Two Higgs Bosons near 125 GeV in the Complex NMSSM and the LHC Run I Data

Stefano Moretti and Shoaib Munir

School of Physics & Astronomy, University of Southampton, Southampton SO17 1BJ, UK
Asia Pacific Center for Theoretical Physics, San 31, Hyoja-dong, Nam-gu, Pohang 790-784, Republic of Korea

Correspondence should be addressed to Shoaib Munir; s.munir@apctp.org

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1. Introduction

The Higgs sector of the NMSSM [1–4] (see, e.g., [5, 6] for reviews) contains two additional neutral mass eigenstates besides the three of the Minimal Supersymmetric Standard Model (MSSM). This is due to the presence of a Higgs singlet superfield besides the two doublet superfields of the MSSM. When all the parameters in the Higgs and sfermion sectors of the NMSSM are real, one of these new Higgs states is a scalar and the other a pseudoscalar. Hence, in total three scalars, \( H_{1,2,3} \), and two pseudoscalars, \( A_{1,2} \), make up the neutral Higgs boson content of the model. This extended Higgs sector of the NMSSM boasts some unique phenomenological possibilities, which are either precluded or experimentally ruled out in the MSSM. For example, in the NMSSM either of the two lightest CP-even Higgs bosons, \( H_1 \) or \( H_2 \), can play the role of the ~125 GeV Standard Model- (SM-) like Higgs boson, \( H_{obs} \), observed at the LHC [7–9].

Of particular interest in the NMSSM is the possibility that the SM-like Higgs boson can obtain a large tree-level mass in a natural way, that is, without requiring large radiative corrections from the supersymmetric sectors. This happens in a specific region of the parameter space, which we refer to as the natural NMSSM, where there is a significant singlet-doublet mixing and \( H_{obs} \) is typically \( H_2 \). This scenario was used to explain [10–12] the enhancement in \( H_{obs} \rightarrow \gamma \gamma \) channel in the early LHC data. However, when the singlet-doublet mixing is too large, the properties of \( H_2 \) can deviate appreciably from an exact SM-like behaviour, resulting in a reduction of its fermionic partial decay widths. An alternative possibility in a very similar parameter space region is that of both \( H_1 \) and \( H_2 \) simultaneously having masses near 125 GeV [13–16]. In that case, the observed excess at the LHC could actually be due to a superposition of these two states, when their individual signal peaks cannot be resolved separately. One of these two Higgs bosons, typically \( H_1 \), is the singlet-like neutral state. Moreover, in [17] it was noted that the lighter one of the two pseudoscalars, \( A_1 \), when it is singlet-like, could also be nearly mass-degenerate with a SM-like \( H_1 \) near 125 GeV, instead of or even along with \( H_2 \). However, such a pseudoscalar can only contribute visibly to the measured signal strength near 125 GeV if it is produced in association with \( b\bar{b} \) pair.

One of the most important yet unresolved issues in particle physics is that of the observed matter-antimatter
asymmetry in the universe. A plausible explanation for this asymmetry is electroweak (EW) baryogenesis [18, 19]. The necessary conditions for successful EW baryogenesis include the following [20]: (1) baryon number violation, (2) CP-violation, and (3) departure from equilibrium at the critical temperature of the EW symmetry breaking (EWSB) phase transition, implying that it is strongly first order. In the SM, a strongly first order EW phase transition is not possible given the measured mass of the Higgs boson at the LHC. Besides, the only source of CP-violation in the SM, the Cabibbo-Kobayashi-Maskawa matrix, is insufficient. Therefore, beyond the SM, a variety of sources of CP-violation have been proposed in the literature (for a review, see [21] and references therein). In the context of supersymmetry (SUSY), a strongly first order phase transition is possible in the MSSM only if the lightest stop has a mass below that of the top quark. This possibility has now been ruled out by SUSY searches at the LHC [22–24]. Also, the MSSM Higgs sector does not violate CP at the tree level but does so only at higher orders [25–32]. The CPV phases, transmitted radiatively to the Higgs sector via couplings to the sfermions, are tightly constrained by the measurements of fermion electric dipole moments (EDMs) [33–35]. However, these EDM constraints can be relaxed under certain conditions [27, 28, 36–41].

The NMSSM has been shown to accommodate a strongly first order EW phase transition without a light stop [42–47]. Additionally, in this model, CP-violation can be invoked explicitly in the Higgs sector even at the tree level by assuming the Higgs self-couplings, λ and κ, to be complex. Beyond the Born approximation, the phase of the SUSY-breaking Higgs-sfermion-sfermion couplings, A_f, where f denotes a SM fermion, is also induced in the Higgs sector, as in the MSSM. In the presence of the associated complex phases, the five neutral Higgs bosons are CP-indefinite states, due to the mixing between the scalar and pseudoscalar interaction eigenstates. CPV phases can therefore influence the phenomenology of the NMSSM Higgs bosons by, for example, modifying their mass spectrum as well as their production and decay rates [48], similarly to the MSSM [49–59]. The impact of these phases in the complex NMSSM (cNMSSM), that is, the CPV NMSSM, on the necessary conditions for successful EW phase transition was also studied some time ago [60]. The consistency of scenarios yielding the correct baryon asymmetry with the LHC Higgs boson data still remains to be studied in depth though. However, even leaving aside these considerations, the possibly distinct phenomenological scenarios that the cNMSSM can yield make it a particularly interesting model for exploration at the Run II of the LHC.

The cNMSSM has therefore been the subject of several studies recently and, in particular, some important theoretical developments have been made in the model. The dominant 1-loop corrections to the neutral Higgs sector from the (s)quark and gauge sectors were studied in [61–64], in the renormalisation group equations-improved effective potential approach. The corrections from the gaugino sector were included in [65] and, more inclusively, recently in [66]. In the Feynman diagrammatic approach, the complete 1-loop Higgs mass matrix was derived in [67] and Ω(α,α) contributions to it were calculated in [68]. As far as the phenomenology of the Higgs bosons in the cNMSSM is concerned, the consistency of several CPV scenarios with the early results on H_{obs} from the LHC data was studied in detail in [48, 67]. Another distinct phenomenological scenario, possible only for nonzero CPV phases, has also been studied in [65].

The CMS and ATLAS collaborations have recently updated their measurements of H_{obs} signal rates in τ+τ− and b̄b channels [69, 70]. The fact that these rates also tend to favour a SM-like H_{obs} is increasingly jeopardising the above-mentioned natural NMSSM scenario with large singlet-doublet mixing but only with one Higgs boson, either H_1 or H_2, around 125 GeV. This makes the scenario with both H_1 and H_2 contributing to the observed ~125 GeV signal all the more important, since it may potentially satisfy better the current Higgs boson data while still leaving plenty of room for new physics. In case of the cNMSSM, since the five neutral Higgs bosons are CP-mixed states, the scenario with mass-degenerate H_1 and H_2 can entail both the corresponding possibilities in the real NMSSM (rNMSSM), that is, mass-degenerate H_1, H_2 or H_1/H_2, A_1.

