Research Article

Effects of the Variation of SUSY Breaking Scale on Yukawa and Gauge Couplings Unification

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The present analysis addresses an interesting primary question on how do the gauge and Yukawa couplings unification scales vary with varying SUSY breaking scales 𝑚𝑠, assuming a single scale for all supersymmetric particles. It is observed that the gauge coupling unification scale increases with 𝑚𝑠 whereas third-generation Yukawa couplings unification scale decreases with 𝑚𝑠. The rising of the unification scale and also the mass of the color triplet multiplets is necessary to increase the proton decay lifetime; the analysis is carried out with two-loop RGEs for the gauge and Yukawa couplings within the minimal supersymmetric SU(5) model, while ignoring for simplicity the threshold effects of the heavy particles, which could be as large as a few percentages.

1. Introduction

The most natural extension of the minimal SU(5) GUT [1] is the supersymmetric SU(5) GUT [2] which has wide predictive power [3, 4]. The most important features are the prediction for weak mixing angle and the unification of the three gauge couplings at very large scale which is called the unification scale 𝑀GUT [3]. It also predicts the unification of the third-generation Yukawa couplings at or below the unification scale and provides a natural solution for the hierarchy problem and an alternative explanation of the electroweak symmetry breaking by the so-called radiative breaking scenario [5–8]. This theory also provides the prediction of proton decay [4] which is caused mainly by 𝐷=5 operator [9–12]. Since the most stringent limit on proton lifetime is provided by the Super Kamiokande experiment [13, 14], with the current lower experimental bound [15] 𝜏𝑝>4×10³³ years, such restrictive value may serve as a criteria to discriminate certain GUT models. This may serve as a direct experimental support to GUT theories.

There are certain arguments against [16, 17] the validity of the SUSY SU(5) GUT model. However there are specific regions in parameter space in minimal renormalizable supersymmetric SU(5) model [18–20] that is consistent with all experimental constraints including gauge couplings unification and the experimental limit on proton lifetime. In the literature there are still some arguments in support of SUSY SU(5) GUT model [21]. In order to suppress the fast 𝐷=5 operator proton decay, we have to rise both the scale of unification and the mass of the color triplet multiplets [18]. Within the SU(5) SUSY GUT, attempts have also been made to suppress 𝐷=5 proton decay operator [22]. In such context there is still enough scope for further investigation in this direction.

In this paper our focus is on the unification of the gauge couplings as well as on the Yukawa couplings in two-loops RGEs within the framework of minimal supersymmetric SU(5) GUT using updated data consistent with the LHC result. We numerically solve the unification scale for three gauge couplings (𝛾₁, 𝛾₂, and 𝛾₃) as well as the three Yukawa
couplings ($h_t$, $h_b$, and $h_\tau$) with varying input values of SUSY breaking scale $m_1$ [18], assuming a single scale for all supersymmetric particles for simplicity of the calculation [23, 24]. There are hints that SUSY particles have a wide spectrum and are not confined to a single energy scale. This kind of assumption is valid as long as the $m_t$ or $m_b < m_1$ [25]. We assume the scale $m_t$ to be somewhere in between 500 GeV and 7 TeV. In the present calculation we also ignore the threshold effects of heavy particles which could be as large as a few percentages [19, 20], and latter would affect the unification scale to some extent.

The paper is organized as follows. In Section 2 we collect the necessary input parameters from [26], which are all given at $m_t$ scale in MS scheme. We then make it evolve up to top quark mass scale ($m_t$) and then converted it into DR scheme. In Section 3, using the values obtained in Section 2, we calculate the Yukawa couplings for top quark, bottom quark, and tau lepton and also the three gauge couplings at $m_t(m_t)$. Using these as the input values and choosing the SUSY breaking scale to be $m_1 = m_1$, we then extrapolate them to very high energy scale and study the unification scenarios. In Section 4 we follow a similar procedure as in Section 3 but instead of $m_1 = m_1$, we choose different $m_t (> m_t)$. Here, we divide the running process into two parts, non-SUSY part (from $m_t$ to $m_t$) and the SUSY part (from $m_t$ to $m_{GUT}$). In Section 5 we summarize our results and we conclude.

2. Evolution of Gauge and Yukawa Couplings with Energy Scales

The most recent experimental data from low energy experiment [26], which would be used for generation of the initial input values at low scales, are given in Table 1.

