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Research Article

Effects of the Variation of SUSY Breaking Scale on Yukawa and Gauge Couplings Unification

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The present analysis addresses an interesting primary question on how do the gauge and Yukawa couplings unification scales vary with varying SUSY breaking scales m_s, assuming a single scale for all supersymmetric particles. It is observed that the gauge coupling unification scale increases with m_s whereas third-generation Yukawa couplings unification scale decreases with m_s. The rising of the unification scale and also the mass of the color triplet multiplets is necessary to increase the proton decay lifetime; the analysis is carried out with two-loop RGEs for the gauge and Yukawa couplings within the minimal supersymmetric SU(5) model, while ignoring for simplicity the threshold effects of the heavy particles, which could be as large as a few percentages.

1. Introduction

The most natural extension of the minimal SU(5) GUT [1] is the supersymmetric SU(5) GUT [2] which has wide predictive power [3, 4]. The most important features are the prediction for weak mixing angle and the unification of the three gauge couplings at very large scale which is called the unification scale M_{GUT} [3]. It also predicts the unification of the third-generation Yukawa couplings at or below the unification scale and provides a natural solution for the hierarchy problem and an alternative explanation of the electroweak symmetry breaking by the so-called radiative breaking scenario [5–8]. This theory also provides the prediction of proton decay [4] which is caused mainly by $D = 5$ operator [9–12]. Since the most stringent limit on proton lifetime is provided by the Super Kamiokande experiment [13, 14], with the current lower experimental bound [15] $\tau_p > 4 \times 10^{33}$ years, such restrictive value may serve as a criteria to discriminate certain GUT models. This may serve as a direct experimental support to GUT theories.

There are certain arguments against [16, 17] the validity of the SUSY SU(5) GUT model. However there are specific regions in parameter space in minimal renormalizable supersymmetric SU(5) model [18–20] that is consistent with all experimental constraints including gauge couplings unification and the experimental limit on proton lifetime. In the literature there are still some arguments in support of SUSY SU(5) GUT model [21]. In order to suppress the fast $D = 5$ operator proton decay, we have to rise both the scale of unification and the mass of the color triplet multiplets [18]. Within the SU(5) SUSY GUT, attempts have also been made to suppress $D = 5$ proton decay operator [22]. In such context there is still enough scope for further investigation in this direction.

In this paper our focus is on the unification of the gauge couplings as well as on the Yukawa couplings in two-loops RGEs within the framework of minimal supersymmetric SU(5) GUT using updated data consistent with the LHC result. We numerically solve the unification scale for three *gauge couplings* (g_1, g_2, and g_3) as well as the three *Yukawa*

Table 1: Experimental input values for fermion masses, gauge couplings, and Weinberg angle at electroweak scale m_z [26].

Mass in GeV	Coupling constant
$m_z(m_z) = 91.1876 \pm 0.0021$	$\alpha_{em}^{-1}(m_z) = 127.944 \pm 0.014$
$m_t(m_t) = 173.5 \pm 0.60$	$\alpha_s(m_z) = 0.1184 \pm 0.007$
$m_b(m_b) = 4.18 \pm 0.030$	
$m_\tau(m_\tau) = 1.7768 \pm 0.0016$	
Weinberg mixing angles	$\sin^2\theta_W(m_z) = 0.23116 \pm 0.00012$

couplings (h_t, h_b, and h_τ) with varying input values of SUSY breaking scale m_s [18], assuming a single scale for all supersymmetric particles for simplicity of the calculation [23, 24]. There are hints that SUSY particles have a wide spectrum and are not confined to a single energy scale. This kind of assumption is valid as long as the m_z or $m_t \ll m_s$ [25]. We assume the scale m_s to be somewhere in between 500 GeV and 7 TeV. In the present calculation we also ignore the threshold effects of heavy particles which could be as large as a few percentages [19, 20], and latter would affect the unification scale to some extent.

The paper is organized as follows. In Section 2 we collect the necessary input parameters from [26], which are all given at m_z scale in $\overline{\text{MS}}$ scheme. We then make it evolve up to top quark mass scale (m_t) and then converted it into $\overline{\text{DR}}$ scheme. In Section 3, using the values obtained in Section 2, we calculate the Yukawa couplings for top quark, bottom quark, and tau lepton and also the three gauge couplings at $m_t(m_t)$. Using these as the input values and choosing the SUSY breaking scale to be $m_s = m_t$, we then extrapolate them to very high energy scale and study the unification scenarios. In Section 4 we follow a similar procedure as in Section 3 but instead of $m_s = m_t$ we choose different m_s ($> m_t$). Here, we divide the running process into two parts, non-SUSY part (from m_t to m_s) and the SUSY part (from m_s to m_{GUT}). In Section 5 we summarize our results and we conclude.

2. Evolution of Gauge and Yukawa Couplings with Energy Scales

The most recent experimental data from low energy experiment [26], which would be used for generation of the initial input values at low scales, are given in Table 1.