In this study we therefore analyse and compare the prospects for scenarios with two mass-degenerate Higgs bosons against those with a single Higgs boson near 125 GeV in the Z_3-invariant cNMSSM. We perform scans of the relevant parameter space [13] of the model using the public program NMSSMCALC [71] to search for all possible ~125 GeV Higgs boson scenarios, with the CPV phase of the coupling κ set to five different values, including 0°, the NMSSM limit, in each case. The condition for mass-degeneracy between two Higgs bosons is imposed by requiring them to lie within 2.5 GeV of each other, which is consistent with the current mass resolution of the LHC [72], taking into account the uncertainties in the theoretical mass prediction. We then use fits to the Higgs boson data from the LHC Run I, both with √s = 7 TeV and √s = 8 TeV, as well as the Tevatron, performed using the program HiggsSignals [73], as the sole criterion for comparing the present likelihood of each of these scenarios. We also discuss how these mass-degenerate Higgs bosons can be identified at the LHC based on the signal rate double ratios introduced in [74].

The paper is organised as follows. In the next section we will briefly revisit the Higgs sector of the cNMSSM. In Section 3 we will provide details of our numerical scans and our procedure for fitting the model predictions for the Higgs boson(s) to the LHC data. In Section 4 we will discuss the results of our analysis and in Section 5 we will present our conclusions.

2. The Higgs Sector of the cNMSSM

The NMSSM contains a singlet Higgs superfield, S, besides the two SU(2)_L doublet superfields,

\[
\begin{align*}
\bar{H}_u &= \begin{pmatrix} \tilde{H}_u^+ \\ \tilde{H}_u^- \end{pmatrix}, \\
\bar{H}_d &= \begin{pmatrix} \tilde{H}_d^+ \\ \tilde{H}_d^- \end{pmatrix}.
\end{align*}
\]
Advances in High Energy Physics

of the MSSM. The superpotential of the NMSSM is written as

\[ W_{\text{NMSSM}} = W_{\text{MSSM}} + \lambda \hat{S} \hat{H}_u \hat{H}_d + \frac{\kappa}{3} S^3, \]  

(2)

where \( \lambda \) and \( \kappa \) are dimensionless Yukawa couplings. This superpotential is scale invariant, since the term \( \mu \hat{H}_u \hat{H}_d \) appearing in the MSSM superpotential has been removed by imposing a discrete \( Z_3 \) symmetry. In this model, an effective \( \mu \)-term, \( \mu_{\text{eff}} = \lambda s \), is instead generated when the singlet field acquires a vacuum expectation value (VEV), \( s \), which is naturally of the order of the SUSY-breaking scale.

The tree-level Higgs potential of the NMSSM, obtained from the superpotential in (2), is written in terms of the neutral scalar components of the Higgs superfields, \( H_u, H_d, \) and \( S \), as

\[ V_0 = \left[ \lambda \left( H_u^+ H_d - H_d^0 H_u^0 \right) + \kappa S^3 \right]^2 + \left( m_{H_u}^2 + |\lambda S|^2 \right) \left( |H_u|^2 - |H_d|^2 \right)^2 + \left( m_{H_d}^2 + |\lambda S|^2 \right) \left( |H_d|^2 - |H_u|^2 \right)^2 + \frac{g^2}{4} \left( |H_u|^2 + |H_d|^2 \right)^2 + \left( \lambda A_\lambda \left( H_u^0 H_d^0 - H_d^0 H_u^0 \right) S + \frac{\kappa}{3} \lambda A_\lambda S^3 + \text{h.c.} \right), \]  

(3)

where \( g^2 \equiv (g_1^2 + g_2^2)/2 \), with \( g_1 \) and \( g_2 \) being the \( U(1)_Y \) and \( SU(2)_L \) gauge couplings, respectively, and \( A_\lambda \) and \( A_\kappa \) are the soft SUSY-breaking Higgs trilinear couplings. The scalar fields \( H_u, H_d, \) and \( S \) are developed around their respective VEVs, \( v_u, v_d, \) and \( s \), as

\[ H_u^0 = \left( \frac{1}{\sqrt{2}} v_u + H_{uR} + i H_{uI} \right), \]

\[ H_d^0 = e^{\phi} \left( \frac{1}{\sqrt{2}} v_d + H_{dR} + i H_{dI} \right), \]

\[ S = \frac{e^{\phi} s}{\sqrt{2}} \left( s + S_R + i S_I \right). \]

(4)

The Higgs coupling parameters appearing in the potential in (3) can very well be complex, implying \( \lambda \equiv |\lambda| e^{i \theta}, \kappa \equiv |\kappa| e^{i \phi}, A_\lambda \equiv |A_\lambda| e^{i \phi_{A_\lambda}}, \) and \( A_\kappa \equiv |A_\kappa| e^{i \phi_{A_\kappa}}. \) As a result, \( V_0 \), evaluated at the vacuum, contains the phase combinations

\[ \phi'_{\lambda} = \phi_{\lambda} + \theta + \phi, \]

\[ \phi'_{\kappa} = \phi_{\kappa} + 3 \phi, \]

\[ \phi_{\lambda}' + \phi_{A_\lambda}, \]

\[ \phi_{\kappa}' + \phi_{A_\kappa}. \]

(5)

For correct EWSB, the Higgs potential should have a minimum at nonvanishing \( v_u, v_d, \) and \( s \), which is ensured by requiring

\[ \left\langle \frac{\delta V_0}{\delta \Phi} \right\rangle = 0 \text{ for } \Phi = H_{dR}, H_{uR}, S_R, H_{dI}, H_{uI}, S_I. \]

(6)

Through the above minimisation conditions the phase combinations \( \phi_{\lambda}' + \phi_{A_\lambda} \) and \( \phi_{\kappa}' + \phi_{A_\kappa} \) can be determined up to a twofold ambiguity by \( \phi_{\lambda}' - \phi_{\kappa}' \). Thus, \( \phi_{\lambda}' - \phi_{\kappa}' \) is the only physical CP phase appearing in the NMSSM Higgs sector at the tree level. Also, using these conditions, the soft mass parameters \( m_{H_u}^2, m_{H_d}^2, \) and \( m_s^2 \) can be traded for the corresponding Higgs field VEVs.