In order to calculate the gauge coupling $\alpha_t(m_t)$ for $U(1)_Y$ and $\alpha_s(m_t)$ for $SU(2)_L$ for the Standard Model $SU(2)_C \times SU(2)_L \times U(1)_Y$, we start with the matching relation and definition of Weinberg mixing angle. Thus,

$$\frac{1}{\alpha_{em}(m_t)} = \frac{5}{3} \frac{1}{\alpha_t(m_t)} + \frac{1}{\alpha_s(m_t)},$$

$$\sin^2 \theta_W(m_t) = \frac{\alpha_{em}(m_t)}{\alpha_s(m_t)}.$$  

Substituting the observed values of coupling constants $\alpha_{em}(m_t)$, $\alpha_t(m_t)$, and $\sin^2 \theta_W$ from Table 1 we obtain the numerical values of $\alpha_t(m_t)$ and $\alpha_s(m_t)$ with uncertainties arising from input value of $\alpha_t(m_t)$,

$$\alpha_t(m_t) = 1.7100^{+0.0012}_{-0.0017} \times 10^{-2},$$  

$$\alpha_s(m_t) = 3.753^{+0.00215}_{-0.00216} \times 10^{-2},$$

respectively. In terms of the normalized coupling constant ($g_i$), $\alpha_t$ can be expressed as $g_i = \sqrt{4\pi} \alpha_t$, where $i = 1, 2, 3$ and it represents electromagnetic, weak, and strong couplings, respectively.

Here we consider two possible scenarios for the unification of the couplings. In the first case we consider the top quark mass $m_t$ to be the starting energy scale for the evolution from which the supersymmetric effect on the couplings has been included. Since the observational data in Table 1 are given only at the $z$-pole mass scale, it is necessary to evolve them up to the top quark mass scale. The evolution equation of the coupling constants at one-loop level [27] is given by

$$\frac{d\alpha_i}{dt} = \frac{b_i}{2\pi} \alpha_i^2,$$

which can be simplified as

$$\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_i(m_t)} - \frac{b_i}{2\pi} \ln \left( \frac{\mu}{m_t} \right),$$

where $\mu$ is the energy scale in the range ($m_t \leq \mu \leq m_t$). For non-SUSY case, we have the coefficient of $\beta$ function of the RGEs [28, 29],

$$b_i = (5.30, -0.50, -4.00).$$

The evolution of the third-generation fermion masses (top, bottom, and tau) is obtained by using the QED-QCD rescaling factor $\eta$ as

$$m_b(m_t) = \frac{m_b(m_b)}{\eta_b},$$

$$m_t(m_t) = \frac{m_t(m_3)}{\eta_\tau},$$

where $\eta_b = 1.530$ and $\eta_\tau = 1.015$ [30, 31].

All the above physical parameters are evaluated in the modified minimal subtraction scheme (MS), without any radiative corrections. The inclusion of radiative correction is achieved by using the method of dimensional regularization through dimensional reduction [32].

Estimation of Yukawa couplings for $t$, $b$, and $\tau$ requires a careful determination of $m_t$, $m_b$, and $m_\tau$ in the DR scheme [28]. However, the effect of running of $m_\tau$ on $h_\tau$ is very small and hence can be neglected. Furthermore, DR technique is used in order to reduce the large uncertainty in the value of $\alpha_t$. 

<table>
<thead>
<tr>
<th>Mass in GeV</th>
<th>Coupling constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_t(m_t)$</td>
<td>$127.944 \pm 0.014$</td>
</tr>
<tr>
<td>$m_b(m_b)$</td>
<td>$118.4 \pm 0.007$</td>
</tr>
<tr>
<td>$m_\tau(m_\tau)$</td>
<td>$4.18 \pm 0.030$</td>
</tr>
<tr>
<td>$m_\mu(m_\mu)$</td>
<td>$1.7768 \pm 0.0016$</td>
</tr>
</tbody>
</table>

Weinberg mixing angles $\sin^2 \theta_W(m_t) = 0.23116 \pm 0.00012$. 

Table 1: Experimental input values for fermion masses, gauge couplings, and Weinberg angle at electroweak scale $m_t$ [26].
Table 2: Numerical values of gauge couplings at top quark mass scale $m_t$.

