Review Article

Lepton Flavor Violation beyond the MSSM

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Most extensions of the Standard Model lepton sector predict large lepton flavor violating rates. Given the promising experimental perspectives for lepton flavor violation in the next few years, this generic expectation might offer a powerful indirect probe to look for new physics. In this review we will cover several aspects of lepton flavor violation in supersymmetric models beyond the Minimal Supersymmetric Standard Model. In particular, we will concentrate on three different scenarios: high-scale and low-scale seesaw models as well as models with $R$-parity violation. We will see that in some cases the LFV phenomenology can have characteristic features for specific scenarios, implying that dedicated studies must be performed in order to correctly understand the phenomenology in nonminimal supersymmetric models.

1. Introduction

The Standard Model (SM) particle content has been recently completed with the discovery of the long-awaited Higgs boson at the CERN Large Hadron Collider (LHC) [1, 2]. This constitutes a well deserved reward after decades of intense search, with great efforts from the theory and experimental communities. Furthermore, it also confirms that the SM must be, at least to a good approximation, a precise description of nature up to the energies explored. In fact, and apart from some phenomenological facts that indeed require some unknown new physics (NP), like the existence of dark matter and neutrino masses, the SM explains to a high level of accuracy all the observations made in a wide variety of experiments.

For the last decades, the progress in theoretical particle physics has been driven by naturalness considerations in the form of the famous hierarchy problem. This has led to many extensions of the SM, all of them attempting to explain why the weak scale has not been pushed to much higher energy scales by some hypothetical NP degrees of freedom. Among the many proposals to address this issue, supersymmetry (SUSY) is certainly the most popular one. However, and similarly to other analogous solutions to the hierarchy problem, the predicted new particles at the weak scale have not been observed at the LHC.

This has of course raised some doubts about the existence of supersymmetry close to the weak scale. Since this proximity is to be expected in case supersymmetry has something to do with the hierarchy problem, the whole idea of weak scale supersymmetry is under some pressure at the moment. However, it is worth keeping in mind that most experimental searches for SUSY focus on the Minimal Supersymmetric Standard Model (MSSM). This model, which constitutes the minimal extension of the SM that incorporates SUSY, has some underlying assumptions that lead to very specific signatures. For example, in the MSSM one assumes the conservation of a discrete symmetry, known as $R$-parity [3, 4], which forbids all renormalizable lepton and baryon number violating operators and leads to the existence of a stable particle, the lightest supersymmetric particle (LSP), which in turn leads to large amounts of missing energy in supersymmetric events at the LHC. There are, however, many known (and well motivated) supersymmetric scenarios with $R$-parity violation and, in fact, several authors have shown that simply by allowing for nonzero $B$-violating terms in the superpotential, the current LHC bounds can be clearly relaxed, allowing for the existence of light squarks and gluinos.
hidden in the huge QCD background [5, 6]. Similarly, one can extend the MSSM in many other directions, often changing the phenomenology at colliders dramatically. This suggests that it might be too soon to give up on SUSY, a framework with many possibilities yet to be fully explored.

As explained above, there are some well-grounded phenomenological issues that cannot be explained within the SM. One of these open problems is the existence of nonzero neutrino masses and mixings, nowadays firmly established by neutrino oscillation experiments [7–9]. In fact, this issue is not addressed in the MSSM either, since neutrinos remain massless in the same way as in the SM. This calls for an extension of the MSSM that extends the lepton sector and accommodates the observations in neutrino oscillation experiments. This can be done in two different ways: (1) high-energy extensions, in which the new degrees of freedom responsible for the generation of neutrino masses live at very high energy scales, and (2) low-energy extensions, with new particles and/or interactions at the SUSY scale.

One of the most generic predictions in neutrino mass models is lepton flavor violation (LFV). In fact, neutrino oscillations are the proof that lepton flavor is not a conserved symmetry of nature, since neutrinos produced with a given flavor change it as they propagate. Therefore, all neutrino mass models built to give an explanation to oscillation experiments violate lepton flavor. However, we have never observed LFV processes involving charged leptons although, in principle, there is no symmetry (besides lepton flavor, which we know to be broken) that forbids processes like $\mu^- \rightarrow e^- \gamma$, $e^- \rightarrow e^- \mu^+ \mu^-$, or $K_L \rightarrow e^- \mu^-$. This fact can be well understood in some minimal frameworks, such as the minimal extension of the SM with Dirac neutrinos. In this case, LFV in the charged lepton sector is strongly suppressed, since neutrino masses are the only source of LFV, leading to unobservable LFV rates, like $BR(\mu^- \rightarrow e^- \gamma) \sim 10^{-35}$ [10]. However, as soon as one extends the SM, this conclusion can be clearly altered [11, 12]. In fact, new sources of LFV can be found in most extensions of the leptonic sector, caused either by new interactions, by new particles, or even by complete new sectors that couple to the SM leptons.

After this discussion on LFV and neutrino masses a clarification is in order. Although neutrino oscillations imply LFV, LFV does not necessarily imply neutrino oscillations. There are models that predict charged lepton LFV without generating a mass for the neutrinos. The simplest example of this class of models is the general Two-Higgs-Doublet of type-III, where neutrinos remain massless but lepton flavor is violated due to the existence of off-diagonal $h \rightarrow \ell_i - \ell_j$ vertices. Another relevant example is the MSSM itself, where neutrinos are also massless, but the slepton soft masses can induce LFV processes if they contain off-diagonal entries. One can actually estimate the branching ratio for the radiative LFV decay $\ell_i \rightarrow \ell_j \gamma$ as [13]

$$BR\left(\ell_i \rightarrow \ell_j \gamma\right) = \frac{48\alpha^2}{G_F^2} \frac{\left|m^2_{ij}\right|^2}{M_{SUSY}^2} BR\left(\ell_i \rightarrow \ell_j \nu_i \nu_j\right),$$

where $G_F$ is the Fermi constant, $\alpha$ is the fine structure constant, $\left|m^2_{ij}\right|$ are the dominant off-diagonal elements of the soft SUSY breaking slepton mass matrices, and $M_{SUSY}$ is the typical mass of the SUSY particles, expected to be in the TeV ballpark. This estimate clearly shows that rather small off-diagonal elements are required to satisfy the experimental bounds [14].

In general, large LFV rates are expected in most models beyond the SM. This observation leads to the so-called flavor puzzle: the nonobservation of LFV is a rather surprising fact, since generic new physics would predict LFV rates clearly above the current experimental limits. This suggests that flavor structures in new physics models cannot be generic, but some organizing principle, such as a flavor symmetry, might be at work. Furthermore, it also motivates the study of LFV as an indirect probe of new physics and, in particular, of supersymmetric models beyond the MSSM. This is the subject of this review. The field of lepton flavor violation beyond the MSSM has been intensely explored for many years and contains a vast literature. In this review I present my personal view of the subject and thus I must apologize for those papers which are not cited. In particular, we will concentrate on three different scenarios: high-scale and low-scale seesaw models as well as models with $R$-parity violation. As we will see, the LFV phenomenology turns out to be very different depending on the exact scenario, implying that lepton flavor violation may be richer than in the MSSM. In some cases the common lore (established in the MSSM) turns out to be wrong, and specific studies must be performed in order to correctly understand the corresponding LFV phenomenology.

Before concluding the introduction, let us clarify the title of this review. As explained above, the MSSM can be made lepton flavor violating by introducing nonzero off-diagonal terms in the soft SUSY breaking terms for the sleptons. These LFV sources will be present in any supersymmetric model that includes the MSSM. In contrast, in this review we will consider a scenario to be beyond the MSSM if it contains additional LFV sources besides those in the MSSM. With this definition, the three specific supersymmetric scenarios discussed in this review fall within this category.

This review is organized as follows: in Section 2 we give an overview of the current experimental situation and briefly discuss some projects that will take place in the near future. Then we review the LFV phenomenology of three different types of models beyond the MSSM: high-scale seesaw models (in Section 3), low-scale seesaw models (in Section 4), and models with $R$-parity violation (in Section 5). Finally, we conclude in Section 6.

2. Current Experimental Situation and Future Projects

The search for LFV is soon going to live a golden age given the upcoming experiments devoted to high-intensity physics (see [15–17] for recent reviews). In addition to the LFV searches already taking place in several experiments, new projects will join the effort in the next few years.
In what concerns the radiative decay $\ell_i \to \ell_j \gamma$, the experiment leading to the most stringent constraints is MEG. This experiment, located at the Paul Scherrer Institute in Switzerland, searches for the radiative process $\mu \to e\gamma$. Recently, the MEG collaboration announced a new limit on the rate for this process based on the analysis of a dataset with $3.6 \times 10^{14}$ stopped muons. The nonobservation of the LFV process led to the limit $\text{BR}(\mu \to e\gamma) < 5.7 \cdot 10^{-13}$ [18], four times more stringent than the previous limit obtained by the same collaboration. Moreover, the MEG collaboration has announced plans for future upgrades. These will allow reaching a sensitivity of about $6 \cdot 10^{-14}$ after 3 years of acquisition time [19]. This is of great importance, as this observable along with the experimental sensitivity currently provides the most stringent limit on LFV parameters in many models.