The neutral Higgs mass matrix is obtained by taking the second derivative of \( V_0 \) evaluated at the vacuum. This \( 5 \times 5 \) matrix, \( M_0 \), in the \( H^T = (H_d, H_u, S_R, H_I, S_I) \) basis, from which the massless Nambu-Goldstone mode has been rotated away, can be diagonalized using an orthogonal matrix, \( O \), as

\[ O^T M_0 O = \text{diag} \left( m_{H_u}^2, m_{H_d}^2, m_s^2, m_{H_I}^2, m_{H_I}^2 \right). \]

This yields the physical tree-level masses corresponding to the five mass eigenstates:

\[ (H_1, H_2, H_3, H_4, H_5)^T = O_{ai} \left( H_{dR}, H_{uR}, S_R, H_I, S_I \right)^T, \]

(7)

such that \( m_{H_1}^2 \leq m_{H_2}^2 \leq m_{H_3}^2 \leq m_{H_4}^2 \leq m_{H_5}^2 \). The elements, \( O_{ai} \), of the mixing matrix then govern the couplings of the Higgs bosons to all the particles in the model.

The tree-level Higgs mass matrix is subject to higher order corrections from the SM fermions, from the gauge and chargino/neutralino sectors and the Higgs sector itself, as well as the sfermion sector, in case of which they are dominated by the stop contributions. Upon the inclusion of these corrections, \( \Delta M^2 \), the Higgs mass matrix gets modified, so that

\[ \Delta M^2 = M_0^2 + \Delta M^2. \]

(8)

Explicit expressions for \( \Delta M^2 \) as well as \( \Delta M^2 \) can be found in [65–67]. Thus, beyond the Born approximation, the CPV phases of the gaugino mass parameters, \( M_{1,2} \), and of \( A_\gamma \) are also radiatively induced in the Higgs sector of the NMSSM.

Therefore, when studying the phenomenology of the Higgs bosons, one needs to take into account also the parameters from the other sectors of the model. However, the most general NMSSM contains more than 130 parameters at the EW scale. Assuming the matrices for the sfermion masses and for the trilinear scalar couplings to be diagonal considerably reduces the number of free parameters. One can further exploit the fact, mentioned above, that the corrections to the Higgs boson masses from the sfermions are largely dominated by the stop sector. For our numerical analysis in the following sections, we will thus impose the following...
supergravity-inspired universality conditions on the model parameters at the EW scale:

\[ M_0 \equiv M_{Q_{1,2,3}} = M_{U_{1,1,1}} = M_{D_{1,2,3}} = M_{L_{1,2,3}} = M_{E_{1,2,3}}, \]

\[ M_{1/2} \equiv 2M_t = M_2 = \frac{1}{3} M_3, \]

\[ A_0 \equiv A_7 = A_9 = A_\gamma, \]

where \( M^2_{Q_{1,2,3}}, M^2_{U_{1,1,1}}, M^2_{D_{1,2,3}}, M^2_{L_{1,2,3}}, \) and \( M_{E_{1,2,3}}^2 \) are the squared soft masses of the sfermions, \( M_{1/2} \) those of the gauginos, and \( A_{7,9,\gamma} \) the soft trilinear couplings. Altogether, the input parameters of the cNMSSM then include \( M_0, |M_{1/2}|, |A_0|, \tan \beta (\equiv v_u/v_d), |\lambda|, |\kappa|, \mu_{\text{eff}}, |A_\lambda|, |A_\mu|, \phi_{1/2}, \phi_7, \phi'_1, \) and \( \phi'_x, \) where \( \phi_{1/2} \) and \( \phi_7 \) are the phases of the unified parameters \( M_{1/2} \) and \( A_0 \), respectively.

### 3. Numerical Analysis

As noted in the Introduction, nonzero CPV phases can modify appreciably the masses and decay widths of the neutral Higgs bosons compared to the CP-conserving case for a given set of the remaining free parameters. In the case of \( H_{\text{obs}} \), candidate in the model, whether \( H_1 \) or \( H_2 \) or even \( H_3 \), the CPV phases are thus strongly constrained by the LHC mass and signal rate measurements. This was analysed in detail in [48], where the scenarios with mass-degenerate Higgs bosons were, however, not taken into account. In the present study we thus test whether the said modifications in the Higgs boson properties with nonzero values of the phase \( \phi_x \) remain valid.

The reason for choosing \( \phi'_x \) as the only variable phase while setting \( \phi_{1/2}, \phi_7 \), and \( \phi'_1 \) to 0° is that it is virtually unconstrained by the measurements of fermionic EDMs [63, 64, 67]. Furthermore, our aim here is to analyse the scenarios with a generic CPV phase and compare them with the rNMSSM limit rather than measure the effect of any of the individual phases. Note however that since only the difference \( \phi'_x - \phi'_1 \) enters the Higgs mass matrix at the tree level, the impact of a variation in \( \phi'_x \) is also quantified by that due to the variation in \( \phi'_1 \) at this level. Higher orders though, a variation in \( \phi'_1 \) has an impact on the sfermion and neutralino/chargino sectors which is independent of \( \phi'_x \).

In our numerical analysis, we used the program NMSSMCALC-v1.03 [71] for computing the Higgs boson mass spectrum and decay branching ratios (BRs) for a given model input point. The public distribution of NMSSMCALC contains two separate packages, one for the rNMSSM only and the other for the cNMSSM. Some supersymmetric corrections to the Higgs boson decay widths are currently only available in the rNMSSM and hence are not included in the cNMSSM package. For consistency among our rNMSSM and cNMSSM results, we therefore set \( \phi_x = 0° \) in the cNMSSM package for the rNMSSM case instead of using the dedicated rNMSSM package. Furthermore, using the cNMSSM code also for the rNMSSM limit makes it convenient to draw a one-on-one correspondence between \( \phi_x = 0° \) case and each of \( \phi_x > 0° \) cases in a given scenario. This is because in the cNMSSM package, even in the rNMSSM limit, the five neutral Higgs bosons are ordered by their masses and not separated on the basis of their CP-identities. Thus, the scenario with mass-degenerate \( H_1, H_2 \), which we will henceforth refer to as the \( H_{\text{obs}} = H_1 + H_2 \) scenario, takes into account both the \( \sim 125 \) GeV \( H_1, H_2 \) and the \( \sim 125 \) GeV \( H_1, A_1 \) solutions of the rNMSSM without distinguishing between them. If one, conversely, uses the rNMSSM package, these two scenarios ought to be considered separately. The same is true also for the \( H_{\text{obs}} = H_2 + H_3 \) scenario, wherein \( H_2, H_3 \) are mass-degenerate.

The program NMSSMCALC allows one the option to include only the complete 1-loop contributions in the Higgs mass matrix or to add also the 2-loop \( \mathcal{O}(\alpha, \kappa) \) corrections to it. In our analysis, for a better theoretical precision, we evaluated the Higgs boson masses at the 2-loop level. In the NMSSMCALC input, one also needs to choose between the modified dimensional regularisation (DR) and on-shell renormalisation schemes for calculating contributions from the top/stop sector in the program. We opted for the DR scheme for each scenario. Note though that further inclusion of \( \mathcal{O}(\alpha_s, \alpha_t) \), \( \mathcal{O}(\alpha_s + \alpha_t + \alpha_t^2) \), and the recently calculated NMSSM-specific \( \mathcal{O}(\alpha_s + \alpha_t^2) \) 2-loop corrections [75] in NMSSMCALC may have a negligibly impact on the Higgs boson masses and observables [76]. We, however, maintain that such contributions will only result in a slight shifting of the parameter configurations yielding solutions of our interest here, but our overall results and conclusions should still remain valid.

