MeV}$, electron capture

process is carried out on heavier and more neutron rich nuclei with $Z < 40$ and $N \geq 40$ [8, 10, 13–15].

In an independent particle picture, the Gamow-Teller transitions (which is the most important in the electron capture cross section calculations) are forbidden for these nuclei [2–4]. However, as it has been demonstrated in several studies, GT transitions in these nuclei are unblocked by finite temperature excitations [21, 22]. At high temperatures, $T \approx 1.5 \text{ MeV}$, GT transitions are thermally unblocked as a result of the excitation of neutrons from the pf-shell into the $g_{9/2}$ orbital.

For astrophysical environment, where the finite temperature and the matter density effects cannot be ignored (the initial nucleus is at finite temperature), in general, the initial nuclear state needs to be a weighted sum over an appropriate energy distribution. Then, assuming Maxwell-Boltzmann distribution of the initial state $|i\rangle$ in (9) [9, 26], the total e^- -capture cross section is given by the expression [10]

$$\sigma(E_e, T) = \frac{G_F^2 \cos^2 \theta_c}{2\pi} \sum_i F(Z, E_e) \frac{(2J_i + 1) e^{-E_i/(kT)}}{G(Z, A, T)} \times \sum_{f, J} (E_e - Q + E_i - E_f)^2 \frac{|\langle i | \hat{O}_J | f \rangle|^2}{(2J_i + 1)}. \quad (11)$$

The sum over initial states in the latter equation denotes a thermal average of levels, with the corresponding partition function $G(Z, A, T)$ [10]. The finite temperature induces the thermal population of excited states in the parent nucleus. In the present work we assume that these excited states in the parent nucleus are all the possible states up to about 2.5 MeV. Calculations involving in addition other states lying at higher energies show that they have no sizeable contribution to the total electron capture cross sections. As mentioned before, for the evaluation of the total electron capture cross sections, the use of a quenched value of g_A is necessary [23, 24, 50, 51]. Since the form factor $F_A(q^2)$ multiplies the four components of the axial-vector operator (see (A.2)–(A.5)), a quenched value of g_A must enter the multipole operators generating the pronounced excitations 0^+ , 1^+ , ..., and so forth. For this reason, in our QRPA calculations we multiplied the free nucleon coupling constant $g_A = 1.262$ by the factor 0.8 [23, 24, 50, 51].

The results coming out of the study of electron capture cross sections under stellar conditions are shown in Figure 3, where the same picture as in the original cross section calculations, but now with larger contribution, is observed.

FIGURE 3: Electron capture cross sections for the ^{66}Zn parent nucleus at high temperature ($T = 0.5$ MeV) in stellar environment obtained assuming Maxwell-Boltzmann statistics for the incident electrons. The total cross section and the dominant individual multipole channels ($J^\pi \leq 5^\pm$) are demonstrated as functions of the incident electron energy E_e .

As discussed before, the dominant multipolarity is the 1^+ , which contributes by more than 40% to the total cross section. In the region of low energies (up to 30 MeV), the total e^- -capture cross section can be described by taking into account only the GT transitions, but at higher incident energies the contributions of other multipolarities become significant and cannot be omitted.

The percentage contributions of various multipolarities (with $J^\pi \leq 5^\pm$) into the total e^- -capture cross section at $T = 0.5$ MeV and for incident electron energy $E_e = 25$ MeV are tabulated in Table 7. In addition, in this table we list the values of the individual e^- -capture cross sections of each multipolarity with $J^\pi \leq 5^\pm$. More specifically, for $E_e = 25$ MeV the 1^+ multipolarity contributes to about 44%, the 0^+ contributes to about 26%, and the 1^- contributes to about 11%. The contributions coming from other multipolarities are less important (smaller than 5%).

In performing state-by-state calculations for the electron capture cross sections, our code has the possibility to provide separately the contribution of the polar-vector, the axial-vector, and the overlap parts induced by the corresponding components of the electron capture operators. In Figure 4 we illustrate the stellar differential cross sections of each individual transition of the polar-vector and axial-vector components.

