Hindawi Publishing Corporation
Advances in High Energy Physics
Volume 2015, Article ID 428013, 6 pages
http://dx.doi.org/10.1155/2015/428013

Research Article

The Study of Thermal Conditions on Weibel Instability

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Received 28 June 2015; Revised 22 August 2015; Accepted 26 August 2015

Academic Editor: Ming Liu

Weibel electromagnetic instability has been studied analytically in relativistic plasma with high parallel temperature, where $|\alpha = (mc^2/T_\parallel)(1+\widehat{p}_\perp^2/m^2c^2)^{1/2}| \ll 1$ and while the collision effects of electron-ion scattering have also been considered. According to these conditions, an analytical expression is derived for the growth rate of the Weibel instability for a limiting case of $|\zeta = \sqrt{\alpha/2}(\omega'/ck)| \ll 1$, where ω' is the sum of the wave frequency of instability and the collision frequency of electrons with background ions. The results show that in the limiting condition $\alpha \ll 1$ there is an unusual situation of the Weibel instability so that $T_\parallel \gg T_\perp$, while in the classic Weibel instability $T_\parallel \ll T_\perp$. The obtained results show that the growth rate of the Weibel instability will be decreased due to an increase in the number of collisions and a decrease in the anisotropic temperature by the increasing of plasma density, while the increase of the parameter $\widehat{\gamma}_\perp = (1+\widehat{p}_\perp^2/m^2c^2)^{1/2}$ leads to the increase of the Weibel instability growth rate.

1. Introduction

The presence of different instabilities and their wrecker effects is one of the important problems in fusion schemes by inertial confinement (during the compression and explosion stages). Besides the Rayleigh-Taylor instabilities and their wrecker effects [1], another group of kinetic and reaction instabilities can be present which acts as a preclude for the transport of sufficient energy to the heart of compressed fuel and the formation of the hot spot. These instabilities, which are known as Weibel-like instabilities, are very considerable because of their potential role in generating high scale magnetic fields. Anisotropy in the velocity space, due to the presence of average free energy in a plasma system, can lead to generation and excitation of these instabilities [2–5]. During recent years, more studies have been done by numerical methods on Weibel instability in the relativistic plasmas [6, 7]; however, there are not enough analytical studies in this context. Hence, in the present paper, there is an attempt to present an analytical model on the Weibel instability in relativistic plasma, in conditions where the contribution of the Coulomb collision between plasma particles is considered.

2. The Mathematical Model

The mathematical formalism in this paper is based on the kinetic theory and the relativistic Boltzmann equation combined with the Maxwell equations:

$$\frac{\partial f}{\partial t} + \vec{V}\cdot\frac{\partial f}{\partial \vec{r}} + q\left(\vec{E} + \frac{\vec{V}}{c}\times\vec{B}\right)\cdot\frac{\partial f}{\partial \vec{p}} = \left(\frac{\partial f}{\partial t}\right)_{\text{coll}}, \quad (1)$$

where

$$\vec{\nabla}\times\vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t},$$
$$\vec{\nabla}\times\vec{B} = -\frac{1}{c}\frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c}\vec{J} \quad (2)$$

and the collision term $(\partial f/\partial t)_{\text{coll}}$ will be replaced by the "Krook" implicitness term, $(\partial f/\partial t)_{\text{coll}} \equiv -\nu_{ei}(f - f_0)$. In these equations, the quantities f_0 and f are the equilibrium and the total electron distribution function at position r and momentum p at time t, where the total distribution is the sum of the equilibrium and perturbed distribution function, $f = f_0 + f_1$. The quantities c, ν_{ei}, $\vec{J}$, $\vec{E}$, and