We performed six sets of scans of the cNMSSM parameter space by linking NMSSMCALC with the MultiNest-v2.18 [77–79] package. MultiNest performs a multimodal sampling of a theoretical model’s parameter space based on Bayesian evidence estimation. However, we use this package not for drawing Bayesian inferences about the various NMSSM scenarios considered but simply to avoid a completely random sampling of the 9-dimensional model parameter space. In the program, we therefore defined a Gaussian likelihood function for \( H_{\text{obs}} \) in a given scan, assuming the experimental measurement of its mass to be \( 125 \) GeV and allowing up to \( \pm 2 \) GeV error in its model prediction. We set the enlargement factor reduction parameter to 0.3 and the evidence tolerance factor to a rather small value of 0.2, so that while the package was sampled more concentratedly near the central mass value, a sufficiently large number of points were collected before the scan converged. In each of the first two sets of scans we required \( H_1 \) to be \( H_{\text{obs}} \). In the third set we imposed this requirement of consistency with \( H_{\text{obs}} \) mass on \( H_2, H_3 \) in the fourth set on \( H_3, \) in the fifth set on both \( H_1 \) and \( H_2, \) and in the sixth set on both \( H_2 \) and \( H_3. \) Each of the six sets further contained five separate scans corresponding to \( \phi_x = 0°, 3°, 10°, 30°, \) and 60°.

The scanned ranges of the nine free parameters (after fixing the phases) of the natural NMSSM, which are uniform across all its five scenarios considered, are given in Table I(a). Only large values of \( \lambda \) and \( \kappa \) are used in this model (with the upper cut-off on them imposed to avoid the Landau pole).
Table 1: Ranges of the NMSSM parameters scanned, with fixed \( \phi_0 \), for (a) each \( H_{\text{obs}} \) scenario in the natural NMSSM and (b) the low-\( \lambda \)-NMSSM scenario.

(a) Parameter | Natural NMSSM range
--- | ---
\( M_0 \) (GeV) | 200–2000
\( M_{1/2} \) (GeV) | 100–1000
\( A_0 \) (GeV) | \(-3000–0\)
tan \( \beta \) | 1–8
\( \lambda \) | 0.4–0.7
\( \kappa \) | 0.3–0.6
\( \mu_{\text{eff}} \) (GeV) | 100–300
\( A_\chi \) (GeV) | \(-1000–1000\)
\( A_\kappa \) (GeV) | \(-1000–1000\)

(b) Parameter | Low-\( \lambda \)-NMSSM range
--- | ---
\( M_0 \) (GeV) | 200–4000
\( M_{1/2} \) (GeV) | 100–2000
\( A_0 \) (GeV) | \(-7000–0\)
tan \( \beta \) | 5–45
\( \lambda \) | 0.001–0.4
\( \kappa \) | 0.001–0.3
\( \mu_{\text{eff}} \) (GeV) | 100–2000
\( A_\chi \) (GeV) | \(-1000–4000\)
\( A_\kappa \) (GeV) | \(-4000–1000\)

Since large radiative corrections from SUSY sectors are not necessary in the natural limit of the NMSSM, the parameters \( M_0, M_{1/2}, \) and \( A_0 \) are not required to take too large values. Note that while \( A_0 \) can in principle be both positive and negative, with a slightly different impact on the physical mass of the SM-like Higgs boson for an identical set of other input parameters in each case, we restricted the scans to its negative values only, in order to increase the scanning efficiency.

In the remaining sixth scan, we considered the complementary parameter space of the NMSSM, with \( \lambda \) and \( \kappa \) kept to relatively smaller (and tan \( \beta \) to larger) values, so as to prevent too large singlet-doublet mixing. In fact, for \( \lambda, \kappa \rightarrow 0 \), when the singlet sector gets effectively decoupled, \( H_1 \), which is by default identified with \( H_{\text{obs}} \), has properties very identical to the lightest Higgs boson of the MSSM. Since \( H_1 \) in such a case does not obtain a maximal tree-level mass that is possible in the most general model, large radiative corrections are needed from the SUSY sector. Hence we used slightly extended ranges of the remaining parameters, which are given in Table 1(b). This scenario, which we refer to as the low-\( \lambda \)-NMSSM scenario henceforth, has been included in our analysis in order to compare the inferences made for the natural NMSSM with an approximate MSSM limit of the model.

Once the scans had been completed, we filtered the points obtained with each by further imposing 123 GeV \( \leq m_{H_{\text{obs}}} \leq 127 \) GeV. Note that, in the \( H_{\text{obs}} = H_1 + H_2 \) and \( H_{\text{obs}} = H_2 + H_3 \) scenarios, this condition was imposed on \( H_2 \), since in both these scenarios it is typically the Higgs boson with SM-like couplings. The total number of points, \( N_{\text{total}} \), remaining after this filter is given in Table 2 for each scenario considered.

All these points were then tested for consistency with the LEP and LHC exclusion limits on the other, non-SM-like, Higgs bosons of the model, using the package HiggsBounds v4.2.0 [80–83]. The points passing the HiggsBounds test were retained as the “good points” for further analysis, and their number, denoted by \( N_{\text{HBR}} \), for each scenario is also given in Table 2. We point out for later reference that in each of the two \( H_{\text{obs}} = H_1 \) scenarios and \( H_{\text{obs}} = H_1 + H_2 \) scenario, the number of surviving good points (where they are available) is very identical across all input values of \( \phi_0 \), implying mutually fairly consistent sample sizes.

Next we carried out fits to \( H_{\text{obs}} \) data for the good points using the public code HiggsSignals v1.3.2 [73]. For obtaining these fits, HiggsSignals requires, along with the masses and BRs of each Higgs boson, \( H_i \), the square of its normalised effective couplings, \( (g_{H_iX}/g_{h_SMX})^2 \), to a given SM particle pair \( X \), with \( h_SM \) being the SM Higgs boson with the same mass as \( H_i \). Note that when \( X \) is a pair of fermions, there is a scalar as well as a pseudoscalar normalised coupling for each \( H_i \), both of which need to be passed separately to HiggsSignals. All these are then used to calculate the normalised cross sections:

\[
\mu_{H_i}^X = \frac{\sigma(pp \rightarrow H_i \rightarrow X)}{\sigma(pp \rightarrow h_SM \rightarrow X)},
\]

(10)
corresponding to a given decay channel, \( X \), in an approximate way. The NMSSMCALC version we used did not provide the normalised Higgs boson couplings as an output. We therefore modified the code to obtain these couplings for adding them as a dedicated block in the SLHA input file for HiggsSignals.