As mentioned before, our code gives separately the partial e^- -capture cross sections of each multipolarity. In order to study the dependence of the differential cross sections on the excitation energy ω throughout the entire pn-QRPA spectrum of the daughter nucleus, a rearrangement of all possible excitations ω in ascending order, with the corresponding

FIGURE 4: Individual contributions of the polar-vector, (Λ_V), and axial-vector, (Λ_A), components and total electron capture rate as functions of the excitation energy ω (^{66}Zn is the parent nucleus).

cross sections, is required. This was performed by using a special code appropriate for matrices [33]. In the model space chosen for ^{66}Zn isotope, for all multipolarities up to $J^\pi = 5^\pm$, we have a number of 447 final states. The differential electron capture cross sections illustrated in Figure 4 present some characteristic clearly pronounced peaks at various excitation energies ω . These peaks correspond mainly to 0^+ , 1^+ , and 2^+ transitions. More specifically, in the ^{66}Cu daughter nucleus the maximum peak corresponds to the 0_1^+ QRPA transition at $\omega = 2.538$ MeV and other characteristic peaks correspond to 1_7^+ , 1_8^+ , and 1_{10}^+ transitions, located at energies $\omega = 3.194$ MeV, $\omega = 3.686$ MeV, and $\omega = 6.555$ MeV, respectively (see Figure 4). The other less important peaks are also shown in Figure 4.

Before closing, it should be mentioned that the e -capture cross sections presented in this work may be useful in estimating neutrino-spectra arising from e -capture on nuclei during supernova phase. The knowledge of ν -spectra at every point and time in the core is quite relevant for simulations of the final collapse and explosion phase of a massive star. As it is known [21], in the collapse phase, neutrinos are mainly produced by e -capture on nuclei and on free protons. The energy spectra of the emerging neutrinos from both reactions are important ingredients in stellar modelling and stellar simulations [21, 27].

Furthermore, in core collapse simulations one defines the reaction rate of electron capture on nuclei given by

$$R_h = \sum_i Y_i \lambda_i, \quad (12)$$

TABLE 7: Total e^- -capture cross sections (in $10^{-42} \text{ MeV}^{-1} \text{ cm}^2$) for $E_e = 25 \text{ MeV}$. The percentage of each multipolarity into the total e^- -capture cross section evaluated with our pn-QRPA code is also tabulated here.

Positive parity transitions			Negative parity transitions		
J^π	$\sigma_e (\times 10^{-42} \text{ cm}^2 \text{ MeV}^{-1})$	Portions (%)	J^π	$\sigma_e (\times 10^{-42} \text{ cm}^2 \text{ MeV}^{-1})$	Portions (%)
0^+	31.164	25.96	0^-	5.288	4.41
1^+	52.779	43.98	1^-	13.409	11.14
2^+	6.921	5.77	2^-	3.262	2.72
3^+	5.499	4.58	3^-	0.905	0.75
4^+	0.244	0.20	4^-	0.299	0.25
5^+	0.208	0.17	5^-	0.042	0.04

where the sum runs over all nuclear isotopes present in the astrophysical environment (Y_i denotes the abundance of a given nuclear isotope and λ_i is the calculated electron capture rate for this isotope). The rates of (12) must be known for a wide range of the parameters: T (temperature) and ρ (nuclear density) of the studied star. Thus, for the calculation of the quantity $Y \cdot \lambda$ of a specific nuclear isotope one needs to know in addition to the nuclear composition Y the electron capture rates λ calculated as we have shown in our present work. The rates of electron capture on various nuclear isotopes and the corresponding emitted neutrino spectra in the range of the parameters (T, ρ, Y_e) describing the star until reaching equilibrium during the core collapse are comprehensively studied in [21, 26, 27] for a great number of nuclear isotopes by using the large scale shell model. We are currently performing similar calculations for a set of isotopes by employing the present pn-QRPA method [52].

Furthermore, the average neutrino energy, $\langle E_\nu \rangle$, of the neutrinos emitted by e -capture on nuclei can be obtained by dividing the neutrino-energy loss rate (defined by an expression similar to (12) by replacing the rate λ_i with the energy loss rate E_j) with the reaction rate for e -capture on nuclei R_h . Assuming, for example, power-law energy distribution for the neutrino spectrum produced by the e -capture in supernova phase, the average neutrino-energy $\langle E_\nu \rangle$ determines a specific supernova-neutrino scenario. In addition, the neutrino emissivity is obtained by multiplying the electron capture rate at nuclear statistical equilibrium with the neutrino-spectra [21, 26, 27]. Finally we note that the rates for the inverse neutrino absorption process are also determined from the electron capture rates obtained as discussed in this section [21].