<table>
<thead>
<tr>
<th>Lower limit</th>
<th>Central value</th>
<th>Upper limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1$</td>
<td>$1.7099 \times 10^{-2}$</td>
<td>$1.7100 \times 10^{-2}$</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>$3.7748 \times 10^{-2}$</td>
<td>$3.3753 \times 10^{-2}$</td>
</tr>
<tr>
<td>$\alpha_3^{DR}$</td>
<td>0.1095</td>
<td>0.1162</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>0.46354</td>
<td>0.46356</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>0.65148</td>
<td>0.65127</td>
</tr>
<tr>
<td>$\alpha_3^{DR}$</td>
<td>1.17294</td>
<td>1.20842</td>
</tr>
</tbody>
</table>

Table 3: $m_b$ in MS and $\overline{DR}$ schemes.

<table>
<thead>
<tr>
<th>Lower limit</th>
<th>Central value</th>
<th>Upper limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>MS $m_b(m_b)$</td>
<td>4.1500</td>
<td>4.1800</td>
</tr>
<tr>
<td>$m_b(m_b)$</td>
<td>2.7605</td>
<td>2.8618</td>
</tr>
<tr>
<td>$m_b(m_b)$</td>
<td>2.6922</td>
<td>2.7860</td>
</tr>
<tr>
<td>$m_b(m_b)$</td>
<td>4.0432</td>
<td>4.0713</td>
</tr>
<tr>
<td>$m_b(m_b)$</td>
<td>2.7265</td>
<td>2.8242</td>
</tr>
<tr>
<td>$m_b(m_b)$</td>
<td>2.6606</td>
<td>2.7512</td>
</tr>
</tbody>
</table>

Except for $m_b$ and $\alpha_3$, all the other parameters are less affected by the radiative correction. So, we consider only $m_b$ and $\alpha_3$ terms neglecting all the others. The equations relating the MS and $\overline{DR}$ scheme [32–35] to $\alpha_i$ and $m_b$ (for $m_z \leq \mu \leq m_t$) are given below as follows:

$$1 \overline{\alpha}_i (\mu)^{DR} = 1 \alpha_i (\mu)^{MS} - \frac{1}{4};$$

$$m_b^{\overline{DR}} (\mu) = m_b^{MS} (\mu) \left(1 - \frac{1}{3\pi} \alpha_i (\mu) - \frac{29}{72\pi} \alpha_i (\mu)^2 \right);$$

$$m_b^{MS} (\mu) = m_b^{MS} (m_b) \frac{F_b (\mu)}{F_b (m_b)};$$

$$F_b (\mu) = \left( \frac{23 \alpha_i (\mu)}{6\pi} \right)^{12/23} \left( 1 + \frac{3731 \alpha_i (\mu)}{3174 \pi} + 1.5007 \left( \frac{\alpha_i (\mu)}{\pi} \right)^2 \right).$$

The values of $\alpha_3$, $\alpha_2$, and $\alpha_3$, evaluated at top quark mass scale using the above equations in $\overline{DR}$ scheme, are shown in Table 2.

The values of $m_b$ at various scales both in the MS and $\overline{DR}$ scheme are shown in Table 3.

3. Effect on the Unification with $m_t$ as the SUSY Breaking Scale ($m_s = m_t$)

With the numerical values of $m_t$, $m_b^{DR}$, and $m_s$ at hand we can now determine the values of Yukawa couplings at top quark mass scale using the following equations [31, 36] from minimal supersymmetric standard model (MSSM):

$$h_i = \frac{m_i (m_t)}{174 \sin \beta} \sqrt{1 + \tan^2 \beta},$$

$$h_b = \frac{m_b (m_t)}{174 \eta_b \cos \beta} \frac{1 + \tan^2 \beta}{174},$$

$$h_t = \frac{m_t (m_t)}{174 \eta_t \cos \beta} \frac{1 + \tan^2 \beta}{174}.$$

Here $h_i, h_b$, and $h_t$ are the third-generation Yukawa couplings for top quark, bottom quark, and tau lepton, respectively. The vacuum expectation value without SUSY is $V/\sqrt{2} = 174$ GeV, and $\tan \beta = V_u/V_d$ is a free parameter in MSSM, where $V_u$ is the VEV for the up-type quarks $V_u = V \sin \beta$ and $V_d$ for the down type quarks $V_d = V \cos \beta$.