The program HiggsSignals compares the computed \( \mu_{H_i}^X \) for each \( H_i \) with the experimentally measured ones, \( \mu_{\text{exp}}^X \), for wide ranges of input Higgs boson masses in a variety of its production and decay channels at the LHC and the Tevatron. We used only the "peak-centred" method and the "latest results" observable set in the program, with the assignment range variable \( A \) set to the default value of 1. It thus performed a fit to a total of 81 Higgs boson peak observables (77 from signal strength and 4 from mass measurements), from the CMS, ATLAS, CDF, and DØ collaborations, for a given model point. We assumed a Gaussian theoretical uncertainty of 2 GeV in the masses of the three lightest neutral Higgs bosons of the model. The default values of the uncertainties in the Higgs boson production cross sections as well as BRs were retained. Further details about the fitting procedure can be found in the manual [73] of the package. The main output of HiggsSignals contains the total \( \chi^2 \) and the p value from the fit, given the number of statistical degrees of freedom, for each model point. Since the aim of this study is a comparison of various \( H_{\text{obs}} \) scenarios rather than the overall goodness of fit for each, we will quantify our results only in terms of \( \chi^2 \) and ignore p value.

As an observable indication of the presence of more than one Higgs boson near 125 GeV, the double ratios

\[
D_1 = \frac{R_{\text{BBF}}^\mu (\gamma\gamma)}{R_{\gamma\gamma}^\mu (\gamma\gamma)} / \frac{R_{\text{BBF}}^\nu (bb)}{R_{\gamma\gamma}^\nu (bb)};
\]
Table 2: Number of scanned points remaining after imposing the mass constraint on $H_{\text{obs}}$ and those passing the HiggsBounds test, for each scenario studied. See text for details.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Low-$\lambda$</th>
<th>$H_{\text{obs}} = H_1$</th>
<th>$H_{\text{obs}} = H_2$</th>
<th>$H_{\text{obs}} = H_1 + H_2$</th>
<th>$H_{\text{obs}} = H_1 + H_3$</th>
<th>$H_{\text{obs}} = H_2 + H_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{\text{total}}$</td>
<td>17886</td>
<td>15675</td>
<td>15072</td>
<td>14431</td>
<td>26045</td>
<td>23736</td>
</tr>
<tr>
<td>$N_{\text{HB}}$</td>
<td>17722</td>
<td>13691</td>
<td>2904</td>
<td>965</td>
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<td>2819</td>
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<td>$N_{\text{total}}$</td>
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<td>15026</td>
<td>14806</td>
<td>27199</td>
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<td>3235</td>
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<td>14810</td>
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<td>14725</td>
<td>247</td>
<td>276</td>
<td>13507</td>
<td>2102</td>
</tr>
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</table>

were proposed in [74]. Each of these ratios should be unity if $H_{\text{obs}}$ consists of only a single Higgs boson, while the contribution of two (or more) Higgs bosons to $H_{\text{obs}}$ signal could result in a deviation of these ratios from 1. In the above expressions, $R_i^h(X) = R_i^{\gamma\gamma}(X) + R_i^{WW}(X)$, where $H_i$ and $H_j$ are the two mass-degenerate Higgs bosons in a given scenario and the subscripts VBF and gg imply the vector boson fusion and the gluon fusion production modes, respectively. $R_i^{\gamma\gamma}(X)$ for each $H_i$ is defined as

$$D_2 = \frac{R_i^{\gamma\gamma}(\gamma\gamma)}{R_i^{\gamma\gamma}(WW)} = \frac{R_i^{\gamma\gamma}(WW)}{R_i^{\gamma\gamma}(VBF)}$$

$$D_3 = \frac{R_i^{WW}(bb)}{R_i^{\gamma\gamma}(VBF)}$$

(11)

with $Y$ being the given production mode and, in the last equality, $R_i^{WW}(bb) = \Gamma(H_i \rightarrow bb)/\Gamma(h_{\text{SM}} \rightarrow b\bar{b})$, the normalised partial decay width of $H_i$ into $bb$ pair. (Note that (12) assumes that $h_{\text{SM}}$-normalised production cross sections for $Y = VBF$ and $gg$ processes can be approximated by the normalised partial decay widths of $H_i$ in $VV$ and $gg$ decay channels, resp.) $\Gamma_{h_1}$ and $\Gamma_{h_3}$ are the total decay widths of $H_1$ and $h_{\text{SM}}$, respectively.

We also evaluated the ratios $D_1, D_2, D_3, D_4$ for the points which give reasonably good fits to the data (to be defined later) in the scenarios with two mass-degenerate Higgs bosons. For this purpose, $R_i^{\gamma\gamma}(X)$ for each $H_i$ was calculated by fixing $\Gamma_{h_{\text{SM}}}$ in (12) to $4.105 \times 10^{-3}$ GeV, which is the value given by the program HDECAY [84] for 125 GeV $h_{\text{SM}}$. A change of $\pm 2$ GeV in the mass of $h_{\text{SM}}$ has only a marginal effect on this width, which we ignore. For calculating the $\Gamma_{h_{\text{SM}}}$ with HDECAY, care was taken that all the partial decay widths of $h_{\text{SM}}$ were evaluated at the same perturbative order as that implemented in NMSSMCALC for computing $\Gamma_{H_1}$. Moreover, $C_i^{H_1}$ is simply the squared normalised coupling of $H_1$ to a vector boson, $V$, pair for the VBF production mode and to a gluon pair for $gg$ mode. Similarly, $C_i^{\gamma\gamma}$ implies $H_1VV$ and $H_1\gamma\gamma$ normalised couplings squared, respectively, for $X = WW$ and $\gamma\gamma$. All these couplings are thus the same ones obtained from NMSSMCALC for passing to HiggsSignals. In the case of $X = b\bar{b}$, though, there is a scalar and pseudoscalar coupling for each $H_i$, as noted above. For this reason, $C_i^{bb}$’s were calculated using the actual $\Gamma(H_i \rightarrow b\bar{b})$ from the NMSSMCALC output for a given model point and $\Gamma(h_{\text{SM}} \rightarrow b\bar{b})$ obtained from HDECAY for $m_{h_{\text{SM}}} = 125$ GeV.