$\vec{B}$ are the velocity of light, the collision frequency between electrons and ions, the current density, and the perturbed electromagnetic fields, respectively. The quantity V is the velocity of plasma particles (electrons) which is related to the momentum and the rest of mass of the electron by $\vec{V} = \vec{p}/m\gamma$, where γ is the relativistic mass factor. Therefore, let us consider an ordinary model in which the electromagnetic waves propagate in the direction of the Z axis, $\vec{K} = k\hat{e}_z$ and the transverse electromagnetic fields $\vec{E}$ and $\vec{B}$ are in the perpendicular direction to the wave propagation. Notice that, to linearize the Boltzmann equation, only the perturbed part of the total distribution function has time and place dependence, and the equilibrium part only has the velocity dependence. Finally, the linear dispersion relation can be obtained according to these assumptions as follows:

$$\begin{aligned} &1 - \frac{c^2k^2}{\omega^2} + \frac{\omega_{pe}^2}{\omega^2} \\ &\cdot \int \frac{d^3p}{\gamma} \frac{p_\perp/2}{(\omega' - kp_\parallel/m\gamma)} \left[\left(\omega - \frac{kp_\parallel}{m\gamma}\right) \frac{\partial f_0}{\partial p_\perp} \right. \\ &\left. + \frac{kp_\perp}{m\gamma} \frac{\partial f_0}{\partial p_\parallel} \right] = 0, \end{aligned} \tag{3}$$

where the quantity ω is the frequency of wave instability. k and ω_{pe} are explained as the wave vector and the electron frequency of plasma, respectively. The quantity ω' is the sum of the collision frequency and the frequency of the wave in the complex plan is $\omega' = \omega_r + i(\omega_i + \nu_{ei})$. The appendixes $\parallel$ and $\perp$ are introduced based on the parallel and perpendicular directions to the wave vector, and γ is the same relativistic mass factor as $\gamma = (1 + p_\parallel^2/m^2c^2 + p_\perp^2/m^2c^2)^{1/2}$.

During recent years, different distribution functions have been introduced which can show the temperature anisotropy for the relativistic plasmas, but it has not yet been specified which one of them is more suitable [8, 9]. Let us consider that all particles move on a surface with perpendicular momentum $p_\perp = \hat{p}_\perp =$ constant and that they are uniformly distributed in parallel momentum; then one form of distribution function can be shown as follows [9]:

$$\begin{aligned} &F\left(p_\perp^2, p_\parallel\right) \\ &= \frac{1}{2\pi p_\perp} \delta\left(p_\perp - \hat{p}_\perp\right) \frac{\exp\left(-\gamma\left(mc^2/T_\parallel\right)\right)}{2\hat{\gamma}_\perp mcK_1\left(\hat{\gamma}_\perp\left(mc^2/T_\parallel\right)\right)}. \end{aligned} \tag{4}$$

$K_1((mc^2/T_\parallel)\hat{\gamma}_\perp)$ is the modified Bessel function of the second kind of order 1. Making use of model distributio (4), it can be shown that (3) is given by the following:

$$\begin{aligned} &1 - \frac{c^2k^2}{\omega^2} - \frac{\omega_{pe}^2/\hat{\gamma}_\perp}{\omega^2} \frac{K_0(\alpha)}{K_1(\alpha)} + \frac{1}{2} \frac{\omega_{pe}^2/\hat{\gamma}_\perp}{\omega^2} \frac{T_\perp}{mc^2\hat{\gamma}_\perp K_0(\alpha)} \\ &\cdot \int_{-\infty}^{+\infty} \left(1 + x^2\right)^{-3/2} \exp\left(-\alpha\left(1 + x^2\right)^{1/2}\right) dx - \frac{ck}{2} \\ &\cdot \frac{T_\perp}{T_\parallel K_0(\alpha)} \frac{\omega_{pe}^2/\hat{\gamma}_\perp}{\omega^2} \\ &\cdot \int_{-\infty}^{+\infty} x\left(1 + x^2\right)^{-3/2} \frac{\exp\left(-\alpha\left(1 + x^2\right)^{1/2}\right)}{\left(\omega' - ck\left(x/\left(1 + x^2\right)^{1/2}\right)\right)} dx \\ &+ \frac{i\nu}{2} \frac{\omega_{pe}^2/\hat{\gamma}_\perp}{\omega^2} \frac{1}{K_1(\alpha)} \\ &\cdot \int_{-\infty}^{+\infty} \frac{\left(1 + x^2\right)^{-1/2} \exp\left(-\alpha\left(1 + x^2\right)^{1/2}\right)}{\left(\omega' - ck\left(x/\left(1 + x^2\right)^{1/2}\right)\right)} dx - \frac{i\nu}{2} \\ &\cdot \frac{\omega_{pe}^2/\hat{\gamma}_\perp}{\omega^2} \frac{T_\perp}{mc^2\hat{\gamma}_\perp K_0(\alpha)} \\ &\cdot \int_{-\infty}^{+\infty} \left(1 + x^2\right)^{-3/2} \frac{\exp\left(-\alpha\left(1 + x^2\right)^{1/2}\right)}{\left(\omega' - ck\left(x/\left(1 + x^2\right)^{1/2}\right)\right)} dx \\ &- \frac{i\nu}{2} \frac{\omega_{pe}^2/\hat{\gamma}_\perp}{\omega^2} \frac{ck}{mc^2\hat{\gamma}_\perp} \frac{T_\perp}{K_0(\alpha)} \\ &\cdot \int_{-\infty}^{+\infty} x\left(1 + x^2\right)^{-2} \frac{\exp\left(-\alpha\left(1 + x^2\right)^{1/2}\right)}{\left(\omega' - ck\left(x/\left(1 + x^2\right)^{1/2}\right)\right)^2} dx \\ &= 0, \end{aligned} \tag{5}$$