In order to calculate the gauge coupling $\alpha_1(m_z)$ for $U(1)_Y$ and $\alpha_2(m_z)$ for $\text{SU}(2)_L$ for the Standard Model $\text{SU}(3)_C \times \text{SU}(2)_L \times U(1)_Y$, we start with the matching relation and definition of Weinberg mixing angle. Thus,

$$\frac{1}{\alpha_{em}(m_z)} = \frac{5}{3}\frac{1}{\alpha_1(m_z)} + \frac{1}{\alpha_2(m_z)}, \tag{1}$$

$$\sin^2\theta_W(m_z) = \frac{\alpha_{em}(m_z)}{\alpha_2(m_z)}. \tag{2}$$

Substituting the observed values of coupling constants $\alpha_{em}(m_z)$, $\alpha_s(m_z)$, and $\sin^2\theta_W$ from Table 1 we obtain the numerical values of $\alpha_1(m_z)$ and $\alpha_2(m_z)$ with uncertainties arising from input value of $\alpha_s(m_z)$,

$$\alpha_1(m_z) = 1.7100^{+0.00012}_{-0.00017} \times 10^{-2}, \tag{3}$$

$$\alpha_2(m_z) = 3.3753^{-0.00215}_{+0.02150} \times 10^{-2}, \tag{4}$$

respectively. In terms of the normalized coupling constant (g_i), α_i can be expressed as $g_i = \sqrt{4\pi\alpha_i}$, where $i = 1, 2, 3$ and it represents electromagnetic, weak, and strong couplings, respectively.

Here we consider two possible scenarios for the unification of the couplings. In the first case we consider the top quark mass m_t to be the starting energy scale for the evolution from which the supersymmetric effect on the couplings has been included. Since the observational data in Table 1 are given only at the z-pole mass scale, it is necessary to evolve them up to the top quark mass scale. The evolution equation of the coupling constants at one-loop level [27] is given by

$$\frac{d\alpha_i}{dt} = \frac{b_i}{2\pi}\alpha_i^2 \tag{5}$$

which can be simplified as

$$\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_i(m_z)} - \frac{b_i}{2\pi}\ln\left(\frac{\mu}{m_z}\right), \tag{6}$$

where μ is the energy scale in the range ($m_z \le \mu \le m_t$). For non-SUSY case, we have the coefficient of β function of the RGEs [28, 29],

$$b_i = (5.30, -0.50, -4.00). \tag{7}$$

The evolution of the third-generation fermion masses (top, bottom, and tau) is obtained by using the QED-QCD rescaling factor η as

$$m_b(m_t) = \frac{m_b(m_b)}{\eta_b}, \qquad m_\tau(m_t) = \frac{m_\tau(m_\tau)}{\eta_\tau}, \tag{8}$$

where $\eta_b = 1.530$ and $\eta_\tau = 1.015$ [30, 31].

All the above physical parameters are evaluated in the modified minimal subtraction scheme ($\overline{\text{MS}}$), without any radiative corrections. The inclusion of radiative correction is achieved by using the method of dimensional regularization through dimensional reduction [32].

Estimation of Yukawa couplings for t, b, and τ requires a careful determination of m_t, m_b, and m_τ in the DR scheme [28]. However, the effect of running of m_τ on h_τ is very small and hence can be neglected. Furthermore, DR technique is used in order to reduce the large uncertainty in the value of α_s.

Table 2: Numerical values of gauge couplings at top quark mass scale m_t.

	Lower limit	Central value	Upper limit
α_1	1.7099×10^{-2}	1.7100×10^{-2}	1.7102×10^{-2}
α_2	3.7748×10^{-2}	3.3753×10^{-2}	3.3732×10^{-2}
α_3^{DR}	0.1095	0.1162	0.1229
g_1	0.46354	0.46356	0.46358
g_2	0.65148	0.65127	0.65107
g_3^{DR}	1.17294	1.20842	1.24278

Table 3: m_b in $\overline{\mathrm{MS}}$ and $\overline{\mathrm{DR}}$ schemes.

		Lower limit	Central value	Upper limit
$\overline{\mathrm{MS}}$	$m_b(m_b)$	4.1500	4.1800	4.2100
	$m_b(m_z)$	2.7605	2.8618	2.9618
	$m_b(m_t)$	2.6922	2.7860	2.8781
$\overline{\mathrm{DR}}$	$m_b(m_b)$	4.0432	4.0713	4.0995
	$m_b(m_z)$	2.7265	2.8242	2.9205
	$m_b(m_t)$	2.6606	2.7512	2.8400