### 4. Results and Discussion

In Figure 1 we show the total $\chi^2$ obtained for the points from our scans for the various $H_{\text{obs}}$ scenarios considered. The green points in the figure correspond to $\phi_1 = 0^\circ$, violet to $\phi_1 = 3^\circ$, blue to $\phi_1 = 10^\circ$, red to $\phi_1 = 30^\circ$, and cyan to $\phi_1 = 60^\circ$. For the scenarios in which only one of the three lightest neutral Higgs bosons is assumed to be $H_{\text{obs}}$, we have made sure that the difference between the mass of $H_{\text{obs}}$ and that of each additional Higgs boson nearest to it is always larger than 2.5 GeV. The lower cut-off in $\chi^2$ in each panel, in this figure and in those that follow, varies depending on the minimum value obtained in the corresponding scenario. The upper cut-off in $\chi^2$ for each scenario is chosen so as to include as many points in the corresponding figures as possible without $\chi^2$ getting more than 10 units larger than the minimum obtained in that scenario (given that there are 9 statistical degrees of freedom).
Figure 1: Total $\chi^2$ as a function of the following: ((a), (b)) $m_{H_1}$ when only $H_1$ is assumed to be $H_{obs}$, for two different sets of scanned ranges of the parameter space (see text for details); (c) $m_{H_2}$ when only $H_2$ is the $H_{obs}$; (d) $m_{H_3}$ when only $H_3$ is $H_{obs}$; (e) $m_{H_2}$ when both $H_1$ and $H_2$ lie near 125 GeV; and (f) $m_{H_2}$ when both $H_1$ and $H_2$ lie near 125 GeV.
Figure 1(a) corresponds to the low-\(\lambda\)-NMSSM scenario. One notices in the figure that, for \(\phi_2 = 0\), \(\chi^2_{\text{min}}\) lies very close to 70 and is thus almost identical to \(\chi^2_{\text{min}} = 69.96\) that is given by HiggsSignals for a SM Higgs boson at a mass of 125.1 GeV, with the same settings as those used by us. The input parameters (with the exception of \(M_0\), \(M_{1/2}\), and \(A_0\), which can be adjusted with much more freedom) and the masses of the three lightest Higgs bosons are given in Table 3. The negligibly small difference in \(\chi^2_{\text{min}}\) value obtained for \(h_{\text{SM}}\) and for the CP-conserving low-\(\lambda\)-NMSSM results from the fact that \(\lambda\) for the corresponding point in the latter is nonvanishing, as seen in the table, so that the singlet sector is not completely decoupled and an exact MSSM limit is not reached. One can notice in the figure and the table a slightly lower value of \(\chi^2_{\text{min}}\) obtained for the sets of points corresponding to nonzero \(\phi_\ast\) values. However, \(\lambda\) for all these points is even larger than in the CP-conserving limit. Note also that, for all \(\phi_\ast\), most of the points give \(\Delta \chi^2 \leq 1\).

In Figure 1(b), which corresponds to \(H_{\text{obs}} = H_2\) scenario in the natural NMSSM, we see that there is a large concentration of points above \(\chi^2\) value which is very similar to \(\chi^2_{\text{min}}\) seen in Figure 1(a), for each corresponding \(\phi_\ast\). For nonzero \(\phi_\ast\), though, one also sees a few scattered points with \(\chi^2\) lower than that for any of the points in the high concentration region. The overall lowest \(\chi^2\) lies very close to 68, for \(\phi_\ast = 30^\circ\), with the mass of \(H_{\text{obs}}\) for the corresponding point lying at 124.5 GeV. However, according to Table 3, the mass of \(H_2\) for this point is within 3 GeV of that of \(H_1\). It is therefore very likely that the relatively better fit for this particular point is a result of the assignment of \(H_2\) instead of or along with \(H_1\) to some of the observables, especially when their experimental mass resolution is relatively poor. This possibility, which implies that our assumption of two Higgs bosons being individually irresolvable if their masses lie within 2.5 GeV of each other is rather robust, will be discussed further later. For \(\phi_\ast = 60^\circ\) none of the points obtained in the scan for this scenario had \(H_{\text{obs}}\) heavier than 123 GeV.

In \(H_{\text{obs}} = H_2\) scenario, a much smaller number of points were passed by HiggsBounds compared to \(H_{\text{obs}} = H_1\) scenario, as seen in Figure 1(c), but \(\chi^2_{\text{min}}\) is even lower for most \(\phi_\ast\) here, including 0°. Only for \(\phi_\ast = 60^\circ\), while plenty of points with \(m_{H_2} = 125\text{ GeV}\) were obtained in the scan, \(\chi^2\) for them is never low enough to appear in the figure. Once again, in Table 3 one can see that, for the points giving the lowest \(\chi^2\) for each \(\phi_\ast\) in this scenario, \(H_1\) always lies within 3-4 GeV of \(H_2\). Hence the slightly better fit for this point is again made possible by a contribution of \(H_1\) to some search channels. In Figure 1(d) for \(H_{\text{obs}} = H_3\) scenario, although very few points with \(\Delta \chi^2 < 10\) appear in this scenario compared to the ones above, \(\chi^2_{\text{min}}\) is very similar, except for \(\phi_\ast = 0^\circ\) case, when it has a fairly high value of around 77.

In Figure 1(e) is shown the total \(\chi^2\) for \(H_{\text{obs}} = H_1 + H_2\) scenario against \(H_2\) mass. One can observe quite a few similarities between this figure and Figure 1(b) seen above (for \(H_{\text{obs}} = H_1\) scenario). There are once again a large concentration of points with \(\chi^2 \geq 69\) for all \(\phi_\ast\) except 60° and also many scattered points below it. Importantly though, there are many points in this scenario which give \(\chi^2\) lower than 68, which is the overall lowest value observed for any other scenario here. Most of these points, including the one with the overall lowest \(\chi^2\) of \(\sim 65\), correspond to \(\phi_\ast = 10^\circ\), although some points for other \(\phi_\ast\) can also be noticed. In Figure 1(f) one sees \(\chi^2_{\text{min}}\) of 68 for \(H_{\text{obs}} = H_2 + H_3\) scenario also but very few points with \(\chi^2 < 71\), in contrast with \(H_{\text{obs}} = H_1\) and \(H_{\text{obs}} = H_1 + H_2\) scenarios but similar to \(H_{\text{obs}} = H_2\) and \(H_{\text{obs}} = H_3\) scenarios.

From the above discussion, it is clear that certain points, or parameter space configurations, in \(H_{\text{obs}} = H_1 + H_2\) scenario give the best fit to the current experimental Higgs boson data. A global \(\chi^2_{\text{min}}\) that is, the lowest \(\chi^2\) value across all scenarios examined here, of around 65 has been observed for \(\phi_\ast = 10^\circ\) in this scenario, with some points corresponding to other values of \(\phi_\ast\) also lying within 1 unit of \(\chi^2\). None of the points obtained for the other scenarios gives \(\chi^2\) lying even within 3 units of this global minimum, despite the number of sampled points for the \(H_{\text{obs}} = H_1\) scenario being typically larger. The reason for a better fit for some points with two nearly degenerate Higgs bosons becomes apparent by looking at the detailed output of HiggsSignals. In the peak-centred method, HiggsSignals assigns to a given observable the Higgs boson with a mass closest to the measured mass provided by the experiment. This mass measurement currently ranges between 124.7 GeV and 126.0 GeV. Thus, when a single Higgs boson is assigned to all the observables, \(\chi^2\) contribution is large from the observables for which the measured mass lies away from the mass of the assigned Higgs boson, and the experimental mass resolution is good. On the other hand, when two Higgs bosons lie close to each other, the one assigned to a given observable is the one for which the difference of the predicted mass from the experimental value is the smallest, so that \(\chi^2\) contribution from this observable is minimal. This is as long as the mass of the other Higgs boson nearby lies outside the experimental mass resolution; otherwise HiggsSignals automatically assigns both the Higgs bosons to an individual observable if it improves the fit.

