We study the prospects of the $B-L$ model with an additional $Z$ boson to be a Higgs boson factory at high-energy and high-luminosity linear electron-positron colliders, such as the ILC and CLIC, through the Higgs-strahlung process $e^+e^→(Z,Z^*)→Zh$, including both the resonant and the nonresonant effects. We evaluate the total cross section of $Zh$ and we calculate the total number of events for integrated luminosities of 500–2000 fb$^{-1}$ and center of mass energies between 500 and 3000 GeV. We find that the total number of expected $Zh$ events can reach $10^6$, which is a very optimistic scenario and it would be possible to perform precision measurements for both $Z^*$ and Higgs boson in future high-energy $e^+e^-$ colliders experiments.

1. Introduction

The discovery of a light scalar boson $H$ of the ATLAS [1] and CMS [2] collaborations at the Large Hadron Collider (LHC) compatible with a SM Higgs boson [3–7] and with mass around $M_H = 125 \pm 0.4$ (stat.) $\pm 0.5$ (syst.) GeV has opened a window to new sectors in the search for physics beyond the Standard Model (SM). The Higgs boson might be a portal leading to more profound physics models and even physics principles. Therefore, another Higgs factory besides the LHC such as the International Linear Collider (ILC) [8–13] and the Compact Linear Collider (CLIC) [14–16] that can study in detail and can precisely determine the properties of the Higgs boson is another important future step in high-energy and high-luminosity physics exploration.

The existence of a heavy neutral ($Z^*$) vector boson is a feature of many extensions of the Standard Model. In particular, one (or more) additional $U(1)'$ gauge group provides one of the simplest extensions of the SM. Additional $Z^*$ gauge bosons appear in Grand Unified Theories (GUTs) [17], Superstring Theories [18], Left-Right Symmetric Models (LRSM) [19–21], and other models such as models of composite gauge bosons [22]. In particular, it is possible to study some phenomenological features associated with this extra neutral gauge boson by considering a $B-L$ (baryon number minus lepton number) model.

The $B-L$ symmetry plays an important role in various physics scenarios beyond the SM. (a) The gauge $U(1)_{B-L}$ symmetry group is contained in a GUT described by a $SO(10)$ group [23]. (b) The scale of the $B-L$ symmetry breaking is related to the mass scale of the heavy right-handed Majorana neutrinos mass terms providing the well-known see-saw mechanism [24] to explain light left-handed neutrino mass. (c) The $B-L$ symmetry and the scale of its breaking are tightly connected to the baryogenesis mechanism through leptogenesis [25].

The $B-L$ model [26, 27] is attractive due to its relatively simple theoretical structure, and the crucial test of the model is the detection of the new heavy neutral ($Z^*$) gauge boson. The analysis of precision electroweak measurements indicates that the new $Z^*$ gauge boson should be heavier than about 1.2 TeV [28]. On the other hand, recent bounds from the LHC indicate that the $Z^*$ gauge boson should be heavier than about 2 TeV [29, 30], while future LHC runs at 13–14 TeV could...
increase the $Z'$ mass bounds to higher values or may be lucky and find evidence for its presence. Further studies of the $Z'$ properties will require a new linear collider [31], which will also allow us to perform precision studies of the Higgs sector. Detailed discussions on the $B - L$ model can be found in the literature [26, 32–38].

The Higgs-strahlung [39–43] process $e^+e^- \rightarrow Zh$ is one of the main production mechanisms of the Higgs boson in the future linear $e^+e^-$ colliders experiments, such as the ILC and CLIC. Therefore, after the discovery of the Higgs boson, detailed experimental and theoretical studies are necessary for checking its properties and dynamics [44–47]. It is possible to search for the Higgs boson in the framework of the $B - L$ model; however the existence of a new gauge boson could also provide new Higgs particle production mechanisms, which could prove its nonstandard origin. In this work, we analyze how the $Z'$ gauge boson of the $U(1)_{B-L}$ model could be used as a factory of Higgs bosons.