4. Summary and Conclusions

The electron capture on nuclei plays crucial role during the presupernova and collapse phase (in the late stage e^- -capture on free protons is also significant). It becomes increasingly possible as the density in the star's center is enhanced and it is accompanied by an increase of the chemical potential (Fermi energy) of the degenerate electron gas. This process reduces the electron-to-baryon ratio Y_e of the matter composition.

In this work, by using our numerical approach based on a refinement of the pn-QRPA that describes reliably all the semileptonic weak interaction processes in nuclei, we

studied in detail the electron capture process on ^{66}Zn isotope and calculated original and stellar e^- -capture cross sections. We tested our nuclear model (the pn-QRPA) through the reproducibility of orbital muon capture rates for this isotope. The agreement with experimental data and other reliable theoretical results of partial and total μ -capture rates and of the percentage contributions of various low-lying excitations is quite good which provides us with high confidence level for the obtained cross sections.

Our future plans are to extend the application of this method and make similar calculations for other interesting nuclei [52]. Also this method could be applied to other semileptonic nuclear processes like beta-decay and charged-current neutrino-nucleus processes important in nuclear astrophysics and neutrino nucleosynthesis.

Appendices

A. Nuclear Matrix Elements

The eight different tensor multipole operators entering the above equations (see Section 3), referred to as Coulomb $\widehat{\mathcal{M}}_{JM}$, longitudinal $\widehat{\mathcal{L}}_{JM}$, transverse electric $\widehat{\mathcal{T}}_{JM}^{\text{el}}$, and transverse magnetic $\widehat{\mathcal{T}}_{JM}^{\text{magn}}$, are defined as follows:

$$\begin{aligned} \widehat{\mathcal{M}}_{JM}(qr) &= \widehat{M}_{JM}^{\text{coul}} + \widehat{M}_{JM}^{\text{coul}5}, & \widehat{\mathcal{L}}_{JM}(qr) &= \widehat{L}_{JM} + \widehat{L}_{JM}^5, \\ \widehat{\mathcal{T}}_{JM}^{\text{el}}(qr) &= \widehat{T}_{JM}^{\text{el}} + \widehat{T}_{JM}^{\text{el}5}, & \widehat{\mathcal{T}}_{JM}^{\text{magn}}(qr) &= \widehat{T}_{JM}^{\text{magn}} + \widehat{T}_{JM}^{\text{magn}5}. \end{aligned} \quad (\text{A.1})$$

These multipole operators contain polar-vector and axial-vector parts and are written in terms of seven independent basic multipole operators as follows:

$$\widehat{M}_{JM}^{\text{coul}}(\mathbf{qr}) = F_1^V(q_\mu^2) M_M^J(\mathbf{qr}), \quad (\text{A.2})$$

$$\widehat{L}_{JM}(\mathbf{qr}) = \frac{q_0}{q} \widehat{M}_{JM}^{\text{coul}}(\mathbf{qr}), \quad (\text{A.3})$$

$$\widehat{T}_{JM}^{\text{el}}(\mathbf{qr}) = \frac{q}{M_N} \left[F_1^V(q_\mu^2) \Delta'^J_M(\mathbf{qr}) + \frac{1}{2} \mu^V(q_\mu^2) \Sigma_M^J(\mathbf{qr}) \right], \quad (\text{A.4})$$

$$i\hat{T}_{JM}^{\text{mag}}(q\mathbf{r}) = \frac{q}{M_N} \left[F_1^V(q_\mu^2) \Delta_M^J(q\mathbf{r}) - \frac{1}{2} \mu^V(q_\mu^2) \Sigma_M'^J(q\mathbf{r}) \right], \quad (\text{A.5})$$