Some caveats are in order here though. \(\Delta \chi^2 = 3\) is statistically quite insignificant for drawing any concrete inferences about the considered scenarios, since the total number of observables and statistical degrees of freedom is quite large. At the same time, the number of points giving \(\Delta \chi^2 \leq 3\) is also fairly small. Moreover, no other experimental constraints have been imposed in our analysis, since the publicly available tools for testing these are so far not compatible with the CNMSSM. It is thus possible that many of the interesting points may have already been ruled out by such constraints. However, the aim of this study is not to disregard one scenario in favour of another, but to simply show that, given the current experimental data, the scenario with two mass-degenerate Higgs bosons in the NMSSM provides as good, if not better, a fit as the scenarios with a single Higgs boson near 125 GeV. This alternative possibility even points towards a source of CP-violation beyond the SM and, therefore, warrants more dedicated analyses as well as experimental probes. In the following we discuss some other interesting aspects of this scenario.

In the left, middle, and right panels of Figure 2 we show the ratios \(D_1\), \(D_2\), and \(D_3\), respectively, as functions of
### Table 3: Input parameters and Higgs boson masses corresponding to the points giving the lowest $\chi^2$ for all $\phi_{\kappa}$ cases in each of $H_{obs}$ scenarios considered.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$\chi^2_{\text{min}}$</th>
<th>$H_{obs} = H_1$</th>
<th>$H_{obs} = H_2$</th>
<th>$H_{obs} = H_3$</th>
<th>$H_{obs} = H_1 + H_2$</th>
<th>$H_{obs} = H_2 + H_3$</th>
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<tr>
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<td>0.582</td>
<td>0.653</td>
<td>0.48</td>
<td>0.597</td>
<td>0.597</td>
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<td>0.305</td>
<td>0.302</td>
<td>0.327</td>
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<td>6.98</td>
<td>2.39</td>
<td>2.07</td>
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<td>680.7</td>
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<td>507.8</td>
<td>124.3</td>
<td>405.8</td>
<td>125.8</td>
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Figure 2: The ratios $D_1$, $D_2$, and $D_3$, defined in (II), as functions of the difference between $H_2$ and $H_1$ masses, in the scenario when $H_{\text{obs}} = H_1 + H_2$. In (a) $\phi_\nu$ is set to 0°, in (b) to 3°, in (c) to 10°, and in (d) to 30°. The heatmap in all the panels corresponds to the total $\chi^2$. 
the mass difference, $m_{H_2} - m_{H_1}$, for various $\phi_\chi$ values in $H_{\text{obs}} = H_1 + H_2$ scenario. The heat map corresponds to the total $\chi^2$ obtained for the points shown in each panel. $\chi^2$ has a uniform upper cut-off of 71 across all panels, as in Figure 1(e), but its lower cut-off varies according to the minimum obtained for $\phi_\chi$ case that a given panel corresponds to. According to Figure 2(a), for $\phi_\chi = 0^\circ$ the three ratios remain largely close to unity, but deviations up to 15–20% can be seen for some points. $D_2$, the ratio dependent on only the bosonic signal strengths, only gets smaller than 1 for some points and its maximum observed deviation is lower than that of $D_1$ and $D_3$, each of which can be both above or below unity. Importantly, the points for which a large deviation of each ratio from 1 is seen are also generally the ones giving a relatively good $\chi^2$ fit to the data.

A similar trend is seen also for other values of $\phi_\chi$. However, deviations of $D_1$ and $D_2$ from unity by up to 40–50% are obtained for $\phi_\chi = 3^\circ$ (Figure 2(b)) and $\phi_\chi = 10^\circ$ (Figure 2(c)), but there are many more points with significantly large deviations of each of the ratios for the latter phase compared to the former one. For $\phi_\chi = 30^\circ$ all the points appearing in Figure 2(d) give $D_1$, $D_2$, and $D_3$ smaller than 1 and the overall deviation is generally smaller than for other nonzero phases but larger than for the rN MSSM limit. Thus, for this phase, the measured signal strengths can provide a clear indication whenever two Higgs bosons are present near 125 GeV instead of one. The reason why the deviations of the three ratios are much smaller overall in the case of $\phi_\chi = 0^\circ$ than for the CPV cases, for points showing the highest consistency with the data, will become clearer below.

As noted earlier, a scenario with two mass-degenerate Higgs bosons in the cN MSSM entails both $H_{\text{obs}} = H_1 + H_2$ and $H_{\text{obs}} = H_1/H_2 + A_1$ possibilities of the rN MSSM. Thus it is interesting to see which one of these two possibilities is favoured more by the data, for a given $\phi_\chi$. In the left panels of Figure 3 we thus show the squared normalised coupling $C_{VV}^{H_1}$ against $C_{VV}^{H_1}$, with the heat map corresponding to the total $\chi^2$. Similarly, in the right panels we have plotted $C_{VV}^{H_2}$ versus $C_{VV}^{H_2}$, while the distribution of $m_{H_i}$ is shown by the heat map. For clarity of observation, we have included in this figure points with a total $\chi^2$ reaching up to 80, which is much higher than for the points shown in the earlier figures for this scenario. Also we have imposed an upper cut-off of 300 GeV on the mass of $H_i$. We expect $C_{VV}^{H_2}$ to either vanish when a given $H_2$ is a pure pseudoscalar (in the rN MSSM limit) or be relatively small when it is pseudoscalar-like (for $\phi_\chi > 0^\circ$). Note that these couplings satisfy the sum rule [63, 64]

$$\sum_{i=1}^{N} C_{VV}^{H_i} = 1,$$

(13)

where $N$ is the total number of neutral Higgs bosons that have a tree-level coupling to the gauge bosons, that is, 5 in the cN MSSM and 3 in the rN MSSM limit. (Note that since $h_{SM}$ is a hypothetical SM Higgs boson with the same mass as a given $H_i$, at the tree level the ratio $C_{XX}^{H_i}$ in fact corresponds to $(g_{H_iX}/g_{h_{SM}X})^2$ and the equality in (13) is exact.) However, since $C_{XX}^{H_i}$ have actually been defined here in terms of the partial decay widths of $H_i$ in $X$ channel, which include higher order effects also, the sum of $C_{VV}^{H_i}$ may deviate slightly from unity.