where $x = p_\parallel/mc\hat{\gamma}_\perp$. Notice that solving this equation may be really difficult. Therefore, let us consider a special condition, in which the parameter $\alpha = mc^2\hat{\gamma}_\perp/T_\parallel$ with $\hat{\gamma}_\perp = (1 + \hat{p}_\perp^2/m^2c^2)^{1/2}$ is much smaller than the unit; $\alpha \ll 1$. In such conditions, the dispersion relation can be corrected as follows:

$$\begin{aligned} &1 - \frac{c^2k^2}{\omega^2} - \frac{\omega_{pe}^2/\hat{\gamma}_\perp}{\omega^2} \frac{K_0(\alpha)}{K_1(\alpha)} + \frac{1}{2} \frac{\omega_{pe}^2/\hat{\gamma}_\perp}{\omega^2} \frac{T_\perp}{mc^2\hat{\gamma}_\perp K_0(\alpha)} \\ &\cdot \sqrt{\frac{2\pi}{\alpha}} \left\{ \frac{3}{2}\left(1 + \frac{1}{\alpha} - \frac{2\alpha}{3}\right) \right. \\ &+ \left(\frac{2}{\alpha}\right)^{1/2} \frac{\zeta}{2} + \left(1 - \frac{1}{2\alpha}\right)\zeta^2 + \frac{2}{\alpha}\zeta^4 + \frac{1}{2}\left(\frac{2}{\alpha}\right)^{7/2}\zeta^5 \\ &\left. + \left(\alpha\zeta - \zeta^3 + \frac{2}{\alpha}\zeta^5\right) Z(\zeta) \right\} - \frac{i\nu}{2} \frac{\omega_{pe}^2/\hat{\gamma}_\perp}{\omega^2} \frac{1}{K_1(\alpha)} \\ &\cdot \frac{\sqrt{\pi}}{ck} \left\{ \left(\frac{5}{\alpha} + 1\right) \frac{2}{\alpha}\zeta \right. \\ &\left. + 2\left(\frac{2}{\alpha}\right)^2 \zeta^3 + \left(1 + \frac{2}{\alpha}\zeta^2 + 2\left(\frac{2}{\alpha}\right)^2 \zeta^4\right) Z(\zeta) \right\} + \frac{i\nu}{2} \\ &\cdot \frac{\omega_{pe}^2/\hat{\gamma}_\perp}{\omega^2} \frac{T_\perp}{mc^2\hat{\gamma}_\perp K_0(\alpha)} \frac{\sqrt{\pi}}{ck} \left\{2\zeta + 2\zeta^2 Z(\zeta)\right\} = 0, \end{aligned} \tag{6}$$

where the quantity ζ is introduced as $\zeta = \sqrt{\alpha/2}(\omega'/ck)$ and $Z(\zeta)$ is the known dispersion function of plasma, $Z(\zeta) = (1/\sqrt{\pi})\int_{-\infty}^{+\infty}(e^{-t^2}/(\zeta - t))dt$.