The most promising improvements in the near future are expected in $\mu \to 3e$ and $\mu \to e$ conversion in nuclei. Regarding the former, the decay $\mu \to 3e$ was searched for long ago by the SINDRUM experiment [20], setting the strong limit $\text{BR}(\mu \to 3e) < 1.0 \cdot 10^{-12}$. The future Mu3e experiment announces a sensitivity of $\sim 10^{-16}$ [21], which would imply an impressive improvement by 4 orders of magnitude. As for the latter, several experiments will compete in the next few years, with sensitivities for the conversion rate ranging from $10^{-14}$ to an impressive $10^{-18}$. These include Mu2e [22–24], DeeMe [25], COMET [26, 27], and the future PRISM/PRIME [28]. In all cases, these experiments will definitely improve on previous experimental limits.

The limits for $\tau$ observables are less stringent, although significant improvements are expected from $B$ factories like Belle II [29, 30]. Finally, although the most common way to search for LFV is in low-energy experiments, colliders can also play a very relevant role looking for LFV processes at high energies. The LHCb collaboration reported recently the first bounds on $\tau \to 3\mu$ ever obtained in a hadron collider [31]. Furthermore, the CMS collaboration recently found an intriguing $2.4 \sigma$ excess in the $h \to \tau\tau$ channel which translates into $\text{BR}(h \to \tau\tau) = (0.84^{+0.39}_{-0.37})\%$ [32]. For reference, in Table 1 we collect present bounds and expected near-future sensitivities for the most popular low-energy LFV observables.

The theoretical understanding of all these processes will be crucial in case a discovery is made. With such a large variety of processes, the determination of hierarchies or correlations in specific models will allow us to extract fundamental information on the underlying physics behind LFV. This goal requires detailed analytical and numerical studies of the different contributions to the LFV processes, in order to get a global picture of the LFV anatomy of the relevant models and be able to discriminate among them by means of combinations of observables with definite predictions [33].

3. High-Scale Seesaw Models

Neutrino mixing is, by itself, a flavor violating effect. Therefore, all neutrino mass models that aim at explaining the observed pattern of neutrino masses and mixings incorporate lepton flavor violation. However, specific predictions can be very different in different models.

Among the huge number of scenarios proposed for neutrino mass generation, the seesaw mechanism is arguably the most popular one. In its conventional form, the seesaw mechanism explains the smallness of neutrino mass by means of a very large energy scale, the seesaw scale $M_{\text{SS}}$, which suppresses neutrino masses as

$$m_\nu \sim \frac{v^2}{M_{\text{SS}}}.$$

Here $(H^0) = v/\sqrt{2} = 174 \text{ GeV}$ is the standard Higgs boson vacuum expectation value (VEV) that determines the weak scale. In order to obtain neutrino masses of about $\sim 0.1 \text{ eV}$, one requires $M_{\text{SS}} \sim 10^{14} \text{ GeV}$. For this reason, this setup is usually called high-scale seesaw. The proximity of the high-energy scale $M_{\text{SS}}$ to the grand unification (GUT) scale (as predicted in the MSSM) $m_{\text{GUT}} = 2 \cdot 10^{16} \text{ GeV}$ suggests an intriguing connection with unification physics, making the seesaw a very well-motivated scenario.

Regarding specific realizations of the seesaw mechanism, it is well-known that, with renormalizable interactions only, three tree-level realizations exist [48]. These are usually called type-I [49–54], type-II [53–59], and type-III [60]. They differ from each other by the nature of the seesaw messengers: in the type-I seesaw these are singlet right-handed neutrinos, in the type-II seesaw scalar $SU(2)_L$ triplets with hypercharge two, and in the type-III seesaw fermionic $SU(2)_L$ triplets with vanishing hypercharge. In all cases they lead to a neutrino mass of the form of (2), where $M_{\text{SS}}$ is proportional to the mass of the heavy mediators, and the induced neutrino masses are of Majorana type, thus breaking lepton number in two units.

Given the large Majorana masses of the seesaw mediators, one may wonder about how to probe the high-scale seesaw. In supersymmetric scenarios this is possible thanks to the sleptons. Even if their soft terms are flavor conserving at some high-energy scale, the renormalization group running down to the SUSY scale will induce nonzero off-diagonal terms due to their interactions with the seesaw mediators [61].

<table>
<thead>
<tr>
<th>LFV process</th>
<th>Present bound</th>
<th>Future sensitivity</th>
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<tbody>
<tr>
<td>$\mu \to e\gamma$</td>
<td>$5.7 \times 10^{-13}$ [18]</td>
<td>$6 \times 10^{-14}$ [19]</td>
</tr>
<tr>
<td>$\tau \to e\gamma$</td>
<td>$3.5 \times 10^{-5}$ [34]</td>
<td>$\sim 3 \times 10^{-7}$ [29]</td>
</tr>
<tr>
<td>$\mu \to e\tau\tau$</td>
<td>$4.4 \times 10^{-8}$ [34]</td>
<td>$\sim 3 \times 10^{-7}$ [29]</td>
</tr>
<tr>
<td>$\mu \to eee$</td>
<td>$1.0 \times 10^{-12}$ [20]</td>
<td>$\sim 10^{-16}$ [21]</td>
</tr>
<tr>
<td>$\tau \to \mu\mu\mu$</td>
<td>$2.1 \times 10^{-8}$ [35]</td>
<td>$\sim 10^{-9}$ [29]</td>
</tr>
<tr>
<td>$\tau \to e'e'e'$</td>
<td>$1.8 \times 10^{-8}$ [35]</td>
<td>$\sim 10^{-9}$ [29]</td>
</tr>
<tr>
<td>$\tau \to eee$</td>
<td>$2.7 \times 10^{-8}$ [35]</td>
<td>$\sim 10^{-9}$ [29]</td>
</tr>
<tr>
<td>$\mu', Ti \to e', Ti$</td>
<td>$4.3 \times 10^{-12}$ [36]</td>
<td>$\sim 10^{-18}$ [37]</td>
</tr>
<tr>
<td>$\mu', Au \to e', Au$</td>
<td>$7 \times 10^{-13}$ [38]</td>
<td>$10^{-15}$–$10^{-18}$</td>
</tr>
<tr>
<td>$\mu', Al \to e', Al$</td>
<td>$4.3 \times 10^{-12}$ [36]</td>
<td>$\sim 10^{-18}$ [37]</td>
</tr>
<tr>
<td>$\mu', SiC \to e', SiC$</td>
<td>$10^{-12}$ [39]</td>
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Table 1: Current experimental bounds and future sensitivities for the most important LFV observables.
These can be probed since the misalignment of the slepton mass matrices with respect to that of the SM charged leptons induces LFV processes such as $\ell_i \rightarrow \ell_j \gamma$, $\ell_i \rightarrow 3\ell_j$ and $\mu - e$ conversion in nuclei. This connection between the phenomenology at low-energies and the high-scale mediators is only possible in supersymmetric models and constitutes an excellent opportunity to test the standard seesaw scenario. In the non-SUSY version of the seesaw mechanism this link between high and low energy scales is lost. In this case probing the origin of neutrino masses becomes a quite challenging task, and only very indirect probes such as neutrinoless double beta decay are possible [62]. However, it is worth pointing out that such clean connection is only possible in the absence of additional sources of LFV. This requires a strong theoretical assumption: universal and flavor conserving boundary conditions for the soft terms at the GUT scale.

In case of high-scale seesaw models, the low-energy theory is simply the MSSM. This allows one to establish definite patterns and hierarchies among the LFV observables. For instance, the branching ratios for the $\ell_i \rightarrow \ell_j \gamma$ and $\ell_i \rightarrow 3\ell_j$ LFV decays follow the approximate relation [63–65],

$$\text{BR}(\ell_i \rightarrow 3\ell_j) = \frac{\alpha}{3\pi} \left( \log \left( \frac{m_j^2}{m_i^2} \right) - \frac{11}{4} \right) \text{BR}(\ell_i \rightarrow \ell_j \gamma).$$

Therefore, in supersymmetric high-scale seesaw models, the most constraining LFV process is $\ell_i \rightarrow \ell_j \gamma$. The relation in (3) is caused by the so-called dipole dominance in high-scale seesaw models. Among the different contributions to the 3-body decay $\ell_i \rightarrow 3\ell_j$, the dipole photon penguins come multiplied by a large logarithmic term, caused by the infrared divergence that would appear in the $m_j \rightarrow 0$ limit, thus becoming the dominant ones and leading to the proportionality between the $\ell_i \rightarrow \ell_j \gamma$ and $\ell_i \rightarrow 3\ell_j$ branching ratios. An exception to this general rule is found for low pseudoscalar masses and large $\tan\beta$ [66]. In this case, Higgs penguins turn out to be dominant in processes involving the second and third generations, like $\tau \rightarrow 3\mu$. However, this region of parameter space is nowadays under some tension due to strong flavor constraints derived from the observation of quark flavor violating processes like $B_s \rightarrow \mu^+\mu^-$ [67].

3.1 Standard High-Scale Seesaw Scenarios. Implementing a high-scale seesaw mechanism in supersymmetric scenarios involves an additional complication. This is related to one of the most appealing features of the MSSM: gauge coupling unification. In case of the type-I seesaw, the introduction of the seesaw mediator does not spoil this attractive feature, since the right-handed neutrino superfields are gauge singlets and do not affect the running of the gauge couplings. In contrast, in the type-II and type-III seesaws, new contributions to the running of the SU(2)$_L$ and U(1)$_Y$ gauge couplings are induced by the seesaw mediators. However, a well-known solution to this problem exists. Unification can be easily restored by embedding the seesaw mediators in full SU(5) multiplets, like 15-plets in the case of type-II [68] or 24-plets [69] in the case of type-III. The contributions from the other members of the multiplet guarantee that these three gauge couplings will eventually meet at a high energy scale, $m_{\text{GUT}}$, although the common value of the coupling changes, $g_{\text{GUT}}$, might be different from that of the MSSM. In addition, note that the 24-plet of SU(5) contains, besides the SU(2)$_L$ triplet, a singlet state which also contributes to neutrino masses. Hence, in this case one actually has a mixture between type-I and type-III seesaws.