Our aim in the present paper is to study the sensitivity of the $Z'$ boson of the $B - L$ model as a Higgs boson factory through the Higgs-strahlung process $e^+e^- \rightarrow (Z, Z') \rightarrow Zh$, including both the resonant and the nonresonant effects at future high-energy and high-luminosity linear $e^+e^-$ colliders, such as the International Linear Collider (ILC) [8] and the Compact Linear Collider (CLIC) [14]. We evaluate the total cross section of $Zh$ and we calculate the total number of events for integrated luminosities of 500–2000 fb$^{-1}$ and center-of-mass energies between 500 and 3000 GeV. We find that the total number of expected $Zh$ events for the $e^+e^-$ colliders is very promising and that it would be possible to perform precision measurements for both the $Z'$ and the Higgs boson in the future high-energy $e^+e^-$ colliders experiments. In addition, we also studied the dependence of the Higgs signal strengths ($\mu$) on the parameters $g'_1$ and $\theta_{B-L}$ of the $U(1)_{B-L}$ model for the Higgs-strahlung process $e^+e^- \rightarrow Zh$.

This paper is organized as follows. In Section 2, we present the theoretical framework. In Section 3, we present the decay widths of the $Z'$ boson in the context of the $B - L$ model. In Section 4, we present the calculation of the process $e^+e^- \rightarrow (Z, Z') \rightarrow Zh$, and, finally, we present our results and conclusions in Section 5.

## 2. Theoretical Framework

We consider an $SU(2)_L \times U(1)_Y \times U(1)_{B-L}$ model consisting of one doublet $\Phi$ and one singlet $\chi$ and briefly describe the lagrangian including the scalar, fermion, and gauge sector. The Lagrangian for the gauge sector is given by [36, 48–50]

$$\mathcal{L}_g = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W^\mu W^{\mu\nu} - \frac{1}{4} Z^\mu Z^{\mu\nu},$$

where $W^\mu, B^\mu$, and $Z^\mu$ are the field strength tensors for $SU(2)_L, U(1)_Y$, and $U(1)_{B-L}$, respectively.

The Lagrangian for the scalar sector of the $SU(2)_L \times U(1)_{B-L}$ model is

$$\mathcal{L}_s = (D^\mu \Phi)^\dagger (D^\mu \Phi) + (D^\mu \chi)^\dagger (D^\mu \chi) - V(\Phi, \chi),$$

where the potential term is [34]

$$V(\Phi, \chi) = m^2 (\Phi^\dagger \Phi) + \mu^2 |\chi|^2 + \lambda_1 (\Phi^\dagger \Phi)^2 + \lambda_2 |\chi|^4 + \lambda_3 (\Phi^\dagger \Phi)^2 |\chi|^2,$$

with $\Phi$ and $\chi$ as the complex scalar Higgs doublet and singlet fields, respectively. The covariant derivatives for the doublet and singlet are given by [32–34]

$$D^\mu \Phi = \partial^\mu \Phi + i \left[ g T^a W^\mu_{\mu} + g_1 Y B^\mu_{\mu} + g_1' Y' B'_{\mu} \right] \Phi,$$

$$D^\mu \chi = \partial^\mu \chi + i \left[ g_1 Y B_{\mu} + g_1' Y' B'_{\mu} \right] \chi,$$

where the doublet and singlet scalars are

$$\Phi = \left( \begin{array}{c} G^+ \\ v + \phi^0 + i G_Z \end{array} \right),$$

$$\chi = \left( \begin{array}{c} v' + \phi^0 + i z' \\ \sqrt{2} \end{array} \right),$$

with $G^+, G_Z$, and $z'$ being the Goldstone bosons of $W^\pm, Z$, and $Z'$, respectively.