In the figure we see the above sum rule being satisfied almost completely by the three lightest neutral Higgs bosons under consideration here, implying that the remaining two doublet-like Higgs bosons are nearly decoupled.

In the case of $\phi_\chi = 0^\circ$ (i.e., in the rN MSSM limit) in the left panel of Figure 3(a), we see two distinct kinds of points. There are some points lying along the diagonal, for which $C_{VV}^{H_i}$ and $D_{VV}^{H_i}$ alone are enough to satisfy the sum rule in (13). It is further evident from the right panel that $C_{VV}^{H_1}$ for these points is exactly 0. $H_1$ and $H_2$ in these points should thus be scalars and $H_3$ should be a pseudoscalar (i.e., $A_1$). But for the majority of the points, lying along either of the axes, $C_{VV}^{H_1}$ is nearly 1, implying it is an almost pure doublet-like scalar, while $C_{VV}^{H_2}$ is exactly 0, implying it is a pseudoscalar, or vice versa. One can then observe in the right panel that for such points $C_{VV}^{H_1}$, with $H_1$ being the singlet-like scalar, is responsible for the sum rule being satisfied. Thus when the doublet-like scalar, whether $H_1$ or $H_2$, has $C_{VV}^{H_i}$ slightly below 1, $C_{VV}^{H_1}$ is slightly above 0. The mixing of the doublet scalar with $H_3$ increases as its mass decreases, as is evident from the heat map in the right panel of the figure. As a result, the largest $C_{VV}^{H_1, g} > 0.8$, is seen for the lowest $m_{H_1}$, obtained, which lies just above the allowed $H_{\text{obs}}$ mass window.

A closer inspection of the heat map in the left panel of Figure 3(a) reveals that the lowest values of $\chi^2$ are obtained for points lying along one of the axes, that is, when the doublet-like scalar is nearly mass-degenerate with the pseudoscalar. For points along the diagonal, $\chi^2$ is in fact always larger than 71. This is the reason for the relatively small deviations of $D_1$, $D_2$, and $D_3$ from 1 seen in Figure 2(a), where only points with $\chi^2$ lower than 71 were shown. For such points, since one of $H_1$ and $H_2$ is a pure pseudoscalar as well as singlet-dominated, its contribution to the combined signal strength in WW channel is null and that in $Y$ and $b\bar{b}$ channels is minimal. Therefore, while the presence of $H_1$ and $H_2$ of the rN MSSM near 125 GeV may possibly cause $D_1$, $D_2$, and $D_3$ to deviate more significantly from 1, the consistency of this scenario with the LHC data is worse than that of $H_1 + A_1$ scenario.

Figure 3(b) shows that, for $\phi_\chi = 3^\circ$, $H_1$ and $H_2$ are almost always scalar-like while $H_3$ is highly pseudoscalar-like with a relatively much smaller $C_{VV}^{H_3}$ generally. However, due to CP-mixing, $C_{VV}^{H_2}$ can reach as high as 0.7 or so when the mass of $H_2$ is close to that of $H_1$ and $H_2$, though this happens for only a few points. A very crucial point to note here is that the total $\chi^2$ in the left panel never falls below 68, which is due to the cut-off on the allowed upper value of $m_{H_2}$. This means that the points which give the overall best fit to the data have a much higher $H_2$ mass, which leads to a much smaller scalar-pseudoscalar mixing and hence negligible $C_{VV}^{H_2}$.

For $\phi_\chi = 10^\circ$ case, illustrated in Figure 3(c), while the maximum $C_{VV}^{H_1}$ obtained is relatively small and hence $C_{VV}^{H_2}$ and $C_{VV}^{H_3}$ do not deviate from the diagonal by much in the left panel, there are many more points, compared to $\phi_\chi = 3^\circ$,
Figure 3: Squared normalised coupling of $H_1$ to the gauge bosons versus that of $H_2$ (left) and of $H_3$ (right) in the scenario when $H_{\text{obs}} = H_1 + H_2$, with $\phi_\epsilon$ set to (a) $0^\circ$, (b) $3^\circ$, (c) $10^\circ$, and (d) $30^\circ$. The heatmap in the left panels shows the distribution of $\chi^2$ and in the right panels that of $m_{H_3}$. 
case above, for which \( C_{VV}^H \) is significant, according to the right panel. Finally, for \( \phi_3 = 30^\circ \), although \( C_{VV}^H \) never completely vanishes, it also stays smaller overall than it is for other phases. The reason for this is that the pseudoscalar-like \( H_3 \) never achieves a mass below 220 GeV or so, as can be noted from the heat map in the right panel of Figure 3(d). In the left panel one therefore sees that \( C_{VV}^H \) and \( C_{VV}^{H_2} \) always remain very close to the diagonal. Hence, for nonzero \( \phi_3 \), the data clearly favours two scalar-like Higgs bosons near 125 GeV, instead of a pair of scalar-like and pseudoscalar-like Higgs bosons.

5. Conclusions

In summary, we have tested the consistency of the real and complex NMSSM with the latest Higgs boson data from the LHC Run I and the Tevatron. In particular, we have focused on scenarios wherein the resonant peak seen by the experiments can be explained in terms of two nearly mass-degenerate Higgs states around 125 GeV. Such scenarios have been verified in the nNMSSM previously and have not been ruled out yet. What we have shown here is that the possibility of such dynamics being available in the NMSSM is somewhat enhanced if some degree of (explicit) CP-violation is allowed in the Higgs sector. This can be done by assuming one or more of the Higgs sector parameters to be complex. By choosing this parameter to be \( \kappa \), one can evade the fermion EDM measurements, which tightly constrain the other possibly complex parameters in the Higgs and soft SUSY sectors of the NMSSM.

In order to achieve the above we have performed detailed numerical scans of the parameter space of the cNMSSM to obtain various possible configurations with \( \sim 125 \) GeV Higgs boson(s) that also give SM-like signal strengths. In these scans we set the phase of \( \kappa \) to five different values, 0°, 3°, 10°, 30°, and 60°. Through a comprehensive analysis of the points obtained from these scans, we have then established that certain parameter configurations yielding two Higgs bosons near 125 GeV are slightly more favoured by the current data compared to scenarios with a single near 125 GeV are slightly more favoured by the current data. Certain parameter configurations yielding two Higgs bosons are CP-mixed states. For this case, one can evade the fermion EDM measurements, which tightly constrain the other possibly complex parameters in the Higgs and soft SUSY sectors of the NMSSM.

In the left panel one therefore sees that \( C_{VV}^H \) and \( C_{VV}^{H_2} \) always remain very close to the diagonal. Hence, for nonzero \( \phi_3 \), the data clearly favours two scalar-like Higgs bosons near 125 GeV, instead of a pair of scalar-like and pseudoscalar-like Higgs bosons.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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