The new superfield content, explicitly denoting gauge conserving boundary conditions for the soft terms at the GUT scale.

(i) Type-I. Three generations of right-handed neutrino superfields, singlets of SU(5), are introduced, $\tilde{N}^c \sim (1, 1, 0)$:

$$W_I = W_{\text{MSSM}} + Y_\nu \tilde{N}^c \tilde{L} \tilde{H}_u + \frac{1}{2} M_R \tilde{N}^c \tilde{N}^c. \quad (4)$$

(ii) Type-II. In this case one needs to introduce a vector-like pair of 15 and 15 of SU(5), decomposed as $\tilde{S} \sim (6, 1, -2/3)$, $\tilde{T} \sim (1, 3, 1)$, and $\tilde{Z} \sim (3, 2, 1/6)$ (as well as the corresponding bar superfields). $\tilde{T}$ and $\tilde{T}$ are the SU(2)$_L$ triplets responsible for neutrino mass generation. Note that in this case only one generation of 15 and 15 is required, since the type-II seesaw can generate three nonzero masses for the light neutrinos with only one SU(2)$_L$ scalar triplet:

$$W_{II} = W_{\text{MSSM}} + \frac{1}{\sqrt{2}} \left( Y_{\tau} \tilde{T} \tilde{H}_d + Y_{\tau} \tilde{S} \tilde{F} \right) + Y_{\tau} \tilde{S} \tilde{T} \tilde{L} + \frac{1}{\sqrt{2}} (\lambda_1 \tilde{T} \tilde{H}_d \tilde{T} \tilde{H}_u + \lambda_2 \tilde{H}_d \tilde{T} \tilde{H}_u) + M_T \tilde{T} \tilde{T} \quad (5)$$

(iii) Type-III. Three generations of 24 of SU(5) are added. They can be decomposed as $\tilde{N}^c \sim (1, 1, 0)$, $\tilde{G} \sim (8, 1, 0)$, $\tilde{S} \sim (1, 3, 0)$, $\tilde{X} \sim (3, 2, -5/6)$, and $\tilde{X} \sim (3, 2, 5/6)$. As explained above, neutrino masses are generated as a combination of a type-I seesaw (mediated by $N^c$) and a type-III seesaw (mediated by the SU(2)$_L$ triplet $\Sigma$):

$$W_{III} = W_{\text{MSSM}} + \tilde{H}_u \left( Y_{\tau} \tilde{S} \tilde{X} \tilde{X} + \frac{3}{10} Y_{\nu} \tilde{N}^c \tilde{L} \right) + Y_{\tau} \tilde{H}_d \tilde{S} \tilde{S} \tilde{F} + \frac{1}{2} M_R \tilde{N}^c \tilde{N}^c + \frac{1}{2} M_G \tilde{G} \tilde{G} + \frac{1}{2} M_Z \tilde{Z} \tilde{Z} + M_S \tilde{S} \tilde{S}. \quad (6)$$

The following notation is used in (4), (5), and (6): $W_{\text{MSSM}}$ is the MSSM superpotential, $\tilde{H}_d$, $\tilde{H}_u$, and $\tilde{L}$ are the down-Higgs, up-Higgs, and lepton SU(2)$_L$ doublet superfields,
Advances in High Energy Physics...the off-diagonal terms are generated at low energies by renormalization group running, thus inducing all kinds of LFV processes. In the case studied in [70] for the three seesaw variants. In the following interplay between the Higgs mass constraint and LFV was type-I seesaw. Figure 1 shows the branching ratios for the III seesaw has been studied in [88–90]. More recently, the tribimaximal mixing. Furthermore, a massless lightest neutrino was also assumed. Figure taken from [40].

Let us first comment on some results for the SUSY type-I seesaw. Figure 1 shows the branching ratios for the \( \ell_i \rightarrow \ell_j \gamma \) and \( \ell_i \rightarrow 3 \ell_j \) as a function of the seesaw scale in the SUSY type-I seesaw. This figure was obtained in the standard SPS1a point, assuming degenerate right-handed neutrinos and fixing the neutrino Yukawas to reproduce tribimaximal mixing. Furthermore, a massless lightest neutrino was also assumed. Figure taken from [40].

respectively, and \( \tilde{F} \) is the right-handed down-type quark superfield.

The LFV phenomenology of SUSY seesaw models has been studied by many authors. For the type-I seesaw, low-energy LFV decays such as \( \ell_i \rightarrow \ell_j \gamma \) and \( \ell_i \rightarrow 3 \ell_j \) have been calculated in [40, 64, 65, 71–80]. Similarly, \( \mu \rightarrow e \) conversion in nuclei has been studied in [81, 82]. The other two seesaw variants have received much less attention. The LFV phenomenology of the SUSY type-II seesaw has been considered in [41, 68, 83–87], whereas the SUSY type-III seesaw has been studied in [88–90]. More recently, the interplay between the Higgs mass constraint and LFV was studied in [70] for the three seesaw variants. In the following we comment on some selected results.

Let us first comment on some results for the SUSY type-I seesaw. Figure 1 shows the branching ratios for the \( \ell_i \rightarrow \ell_j \gamma \) and \( \ell_i \rightarrow 3 \ell_j \) as a function of the seesaw scale (the mass of the right-handed neutrino mass). This figure was obtained in [40], using the standard SPS1a point [42], assuming degenerate right-handed neutrinos and a massless lightest neutrino and fixing the neutrino Yukawas to reproduce tribimaximal mixing. Although this parameter choice is nowadays excluded for several reasons (the SUSY spectrum is too light to pass the constraints from LHC searches and tribimaximal mixing is now excluded after \( \theta_{13} \) has been measured), it serves to illustrate the dipole dominance discussed above. Indeed, one sees a perfect correlation between the branching ratios of \( \ell_i \rightarrow \ell_j \gamma \) and \( \ell_i \rightarrow 3 \ell_j \), with \( \text{BR}(\ell_i \rightarrow 3 \ell_j) \approx \text{BR}(\ell_i \rightarrow \ell_j \gamma) \). As already discussed, this is due to the fact that the photonic dipole operator \( \langle \tilde{F}_\mu \sigma^\mu \ell_j \rangle \) dominates both processes.

We now turn to the SUSY type-II seesaw. In the type-II seesaw, the neutrino mass matrix is proportional to the \( Y_T \) Yukawa matrix,

\[
 m_\nu = \frac{\lambda^2}{2 M_T^2} Y_T. \tag{7}
\]

This is derived from the superpotential term \( Y_T \tilde{L} \tilde{H} \tilde{L} \) in (5). This direct relation has important consequences for the phenomenology, since it forces the flavor structure of \( Y_T \) to be the same as that of \( m_\nu \), the latter being measured in neutrino oscillation experiments. In contrast, in the type-I and type-III seesaws the analogous relation is quadratic in the Yukawa coupling. This introduces extra freedom in the determination of the seesaw parameters (usually encoded in the so-called \( R \) matrix [91]) and makes it impossible to predict the Yukawa flavor structure only from neutrino oscillation data. In other words, if all the neutrino masses, angles, and phases were known, \( Y_T \) would be completely fixed (up to an overall constant). Since \( Y_T \) determines the LFV phenomenology, this implies correlations between the neutrino oscillation parameters and LFV observables.

A clear illustration of the previous point is shown in Figure 2, borrowed from [41]. By computing the ratios \( \text{BR}(\ell_i \rightarrow e \gamma)/\text{BR}(\ell_m \rightarrow e \gamma) \) one gets rid of the unknown overall factor in the \( Y_T \) Yukawas, thus obtaining direct predictions in terms of neutrino parameters. In this case, the figure shows the dependence of these ratios on the mixing angle \( \theta_{13} \) and the Dirac CP violating phase \( \delta \). We see that this scenario is extremely predictive. For example, finding experimentally \( \text{BR}(\tau \rightarrow e \gamma) > \text{BR}(\mu \rightarrow e \gamma) \) would immediately rule out the model, at least in its minimal form. One way to spoil these strict predictions is to introduce a second \( SU(2)_L \) triplet \( T' \). In this case \( m_\nu \) would receive contributions from \( T \) and \( T' \), \( m_\nu = m_\nu^T + m_\nu^{T'} \), and the proportionality in (7) would be lost.

Additional ways to test high-scale SUSY seesaws include slepton mass splittings [92] (directly related to LFV) and the study of the SUSY spectrum, usually deformed with respect to the standard spectra in constrained (CMSSM) scenarios. In particular, one can construct certain invariants that contain information about the high-energy scale; see, for example, [69, 93, 94]. See also [95] for related ideas.