After spontaneous symmetry breaking the two scalar fields can be written as

$$\Phi = \left( \begin{array}{c} 0 \\ \sqrt{2} \end{array} \right),$$

$$\chi = \left( \begin{array}{c} v' + \phi^0 \\ \sqrt{2} \end{array} \right),$$

with $v$ and $v'$ being real and positive. Minimization of (3) gives

$$m^2 + 2\lambda_1 v^2 + \lambda_3 vv'^2 = 0,$$

$$\mu^2 + 4\lambda_2 v'^2 + \lambda_3 vv'^2 = 0.$$

To compute the scalar masses, we must expand the potential in (3) around the minima in (6). Using the minimization conditions, we have the following scalar mass matrix:

$$M = \begin{pmatrix} \lambda_1 v^2 & \frac{\lambda_3 vv'}{2} \\ \frac{\lambda_3 vv'}{2} & \lambda_2 v'^2 \end{pmatrix} = \begin{pmatrix} \mathcal{M}_{11} & \mathcal{M}_{12} \\ \mathcal{M}_{21} & \mathcal{M}_{22} \end{pmatrix}.$$

The expressions for the scalar mass eigenvalues ($m_{\Phi'} > m_{\chi'}$) are

$$m_{\Phi'}^2 = \frac{(\mathcal{M}_{11} + \mathcal{M}_{22}) \pm \sqrt{(\mathcal{M}_{11} - \mathcal{M}_{22})^2 + 4\mathcal{M}_{12}^2}}{2},$$

$$m_{\chi'}^2 = \frac{(\mathcal{M}_{11} + \mathcal{M}_{22}) \mp \sqrt{(\mathcal{M}_{11} - \mathcal{M}_{22})^2 + 4\mathcal{M}_{12}^2}}{2}.$$
and the mass eigenstates are linear combinations of $\phi^0$ and $\phi^{0'}$ and written as
\[
\begin{pmatrix}
  h \\
  H'
\end{pmatrix} = \begin{pmatrix}
  \cos \alpha & -\sin \alpha \\
  \sin \alpha & \cos \alpha
\end{pmatrix} \begin{pmatrix}
  \phi^0 \\
  \phi^{0'}
\end{pmatrix},
\]
where $h$ is the SM-like Higgs boson. The scalar mixing angle $\alpha$ can be expressed as
\[
\tan(2\alpha) = \frac{2\mathcal{M}_{12}}{\mathcal{M}_{11} - \mathcal{M}_{22}} = \frac{\lambda_3 v v'}{\lambda_1 v^2 - \lambda_2 v'^2}.
\]

In the Lagrangian of the SU(2)$_L \times$U(1)$_Y \times$U(1)$_{B-L}$ model, the terms for the interactions between neutral gauge bosons $Z, Z'$ and a pair of fermions of the SM can be written in the form [36, 37]
\[
\mathcal{L}_{NC} = \frac{-ig}{\cos \theta_W} \sum_f \bar{f} Y^f \sigma^\mu \left( \frac{1}{2} (g^f_\mu - g_A^f Y^5) f Z \right) \\
+ \frac{-ig}{\cos \theta_W} \sum_f \bar{f} Y^f \sigma^\mu \left( \frac{1}{2} (g^f_\mu - g_A^f Y^5) f Z' \right)
\]
From this Lagrangian we determine the expressions for the new couplings of the $Z, Z'$ bosons with the SM fermions, which are given by
\[
g^f_\mu = T_3^f \cos \theta_{B-L} - 2Q_f \sin^2 \theta_W \cos \theta_{B-L} \\
+ \frac{2g_1^f}{g} \cos \theta_W \sin \theta_{B-L} \\
g_A^f = T_3^f \cos \theta_{B-L},
\]
\[
g_B^f = -T_3^f \sin \theta_{B-L} - 2Q_f \sin^2 \theta_W \sin \theta_{B-L} \\
+ \frac{2g_1^f}{g} \cos \theta_W \cos \theta_{B-L},
\]
\[
g_A^f = -T_3^f \sin \theta_{B-L},
\]
where $g = e/\sqrt{2}$ and $\theta_{B-L}$ is the $Z - Z'$ mixing angle. The current bound on this parameter is $|\theta_{B-L}| \leq 10^{-3}$ [51]. In the decoupling limit, that is to say, when $g_1^f = 0$ and $\theta_{B-L} = 0$, the couplings of the SM are recovered.