The Fried-Conte plasma dispersion function (the dispersion function of plasma) can be written as [10]

$$\sqrt{\pi}Z(\zeta) \equiv \begin{cases} \displaystyle\int_{-\infty}^{+\infty} dt \frac{e^{-t^2}}{t-\zeta} & I_m\{\zeta\} > 0, \\ \displaystyle P\int_{-\infty}^{+\infty} dt \frac{e^{-t^2}}{t-\zeta} + \pi i e^{-\zeta^2} & I_m\{\zeta\} = 0, \\ \displaystyle\int_{-\infty}^{+\infty} dt \frac{e^{-t^2}}{t-\zeta} + 2\pi i e^{-\zeta^2} & I_m\{\zeta\} < 0. \end{cases} \quad (7)$$

Here, P is the Cauchy principal value operator that defines the integral on the singularity at $t = \zeta$ when ζ is real. While the definition of $Z(\zeta)$ might appear to be discontinuous at $I_m\{\zeta\} = 0$, it is in fact continuous there. Its continuity there can be verified by taking the $I_m\{\zeta\} \to 0$ limit for $\mathrm{Re}\{\zeta\} \neq 0$ of the forms given above for $I_m\{\zeta\} > 0$ and $I_m\{\zeta\} < 0$ and showing that they are identical to the $I_m\{\zeta\} = 0$ definition.

A wave propagates in the plasma environment when its frequency is higher than the frequency of the plasma. The Weibel unstable wave is a low-frequency wave ($|\omega| \ll ck$); therefore, the specific limiting condition, $|\zeta| \gg 1$, can not be suitable and the limiting $|\zeta| \ll 1$ is suitable for discussion.

For the limiting condition $|\zeta| \ll 1$, the corrected configuration of the plasma dispersion function can be obtained as follows (by the Plemelj formula):

$$Z(\zeta) = -2\zeta + \cdots + i\sqrt{\pi}e^{-\zeta^2}. \quad (8)$$

On setting the value of $Z(\zeta)$ in (6) and ignoring the second and higher order of ζ and ω^2 in contrast to c^2k^2, (6) reduces to

$$\begin{aligned} &-c^2k^2 - \frac{\omega_{pe}^2}{\widehat{\gamma}_\perp}\frac{K_0(\alpha)}{K_1(\alpha)} + \frac{1}{4}\frac{T_\perp}{T_\parallel}K_0(\alpha)\frac{\omega_{pe}^2}{\widehat{\gamma}_\perp}\left(\frac{2}{\alpha}\right)^{5/2}\frac{3\sqrt{\pi}}{4} \\ &\quad + i\frac{\pi}{\sqrt{2\alpha}}\frac{T_\perp}{T_\parallel K_0(\alpha)}\frac{\omega_{pe}^2}{\widehat{\gamma}_\perp}\zeta = 0. \end{aligned} \quad (9)$$

Based on the definitions, the growth rate of the Weibel instability is equal to the imaginary part of the wave frequency; $\delta = I_m\omega = \omega_i$; then the growth rate will be obtained from the above dispersion relation as:

$$\begin{aligned} \delta = &\frac{3ck}{8\sqrt{\pi}}\left(\frac{2}{\alpha}\right)^{5/2} \\ &\cdot\left(1 - \frac{\left(K_0(\alpha)/K_1(\alpha) + c^2k^2/\left(\omega_{pe}^2/\widehat{\gamma}_\perp\right)\right)}{\mu\left(T_\perp/T_\parallel\right)}\right) \\ &- \nu_{ei}, \end{aligned} \quad (10)$$

where the quantity $\mu = 3\sqrt{2\pi}/2\alpha^{5/2}K_0(\alpha)$ is the contribution of relativistic effects of distribution governed by the particles.