With the values of three gauge couplings in Table 2 and Yukawa couplings in (10) as input values, we estimate the nature of variation of gauge and Yukawa couplings from top quark mass scale $m_t$ up to the point of unification using 2-loops RGEs [29, 31, 37, 38] defined as

$$\frac{dg_i}{dt} = \frac{b_i}{16\pi^2} g_i^3,$$

$$+ \left( \frac{1}{16\pi^2} \right)^2 \left[ \sum_{j=1}^{3} b_j g_j^3 g_j^3 - \sum_{j=1}^{3} a_{ij} g_i^3 h_j^2 \right],$$

where $t = \ln \mu$ and $b_i, b_j, a_{ij}$ are $\beta$ function coefficients in MSSM,

$$b_i = (6.6, 1.0, -3.0),$$

$$b_j = \begin{pmatrix} 7.96 & 5.40 & 17.60 \ 1.80 & 25.00 & 24.00 \ 2.20 & 9.00 & 14.00 \ \end{pmatrix},$$

$$a_{ij} = \begin{pmatrix} 5.2 & 2.8 & 3.6 \ 6.0 & 6.0 & 2.0 \ 4.0 & 4.0 & 0.0 \ \end{pmatrix},$$
and for Yukawa couplings at 2-loop level [29–31],

\[
\frac{dh_t}{dt} = \frac{h_t}{16\pi^2} \left( 6h_t^2 + h_b^2 - \sum_{i=1}^{3} c_i g_i^2 \right) + \frac{h_t}{16\pi^2} \left[ \sum_{i=1}^{3} \left( c_i b_i + \frac{c_i^2}{2} \right) g_i^4 + \frac{136}{45} g_1 g_2 g_3 \right]
\]

\[
\frac{dh_b}{dt} = \frac{h_b}{16\pi^2} \left( 6h_b^2 + h_t^2 + h_c^2 + \sum_{i=1}^{3} c_i' g_i^2 \right) + \frac{h_b}{16\pi^2} \left[ \sum_{i=1}^{3} \left( c_i' b_i + \frac{c_i^2}{2} \right) g_i^4 + \frac{8}{9} g_1^2 g_3 \right]
\]

\[
+ 8g_2^2 g_3^2 + \left( \frac{2}{5} g_1^2 + \frac{4}{5} g_2^2 \right) h_b^2 + \frac{4}{5} g_1^2 h_t^2
\]

\[
+ \frac{6}{5} g_1^2 h_t^2 - 22h_b^4 - 5h_t^4 - 5h_b^4 - 5h_t h_b^2 - 5h_b h_t^2
\]

\[
\frac{dh_c}{dt} = \frac{h_c}{16\pi^2} \left( 4h_c^2 + 3h_b^2 - \sum_{i=1}^{3} c_i'' g_i^2 \right) + \frac{h_c}{16\pi^2} \left[ \sum_{i=1}^{3} \left( c_i'' b_i + \frac{c_i^2}{2} \right) g_i^4 + \frac{9}{5} g_1^2 g_2^2 \right]
\]

\[
+ \frac{6}{5} g_1^2 + 6g_2^2 h_c^2 + \left( \frac{2}{5} g_1^2 + 16g_3^2 \right) h_b^2 + 9h_b^4
\]

\[
- 10h_t^2 - 3h_t h_b^2 - 3h_t h_c^2
\]

where

\[
c_i = \left( \frac{13}{15}, 3, \frac{16}{13} \right),
\]

\[
c_i' = \left( \frac{7}{15}, 3, \frac{16}{3} \right),
\]

\[
c_i'' = \left( \frac{9}{5}, 3, 0 \right).
\]

With the central value of \( g_3^{DR} \) there is an approximate gauge couplings unification around \( 2.58 \times 10^{16} \) GeV but a sharp Yukawa couplings unification at \( 3.88 \times 10^{13} \) GeV as shown in Table 4. However, if we vary \( g_3^{DR} \) within the experimental bound \( 1.2084^{+0.0344}_{-0.0355} \), it is possible for both gauge couplings and Yukawa couplings to have a sharp unification scale at their respective \( \tan \beta \) values as shown in Table 5 along with their graphical representation in Figures 1 and 2.

### Table 4: Approximate unification points for gauge couplings and Yukawa couplings for \( g_3^{DR} = 1.2084 \) and \( m_t = m_t \).

<table>
<thead>
<tr>
<th>Unification points (in GeV)</th>
<th>( \tan \beta )</th>
<th>( g_3 )</th>
<th>Gauge</th>
<th>Yukawa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Central value</td>
<td>59.9905</td>
<td>1.2084</td>
<td>2.59 \times 10^{16}</td>
<td>1.997 \times 10^{12}</td>
</tr>
</tbody>
</table>

### Table 5: Exact unification points for gauge couplings and Yukawa couplings for input values of \( g_3^{DR} \) in the range \( 1.2084^{+0.0344}_{-0.0355} \) and \( m_t = m_t \).