Except for m_b and α_s, all the other parameters are less affected by the radiative correction. So, we consider only m_b and α_s terms neglecting all the others. The equations relating the $\overline{\mathrm{MS}}$ and $\overline{\mathrm{DR}}$ scheme [32–35] to α_s and m_b (for $m_z \le \mu \le m_t$) are given below as follows:

$$\begin{aligned}
&\frac{1}{\overline{\alpha}_s(\mu)^{\mathrm{DR}}} = \frac{1}{\alpha_s(\mu)^{\mathrm{MS}}} - \frac{1}{4};\\
&m_b^{\overline{\mathrm{DR}}}(\mu) = m_b^{\overline{\mathrm{MS}}}(\mu)\left(1 - \frac{1}{3\pi}\alpha_s(\mu) - \frac{29}{72\pi}\alpha_s(\mu)^2\right);\\
&m_b^{\overline{\mathrm{MS}}}(\mu) = m_b^{\overline{\mathrm{MS}}}(m_b)\frac{F_b(\mu)}{F_b(m_b)},\\
&F_b(\mu) = \left(\frac{23\alpha_s(\mu)}{6\pi}\right)^{12/23}\\
&\quad\cdot\left(1 + \frac{3731}{3174}\frac{\alpha_s(\mu)}{\pi} + 1.5007\left(\frac{\alpha_s(\mu)}{\pi}\right)^2\right).
\end{aligned} \tag{9}$$

The values of α_1, α_2, and α_3, evaluated at top quark mass scale using the above equations in $\overline{\mathrm{DR}}$ scheme, are shown in Table 2.

The values of m_b at various scales both in the $\overline{\mathrm{MS}}$ and $\overline{\mathrm{DR}}$ scheme are shown in Table 3.

3. Effect on the Unification with m_t as the SUSY Breaking Scale ($m_s = m_t$)

With the numerical values of m_t, m_b^{DR}, and m_τ at hand we can now determine the values of Yukawa couplings at top quark mass scale using the following equations [31, 36] from minimal supersymmetric standard model (MSSM):

$$\begin{aligned}
h_t &= \frac{m_t(m_t)}{174\sin\beta} = \frac{m_t(m_t)\sqrt{1+\tan^2\beta}}{174\tan\beta},\\
h_b &= \frac{m_b(m_b)}{174\eta_b\cos\beta} = \frac{m_b(m_t)\sqrt{1+\tan^2\beta}}{174},\\
h_\tau &= \frac{m_\tau(m_\tau)}{174\eta_\tau\cos\beta} = \frac{m_\tau(m_t)\sqrt{1+\tan^2\beta}}{174}.
\end{aligned} \tag{10}$$

Here h_t, h_b, and h_τ are the third-generation Yukawa couplings for top quark, bottom quark, and tau lepton, respectively. The vacuum expectation value without SUSY is $V/\sqrt{2} = 174$ GeV, and $\tan\beta = V_u/V_d$ is a free parameter in MSSM, where V_u is the VEV for the up-type quarks $V_u = V\sin\beta$ and V_d for the down type quarks $V_d = V\cos\beta$.

With the values of three gauge couplings in Table 2 and Yukawa couplings in (10) as input values, we estimate the nature of variation of gauge and Yukawa couplings from top quark mass scale m_t up to the point of unification using 2-loops RGEs [29, 31, 37, 38] defined as

$$\begin{aligned}
\frac{dg_i}{dt} &= \frac{b_i}{16\pi^2}g_i^3\\
&\quad + \left(\frac{1}{16\pi^2}\right)^2\left[\sum_{j=1}^{3} b_{ij}g_i^3g_j^2 - \sum_{j=t,b,\tau} a_{ij}g_i^3h_j^2\right],
\end{aligned} \tag{11}$$

where $t = \ln\mu$ and b_i, b_{ij}, a_{ij} are β function coefficients in MSSM,

$$\begin{aligned}
b_i &= (6.6, 1.0, -3.0),\\
b_{ij} &= \begin{pmatrix} 7.96 & 5.40 & 17.60\\ 1.80 & 25.00 & 24.00\\ 2.20 & 9.00 & 14.00 \end{pmatrix},\\
a_{ij} &= \begin{pmatrix} 5.2 & 2.8 & 3.6\\ 6.0 & 6.0 & 2.0\\ 4.0 & 4.0 & 0.0 \end{pmatrix},
\end{aligned} \tag{12}$$