3.2. Extended High-Scale Seesaw Scenarios. We now turn our attention to extended high-scale SUSY seesaw scenarios beyond the classical type-I, type-II, and type-III seesaws. However, before we concentrate on the extended models, let us make a general observation. As already discussed, flavor violating entries in the slepton soft terms \( m_\tilde{L}^2 \) and \( m_\tilde{E}^2 \) (the left and right slepton squared soft masses, resp.) are induced due to their interactions with the seesaw mediators. Even if they are flavor diagonal at the unification scale, off-diagonal terms are generated at low energies by renormalization group running, thus inducing all kinds of LFV processes. In the case
of the radiative $\ell_i \to \ell_j \gamma$, the effective dipole operator that contributes to the decay can be written as

$$\mathcal{L}_{\text{dipole}} = e \frac{m_{\ell_i}}{2} \eta_{\mu
u} p^\mu p^\nu \left( A_L^{ij} p_L + A_R^{ij} p_R \right) \epsilon_j + \text{h.c.},$$

(8)

where $p_{L,R} = (1/2)(1 \mp \gamma_5)$ are the usual chirality projectors and $e$ is the electric charge. The Wilson coefficients $A_L$ and $A_R$ are generated by loops with left and right sleptons, respectively. One finds

$$A_L^{ij} \sim \frac{m_{\ell_i}^2}{M_{\text{SUSY}}^4},$$

$$A_R^{ij} \sim \frac{m_{\ell_i}^2}{M_{\text{SUSY}}^4},$$

(9)

where it has been assumed that $A$-terms mixing left-right transitions are negligible. $\text{BR}(\ell_i \to \ell_j \gamma)$ can be computed in terms of $A_L$ and $A_R$ as

$$\text{BR}(\ell_i \to \ell_j \gamma) = \frac{48\pi^2}{G_F} \left( \left| A_L^{ij} \right|^2 + \left| A_R^{ij} \right|^2 \right) \text{BR}(\ell_i \to \ell_j \gamma).$$

(10)

The straightforward combination of (9) and (10) leads to (1).

In the minimal SUSY seesaw models discussed above, the seesaw mediators only couple to the left sleptons. For instance, in the type-I case this interaction is given by the superpotential coupling $Y_L \tilde{L}_i \tilde{H}_D \tilde{N}_i$, whereas in the type-II case it is given by the $Y_L \tilde{L} \tilde{T}_L \tilde{N}_i$ term. For this reason, negligible off-diagonal entries in $m_\nu^2$ are induced, implying that minimal SUSY seesaw models predict $A_R \approx 0$. As we will see below, this has an impact on some low-energy observables that allow, in principle, testing the minimality of the high-scale seesaw mechanism.

### 3.2.1. Supersymmetric Models with Nonminimal Seesaw Mechanisms

As an example supersymmetric model with non-minimal seesaw mechanisms, we consider the left-right symmetric model of [96, 97] (in the following simply called “the LR model”). The LFV and dark matter phenomenology of this model has been studied in detail in [43, 98].

The model is defined below the GUT scale (the model implicitly assumes the existence of a GUT model at higher energies; at $m_{\text{GUT}}$, the gauge couplings and soft terms unify), where the gauge group is $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. In addition, we assume that parity is conserved. The matter content of the model is given in Table 2. Here $\tilde{Q}$, $\tilde{Q}^c$, $\tilde{L}$, and $\tilde{L}^c$ are the quark and lepton superfields of the MSSM with the addition of (three) right-handed neutrino superfields to complete the $\tilde{L}^c$-SU(2)$_L$ doublets.

Two $\tilde{\Phi}$ superfields, bidoublets under SU(2)$_L \times SU(2)_R$, are introduced. Among their components, they contain the standard $\tilde{H}_d$ and $\tilde{H}_u$ MSSM Higgs doublets. Finally, the rest of the superfields in Table 2 are introduced to break the LR symmetry.

With the representations in Table 2, the most general superpotential compatible with the gauge symmetry and parity is

$$W_{\text{LR}} = Y_Q \tilde{Q} \tilde{\Phi} \tilde{Q}^c + Y_L \tilde{L} \tilde{\Phi} \tilde{L}^c - \frac{\mu}{2} \tilde{\Phi} \tilde{\Phi} + f \bar{\tilde{L}} \tilde{\Delta} \tilde{\Delta}^c + a \tilde{\Delta} \tilde{\Delta} \tilde{\Delta}^c \tilde{\Delta} \tilde{\Delta} + a \tilde{\Delta} \tilde{\Phi} \tilde{\Phi}$$

$$+ \alpha^* \tilde{\Phi} \tilde{\Phi} + M_\Delta \tilde{\Delta} \tilde{\Delta}^c \tilde{\Delta} \tilde{\Delta} + M_\Omega \tilde{\Omega} \tilde{\Omega}^c + M_{\Omega} \tilde{\Omega} \tilde{\Omega}$$

(II)

Family and gauge indices have been omitted in (II); more detailed expressions can be found in [96]. Note that this
Table 2: LR model. Matter content between the GUT scale and the SU(2) breaking scale. The electric charge operator is defined as \( Q = I_{3L} + I_{3R} + (B - L)/2 \).

<table>
<thead>
<tr>
<th>Superfield</th>
<th>Generations</th>
<th>SU(3)</th>
<th>SU(2)_L</th>
<th>SU(2)_R</th>
<th>U(1)_{B-L}</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tilde{Q} )</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>1/3</td>
</tr>
<tr>
<td>( \tilde{Q}^c )</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1/3</td>
</tr>
<tr>
<td>( \tilde{L} )</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>( \tilde{L}^c )</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>( \tilde{\Phi} )</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>( \tilde{\Lambda} )</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>( \tilde{\Delta} )</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>-2</td>
</tr>
<tr>
<td>( \tilde{\Delta}^c )</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>-2</td>
</tr>
<tr>
<td>( \tilde{\Omega} )</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>( \tilde{\Omega}^c )</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>0</td>
</tr>
</tbody>
</table>

Superpotential is invariant under the parity transformations \( \tilde{Q} \leftrightarrow (\tilde{Q}^c)^* \), \( \tilde{L} \leftrightarrow (\tilde{L}^c)^* \), \( \tilde{\Phi} \leftrightarrow \tilde{\Phi}^c \), \( \tilde{\Delta} \leftrightarrow (\tilde{\Delta}^c)^* \), and \( \tilde{\Omega} \leftrightarrow (\tilde{\Omega}^c)^* \). This discrete symmetry reduces the number of free parameters of the model.

The breaking of the left-right gauge group to the MSSM gauge group takes place in two steps: \( SU(2)_R \times U(1)_{B-L} \rightarrow U(1)_R \times U(1)_{B-L} \rightarrow U(1)_Y \). In the first step, the neutral component of the triplet \( \Omega \) takes a VEV,

\[
\langle \Omega^0 \rangle = \frac{v_R}{\sqrt{2}}
\]

which breaks \( SU(2)_R \). However, since \( I_{3R}(\Omega^0) = 0 \) there is a \( U(1)_R \) symmetry left over. Next, the group \( U(1)_R \times U(1)_{B-L} \) is broken by

\[
\langle \Delta^0 \rangle = \frac{v_{RL}}{\sqrt{2}},
\]

\[
\langle \Delta^c_0 \rangle = \frac{v_{RL}}{\sqrt{2}}
\]

The remaining symmetry is now \( U(1)_Y \) with hypercharge defined as \( Y = I_{3R} + (B - L)/2 \).

Regarding neutral masses, assuming that the left triplets \( \Delta \) and \( \Delta^c \) have vanishing VEVs, one induces neutrinos masses from a type-I seesaw only thanks to the presence of the right-handed neutrinos \[96\].

Before discussing how to test this scenario with lepton flavor violation, let us mention some other nonminimal SUSY seesaw models. The phenomenological study in \[99\] is based on a model very similar to the discussed here, without \( \tilde{\Omega} \) superfields. See also \[100\] for a comprehensive study of supersymmetric models with extended gauge groups at intermediate steps. Finally, the seesaw mechanism can also be embedded in SUSY GUTs, usually leading to very predictive scenarios \[101–106\].

3.2.2. Probing Nonminimal Seesaw Mechanisms. As already discussed, a pure seesaw model predicts \( A_R = 0 \) simply because the right sleptons do not couple to the seesaw mediators. However, in models with nonminimal seesaw mechanisms, new interactions between the right sleptons and the members of the extended particle content at high energies might exist. When this is the case, nonzero \( A_R \) coefficients can be induced.

Let us consider an example. In the LR model, the left-right symmetry implies that, above the parity breaking scale, the flavor violating entries generated in \( m_\tau^2 \) are exactly as large as the ones in \( m_\tau^2 \). As a consequence of this, \( A_R \neq 0 \) is obtained at low energies. In fact, one can even get a handle on the symmetry breaking pattern at high energies. Below the \( SU(2)_R \) breaking scale, parity is broken and left and right slepton soft masses evolve differently. The left ones keep running from the \( SU(2)_R \) breaking scale to the \( U(1)_{B-L} \) scale due to the left slepton couplings with the right-handed neutrinos. One thus expects larger flavor violating effects in the left slepton sector, and the difference between left and right must correlate with the ratio \( v_{RL}/v_R \), which measures the hierarchy between the two breaking scales.

The question is how to measure this difference. For this purpose one can use the positron polarization asymmetry, defined as

\[
\mathcal{A}(\mu^+ \rightarrow e^+\gamma) = \frac{|A_L|^2 - |A_R|^2}{|A_L|^2 + |A_R|^2}.
\] (14)

If MEG observes \( \mu^+ \rightarrow e^+\gamma \) events, the angular distribution of the outgoing positrons can be used to discriminate between left- and right-handed polarized states and measure \( \mathcal{A} [107, 108] \). And this can in turn be used to get information on \( A_L \) and \( A_R \).