3. Results and Discussion

Our aim in this paper is the study and presentation of the growth rate and the conditions governed by the growth of the Weibel instability based on the analytical model. The growth rate is explained in (10). In this equation, one sentence is explained as the growth rate of instability when the Debye region of plasma is less in population, and the free path times of the particle are much greater than those of the interparticle interactions. In other words, the collision effects are very small and dispensable. In such situations, the growth condition is calculated as

$$\mu\eta\frac{T_\perp}{T_\parallel} \gg \left(\frac{K_0(\alpha)}{K_1(\alpha)} + \frac{c^2k^2}{\omega_{pe}^2/\widehat{\gamma}_\perp}\right), \quad (11)$$

where $\eta = 1 - (8\sqrt{\pi}/3ck)(\alpha/2)^{5/2}\nu_{ei}$ is included in the collision effects. Notice that the obtained results are based on the specific limiting condition $\alpha \ll 1$. Here, the limiting (asymptotic) form of the modified Bessel functions consist in

$$\begin{aligned} K_0(\alpha) &\longrightarrow -\left[\ln\left(\frac{\alpha}{2}\right) + 0.577\cdots\right], \\ K_1(\alpha) &\longrightarrow \frac{1}{\alpha}. \end{aligned} \quad (12)$$

According to these equations, the relation $K_0(\alpha)/K_1(\alpha)$ will always be positive. Therefore, the quantity η is valid in (11) only when it has a variation range between zero and one; $0 < \eta < 1$.

The electrons' momentum decreases due to collision with heavy ion background. Since it is suitable to know, the collisions as processes, the system will return back to the thermal equilibrium [11, 12]. But, in a relativistic system, the return to the thermal equilibrium will be less expected, because in such systems, the number of collisions will be less than a nonrelativistic case due to the high temperature of plasma. One case of collision frequency in relativistic plasma is given as [13]

$$\nu_{ei} = \begin{cases} \dfrac{z\pi^{3/2}e^4n_e}{2^{5/2}m_0^2c^3}\dfrac{3}{(\gamma^2-1)}\ln\Lambda^* & I_B \gg I_r \\ \dfrac{z\pi^{3/2}e^4n_e}{2m_0^{1/2}}\dfrac{1}{(\gamma^2-1)^{3/2}}\ln\Lambda & I_B \ll I_r, \end{cases} \quad (13)$$

where I_B and I_r are the Bremsstrahlung and reabsorption intensity, respectively. For the case of relativistic Bremsstrahlung reabsorption, the term Λ^* or Λ of the Coulombs logarithm ($\ln\Lambda^*$ or $\ln\Lambda$) is modified as follows:

$$\begin{aligned} \Lambda^* &= \frac{3\left(m_0c^2\right)^{3/2}}{2\pi^{1/2}ze^3n_e^{1/2}}(\gamma-1)^{3/2}, \\ \Lambda &= \frac{3}{2\pi^{1/2}ze^3}\left(\frac{K_B^3T_\perp^2T_\parallel}{n_e}\right)^{1/2}. \end{aligned} \quad (14)$$

The variation curves of the normalized growth rate, $(\delta/\omega_{pe})\widehat{\gamma}_\perp^{1/2}$, according to the variation of $(ck/\omega_{pe})\widehat{\gamma}_\perp^{1/2}$ for

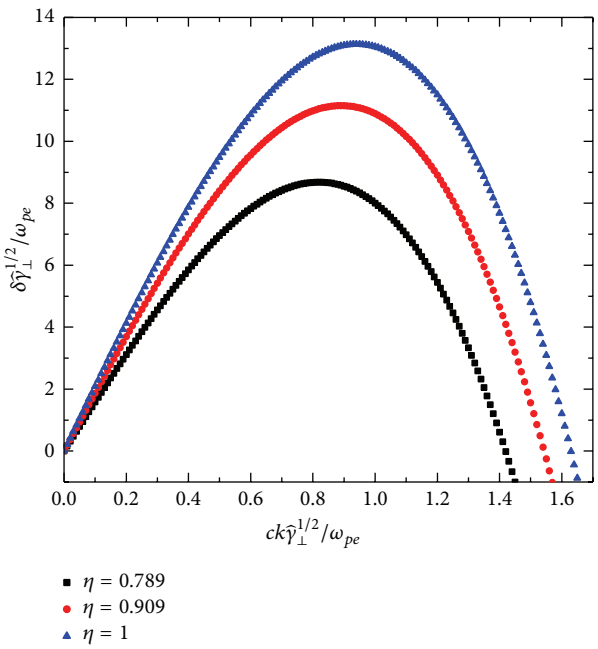

FIGURE 1: Variation of the normalized growth rate, $(\delta/\omega_{pe})\hat{\gamma}_\perp^{1/2}$, according to $(ck/\omega_{pe})\hat{\gamma}_\perp^{1/2}$ for difference η in limits $\alpha = 0.3$ and $\hat{\gamma}_\perp = 3$.