<table>
<thead>
<tr>
<th>Unification points (energy in GeV)</th>
<th>( \tan \beta )</th>
<th>( g_3^{DR} )</th>
<th>Gauge</th>
<th>Yukawa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Central value</td>
<td>60.1380</td>
<td>1.2240</td>
<td>2.95 \times 10^{16}</td>
<td>3.88 \times 10^{12}</td>
</tr>
</tbody>
</table>

### 4. Unification Based on Variation of SUSY Breaking Scale from Recent LHC Data

\( m_{\chi} > m_t \)

Following Section 2, here we will consider the second case where SUSY breaking scale is being pushed higher up to 7 TeV. To be precise we consider some viable points, namely, 500 GeV, 1 TeV, 2 TeV, 3 TeV, 5 TeV, and 7 TeV assuming the supersymmetric effect to start somewhere in between.

The technique is almost similar to the previous section. The RGEs governing the evolution of the gauge couplings and the Yukawa couplings are the same as those given in (II) and (13) with the only difference in the values of the energy scale and the coefficients of the beta function, that is, \( b_i \) and \( c_i \).

Because of the difference in the intermediate energy level, one more step is needed. In the previous section (Section 2) we elevate the physical parameters from \( m_t \) scale up to \( m_t \) scale and then to unification point using (II) and (13). Here in this case we will be doing the same but with one more step as shown below.

1. Evolution from \( m_t \) scale up to \( m_t \) using (I) for the energy range \( m_{\chi} \leq \mu \leq m_t \).

2. Evolution from \( m_t \) to \( m_t \), where \( m_{\chi} = 500 \text{ GeV} \), 1 TeV, 3 TeV, 5 TeV, and 7 TeV. Using beta function coefficients for non-SUSY case in (II) and two-loop RGE for third-generation Yukawa couplings in non-SUSY (15), given by

\[
\frac{dh_t}{dt} = \frac{h_t}{16\pi^2} \left( \frac{3}{2} h_t^2 - \frac{3}{2} h_b^2 + Y_2(S) - \sum_{i=1}^{3} c_i g_i^2 \right)
\]

\[
+ \frac{h_t}{(16\pi^2)^2} \left( \frac{1187}{600} g_1^4 - \frac{23}{4} g_2^4 - 108g_3^4 - \frac{9}{20} g_1^2 g_2^2 \right).
\]
\begin{equation}
\begin{aligned}
&+ \frac{19}{15} g_1^2 g_2^2 + 9 g_2^2 g_3^2 + \left( \frac{223}{80} g_1^3 + \frac{135}{16} g_2^3 + 16 g_3^3 \right) h_i^2 \\
&- \left( \frac{43}{80} g_1^2 - \frac{7}{8} g_2^2 + 16 g_3^2 \right) h_b^2 + \frac{5}{2} Y_4 (S) \\
&- 2 \lambda \left( 3 h_i^2 + h_b^2 \right) + \frac{3}{4} h_i^2 h_b^2 + \frac{11}{4} h_b^4 \\
&+ Y_2 (S) \left( \frac{5}{4} h_b^2 - \frac{9}{4} h_t^2 \right) - X_4 (S) \left( \frac{3}{2} \lambda^2 \right)
\end{aligned}
\end{equation}