In a pure SUSY seesaw model one expects \( \mathcal{A} = +1 \) to a very good accuracy. However, in models with nonminimal seesaw mechanisms \( \mathcal{A} \) can significantly depart from +1. For example, the LR model typically leads to significant departures from this expectation, giving an interesting signature of the high-energy restoration of parity. This is shown in Figure 3, extracted from \[43\]. First of all, it is clear that the polarization asymmetry \( \mathcal{A}(\mu^+ \rightarrow e^+\gamma) \) is well correlated with the quantity \( \log(v_R/m_{GUT})/\log(v_{RL}/m_{GUT}) \). One finds that as \( v_{RL} \) and \( v_R \) become very different, \( \mathcal{A} \) approaches +1. In contrast, when the two breaking scales are close, \( v_{RL}/v_R \sim 1 \), this effect disappears and the positron polarization asymmetry approaches \( \mathcal{A} = 0 \). Note that a negative value for \( \mathcal{A} \) is not possible in this model, since the LFV terms in the right slepton sector never run more than the corresponding terms in the left one.

There are alternative ways to test nonminimal high-scale SUSY seesaw scenarios. These include the study of the SUSY spectrum and, in particular, of the invariants pointed out for minimal seesaw models. In this case, they contain information about the high-energy intermediate scales \[100, 109\].

4. Low-Scale Seesaw Models

The high-scale seesaw has an important drawback: the heaviness of the seesaw mediators precludes any chance of direct
tests. Only indirect tests, based on low-energy processes which may have an imprint of the high seesaw scale $M_{SS}$, are possible, as explained in Section 3. In contrast to high-scale models, low-scale seesaw models [110] offer a richer phenomenological perspective since the seesaw mediators are allowed to be light. In this type of neutrino mass models, instead of (2), neutrino masses are given by

$$m_\nu \sim \mu_\nu \sqrt{\frac{\mu}{M_{SS}}}$$

(15)

where $M_{SS}$ is again given by the mass scale of the seesaw mediators and $\mu_\nu$ (not to be confused with the $\mu$ parameter of the MSSM) is a small dimensionful parameter; $\mu_\nu \ll \nu_\ell M_{SS}$. In this case, the smallness of neutrino masses is not obtained with a large $M_{SS}$ scale, but with a tiny $\mu_\nu$ parameter. Indeed, if $M_{SS} \sim \text{TeV}$, (15) implies a $\mu_\nu$ parameter of the order of the eV in order to get $m_\nu$ in the ~0.1 eV ballpark. Therefore, one can simultaneously obtain the correct size of neutrino masses while having seesaw mediators at the TeV scale. This leads to a plethora of new effects, not present in high-scale seesaw models, induced by the light seesaw mediators. In particular, novel (and sizable) contributions to LFV processes are possible, sometimes breaking the relation in (3).

The $\mu_\nu$ parameter is intimately related to the breaking of lepton number. In fact, in the $\mu_\nu \to 0$ limit, lepton number is restored and the Majorana neutrino masses in (15) vanish. This makes the smallness of the $\mu_\nu$ parameter natural, in the sense of 't Hooft [III], since the symmetry of the Lagrangian gets increased when the parameter is set to zero. For this reason, low-scale seesaw models are also said to have almost conserved or slightly broken lepton number.

The collider phenomenology of low-scale seesaw models is much richer than that of high-scale ones. The seesaw mediators can in principle be produced and, through their decays, one may be able to test the mechanism behind neutrino masses. At the LHC, one typically expects multilepton final states, often including missing energy carried away by undetected neutrinos. In addition, the LFV signatures can be as frequent as the flavor conserving ones. For an incomplete list of references on the phenomenology of low-scale seesaw models see [112–135]. This list includes phenomenological studies on the production of right-handed neutrinos [112, 113, 117, 122–125, 128, 130, 133, 135] and related processes at colliders [120, 121, 124, 125, 129], works where other low-scale seesaw mediators are considered [112, 126, 132], sneutrino dark matter studies in low-scale seesaw scenarios [118, 119, 123, 129], papers that explore the impact of light right-handed neutrinos on the unitarity of the lepton mixing matrix [114], some works on the way the supersymmetric spectrum is altered in the presence of light right-handed neutrino superfields [115, 116, 120], and papers discussing other phenomenological issues in extended frameworks [115, 131]. The phenomenology of light right-handed neutrinos is also reviewed in detail in [127, 134]. In the case of a type-I seesaw, the seesaw mediator is a fermionic gauge singlet. This usually suppresses its production in hadronic colliders. However, sizable right-handed neutrino production cross sections are possible in some type-I seesaw realizations due to the mixing with the left-handed neutrinos, which serves as a portal to the gauge sector. Furthermore, when the type-I seesaw is embedded in a left-right symmetric scenario [136–138] new production mechanisms are possible thanks to the new charged currents mediated by the $W_R^\pm$ gauge bosons. This allows for further collider tests or the model, including searches for lepton number violation; see, for example, [139–145].

We now present the most popular representative of the low-scale seesaw models: the inverse seesaw. For other low-scale seesaw models and their LFV phenomenology see [146–151].

4.1. The Supersymmetric Inverse Seesaw. In the supersymmetric inverse seesaw (ISS) [152–154], the MSSM particle content is extended with 3 generations of right-handed neutrino superfields $\bar{N}_c^c$ and 3 generations of singlet superfields $\bar{X}$. More minimal realizations of the ISS are possible [155–159]. However, for simplicity, we will stick to the most common version with 3 + 3 singlet superfields. The superpotential takes the form

$$W = W_{\text{MSSM}} + Y_c \bar{N}_c^c \tilde{L} + M_R \bar{N}_c^c \bar{X} + \frac{1}{2} \mu_\nu \bar{X} \bar{X},$$

(16)

where we have omitted family indices. $Y_c$ and $M_R$ are general 3×3 complex mass matrices and $\mu_\nu$ is a complex symmetric 3×3 matrix. One can easily check that the superpotential in (16) violates lepton number by two units. In this case, all lepton number assignments are arbitrary. However, they serve to illustrate the violation of lepton number. For example, it is common practice to assign lepton numbers −1 and +1 to the $\bar{N}_c^c$ and $\bar{X}$ superfields, respectively. With this lepton number assignment, while $M_R$ generates a lepton number conserving Dirac mass term for the fermion singlets, $\mu_\nu$ violates lepton number by two units. This Majorana mass term also leads to a small mass splitting in the heavy neutrino sector,
which is then composed by three quasi-Dirac neutrinos. The corresponding soft SUSY breaking Lagrangian is given by
\[ -\mathcal{L}_{\text{soft}} = -\mathcal{L}_{\text{MSSM}} + \frac{1}{2} \mathcal{N} \mathcal{N}^c + \tilde{\nu} \tilde{\nu}^c + \tilde{\nu} \tilde{\nu}^c + B_{\mu} \tilde{\nu} \tilde{\nu}^c + \tilde{\nu} \tilde{\nu}^c + h.c. \]
\[ + \left( T, \mathcal{N} \mathcal{L} \mathcal{H}_u + B_{M_{\nu}} \mathcal{N} \mathcal{N} + \frac{1}{2} B_{\mu} \tilde{\nu} \tilde{\nu}^{c} \right), \]  
(17)
where \( B_{M_{\nu}} \) and \( B_{\mu} \) are the new parameters involving the scalar superpartners of the singlet neutrino states. Notice that, with the previous lepton number assignment, while the former conserves lepton number, the latter violates lepton number by two units. Finally, \( \mathcal{L}_{\text{MSSM}} \) contains the soft SUSY breaking terms of the MSSM.

The scalar potential of the model is such that the neutral components of the Higgs superfields get nonzero VEVs,
\[ \langle H_u^0 \rangle = \frac{v_u}{\sqrt{2}}, \]
\[ \langle H_u^0 \rangle = \frac{v_u}{\sqrt{2}}, \]
(18)
triggering electroweak symmetry breaking (EWSB). This induces mixings in the neutrino sector. In the basis \( \nu = (\nu_1, \mathcal{N}^c, X) \), the \( 9 \times 9 \) neutrino mass matrix is given by
\[ M_{\text{ISS}} = \begin{pmatrix} 0 & m_D^T & 0 \\ m_D & 0 & M_{\nu} \\ 0 & M_{\nu}^T & \mu_{\nu} \end{pmatrix}, \]
(19)
where \( m_D = (1/\sqrt{2})Y_{\nu_1}v_u \). Assuming the hierarchy \( \mu_{\nu} \ll m_D \ll M_{\nu} \), the mass matrix \( M_{\text{ISS}} \) can be approximately block-diagonalized to give the effective mass matrix for the light neutrinos [160]
\[ m_{\text{light}} = m_D^T M_{\nu}^{-1} \mu_{\nu} M_{\nu}^{-1} m_D. \]
(20)

On the other hand, the other neutrino states form three heavy quasi-Dirac pairs, with masses corresponding approximately to the entries of \( M_{\nu} \).

Equation (20) has the same form as (15), with \( M_{\text{SS}} \sim M_{\nu} \). Therefore, by taking a small \( \mu_{\nu} \) parameter, the model allows for small neutrino masses, sizable \( Y_{\nu} \), Yukawa couplings, and singlet neutrinos at the TeV scale (or below).