η difference are illustrated in Figure 1. It is shown that the Weibel instability growth rate decreases by increasing the collision frequency.

As defined, additional collision effects, two quantities $T_\perp/T_\parallel$ and μ, that are strongly dependent on the parameter α are effective on the growth rate and the growth condition of the instability. According to the definition of temperature, by using the distribution function (4), it can be shown as

$$\begin{aligned} \frac{T_\parallel}{mc^2\hat{\gamma}_\perp} &= \int \frac{d^3p}{m^2c^2\hat{\gamma}_\perp\gamma} p_\parallel^2 F\left(p_\perp^2, p_\parallel\right) = \frac{1}{\alpha}, \\ T_\perp &= \int d^3p \frac{p_\perp^2}{2m\gamma} F\left(p_\perp^2, p_\parallel\right) \\ &= \frac{1}{2} mc^2\hat{\gamma}_\perp \left(\frac{\hat{p}_\perp}{mc\hat{\gamma}_\perp}\right)^2 \frac{K_0(\alpha)}{K_1(\alpha)}. \end{aligned} \tag{15}$$

It must be noticed that, in the limiting condition $\alpha \ll 1$, we have $T_\parallel \gg T_\perp$ (Figures 2 and 3) while the normal Weibel instability is defined for $T_\perp \gg T_\parallel$. In fact, in this limit, an unusual situation of the Weibel instability is governed in the system. The obtained results show that, in such situation, the decrease of the temperature anisotropic parameter, α, leads to the increase of the instability growth rate (Figure 4). Also, Figure 5 shows that the growth rate of instability will increase by increasing the quantity $\hat{\gamma}_\perp$ that is equivalent to the increase of temperature anisotropy.

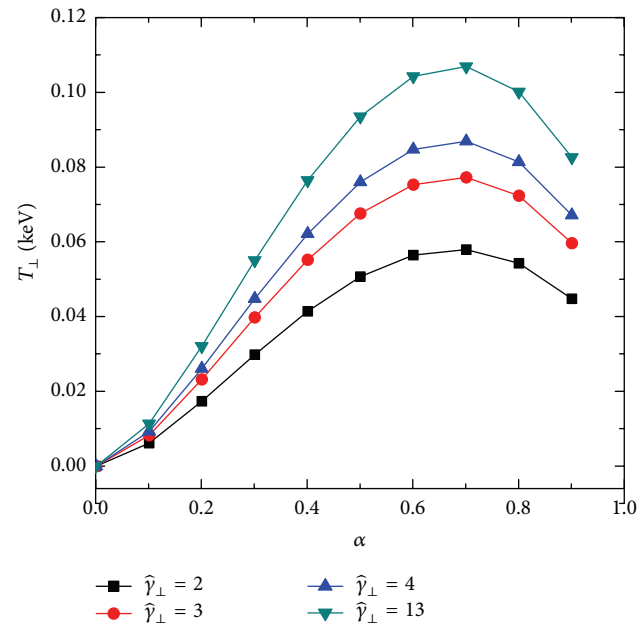

FIGURE 2: The variation curves of $T_\perp$ according to the quantity α for different values of $\hat{\gamma}_\perp$.