3. The Decay Widths of $Z'$ in the $B - L$ Model

In this section we present the new decay widths of the $Z'$ boson [28, 52–54] in the context of the $B - L$ model which we need in the calculation of the cross section for the process $e^+ e^\to Zh$. The $Z'$ partial decay widths involving vector bosons and the scalar boson are
\[
\Gamma(Z' \to W^+ W^-) = \frac{G_F M_W^2}{24 \pi \sqrt{2}} \cos^2 \theta_W \sin^2 \theta_{B-L} \\
\cdot M_{Z'} (\frac{M_{Z'}}{M_Z})^4 \left(1 - \frac{M_W^2}{M_{Z'}}\right)^{1/3} \\
\cdot \left[1 + 20 \frac{M_Z^2}{M_{Z'}} + 12 \frac{M_W^2}{M_{Z'}}\right],
\]
\[
\Gamma(Z' \to Zh) = \frac{G_F M_Z^2 M_{Z'}}{24 \pi \sqrt{2} \lambda} \left[\lambda + 2\frac{M_Z^2}{M_{Z'}}\right] \\
\cdot [f(\theta_{B-L}) \cos \alpha - g(\theta_{B-L}) \sin \alpha]^2,
\]
where
\[
\lambda = 1 + \left(\frac{M_Z^2}{M_{Z'}}\right) \left(\frac{M_{h_2}^2}{M_{h_2}^{Z'}}\right) - 2 \left(\frac{M_{h_1}^2}{M_{h_1}^{Z'}}\right) - 2 \left(\frac{M_{h_1}^0}{M_{h_1}^{Z'}}\right),
\]
\[
f(\theta_{B-L}) = \left(\frac{4M_Z^2}{v'^2} - g_1^2\right) \sin(2\theta_{B-L}) \\
+ \left(\frac{4g_1^f M_Z}{v'}\right) \cos(2\theta_{B-L}),
\]
\[
g(\theta_{B-L}) = 4g_1^f \left(\frac{\nu'}{v}\right) \sin(2\theta_{B-L}).
\]
The vacuum expectation value $v'$ is taken as $v' = 2$ TeV, while $\alpha = \pi/9$ for the Higgs mixing parameter in correspondence with [1, 2, 48, 55]. In our analysis we take $v = 246$ GeV and constrain the other scale, $v'$, by the lower bounds imposed on the mass of the extra neutral gauge boson $Z'$. The mass of the $Z'$ and of the heavy neutrinos depends on $v'$ and should be related to it, while the Higgs masses depend on the angle $\alpha$, the value of which is completely arbitrary.

Finally, the decay width of the $Z'$ boson to fermions is given by
\[
\Gamma(Z' \to \ell \ell) = \frac{2G_F}{3 \pi \sqrt{2}} \\
\cdot N_f M_Z^2 M_{Z'} \left[1 - 4 \left(\frac{M_{Z'}^2}{M_{Z}^2}\right) \left([g_B^f]^2 + 2 \left(\frac{M_{Z'}^2}{M_{Z}^2}\right) \left(1 - 4 \left(\frac{M_{Z'}^2}{M_{Z}^2}\right) \left(\frac{M_{h_1}^0}{M_{h_1}^{Z'}}\right)\right)\right)\right],
\]
where $N_f$ is the color factor ($N_f = 1$ for leptons, $N_f = 3$ for quarks) and the couplings $g_{tV}^t$ and $g_{A}^t$ of the $Z'$ boson with the SM fermions are given in (14).

4. The Total Cross Section of $e^+e^- \rightarrow Zh$ in the $B-L$ Model

In this section, we calculate the Higgs production cross section via the process $e^+e^- \rightarrow Zh$ in the context of the $B-L$ model at future high-energy and high-luminosity linear electron-positron colliders, such as the ILC and CLIC.

The Feynman diagrams contributing to the process $e^+e^- \rightarrow (Z,Z') \rightarrow Zh$ are shown in Figure 1. The expressions for the total cross section of the Higgs-strahlung process for the different contributions, that is to say SM, $B-L$, and SM – ($B-L$), respectively, can be written in the following compact form:

$$
\sigma(e^+e^- \rightarrow Zh)_{tot} = G_F^2 M_Z^2 \left[ \frac{g_Y^2}{24\pi} + (g_A^t)^2 \right] + \frac{s\sqrt{\lambda}}{M_{Z'}^2} \left[ \left( s-M_Z^2 \right)^2 + M_Z^2 \Gamma_Z^2 \right] \left( g_Y^2 + (g_A^t)^2 \right)
$$