4.2. LFV in the Supersymmetric Inverse Seesaw. The presence of light singlet neutrino states allows all sorts of effects. Here we will concentrate on their contributions to LFV processes. For some recent works on phenomenological aspects of light singlet neutrinos see [124, 127, 130, 134, 135, 161–169].

There is a vast literature on LFV in models with light singlet neutrinos. Potentially large enhancements, with respect to the usual high-scale models, were already pointed out in early studies [63, 77, 82, 154]. More recently, several works have explored in detail the LFV anatomy of these models, highlighting the relevance of (non-SUSY) box diagrams induced by singlet neutrinos [149, 170–172], computing Higgs penguin contributions [173], and showing enhancements in the usual photon penguin contributions [163]. Regarding purely supersymmetric contributions, the relevance of \( Z \) penguins with right sneutrinos was recently readdressed in [174], solving an inconsistency in the analytical results of [65] and [175]. Finally, [44] constitutes the first complete analysis of LFV in the supersymmetric inverse seesaw, taking into account all possible contributions, supersymmetric as well as nonsupersymmetric.

We will now present the main results in [44]. These were obtained using FlavorsKit [176], a tool that combines the analytical power of SARAH [177–181] with the numerical routines of SPheno [182, 183] to obtain predictions in a wide range of models, based on the automatic computation of the lepton flavor violating observables. See [184] for a comprehensive and pedagogical review of this set of tools.

In the following, we will discuss numerical results obtained using universal boundary conditions at the gauge coupling unification scale, \( m_{\text{GUT}} = 2 \times 10^{16} \text{GeV} \), setting \( M_{\text{SUSY}} = m_0 = M_{1/2} = -A_0 \). In addition, we fixed tan \( \beta = 10 \), \( \mu > 0 \) and considered a degenerate singlet spectrum (\( M_{\nu}^R = M_{\nu} \) with \( i = 1, 2, 3 \)). We also fixed the \( B_{\mu_\nu} \) and \( B_{M_{\nu}} \) bilinear parameters to \( B_{\mu_\nu} = 100 \mu_{\nu} \) and \( B_{M_{\nu}} = 100 M_{\nu} \), although we explicitly checked that they have small impact on the LFV phenomenology. Furthermore, we fixed the \( Y_{\nu} \), Yukawa couplings using a modified Casas-Ibarra parameterization [91], adapted for the inverse seesaw [161, 185]. With this help of this parameterization, we were able to reproduce the best-fit values for the neutrino squared mass differences and mixing angles determined in the global fit [186] (see also [7] for an update). For simplicity, we considered a Casas-Ibarra matrix \( R \) equal to the unit matrix, normal hierarchy for the light neutrinos and a lightest neutrino mass \( m_{\nu_1} = 10^{-4} \text{eV} \).

A general conclusion one can draw from [44] is that the LFV phenomenology strongly depends on \( M_{\nu} \) and \( M_{\text{SUSY}} \). The first scale determines the mass of the singlet neutrinos, whereas the second one sets the superparticle masses and their relative size determines the phenomenology. This can be seen in Figure 4, where \( \mathcal{BR}(\mu \rightarrow e \gamma) \) is shown as a function of \( M_{\text{SUSY}} \) and \( M_{\nu} \). The results are displayed in three curves: the full observable, the SUSY contributions, and the non-SUSY ones. The latter consist of contributions from \( \gamma-W^\pm \) and \( \gamma-H^\pm \) loop diagrams, thus involving the singlet neutrinos in combination with the W boson or a charged Higgs. One finds that the relative weight of SUSY and non-SUSY contributions is given by the hierarchy between these two mass scales. For \( M_{\text{SUSY}} \gg M_{\nu} \), non-SUSY contributions induced by the singlet neutrinos dominate the \( \mu \rightarrow e \gamma \) amplitude, whereas for \( M_{\text{SUSY}} \ll M_{\nu} \), the usual MSSM contributions generated by chargino/sneutrino and neutralino/slepton loops turn out to be dominant. Moreover, we find that non-SUSY contributions can have strong cancellations.

A similar behavior is found in case of the 3-body decays \( \ell_i \rightarrow 3\ell_j \). In Figure 5 we display numerical results for \( \mathcal{BR}(\mu \rightarrow 3e) \) as well as for various contributions to this observable. The anatomy of this decay is more involved,
Figure 4: BR(μ → eγ) as a function of $M_{\text{SUSY}}$ and $M_R$. In (a) $M_{\text{SUSY}} = 1$ TeV is fixed, whereas in (b) we set $M_R = 2$ TeV. The other parameters are given in the text. The gray area roughly corresponds to the parameter space excluded by the LHC SUSY searches. Figure taken from [44].

Figure 5: As Figure 4, but with BR(μ → 3e) as a function of $M_{\text{SUSY}}$ and $M_R$. Figure taken from [44].

since more types of Feynman diagrams contribute to the amplitude: SUSY and non-SUSY photon, $Z$ and Higgs penguins, and box diagrams. As for $\ell_i \rightarrow \ell_f \gamma$, we observe that non-SUSY contributions dominate for low $M_R (<M_{\text{SUSY}})$. In particular, we see on the left-hand side of Figure 5 that non-SUSY boxes become completely dominant as soon as one goes to $M_R$ values below $\sim 2$ TeV. This generic feature was already noted in [149, 170–172]. For higher values of $M_R$ SUSY contributions, and in particular the standard photon dipole penguin, dominate. On the right-hand side we find complementary information, with the different contributions as a function of $M_{\text{SUSY}}$ for a fixed $M_R$. It is worth noticing that supersymmetric $Z$ penguins never dominate.

Analogous results are obtained for the $\mu - e$ conversion in nuclei. Interestingly, the large non-SUSY boxes found at low $M_R$ break the dipole dominance, leading to a clear departure from (3). This is illustrated in Figure 6, where BR(μ → eγ), BR(μ → 3e), and the $\mu - e$ conversion rates in Ti and Al are shown as a function of $M_R$. Indeed, for $M_R \lesssim 500$ GeV, the rates for all LFV processes have similar sizes. In this scenario, experiments looking for $\mu \rightarrow 3e$ and $\mu-e$ conversion in nuclei will soon provide the most stringent constraints in this model.

Finally, although we have assumed a degenerate right-handed neutrino spectrum in all the results presented in this section, we note that the qualitative picture would be exactly the same in scenarios with nondegenerate right-handed neutrinos. Furthermore, we emphasize once more that low-scale seesaw models have a rich collider phenomenology, complementary to LFV searches. We refer to the beginning of this section for a discussion and references.

5. $R$-Parity Violating Models

The particle content and symmetries of the MSSM allow for the following superpotential terms

$$W_R = \frac{1}{2} \lambda_{ijk} \bar{T}_i \tilde{T}_j \tilde{T}_k + \lambda'_{ijk} \bar{L}_i \tilde{Q}_j \tilde{E}_k + e_{ij} \bar{L}_i H_u + \frac{1}{2} \lambda''_{ijk} \bar{U}_i \tilde{D}_j \tilde{D}_k,$$

(21)
where we have explicitly introduced family indices. Here $\tilde{u}^c$ and $\tilde{e}^c$ are the right-handed up-quark and charged lepton superfields, respectively, and the rest of the superfields have been already defined. The first three terms in $W^P_3$ break lepton number ($L$) whereas the last one breaks baryon number ($B$). In principle, these couplings are not welcome, since they give rise to many lepton and baryon number violating processes, never observed in nature. For example, the simultaneous presence of the $\lambda'$ and $\lambda''$ couplings would lead to proton decay [187, 188]. This phenomenological problem is solved in the MSSM by introducing by hand a new discrete symmetry that forbids all terms in $W^P_3$. This symmetry is known as $R$-parity [3, 4] and is defined as

$$ R_p = (-1)^{3(B-L) + 2s}. $$

(22)

Here $s$ is the spin of the particle. It is straightforward to verify that all terms in (21) break $R$-parity and thus they are forbidden once this symmetry is imposed. The MSSM is defined as $R$-parity conserving.

However, several arguments can be raised against $R$-parity:

(i) **$R$-parity is imposed by hand.** Unlike the SM, where $L$ and $B$ conservation is automatic, in the MSSM this has to be forced by introducing a new symmetry, not derived from first principles. This is clearly a step back from the SM. However, it is worth pointing out that in the absence of $R$-parity some mechanism must be introduced in order to suppress the dangerous $R_p$ parameters.

(ii) **$R$-parity does not solve fast proton decay.** It is well known that $R$-parity does not forbid some dangerous dimension-5 operators that lead to proton decay [189–191]. For example, the operator $\mathcal{O}_5 = (f/M)QQQL$ has $R_p(\mathcal{O}_5) = +1$ and thus conserves $R$-parity. The bounds obtained from the nonobservation of proton decay imply that, even for $M = M_{\text{Planck}}, f$ must be smaller than $10^{-7}$ [192]. In order to forbid $\mathcal{O}_5$ and other similar dimension-5 operators, one may resort to additional flavor symmetries [193].

(iii) **There is no reason to forbid all the $L$ and $B$ violating operators.** Proton decay requires the simultaneous presence of $L$ and $B$ violating couplings. Therefore, it is sufficient to impose the conservation of just one of these two symmetries in order to forbid proton decay. This has led to the consideration of alternative discrete symmetries which allow for either $L$ or $B$ violation while protecting the proton. An example of such symmetries is baryon triality ($Z_2^B$) [189, 194].