$$i\hat{M}_{JM}^5(q\mathbf{r}) = \frac{q}{M_N} \left[F_A(q_\mu^2) \Omega_M^J(q\mathbf{r}) + \frac{1}{2} (F_A(q_\mu^2) + q_0 F_P(q_\mu^2)) \Sigma_M''^J(q\mathbf{r}) \right], \quad (\text{A.6})$$

$$-i\hat{L}_{JM}^5(q\mathbf{r}) = \left[F_A(q_\mu^2) - \frac{q^2}{2M_N} F_P(q_\mu^2) \right] \Sigma_M''^J(q\mathbf{r}), \quad (\text{A.7})$$

$$-i\hat{T}_{JM}^{\text{el}5}(q\mathbf{r}) = F_A(q_\mu^2) \Sigma_M'^J(q\mathbf{r}), \quad (\text{A.8})$$

$$\hat{T}_{JM}^{\text{mag}5}(q\mathbf{r}) = F_A(q_\mu^2) \Sigma_M^J(q\mathbf{r}), \quad (\text{A.9})$$

where the form factors F_X , $X = 1, A, P$, and μ^V are functions of the 4-momentum transfer q_μ^2 and M_N is the nucleon mass.

These multipole operators, due to the Conserved Vector Current (CVC) theory, are reduced to seven new basic operators expressed in terms of spherical Bessel functions, spherical harmonics, and vector spherical harmonics (see [24, 31, 44]). The single particle reduced matrix elements of the form $\langle j_1 \| T_i^J \| j_2 \rangle$, where T_i^J represents any of the seven basic multipole operators (M_M^J , Ω_M^J , Σ_M^J , $\Sigma_M'^J$, $\Sigma_M''^J$, Δ_M^J , $\Delta_M'^J$) of (A.2)–(A.9), have been written in closed compact expressions as follows [31]:

$$\langle (n_1 l_1) j_1 \| T^J \| (n_2 l_2) j_2 \rangle = e^{-y} y^{\beta/2} \sum_{\mu=0}^{n_{\max}} P_\mu^J y^\mu, \quad (\text{A.10})$$

where the coefficients P_μ^J are given in [31]. In the latter summation the upper index $n_{\max}$ represents the maximum harmonic oscillator quanta included in the active model space chosen as $n_{\max} = (N_1 + N_2 - \beta)/2$, where $N_i = 2n_i + l_i$, $i = 1, 2$, and β is related to the rank of the above operators [31].

In the context of the pn-QRPA, the required reduced nuclear matrix elements between the initial $|0_{gs}^+\rangle$ and the final $|f\rangle$ state entering the rates of (6) are given by

$$\langle f \| \hat{T}^J \| 0_{gs}^+ \rangle = \sum_{j_2 \geq j_1} \frac{\langle j_2 \| \hat{T}^J \| j_1 \rangle}{[J]} [X_{j_2 j_1} u_{j_2}^p v_{j_1}^n + Y_{j_2 j_1} v_{j_2}^p u_{j_1}^n], \quad (\text{A.11})$$

where u_j and v_j are the probability amplitudes for the j -level to be unoccupied or occupied, respectively (see the text) [31, 32].

These matrix elements enter the description of various semileptonic weak interaction processes in the presence of nuclei [31–38, 44, 53, 54].

B. Nuclear Form Factors

In (A.2)–(A.9) the standard set of free nucleon form factors $F_X(q_\mu^2)$, $X = 1, A, P$ and $\mu^V(q_\mu^2)$ reads

$$\begin{aligned} F_1^V(q_\mu^2) &= 1.000 \left[1 + \left(\frac{q}{840 \text{ MeV}} \right)^2 \right]^{-2}, \\ \mu^V(q_\mu^2) &= 4.706 \left[1 + \left(\frac{q}{840 \text{ MeV}} \right)^2 \right]^{-2}, \\ F_A(q_\mu^2) &= g_A \left[1 + \left(\frac{q}{1032 \text{ MeV}} \right)^2 \right]^{-2}, \\ F_P(q_\mu^2) &= \frac{2M_N F_A(q_\mu^2)}{q^2 + m_\pi^2}, \end{aligned} \quad (\text{B.1})$$

where M_N is the nucleon mass and g_A is the axial vector free nucleon coupling constant (see the text).

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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