$$
\cdot \sin\alpha^2 + \frac{G_F^2 M_Z^2}{12\pi} \left[ g_Y v_Y^t + g_A^t v_A^t \right] + \frac{1}{M_{Z'}^2} \left( \lambda + 6 \left( M_{Z'}^2 - M_Z^2 \right) \right)
$$

$$
+ \frac{1}{M_{Z'}^2} \left( \lambda + 6 \left( M_{Z'}^2 - M_Z^2 \right) \right)
$$

$$
+ \frac{1}{M_{Z'}^2} \left( \lambda + 6 \left( M_{Z'}^2 - M_Z^2 \right) \right)
$$

$$
\cdot \cos\alpha - g(\theta_{B-L}) \sin\alpha
$$

(18)

is the usual two-particle phase space function, while $g_{tV}^t$, $g_{A}^t$, $g_Y^t$, $g_A^t$, $f(\theta_{B-L})$, and $g(\theta_{B-L})$ are given in (13), (14), and (16), respectively.

The expression given in the first term of (18) corresponds to the cross section with the exchange of the $Z$ boson, while the second and third terms come from the contributions of the $B-L$ model and of the interference, respectively. The SM expression for the cross section of the reaction $e^+e^- \rightarrow Zh$ can be obtained in the decoupling limit, that is to say, when $\theta_{B-L} = 0$ and $g_Y^t = 0$; in this case the terms that depend on $\theta_{B-L}$ and $g_Y^t$ in (18) are zero and (18) is reduced to the expression given in [39, 43] for the standard model.

5. Results and Conclusions

5.1. $Z'$ Resonance and Associated $Zh$ Production in the $B-L$ Model

In this section we evaluate the total cross section of the Higgs-strahlung process $e^+e^- \rightarrow (Z,Z') \rightarrow Zh$ in the context of the $B-L$ model at next generation linear $e^+e^-$ colliders such as the ILC and CLIC. Using the following values for numerical computation [51], $\sin^2\theta_W = 0.23126 \pm 0.00022$, $m_e = 1776.82 \pm 0.16 \text{MeV}$, $m_h = 4.6 \pm 0.18 \text{GeV}$, $m_e = 172 \pm 0.9 \text{GeV}$, $M_W = 80.389 \pm 0.023 \text{GeV}$, $M_Z = 91.1876 \pm 0.0021 \text{GeV}$, $\Gamma_Z = 2.4952 \pm 0.0023 \text{GeV}$, and $M_h = 125 \pm 0.4$, and considering the most recent limit from LEP [56]

$$
\frac{M_{Z'}}{g_Y^t} \geq 7 \text{ TeV},
$$

(20)

in our numerical analysis, we obtain the total cross section $\sigma_{tot} = \sigma_{tot}(\sqrt{s}, M_{Z'}, g_Y^t)$. Thus, in our numerical computation, we will assume $\sqrt{s}$, $M_{Z'}$, and $g_Y^t$ as free parameters.

We do not consider the process $e^+e^- \rightarrow (Z,Z') \rightarrow ZH'$ [35] in our study since in major parts of the $U(1)_{B-L}$ model parameter space the Higgs boson $H'$ is quite heavy, and it is difficult to detect the process $e^+e^- \rightarrow ZH'$ when the relevant mechanism is $e^+e^- \rightarrow Zh$.

In Figures 2 and 3 we present the total decay width of the $Z'$ boson as a function of $M_{Z'}$ and the new $U(1)_{B-L}$ gauge coupling $g_Y^t$, respectively, with the other parameters held fixed to three different values. From Figure 2, we see that the total width of the $Z'$ new gauge boson varies from a few to hundreds of GeV over a mass range of 500 GeV $\leq M_{Z'} \leq 3000$ GeV, depending on the value of $g_Y^t$. In the case of Figure 3, a similar behavior is obtained in the range $0 \leq g_Y^t \leq 1$ and depends on the value of $M_{Z'}$. The branching ratios versus $Z'$ mass are given in Figure 4 for different channels, that is to say, $\text{BR}(Z' \rightarrow f \bar{f})$, $\text{BR}(Z' \rightarrow Zh)$, and $\text{BR}(Z' \rightarrow W^+W^-)$, respectively. In this figure the $\text{BR}(Z' \rightarrow f \bar{f})$ is the sum of all BRs for the decays into fermions. We consider $\theta_{B-L} = 10^{-3}$, $g_Y^t = 0.5$, and 500 GeV $\leq M_{Z'} \leq 3000$ GeV.