and for Yukawa couplings at 2-loop level [29–31],

$$\begin{aligned}
\frac{dh_t}{dt} &= \frac{h_t}{16\pi^2}\left(6h_t^2+h_b^2-\sum_{i=1}^{3}c_i g_i^2\right)\\
&+\frac{h_t}{16\pi^2}\left[\sum_{i=1}\left(c_i b_i+\frac{c_i^2}{2}\right)g_i^4+g_1^2g_2^2+\frac{136}{45}g_1^2g_3^2\right.\\
&+8g_2^2g_3^2+\left(\frac{6}{5}g_1^2+6g_2^2+16g_3^2\right)h_t^2+\frac{2}{5}g_1^2h_b^2-22h_t^4\\
&\left.-5h_b^4-5h_t^2h_b^2-h_b^2h_\tau^2\right]\\
\frac{dh_b}{dt} &= \frac{h_b}{16\pi^2}\left(6h_b^2+h_\tau^2+h_t^2-\sum_{i=1}^{3}c_i' g_i^2\right)\\
&+\frac{h_b}{16\pi^2}\left[\sum_{i=1}\left(c_i' b_i+\frac{c_i'^2}{2}\right)g_i^4+g_1^2g_2^2+\frac{8}{9}g_1^2g_3^2\right.\\
&+8g_2^2g_3^2+\left(\frac{2}{5}g_1^2+6g_2^2+16g_3^2\right)h_b^2+\frac{4}{5}g_1^2h_t^2\\
&\left.+\frac{6}{5}g_1^2h_\tau^2-22h_b^4-3h_\tau^4-5h_t^4-5h_b^2h_t^2-3h_b^2h_\tau^2\right]\\
\frac{dh_\tau}{dt} &= \frac{h_\tau}{16\pi^2}\left(4h_\tau^2+3h_b^2-\sum_{i=1}^{3}c_i'' g_i^2\right)\\
&+\frac{h_\tau}{16\pi^2}\left[\sum_{i=1}\left(c_i'' b_i+\frac{c_i''^2}{2}\right)g_i^4+\frac{9}{5}g_1^2g_2^2\right.\\
&+\left(\frac{6}{5}g_1^2+6g_2^2\right)h_\tau^2+\left(\frac{-2}{5}g_1^2+16g_3^2\right)h_b^2+9h_b^4\\
&\left.-10h_\tau^4-3h_b^2h_t^2-9h_b^2h_\tau^2\right],
\end{aligned} \tag{13}$$

where

$$\begin{aligned}
c_i &= \left(\frac{13}{15},3,\frac{16}{13}\right),\\
c_i' &= \left(\frac{7}{15},3,\frac{16}{3}\right),\\
c_i'' &= \left(\frac{9}{5},3,0\right).
\end{aligned} \tag{14}$$

With the central value of g_3^{DR} there is an approximate gauge couplings unification around 2.58×10^{16} GeV but a sharp Yukawa couplings unification at 3.88×10^{11} GeV as shown in Table 4. However, if we vary g_3^{DR} within the experimental bound $1.2084^{+0.0344}_{-0.0355}$, it is possible for both gauge couplings and Yukawa couplings to have a sharp unification scale at their respective $\tan\beta$ values as shown in Table 5 along with their graphical representation in Figures 1 and 2.

Table 4: Approximate unification points for gauge couplings and Yukawa couplings for $g_3^{\mathrm{DR}}=1.2084$ and $m_s=m_t$.

At experimental	$\tan\beta$	g_3	Unification points (in GeV) Gauge U_{g_1,g_2,g_3}	Yukawa U_{h_t,h_b,h_τ}
Central value	59.9905	1.2084	$\sim2.59\times10^{16}$	1.997×10^{12}

Table 5: Exact unification points for gauge couplings and Yukawa couplings for input values of g_3^{DR} in the range $1.2084^{+0.0344}_{-0.0355}$ and $m_s=m_t$.

At experimental	$\tan\beta$	g_3^{DR}	Unification points (energy in GeV) Gauge U_{g_1,g_2,g_3}	Yukawa U_{h_t,h_b,h_τ}
Central value	60.1380	1.2240	2.95×10^{16}	3.88×10^{11}

4. Unification Based on Variation of SUSY Breaking Scale from Recent LHC Data ($m_s>m_t$)

Following Section 2, here we will consider the second case where SUSY breaking scale is being pushed higher up to 7 TeV. To be precise we consider some viable points, namely, 500 GeV, 1 TeV, 2 TeV, 3 TeV, 5 TeV, and 7 TeV assuming the supersymmetric effect to start somewhere in between.

The technique is almost similar to the previous section. The RGEs governing the evolution of the gauge couplings and the Yukawa couplings are the same as those given in (11) and (13) with the only difference in the values of the energy scale and the coefficients of the beta function, that is, b_i and c_i's.

Because of the difference in the intermediate energy level, one more step is needed. In the previous section (Section 2) we elevate the physical parameters from m_z scale up to m_t scale and then to unification point using (11) and (13). Here in this case we will be doing the same but with one more step as shown below.

(1) Evolution from m_z scale up to m_t using (1) for the energy range $m_z\le\mu\le m_t$.