Furthermore, there are several good motivations to consider $R$-parity violating ($R_p$) scenarios. The violation of lepton number by any of the first three couplings in (21) automatically leads to nonzero neutrino masses [195–197]. Moreover, the presence of $R_p$ couplings leads to a rich collider phenomenology due to the decay of the LSP. This can be translated into longer decay chains, changing the expected signatures at the LHC [198, 199]. In fact, $R_p$ has also been considered as a way to relax the stringent bounds on the squark and gluino masses; see, for example, [5, 6, 200, 201].

Finally, in $R_p$ the standard neutralino LSP is lost as a dark matter candidate. Therefore, alternative candidates must be considered. Examples in the literature include (i) gravitinos [202–204], (ii) the axion [205, 206], or (iii) its superpartner, the axino [207, 208]. For general reviews on $R$-parity violation and collections on bounds on the $R_p$ couplings see [209–212].

We will now discuss separately the LFV phenomenology of two very different supersymmetric scenarios with $R$-parity violation: explicit $R$-parity violation ($e-R_p$) and spontaneous $R$-parity violation ($s-R_p$).

### 5.1. Explicit $R$-Parity Violation

The most characteristic signatures of $R$-parity violating models are, of course, processes with $L$ or $B$ violation. Nevertheless, processes that violate lepton flavor can provide interesting signatures as well and, in fact, they can be more attractive due to the large number of upcoming LFV experiments.

#### 5.1.1. Higgs LFV Decays

After the historical discovery of the Higgs boson [1, 2], a lot of effort has been put into the determination of its properties. In particular, the Higgs boson decays may contain a lot of valuable information, with potential indications of new physics. Recently, the CMS collaboration reported on an intriguing 2.4σ excess in the $h \rightarrow \tau\mu$ channel [32]. This hint, which translates into $\text{BR}(h \rightarrow \tau\mu) = (0.84^{+0.39}_{-0.32})$, is based on the analysis of the 2012 dataset, taken at $\sqrt{s} = 8\text{ TeV}$ and an integrated luminosity of 19.7fb$^{-1}$. This large Higgs LFV (For pioneer works on Higgs LFV decays see [213, 214],) branching ratio is quite challenging and most NP models cannot accommodate it [215]. In fact, the flavor conserving Higgs decay $h \rightarrow \tau\tau$ has a branching ratio of only $\sim 6\%$, not much higher than the LFV one found by CMS. Although independent confirmation by ATLAS, as well as additional statistics in CMS, would be required in order to confirm that this hint is not just a
fluctuation but an evidence of new physics, it is interesting to explore different models in order to determine what type of frameworks can accommodate this signal.

Regarding supersymmetric models, several scenarios have been recently explored, some of them even before the CMS hint was announced. In particular, the authors of [45, 216] considered an extension of the MSSM including all $L$ violating couplings in (21). The particles-sparticles mixing due to the $\kappa_p$ couplings induce Higgs LFV decays at tree-level, thus potentially being able to reach branching ratios as high as the one found by the CMS collaboration. Two specific examples are shown in Figure 7. However, the existing experimental bounds on the relevant combinations of $\kappa_p$ parameters are strongly constrained. In case of $B_1$, the $\kappa_p$ mixings between the Higgs boson and the sneutrinos ($\mathcal{S} \supset B_1 \tilde{L}, H_u$), they have strong bounds since they induce nonzero neutrino masses [217–219]. Moreover, the $\lambda$ couplings are constrained by charged current experiments [209]. Once these constraints are taken into account, the maximum $\text{BR}(h \rightarrow \tau \mu)$ one can get is not very impressive. This is illustrated in Figure 8, where $\text{BR}(h \rightarrow \tau \mu)$ contours are drawn on the $B_2 - \lambda_{232}$ plane. From this figure one concludes that $\text{BR}(h \rightarrow \tau \mu)$ can reach, at most, a few $\times 10^{-5}$, clearly below the CMS hint. Similar conclusions are obtained when other combinations of $\kappa_p$ parameters are considered.

The supersymmetric inverse seesaw has also been considered as a possible setup to reproduce a Higgs LFV branching ratio into $\tau \mu$ at the 1% level [220]. In this case, $h \rightarrow \tau \mu$ takes place at 1-loop, naturally suppressing the branching ratio. As a result of this, as well as due to the constraints from other LFV processes such as $\ell_i \rightarrow \ell_j \gamma$, one finds that the maximum allowed $\text{BR}(h \rightarrow \tau \mu)$ is about $10^{-5}$. Therefore, this model cannot account for a branching ratio as obtained by CMS either. In order to conclude this discussion on $h \rightarrow \tau \mu$ with a positive note, let us mention that known models that can account for $\text{BR}(h \rightarrow \tau \mu) \sim 1\%$ exist in the literature. They all involve extended Higgs sectors. In particular, it has been shown that Two-Higgs-Doublet models of type-III, in which both Higgs doublets can couple to up- and down-type fermions, can easily accommodate the CMS signal [215, 221–225]. The MSSM, being Two-Higgs-Doublet models of type-II in which one Higgs doublet couples to up- and the other to down-type fermions, cannot. In fact, several studies have shown that one cannot accommodate $\text{BR}(h \rightarrow \tau \mu) \sim 1\%$ when the low-energy theory is the MSSM [226, 227]. The same conclusion applies to heavy Higgs LFV decays [228].

5.1.2. Trilinear R-Parity Violation and LFV. In principle, the usual LFV processes studied in R-parity conserving models can be studied in the R-parity violating ones and, in some cases, they get additional $\kappa_p$ contributions. This is the case of $\ell_i \rightarrow 3\ell_j \gamma, M \rightarrow \ell_i \ell_j$, and $\tau \rightarrow M \ell_j$, where $M$ is a neutral meson, which, in the presence of trilinear $\kappa_p$ couplings, can be induced at tree-level [229]. This is represented in Figure 9, where two examples are shown: $\ell_i \rightarrow 3\ell_j$ induced by $\lambda$ couplings and sneutrino exchange and $\tau \rightarrow M \ell_j$ induced by $\lambda'$ couplings and squark exchange. This allows one to derive a large collection of bounds on the size of the trilinear...

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Figure 7: Tree-level $\kappa_p$ contributions to $h \rightarrow \tau \mu$. (a) A $\lambda \lambda$ contribution. (b) A $\epsilon_i^3$ (denoted as $\mu_i^3$ in [45]) contribution. Figure borrowed from [45].

Figure 8: $\text{BR}(h \rightarrow \mu \tau)$ contours on the $B_2 - \lambda_{232}$ plane. The continuous horizontal and vertical lines show approximate limits due to neutrino masses (in case of $B_2$) and charged current experiments (in case of $\lambda_{232}$). Figure borrowed from [45].
couplings and the masses of the superparticles mediating the LFV decays [230, 231].

Let us consider the right-hand side of Figure 9. After dressing the quarks in the final state, this Feynman diagram induces $\tau \to M\mu$ at tree-level. Since the exchanged particle, a down-type squark in this case, is much heavier than the rest of particles, this process can be well described by the 4-fermion effective Lagrangian

$$\mathcal{L}_{\text{eff}} = -\lambda'^*_{3ij}\lambda_{2kj} m_d^2 \bar{u}_i P_L u_j (\bar{d}_k P_R \mu^c),$$

obtained after integrating out the heavy squark. One can now use this Lagrangian and, together with the relevant hadronic form factors, compute rates for processes such as $\tau \to \mu \pi^+ \pi^-$. The authors of [231] followed this method and used Belle results on searches for $\tau$ LFV decays [232–234] to obtain the limit

$$\lambda'^*_{3ij}\lambda_{2kj} < 2.1 \cdot 10^{-4} \left(\frac{m_d^2}{100 \text{ GeV}}\right)^2.$$  

One can exploit this idea using other LFV observables involving mesons. We refer to [230, 231] for a more complete list of constraints.

Similarly, trilinear $R_p$ couplings can also trigger $\mu - e$ conversion in nuclei, induced by diagrams very similar to the one on the right-hand side of Figure 9. Interestingly, in this case $\mu - e$ conversion in nuclei would take place at tree-level, while the more popular $\mu \to e\gamma$ would take place at 1-loop. This has been recently pointed out by the authors of [235], who argue that experiments looking for $\mu - e$ conversion in nuclei might be the first (and perhaps the only ones) to observe a nonzero signal in the next round of experiments.

5.1.3. Other Results on LFV in $R_p$ Scenarios. Before concluding, let us briefly comment on other aspects of LFV in $R_p$ models. An interesting feature of $R_p$ models is that some lepton number violating processes at colliders might look like lepton flavor violating ones. This is, for example, the case of sneutrino decay in bilinear $R$-parity violation [46], as shown in Figure 10. This process is possible thanks to the mixing between the MSSM charginos and the standard charged leptons and, being lepton number violating, it also violates lepton flavor. However, at the LHC, if the sneutrinos are directly produced, the process would look simply like a LFV one.

Finally, for other recent works on LFV in $R_p$ models see [236–238].