To illustrate our results on the sensitivity of the $Z'$ gauge boson of the $B-L$ model as a Higgs boson factory through the Higgs-strahlung process $e^+e^- \rightarrow (Z,Z') \rightarrow Zh$, including both the resonant and the nonresonant effects at future high-energy and high-luminosity linear $e^+e^-$ colliders, such as the International Linear Collider (ILC) and the Compact Linear Collider (CLIC), we present the total cross section in Figures 5–11.
In Figure 5, we show the cross section $\sigma(e^+e^-\to Zh)$ for the different contributions as a function of the center-of-mass energy $\sqrt{s}$ for $\theta_{B-L} = 10^{-3}$ and $g_1' = 0.5$: the solid line corresponds to the first term of (18), where in the $U(1)_{B-L}$ model the couplings $g_1'$ and $g_A$ of the SM gauge boson $Z$ to electrons receive contributions of the $U(1)_{B-L}$ model. The dashed line corresponds to the second term of (18), that is to say, is the pure $B - L$ contribution. Finally, the dot-dashed line corresponds to the total cross section of the process $\sigma(e^+e^-\to Zh)$. From Figure 5, we can see that the cross section corresponding to the first term of (18) decreases for large $\sqrt{s}$, whereas, in the case of the cross section of the $B - L$ model and the total cross section, respectively, these are increased for large values of the center-of-mass energy, reaching its maximum value at the resonance $Z'$ boson; that is to say, $\sqrt{s} = 1500$ GeV.

To see the effects of $g_1'$, the free parameter of the $B - L$ model on the process $e^+e^-\to (Z,Z')\to Zh$, we plot the relative correction $\delta \sigma/\sigma_{SM} = (\sigma_{tot} - \sigma_{SM})/\sigma_{SM}$ as a function of $g_1'$ for $M_{Z'} = 1500, 2000, 2500$ GeV and $\sqrt{s} = 1500, 2000, 2500$ GeV in Figure 6. We can see that the relative correction reaches its maximum value between $0.1 \leq g_1' \leq 2.5$ and remains almost constant as $g_1'$ increases.

The deviation of the cross section in our model from the SM one $\delta \sigma/\sigma_{SM}$ is depicted in Figure 7 as a function of $M_{Z'}$ for $\sqrt{s} = 1500$ GeV and three values of the $g_1'$, new gauge coupling. Figure 7 shows that the relative correction is very sensitive to the gauge boson mass $M_{Z'}$ and for the gauge parameter $g_1' = 0.2, 0.5, 0.8$ the peak of the total cross section emerges when the heavy gauge boson mass approximately equals $M_{Z'} = 1500, 1450, 1300$ GeV, respectively. Thus, in a sizeable parameter region of the $B - L$ model, the new heavy gauge boson $Z'$ can produce a significant signal, which can be detected in future ILC and CLIC experiments.
We plot the total cross section of the reaction $e^+e^- \rightarrow Zh$ in Figure 8 as a function of the center-of-mass energy, $\sqrt{s}$ for the values of the heavy gauge boson mass of $M_{Z'} = 1500, 2000, 2500$ GeV and $\theta_{B-L} = 10^{-3}$, $g_1' = 0.2$, respectively. In this figure we observed that, for $\sqrt{s} = M_{Z'}$, the resonant effect dominates the Higgs particle production. A similar analysis was performed in Figure 9, but in this case $\theta_{B-L} = 10^{-3}$ and $g_1' = 0.5$. In both figures we show that the cross section is sensitive to the free parameters. Comparing Figures 8 and 9, we observe that the height of the resonances is the same in both figures, but the resonances are broader for larger $g_1'$ values, as the total width of the $Z'$ boson increases with $g_1'$, as it is shown in Figure 2.