(2) Evolution from m_t to m_s, where m_s = 500 GeV, 1 TeV, 3 TeV, 5 TeV, and 7 TeV. Using beta function coefficients for non-SUSY case in (11) and two-loop RGE for third-generation Yukawa couplings in non-SUSY (15), given by

$$\begin{aligned}
\frac{dh_t}{dt} &= \frac{h_t}{16\pi^2}\left(\frac{3}{2}h_t^2-\frac{3}{2}h_b^2+Y_2(S)-\sum_{i=1}^{3}c_i g_i^2\right)\\
&+\frac{h_t}{(16\pi^2)^2}\left[\frac{1187}{600}g_1^4-\frac{23}{4}g_2^4-108g_3^4-\frac{9}{20}g_1^2g_2^2\right.
\end{aligned}$$

$$
\begin{aligned}
&+\frac{19}{15}g_1^2g_3^2+9g_2^2g_3^2+\left(\frac{223}{80}g_1^2+\frac{135}{16}g_2^2+16g_3^2\right)h_t^2\\
&-\left(\frac{43}{80}g_1^2-\frac{9}{16}g_2^2+16g_3^2\right)h_b^2+\frac{5}{2}Y_4(S)\\
&-2\lambda\left(3h_t^2+h_b^2\right)+\frac{3}{2}h_t^4-\frac{5}{4}h_t^2h_b^2+\frac{11}{4}h_b^4\\
&+Y_2(S)\left(\frac{5}{4}h_b^2-\frac{9}{4}h_t^2\right)-\chi_4(S)+\frac{3}{2}\lambda^2\Big]\\
\frac{dh_b}{dt}&=\frac{h_b}{16\pi^2}\left(\frac{3}{2}h_b^2-\frac{3}{2}h_t^2+Y_2(S)-\sum_{i=1}^{3}c_i'g_i^2\right)\\
&+\frac{h_b}{(16\pi^2)^2}\Big[-\frac{127}{600}g_1^4-\frac{23}{4}g_2^4-108g_3^4-\frac{27}{20}g_1^2g_2^2\\
&+\frac{31}{15}g_1^2g_3^2+9g_2^2g_3^2-\left(\frac{79}{80}g_1^2-\frac{9}{16}g_2^2+16g_3^2\right)h_t^2\\
&+\left(\frac{187}{80}g_1^2+\frac{135}{16}g_2^2+16g_3^2\right)h_b^2+\frac{5}{2}Y_4(S)\\
&-2\lambda\left(h_t^2+3h_b^2\right)+\frac{3}{2}h_b^4-\frac{5}{4}h_t^2h_b^2+\frac{11}{4}h_t^4\\
&+Y_2(S)\left(\frac{5}{4}h_t^2-\frac{9}{4}h_b^2\right)-\chi_4(S)+\frac{3}{2}\lambda^2\Big]\\
\frac{dh_\tau}{dt}&=\frac{h_\tau}{16\pi^2}\left(\frac{3}{2}h_\tau^2+Y_2(S)-\sum_{i=1}^{3}c_i''g_i^2\right)\\
&+\frac{h_\tau}{(16\pi^2)^2}\Big[\frac{1371}{200}g_1^4-\frac{23}{4}g_2^4-\frac{27}{20}g_1^2g_2^2\\
&+\left(\frac{387}{80}g_1^2+\frac{135}{16}g_2^2\right)h_\tau^2+\frac{5}{2}Y_4(S)-6\lambda h_t^2+\frac{3}{2}h_\tau^4\\
&-\frac{9}{4}Y_2(S)h_\tau^2-\chi_4(S)+\frac{3}{2}\lambda^2\Big]\\
\frac{d\lambda}{dt}&=\frac{1}{16\pi^2}\Big[\frac{9}{4}\left(\frac{3}{25}g_1^4+\frac{2}{5}g_1^2g_2^2+g_2^4\right)\\
&-\left(\frac{9}{5}g_1^2+9g_2^2\right)\lambda+4Y_2(S)\lambda-4H(S)+12\lambda^2\Big]\\
&+\frac{1}{(16\pi^2)^2}\Big[-78\lambda^3+18\left(\frac{3}{5}g_1^2+3g_2^2\right)\lambda^2\\
&+\left(-\frac{73}{8}g_2^4+\frac{117}{20}g_1^2g_2^2+\frac{1887}{200}g_1^4\right)\lambda+\frac{305}{8}g_2^6\\
&-\frac{867}{120}g_1^2g_2^4-\frac{1677}{200}g_1^4g_2^2-\frac{3411}{1000}g_1^6\\
&-64g_3^2\left(h_t^4+h_b^4\right)-\frac{8}{5}g_1^2\left(2h_t^4-h_b^4+3h_\tau^4\right)\\
&-\frac{3}{2}g_2^4Y_2(S)+10\lambda Y_4(S)+\frac{3}{5}g_1^2\left(-\frac{57}{10}g_1^2+21g_2^2\right)h_t^2\\
&+\left(\frac{3}{2}g_1^2+9g_2^2\right)h_b^2+\left(-\frac{15}{2}g_1^2+11g_2^2\right)h_\tau^2\\
&-24\lambda^2Y_2(S)-\lambda H(S)+6\lambda h_t^2h_b^2\\
&+20\left(3h_t^6+3h_b^6+h_\tau^6\right)-12\left(h_t^4h_b^2+h_t^2h_b^4\right)\Big],
\end{aligned}
\tag{15}
$$

Table 6: Approximate gauge unification points and Yukawa unification points for central value of $g_3^{\mathrm{DR}}=1.2084$.