\begin{equation}
\begin{aligned}
d\lambda &= \frac{1}{16 \pi^2} \left[ \frac{9}{4} \left( \frac{3}{25} g_1^4 + \frac{2}{5} g_1^2 g_2 + g_2^2 \right) \\
&- \left( \frac{9}{5} g_1^2 + 9 g_2^2 \lambda + 4 Y_2 (S) \lambda - 4 H (S) + 12 \lambda^2 \right) \\
&+ \frac{1}{(16 \pi^2)^2} \left[ -78 \lambda^3 + 18 \left( \frac{3}{5} g_1^2 + 3 g_2^2 \right) \lambda^2 \\
&+ \left( \frac{73}{8} g_1^2 + 117 g_2^2 + \frac{1887}{200} g_1^4 \lambda + 305 \right) g_2^6 \\
&- \frac{867}{120} g_1^2 g_2^4 - \frac{1677}{200} g_1^4 g_2^2 - \frac{3411}{1000} g_1^6 \\
&- 64 g_2^6 \left( h_t^2 + h_b^2 \right) - \frac{8}{5} g_1^2 \left( 2 h_t^2 - h_b^2 + 3 h_t^2 \right) \\
&- \frac{3}{2} g_1^2 Y_2 (S) + 10 \lambda Y_4 (S) + \frac{3}{5} g_1^2 \left( - \frac{57}{10} g_1^2 + 21 g_2^2 \right) h_t^2 \\
&+ \left( \frac{3}{2} g_1^2 + 9 g_2^2 \right) h_b^2 + \left( - \frac{15}{2} g_1^2 + 11 g_2^2 \right) h_t^2 \\
&- 24 A \lambda Y_2 (S) \lambda H (S) + 6 A h_t^2 h_b^2 \\
&+ 20 \left( 3 h_b^2 + 3 h_b^2 + h_t^2 \right) - 12 \left( h_b^2 h_t^2 + h_t^4 h_b^2 \right)
\end{aligned}
\end{equation}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
SUSY breaking & tan \beta & Unification points (energy in GeV) & Yukawa \\
\hline
scale \((m_t)\) & & & \\
\hline
500 GeV & 60.9070 & 3.7447 \times 10^{16} & 1.9315 \times 10^{11} \\
1 TeV & 61.4656 & 4.1134 \times 10^{16} & 8.6171 \times 10^{10} \\
3 TeV & 62.4100 & 4.8372 \times 10^{16} & 2.5719 \times 10^{10} \\
5 TeV & 62.8150 & 5.1843 \times 10^{16} & 1.6339 \times 10^{10} \\
7 TeV & 63.0523 & 5.4012 \times 10^{16} & 1.2611 \times 10^{10} \\
\hline
\end{tabular}
\caption{Approximate gauge unification points and Yukawa unification points for central value of \(g_3^{DR} = 1.2084\).}
\end{table}

where

\begin{equation}
Y_2 (S) = 3 h_t^2 + 3 h_b^2 + h_t^4 \\
Y_4 (S) = \frac{1}{3} \left[ 3 \sum c_1 g_1^2 h_t^2 + 3 \sum c'_1 g_1^2 h_b^2 + 3 \sum c''_1 g_1^2 h_t^2 \right] \\
\chi_4 (S) = \frac{9}{4} \left[ 3 h_t^4 + 3 h_b^4 + h_t^2 + \frac{2}{3} h_b^2 h_t^2 \right] \\
H (S) = 3 h_t^4 + 3 h_b^4 + h_t^4 \\
\lambda = \frac{m_h^2}{V^2} \text{ is the Higgs self-coupling (} m_h \text{)}
\end{equation}

with the values of beta function coefficients for non-SUSY case [29, 30],

\begin{align}
b_f &= (4.100, -3.167, -7.000), \\
g_{ij} &= (3.98, 2.70, 8.8), \\
\alpha_{ij} &= (0.85, 0.5, 0.5), \\
c_i &= (0.15, 0.5, 0.5), \\
c_i' &= (0.25, 2.25, 8.00), \\
c_i'' &= (2.25, 2.25, 0.00)
\end{align}

for the energy range \(m_s \leq \mu \leq m_t\).

(3) Evolution from \(m_t\), where \(m_t = 500 \text{ GeV, } 1 \text{ TeV, } 3 \text{ TeV, } 5 \text{ TeV, and } 7 \text{ TeV to } m_{GUT} \text{ GeV, using (II) and (13) with the same values of } \beta_1, \beta_2, \beta_3, \text{ and } \alpha_{ij}, \text{ and } c_i, c_i', \text{ and } c_i'' \text{ used in Section 3 (} m_s \leq \mu \leq m_t\).