Finally, in Figure 10 we use the currents values of $M_{Z'}$ and $\theta_{B-L}$, as well as the value of the coupling constant $g_1'$ and center-of-mass energy $\sqrt{s}$ of the collider to obtain contour plot 3D for the total cross section $\sigma_{\text{tot}} = \sigma_{\text{tot}}(\sqrt{s}, M_{Z'}, g_1')$ of the process $e^+e^- \rightarrow Zh$ for $M_h = 125$ GeV and $\theta_{B-L} = 10^{-3}$. In this figure the resonance peaks for the boson $Z'$ are evident for the entire range of allowed parameters of the $U(1)_{B-L}$ model.

From Figures 5–10, it is clear that the total cross section is sensitive to the value of the gauge boson mass $M_{Z'}$, center-of-mass energy $\sqrt{s}$, and $g_1'$; the new $U(1)_{B-L}$ gauge coupling increases with the collider energy, reaching a maximum at the resonance of the $Z'$ gauge boson. As an indicator of the order of magnitude, we present the $Zh$ number of events in Table 1, for several gauge boson masses, center-of-mass energies, and $g_1'$ values and for a luminosity of $\mathcal{L} = 500, 1000, 2000$ fb$^{-1}$. We find that the possibility of observing the process $e^+e^- \rightarrow Zh$ is very promising as shown in Table 1, and it would be possible to perform precision measurements for both the $Z'$ and the Higgs boson in the future high-energy linear $e^+e^-$ colliders experiments. We observed in Table 1 that the cross section rises once the threshold for $Zh$ production is reached, with the energy, until the $Z'$ is produced resonantly at $\sqrt{s} = 1500, 2000$, and $2500$ GeV, respectively, for the three cases. Afterwards it decreases with rising energy due to the $Z$ and $Z'$ propagators. Another promising production mode for
Table 1: Total production of $ZH$ in the $B-L$ model for $M_{Z'} = 1500, 2000, 2500$ GeV, $\sqrt{s} = 500, 1000, 2000$ fb$^{-1}$ (1st, 2nd, and 3rd numbers, respectively, in the last 3 columns), $M_H = 125$ GeV, $g_1^f = 0.5$, and $\theta_{B-L} = 10^{-3}$.

<table>
<thead>
<tr>
<th>$\sqrt{s}$ (GeV)</th>
<th>$M_{Z'} = 1500$ GeV</th>
<th>$M_{Z'} = 2000$ GeV</th>
<th>$M_{Z'} = 2500$ GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>85 131; 170 263; 340 526</td>
<td>44 609; 89 219; 178 439</td>
<td>34 747; 69 493; 138 987</td>
</tr>
<tr>
<td>1000</td>
<td>155 482; 310 964; 621 928</td>
<td>33 523; 67 047; 134 094</td>
<td>15 339; 30 678; 61 355</td>
</tr>
<tr>
<td>1500</td>
<td>1 234 000; 2 460 000; 4 930 000</td>
<td>75 192; 150 384; 300 768</td>
<td>18 004; 36 008; 72 016</td>
</tr>
<tr>
<td>2000</td>
<td>92 640; 185 282; 370 564</td>
<td>396 490; 792 980; 1 580 000</td>
<td>42 224; 84 449; 168 899</td>
</tr>
<tr>
<td>2500</td>
<td>20 276; 41 534; 83 069</td>
<td>52 144; 104 288; 208 577</td>
<td>163 538; 327 076; 654 151</td>
</tr>
<tr>
<td>3000</td>
<td>8 243; 16 487; 32 974</td>
<td>12 721; 25 442; 50 885</td>
<td>32 173; 64 346; 128 693</td>
</tr>
</tbody>
</table>

Figure 10: Contour plot 3D for the total cross section $\sigma_{tot} = \sigma_{tot}(\sqrt{s}, M_{Z'}, g_1^f)$ of the process $e^+ e^- \rightarrow ZH$ for $M_h = 125$ GeV and $\theta_{B-L} = 10^{-3}$.

studying the $Z'$ boson and Higgs boson properties is $e^+ e^- \rightarrow (\gamma, Z, Z' \rightarrow \ell \ell H$ [57].