SUSY breaking scale (m_s)	$\tan\beta$	Unification points (energy in GeV) Gauge U_{g_1,g_2,g_3}	Yukawa U_{h_t,h_b,h_τ}
500 GeV	60.9070	3.7447×10^{16}	1.9315×10^{11}
1 TeV	61.4656	4.1134×10^{16}	8.6171×10^{10}
3 TeV	62.4180	4.8372×10^{16}	2.5719×10^{10}
5 TeV	62.8150	5.1843×10^{16}	1.6339×10^{10}
7 TeV	63.0523	5.4012×10^{16}	1.2611×10^{10}

where

$$
\begin{aligned}
Y_2(S)&=3h_t^2+3h_b^2+h_\tau^2\\
Y_4(S)&=\frac{1}{3}\left[3\sum c_ig_i^2h_t^2+3\sum c_i'g_i^2h_b^2+3\sum c_i''g_i^2h_\tau^2\right]\\
\chi_4(S)&=\frac{9}{4}\left[3h_t^4+3h_b^4+h_\tau^4-\frac{2}{3}h_t^2h_b^2\right]\\
H(S)&=3h_t^4+3h_t^4+h_\tau^4
\end{aligned}
\tag{16}
$$

$\lambda=\dfrac{m_h^2}{V^2}$ is the Higgs self-coupling (m_h = Higgs mass),

with the values of beta function coefficients for non-SUSY case [29, 30],

$$
\begin{aligned}
b_i&=(4.100,-3.167,-7.000),\\
g_{ij}&=\begin{pmatrix}3.98 & 2.70 & 8.8\\ 0.90 & 5.83 & 12.0\\ 1.10 & 4.50 & -26.0\end{pmatrix},\\
a_{ij}&=\begin{pmatrix}0.85 & 0.5 & 0.5\\ 1.50 & 1.5 & 0.5\\ 2.00 & 2.0 & 0.0\end{pmatrix},\\
c_i&=(0.85,2.25,8.00),\\
c_i'&=(0.25,2.25,8.00),\\
c_i''&=(2.25,2.25,0.00)
\end{aligned}
\tag{17}
$$

for the energy range $m_t\le\mu\le m_s$.

(3) Evolution from m_s, where $m_s=500$ GeV, 1 TeV, 3 TeV, 5 TeV, and 7 TeV to m_{GUT} GeV, using (11) and (13) with the same values of b_i, b_{ij}, and a_{ij}, and c_i, c_i', and c_1'' used in Section 3 ($m_t\le\mu\le M_u$).

Here, we obtained a similar result to that of Section 3. At the central value of g_3^{DR} there is an approximate gauge couplings unification but a sharp Yukawa couplings unification (Table 6). However, if we vary g_3^{DR} within the experimental

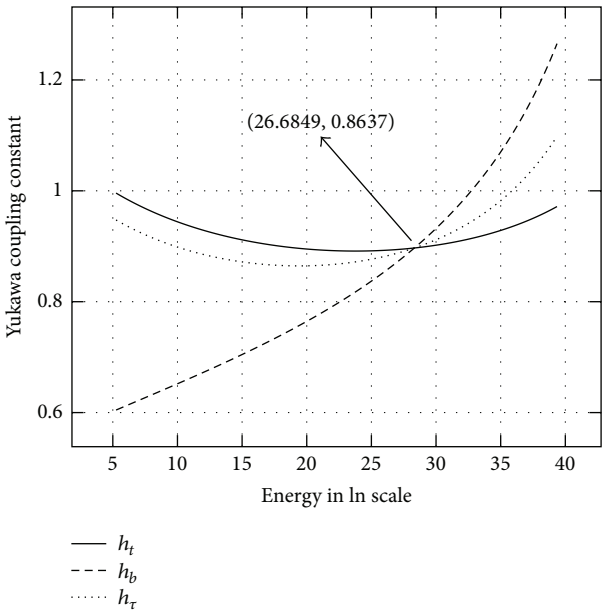

Figure 1: Third-generation Yukawa couplings unification for $m_s = m_t = 173.5$ GeV, ($e^{26.6849} = 3.8828 \times 10^{11}$) GeV.