Here, we obtained a similar result to that of Section 3. At the central value of \(g_3^{DR}\) there is an approximate gauge couplings unification but a sharp Yukawa couplings unification (Table 6). However, if we vary \(g_3^{DR}\) within the experimental
The recent data and the two-loop renormalization group equations [3, 4]. From our study we have found that (in Section 3, where $m_1 = m_t$) with the central value of $g_3^{DR}$ there is an approximate gauge couplings unification and a sharp Yukawa couplings unification as given in Table 4. However, if we vary $g_3^{DR}$ within the experimental bounds ($1.2084^{+0.0344}_{-0.0355}$), it is possible to obtain a sharp unification scale for both the gauge couplings and Yukawa couplings at their respective $m_t$ and $\tan\beta$ values as shown in Figures 1 and 2 and in Table 5 (gauge unification at $2.9518 \times 10^{10}$ GeV and Yukawa unification at $3.8828 \times 10^{11}$ GeV). A similar result is found in Section 4 where there are approximate (Table 6) and sharp unification scales for gauge couplings and Yukawa couplings at central value of $g_3^{DR}$ (Table 6). But with the variation of $g_3^{DR}$ within the experimental range $1.2084^{+0.0344}_{-0.0355}$ we obtained a single unification scale for the gauge couplings at $5.4175 \times 10^{10}$ GeV and for Yukawa couplings at $5.0175 \times 10^{9}$ GeV (Figures 3 and 4 and Table 7). Here we have shown only the graph for $m_t = 7$ TeV case as all the other graphs for different $m_t$ have the similar pattern with the only difference in their unification scale. When we note down the unification points for both the gauge couplings and the Yukawa couplings for different values of $m_t$, a pattern emerged as shown in Figures 5 and 6. For gauge couplings, the unification point increases with the increase in the SUSY breaking scale $m_t$. But for Yukawa couplings the unification points vary in the reverse order compared to the gauge couplings; that is, unification points decrease with the increase in $m_t$. Finally the present analysis addresses an important question on the how do the gauge and Yukawa couplings unification scales vary with the varying SUSY breaking scale.

The present analysis is based on an extremely simplified assumption of a single scale for all SUSY particles. There are strong hints that this is not the case and the SUSY spectrum is more spread than being at a single scale [18]. Such simplified assumption makes the present analysis possible at the cost of exact numerical accuracy. We also neglect the threshold corrections [19, 20] from various factors like (i) threshold correction from the two-loop contribution in the running of coupling constants (ii) light threshold correction from all superpartners in the SUSY sector, and (iii) threshold correction from particles of mass of the unification scale.

5. Results and Discussion

To summarize, we have studied the unification scenario in supersymmetric SU(5) grand unified theory [18–20] using the recent data and the two-loop renormalization group equations [3, 4]. From our study we have found that (in Section 3, where $m_1 = m_t$) with the central value of $g_3^{DR}$ there is an approximate gauge couplings unification and a sharp Yukawa couplings unification as given in Table 4. However, if we vary $g_3^{DR}$ within the experimental bounds ($1.2084^{+0.0344}_{-0.0355}$), it is possible to obtain a sharp unification scale for both the gauge couplings and Yukawa couplings at their respective $m_t$ and $\tan\beta$ values as shown in Figures 1 and 2 and in Table 5 (gauge unification at $2.9518 \times 10^{10}$ GeV and Yukawa unification at $3.8828 \times 10^{11}$ GeV). A similar result is found in Section 4 where there are approximate (Table 6) and sharp unification scales for gauge couplings and Yukawa couplings at central value of $g_3^{DR}$ (Table 6). But with the variation of $g_3^{DR}$ within the experimental range $1.2084^{+0.0344}_{-0.0355}$ we obtained a single unification scale for the gauge couplings at $5.4175 \times 10^{10}$ GeV and for Yukawa couplings at $5.0175 \times 10^{9}$ GeV (Figures 3 and 4 and Table 7). Here we have shown only the graph for $m_t = 7$ TeV case as all the other graphs for different $m_t$ have the similar pattern with the only difference in their unification scale. When we note down the unification points for both the gauge couplings and the Yukawa couplings for different values of $m_t$, a pattern emerged as shown in Figures 5 and 6. For gauge couplings, the unification point increases with the increase in the SUSY breaking scale $m_t$. But for Yukawa couplings the unification points vary in the reverse order compared to the gauge couplings; that is, unification points decrease with the increase in $m_t$. Finally the present analysis addresses an important question on the how do the gauge and Yukawa couplings unification scales vary with the varying SUSY breaking scale.

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The first assumption is valid so long as \( m_\tau \gg m_t \) or \( m_\tau \) \([38]\). These two assumptions when properly taken into account will affect the result by a few percentages. The above issues are crucial to give a realistic numerical estimation of the unification point which directly controls the enhancement of the proton decay rate, and it will be addressed in a separate communication.

**Conflict of Interests**

The authors declare that there is no conflict of interests regarding the publication of this paper.

**References**