5.2. The Higgs Signal Strengths in the $B-L$ Model. Considering the Higgs boson decay channels, the Higgs signal strengths can be defined as

$$\mu_i = \frac{\sigma_{B-L} \times \text{BR}(h \rightarrow i)_{B-L}}{\sigma_{SM} \times \text{BR}(h \rightarrow i)_{SM}},$$

(21)

where $i$ denotes a possible final state of the Higgs boson decay, for example, $b\bar{b}$, $W^+ W^-$, $ZZ$, $gg$, and $\gamma\gamma$.

Fixing the Higgs boson mass to the measured value and considering the decays $h \rightarrow \gamma\gamma$, $h \rightarrow ZZ$, $h \rightarrow W^+ W^-$, $h \rightarrow b\bar{b}$, and $h \rightarrow t^+ t^-$, the ATLAS collaboration reports [58] a signal strength of

$$\mu = 1.18^{+0.15}_{-0.14}.$$  \hspace{1cm} (22)

The corresponding CMS collaboration result [59] is

$$\mu = 1.00 \pm 0.13.$$  \hspace{1cm} (23)

Good consistency is found, for both experiments, across different decay modes and analyses categories related to different production modes.

In the $B-L$ model, the modifications of the $hf\bar{f}$ (the SM fermions pair) and $hVV$ ($V = W, Z$) couplings can give the extra contributions to the Higgs boson production processes. On the other hand, the loop-induced couplings, such as $h\gamma\gamma$ and $hgg$, could also be affected. Finally, besides the effects already seen in the Higgs-strahlung channel due to the couplings equations (13) and (14) and the functions given by (16), the exchange of s-channel heavy neutral gauge boson $Z'$ also affected the production cross section. All effects can modify the signal strengths in a way that may be detectable at the future ILC/CLIC experiments.

In Figure 11, we show the dependence of the Higgs signal strengths $\mu_i (i = b\bar{b}, \gamma\gamma)$ on the parameters $g_1^f$ and $\theta_{B-L}$ for the Higgs-strahlung process $e^+ e^- \rightarrow (Z, Z') \rightarrow Zh$, where (a) and (b) denote the Higgs signal strengths $\mu_{BF}$ and $\mu_{\gamma\gamma}$, respectively.

Using $\theta_{B-L} = 10^{-3}$ for the mixing angle and $M_h = 125$ GeV for the Higgs boson mass, the following bound on the signal strength is obtained:

$$\mu = 1.2^{+0.12}_{-0.16}.$$  \hspace{1cm} (24)

which is consistent with that obtained for the ATLAS [58] and CMS [59] collaborations, (22) and (23), respectively.

In conclusion, we consider the $Z'$ heavy gauge boson of the $B-L$ model as a Higgs boson factory, through the Higgs-strahlung process $e^+ e^- \rightarrow (Z, Z') \rightarrow Zh$. We find that the future linear $e^+ e^-$ colliders experiments such as the ILC and CLIC could test the $B-L$ model by measuring the cross section of the process $e^+ e^- \rightarrow Zh$, and it would be possible to perform precision measurements of the $Z'$ gauge boson and of the $h$ Higgs boson, as well as of the parameters of the model $\theta_{B-L}$ and $g_1^f$, complementing other studies on the $B-L$ model and on the Higgs-strahlung process. The SM expression for the cross section of the reaction $e^+ e^- \rightarrow Zh$ can be obtained in the decoupling limit; that is to say, when $\theta_{B-L} = 0$ and $g_1^f = 0$, in this case the terms that depend on $\theta_{B-L}$ and $g_1^f$ in (18) are zero and (18) is reduced to the expression given in [39, 43] for the standard model. We also studied the dependence of the Higgs signal strengths $\mu$ on the parameters $g_1^f$ and $\theta_{B-L}$ of the $U(1)_{B-L}$ model for the Higgs-strahlung process $e^+ e^- \rightarrow Zh$. We obtain a bound on $\mu$, which is consistent with that obtained for the ATLAS [58].
Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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