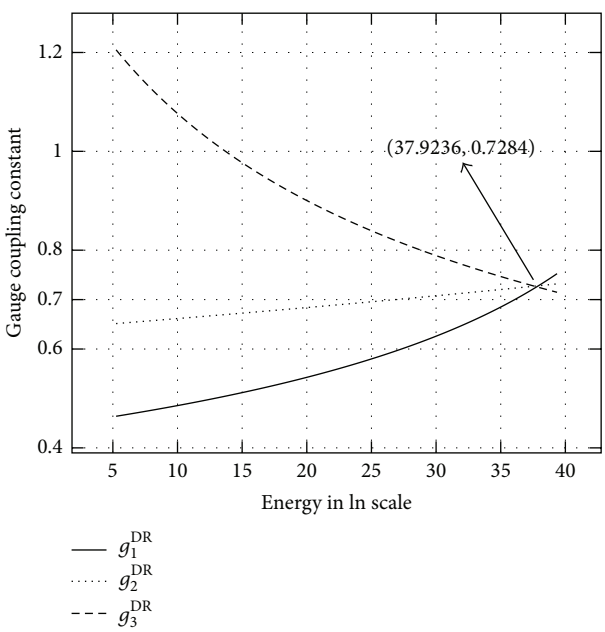

Figure 2: Gauge coupling unification for $m_s = m_t = 173.5$ GeV, ($e^{37.9236} = 2.9515 \times 10^{16}$) GeV.

bounds, it is possible for both gauge couplings and Yukawa couplings to have a sharp single unification scale at their respective energy scale and $\tan\beta$ values as shown in Table 7 and Figures 3 and 4.

5. Results and Discussion

To summarize, we have studied the unification scenario in supersymmetric SU(5) grand unified theory [18–20] using the recent data and the two-loop renormalization group equations [3, 4]. From our study we have found that (in Section 3, where $m_s = m_t$) with the central value of g_3^{DR} there is an approximate gauge couplings unification and a sharp Yukawa couplings unification as given in Table 4. However, if we vary g_3^{DR} within the experimental bounds ($1.2084^{+0.0344}_{-0.0355}$), it is possible to obtain a sharp unification scale for both the gauge couplings and Yukawa couplings at their respective m_s and $\tan\beta$ values as shown in Figures 1 and 2 and in Table 5 (gauge unification at 2.9518×10^{16} GeV and Yukawa unification at 3.8828×10^{11} GeV). A similar result is found in Section 4 where there are approximate (Table 6) and sharp unification scales for gauge couplings and Yukawa couplings at central value of g_3^{DR} (Table 6). But with the variation of g_3^{DR} within the experimental range $1.2084^{+0.0344}_{-0.0355}$, we obtained a single unification scale for the gauge couplings at 5.4175×10^{16} GeV and for Yukawa couplings at 5.0175×10^{9} GeV (Figures 3 and 4 and Table 7). Here we have shown only the graph for $m_s = 7$ TeV case as all the other graphs for different m_s have the similar pattern with the only difference in their unification scale. When we note down the unification points for both the gauge couplings and the Yukawa couplings for different values of m_s, a pattern emerged as shown in Figures 5 and 6. For gauge couplings, the unification point increases with the increase in the SUSY breaking scale m_s. But for Yukawa couplings the unification points vary in the reverse order compared to the gauge couplings; that is, unification points decrease with the increase in m_s. Finally the present analysis addresses an important question on the how do the gauge and Yukawa couplings unification scales vary with the varying SUSY breaking scale.

Table 7: Exact unification points for gauge couplings and Yukawa couplings for input values of g_3^{DR} in the range $1.2084^{+0.0344}_{-0.0355}$.

SUSY breaking scale (m_s)	$\tan\beta$	g_3^{DR}	Unification points (energy in GeV) Gauge U_{g_1,g_2,g_3}	Yukawa U_{h_t,h_b,h_τ}
175.5 GeV	60.1633	1.2248	2.9762×10^{16}	1.7118×10^{11}
500 GeV	61.0600	1.2151	3.7526×10^{16}	1.1423×10^{11}
1 TeV	61.6230	1.2091	4.1142×10^{16}	8.2059×10^{10}
3 TeV	62.5450	1.1992	4.8237×10^{16}	4.6046×10^{10}
5 TeV	62.9750	1.1952	5.1569×10^{16}	3.6490×10^{10}
7 TeV	63.2500	1.1928	5.3722×10^{16}	3.1866×10^{10}

The present analysis is based on an extremely simplified assumption of a single scale for all SUSY particles. There are strong hints that this is not the case and the SUSY spectrum is more spread than being at a single scale [18]. Such simplified assumption makes the present analysis possible at the cost of exact numerical accuracy. We also neglect the threshold corrections [19, 20] from various factors like (i) threshold correction from the two-loop contribution in the running of coupling constants (ii) light threshold correction from all superpartners in the SUSY sector, and (iii) threshold correction from particles of mass of the unification scale.