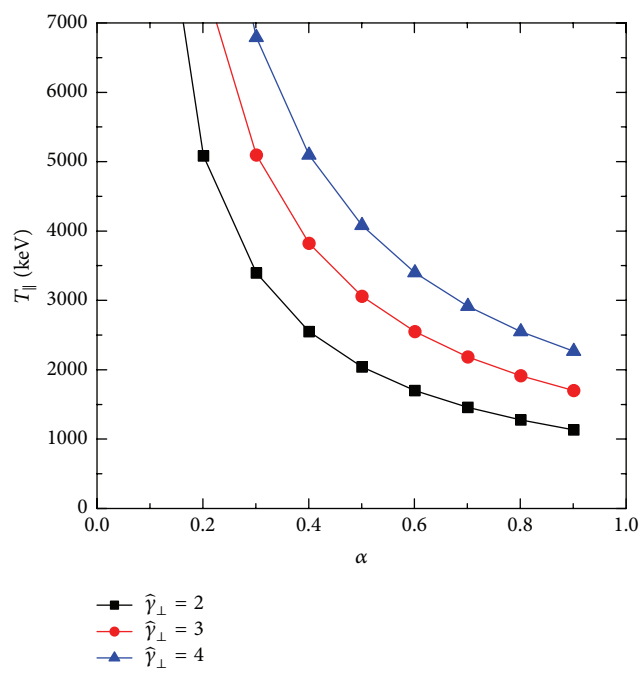

FIGURE 3: The variation curves of $T_\parallel$ according to the quantity α for different values of $\hat{\gamma}_\perp$.

4. Conclusions

In comparison to other results obtained in the context of relativistic Weibel instability, in this study, results are limited in the specific condition, $\alpha \ll 1$, in presence of background coulomb collisions. Obtained results show that an unusual situation of the Weibel instability is governed on the system, where, in comparison to the classical definition, the parallel

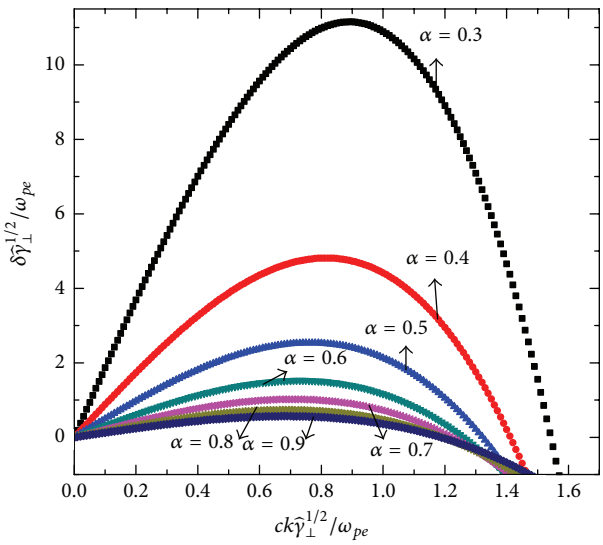

FIGURE 4: Variation of the normalized growth rate, $(\delta/\omega_{pe})\hat{\gamma}_\perp^{1/2}$, according to $(ck/\omega_{pe})\hat{\gamma}_\perp^{1/2}$ for difference α in limits $\eta = 0.909$ and $\hat{\gamma}_\perp = 3$.

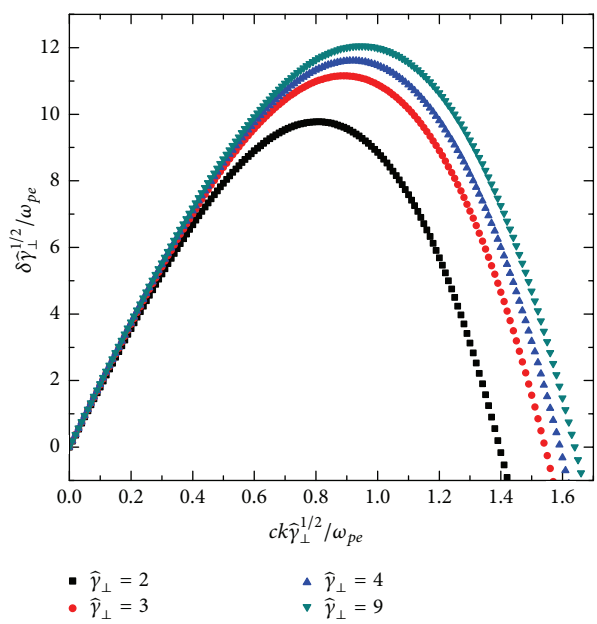

FIGURE 5: Variation of the normalized growth rate, $(\delta/\omega_{pe})\hat{\gamma}_\perp^{1/2}$, according to $(ck/\omega_{pe})\hat{\gamma}_\perp^{1/2}$ for difference $\hat{\gamma}_\perp$ in limits $\eta = 0.909$ and $\alpha = 0.3$.

temperature is higher than the perpendicular one. The Weibel instability growth rate is decreased by increasing the collision frequency, while, in the defined situation, decreasing of the temperature anisotropic parameter, α, leads to the increasing of the instability growth rate.