5.2. Spontaneous $R$-Parity Violation. A very attractive scenario for LFV is that of spontaneous $R$-parity violation. When a scalar field oddly charged under $R$-parity gets a VEV in a theory with $R$-parity conserving Lagrangian, $R$-parity gets spontaneously broken. Here we will concentrate on spontaneous $L$ and $R_p$ violation. Although $R$-parity is a discrete symmetry, its breaking comes along with the breaking of the continuous global symmetry $U(1)_L$. This implies the existence of a massless Goldstone boson, usually called the majoron ($f$) [239, 240].
The nature of the majoron is crucial for the phenomenological success of the model. In fact, in the first model with $sR_p$ [241], the breaking of $R$-parity was triggered by the VEV of a left-handed sneutrino. This simple setup was eventually excluded since the doublet nature of the majoron leads to conflict with LEP bounds on the $Z$ boson invisible decay width and astrophysical data [206, 242]. However, more refined models where the violation of lepton number is induced by a gauge singlet perfectly valid possibilities. As a benchmark example of this family we will consider here the model introduced in [243]. For alternative models with gauged lepton number see, for example, [244–246].

In the model of [243], the particle content is extended with three additional singlet superfields, namely, $\tilde{\gamma}, \tilde{S},$ and $\tilde{\Phi}$, with lepton number assignments of $L = -1, 1, 0,$ respectively. By assumption, the Lagrangian of the theory conserves lepton number. Therefore, the superpotential can be written as

$$W_{SRPV} = W_{\text{MSSM}} + Y_s \tilde{\gamma} \tilde{N}^c \tilde{H}_u - y_\tilde{\gamma} \tilde{H}_u \tilde{\Phi} + h\Phi \tilde{N}^c \tilde{S}$$

(25)

For simplicity, one can simply consider one generation of $\tilde{N}^c$ and $\tilde{S}$ superfields. Several scalar fields acquire VEVs after electroweak symmetry breaking. In addition to the usual MSSM Higgs boson VEVs, $v_d$ and $v_u$, these are $\langle \Phi \rangle = v_\Phi / \sqrt{2}$, $\langle \tilde{\gamma} \rangle = v_\tilde{\gamma} / \sqrt{2}$, $\langle \tilde{S}\rangle = v_{\tilde{S}} / \sqrt{2}$, and $\langle \tilde{N}_e \rangle = v_{\tilde{N}_e} / \sqrt{2}$. This vacuum configuration breaks lepton number and $R$-parity. In fact, we notice that $v_\tilde{\gamma} \neq 0$ generates the effective bilinear $R_p$ terms $e_i = Y_i v_\tilde{\gamma} / \sqrt{2}$. Furthermore, neglecting $v_\tilde{s} \ll v_\tilde{\gamma}, v_\tilde{N}_e,$ one finds the resulting majoron profile

$$J = \text{Im} \left( \frac{v_\tilde{\gamma}}{V} \tilde{S} - \frac{v_\tilde{R}}{V} \tilde{N}^c \right),$$

(26)

where $V = \sqrt{v_d^2 + v_u^2}$. Equation (26) shows that the majoron inherits the singlet nature of the scalar fields that break lepton number with their VEVs, thus suppressing the couplings to the $Z$ boson and evading the stringent LEP bound.

Here we are interested in novel LFV features due to the presence of the majoron. Another interesting signature present in majoron models is the invisible decay of the Higgs boson, $h \to J J$ [247, 248]. This new massless state dramatically changes the phenomenology both at collider and low-energy experiments [47, 249]. In particular, it leads to new LFV processes, such as $\mu \to e\gamma$ or $\mu \to e\gamma^*$. The exotic muon decay $\mu \to e\gamma$ was first studied in [250] and later revisited in [47], where the decay with an additional photon was also considered. Furthermore, the impact of the majoron on $\mu - e$ conversion in nuclei was discussed in [251] (see also [252] for similar LFV processes in the context of invisible axions).

The rate of the $\mu \to e\gamma$ decay is determined by the $e-\mu-J$ coupling, $O_{\mu J}$, which, in the model under consideration, is of the form $O_{\mu J} \sim 1/vR^2$ RPV parameters. This makes us conclude that, in general, one expects large partial muon decay widths to majorons for low $v_R$. However, currently there are no experiments looking for $\mu \to e\gamma$ and the current best limit on the branching ratio, $BR(\mu \to e\gamma) \lesssim 10^{-5}$, dates back to 1986 [253]. Regarding the decay including a photon, $\mu \to e\gamma^*$, one can profit from the MEG experiment and its improvements for the presence of the MEG experiment and its improvement for the full range $\mu \to e\gamma$. The two branching ratios are related by

$$BR(\mu \to e\gamma) = \frac{\alpha}{2\pi} \mathcal{J}(x_{\text{min}}, y_{\text{min}}) BR(\mu \to e\gamma).$$

(27)

Here $\mathcal{J}(x_{\text{min}}, y_{\text{min}})$ is a 3-body phase space integral defined as

$$\mathcal{J}(x_{\text{min}}, y_{\text{min}}) = \int dx \, dy \frac{(x - 1)(2 - xy - y)}{y^2 (1 - x - y)},$$

(28)

different parameter values $x, y$ are defined as

$$x = \frac{2E_e}{m_\mu},$$

$$y = \frac{2E_\gamma}{m_\mu},$$

(29)

and $x_{\text{min}}$ and $y_{\text{min}}$ are the minimal electron and photon energies that a given experiment can measure. Indeed, the integral in (28), which would contain infrared and collinear divergences, is regularized by the specific choices made by an experiment.

As explained above, the main advantage of $\mu \to e\gamma$ is the existence of the MEG experiment. However, the question is whether it is sensitive to this exotic LFV process or not. Figure 11 shows the value of the phase space integral $\mathcal{J}(x_{\text{min}}, y_{\text{min}})$ as a function of $x_{\text{min}}$ for three different values of $y_{\text{min}}$ and for two choices of $\cos\theta_{e\gamma}$ (the relative angle between the electron and photon directions). Unfortunately, the MEG experiment is specifically designed for a single search. In fact, the cuts used in the search for $\mu \to e\gamma$ are very restrictive: $x_{\text{min}} \gtrsim 0.995, y_{\text{min}} \gtrsim 0.99$, and $|\pi - \theta_{e\gamma}| \lesssim 8.4$ mrad. For these exact values one finds a tiny phase space integral, $\mathcal{J} = 6 \times 10^{-16}$. As a consequence of this, a limit for $BR(\mu \to e\gamma)$ of the order of $\lesssim 10^{-13}$ would translate into the useless limit $BR(\mu \to e\gamma) < 0.14$. To improve upon this bound, it is necessary to relax the cuts. For example, by relaxing the cut on the opening angle to $\cos\theta_{e\gamma} = -0.99$. However, this is prone to induce additional unwanted background events, in particular, accidental background from muon annihilation in flight. Therefore, although one could in principle increase the value of the phase space integral $\mathcal{J}(x_{\text{min}}, y_{\text{min}})$, the background in that case would make the search for a positive signal impossible. This discussion suggests that a better timing resolution of the experiment would be welcome in order to reduce the background and be sensitive to final states including majorons.

6. Summary and Conclusions

In summary, we have reviewed the lepton flavor violating phenomenology of several nonminimal supersymmetric models: high-scale and low-scale seesaw models as well as models with explicit or spontaneous $R$-parity violation.
The main conclusion from this overview is that the lepton flavor violating signatures can be very different from those found in the MSSM. This translates into two important messages:

(i) For the Theorists. Lepton flavor violation might be much more intricate than what minimal models predict. Therefore, we should be careful when extrapolating our expectations (derived from the MSSM) to extended frameworks.

(ii) For the Experimentalists. Although minimal models are of course well motivated, lepton flavor violation might show up in nonstandard channels. We must be ready to avoid missing a relevant signal.

The comparison between SUSY and non-SUSY LFV is not straightforward. In general, one can use LFV to distinguish between two models, but it is often impossible to tell whether the underlying physics is supersymmetric or not. There are two main reasons for this. First, nonminimal SUSY models typically contain non-SUSY contributions to LFV observables, making hard a clear distinction. The discussion in Section 4 is a clear example of this interplay. And second, there are many non-SUSY models with LFV phenomenologies that resemble the standard phenomenology in SUSY models (for instance, due to the dominance of dipole operators). Perhaps, the only scenario where a clear distinction can be made is that of high-scale seesaw models: if they are nonsupersymmetric no sizable LFV is induced at low energies, whereas sizable LFV rates at low energies are in principle expected if they are supersymmetric.

The connection between neutrino masses and lepton flavor is one of the main motivations to search for LFV processes. As we have seen in this review, different neutrino mass models typically lead to different LFV phenomenologies. This can in principle be used to unravel the origin of neutrino masses by exploring this link with LFV once one or several positive discoveries are made in the next round of LFV experiments. Although precise numerical predictions are impossible due to the existence of many free parameters in most neutrino mass models, correlations and patterns can favor specific scenarios. For example, the discovery of a clear departure from a dipole dominated scenario could point towards the existence of light singlet neutrinos.

Nevertheless, it has been already emphasized in this review that LFV can take place even in the absence of neutrino masses. Similarly, although all neutrino mass models discussed in this review include Majorana neutrinos and lepton number violation, this aspect is not particularly relevant for our discussion on LFV. Indeed, Majorana neutrino masses are related to the breaking of lepton number, which is conceptually different to the breaking of lepton flavor.

To conclude, let us emphasize once more that properly identifying the underlying physics will be crucial in case a positive observation in one or several LFV experiments is made. This problem might soon have to be addressed, given the exciting projects that are currently going on or soon starting their search for LFV. Hopefully, this review, as well as the many phenomenological studies in the bibliography, will help shedding some light on this matter.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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