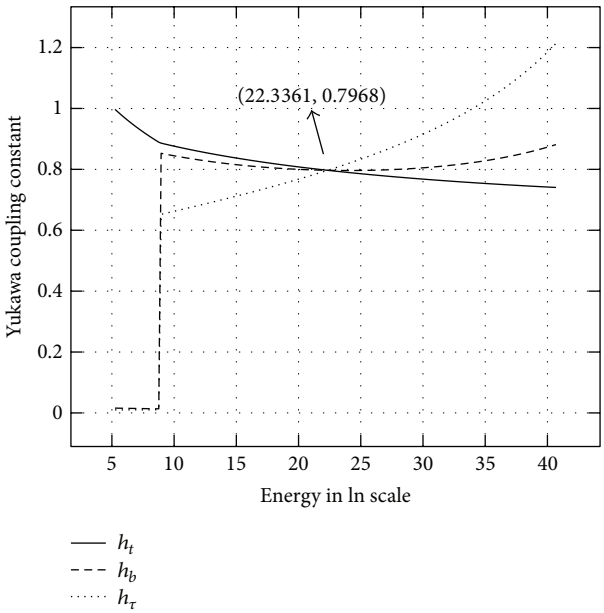

Figure 3: Third-generation Yukawa couplings unification for m_s = 7 TeV, ($e^{22.3361} = 5.0175 \times 10^{11}$) GeV.

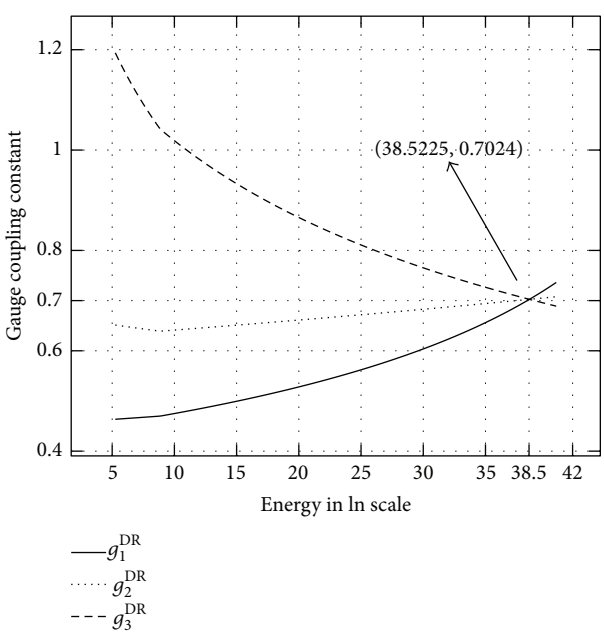

Figure 4: Gauge couplings unification for m_s = 7 TeV, ($e^{38.5225} = 5.3722 \times 10^{16}$) GeV.

The first assumption is valid so long as $m_s \gg m_t$ or m_z [38]. These two assumptions when properly taken into account will affect the result by a few percentages. The above issues are crucial to give a realistic numerical estimation of the unification point which directly controls the enhancement of the proton decay rate, and it will be addressed in a separate communication.

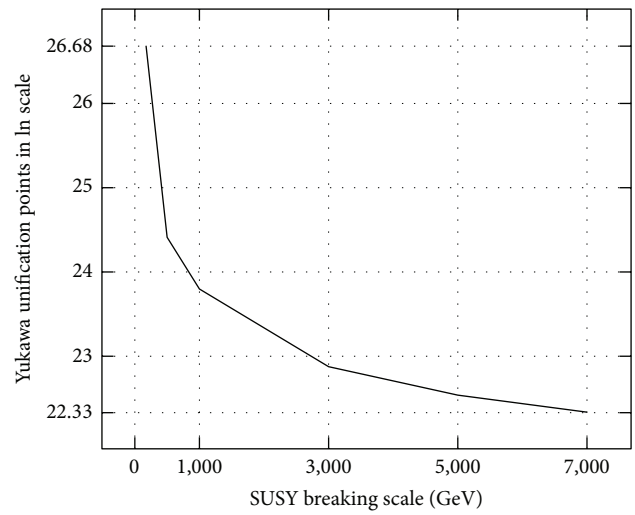

Figure 5: Nature of variation of Yukawa unification points with varying m_s.

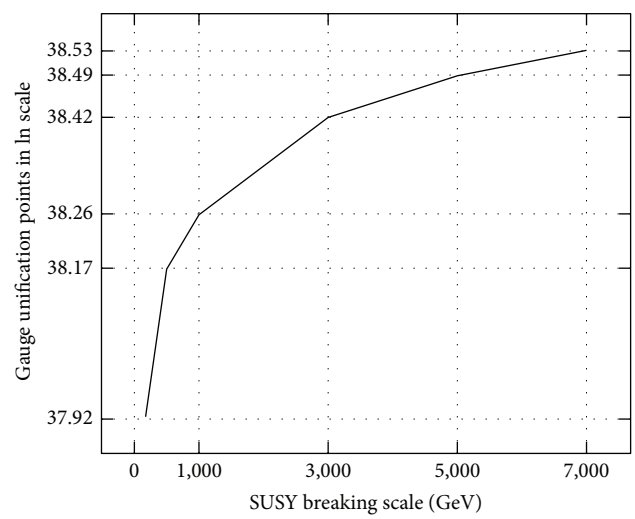

Figure 6: Nature of variation of gauge unification points with varying m_s.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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