In laser produced plasmas and for not very strong laser fields, the effective collision frequency of particles' plasma in the parallel direction of the laser field is expected to be larger than that of the perpendicular direction. This implies that when it occurs, parallel plasma electron degrees of freedom get heated more efficiently than the perpendicular degrees of freedom. The outcome is anisotropic heating which has far-reaching consequences on an entire series of physical processes. Therefore, the given calculations can be useful for studying electromagnetic and electrostatic instabilities in such plasmas.

Appendix

In the limiting condition $\alpha \ll 1$,

$$\int_{-\infty}^{+\infty} \left(1+x^2\right)^{-3/2} \exp\left(-\alpha\left(1+x^2\right)^{1/2}\right) dx = \sqrt{\frac{2\pi}{\alpha}},$$

$$\int_{-\infty}^{+\infty} x\left(1+x^2\right)^{-3/2} \frac{\exp\left(-\alpha\left(1+x^2\right)^{1/2}\right)}{\left(\omega' - ck\left(x/\left(1+x^2\right)^{1/2}\right)\right)} dx$$
$$= -\frac{3\sqrt{\pi}}{8}\left(\frac{2}{\alpha}\right)^{5/2} - \left(\sqrt{\frac{2}{\alpha}}\zeta - \frac{(2/\alpha)^{3/2}\zeta^3}{2}\right.$$
$$\left. + \left(\frac{2}{\alpha}\right)^{5/2}\frac{\zeta^5}{2}\right)\left(\sqrt{\pi}Z(\zeta)\right) - \sqrt{\frac{2\pi}{\alpha}}\left(1 + \frac{1}{\alpha}\zeta^2\right.$$
$$\left. + \frac{2}{\alpha^2}\zeta^4 - \left(\frac{2}{\alpha}\right)^{5/2}\zeta^5\right) - \frac{\sqrt{\pi}}{4}\left(\frac{2}{\alpha}\right)^{3/2}\left(1 - \sqrt{\frac{2}{\alpha}}\zeta\right.$$
$$\left. + \left(\frac{2}{\alpha}\right)\zeta^2 - \left(\frac{2}{\alpha}\right)^{3/2}\zeta^3\right),$$

$$\int_{-\infty}^{+\infty} \left(1+x^2\right)^{-3/2} \frac{\exp\left(-\alpha\left(1+x^2\right)^{1/2}\right)}{\left(\omega' - ck\left(x/\left(1+x^2\right)^{1/2}\right)\right)} dx \tag{A.1}$$
$$= -\frac{\sqrt{\pi}}{ck} Z(\zeta),$$

$$\int_{-\infty}^{+\infty} \frac{\left(1+x^2\right)^{-1/2} \exp\left(-\alpha\left(1+x^2\right)^{1/2}\right)}{\left(\omega' - ck\left(x/\left(1+x^2\right)^{1/2}\right)\right)} dx$$
$$= -\frac{\sqrt{\pi}}{ck}\left\{Z(\zeta)\left(1 + \frac{2}{\alpha}\zeta^2 + 2\left(\frac{2}{\alpha}\right)^2\zeta^4\right)\right.$$
$$\left. + \left(\left(\frac{2}{\alpha}\right)\zeta\right)\left(1 + \frac{5}{\alpha}\right) + 2\left(\frac{2}{\alpha}\right)^2\zeta^3\right\},$$

$$\int_{-\infty}^{+\infty} x\left(1+x^2\right)^{-2} \frac{\exp\left(-\alpha\left(1+x^2\right)^{1/2}\right)}{\left(\omega' - ck\left(x/\left(1+x^2\right)^{1/2}\right)\right)^2} dx$$
$$= -\frac{\sqrt{\pi}}{c^2k^2}\left\{\left(2\zeta^2 - 1\right)Z(\zeta) + 2\zeta\right\}.$$

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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