Research Article

Holographic Polytrropic $f(T)$ Gravity Models

Surajit Chattopadhyay,¹ Abdul Jawad,² and Shamaila Rani²

¹Pailan College of Management and Technology, Bengal Pailan Park, Kolkata 700 104, India

²Department of Mathematics, COMSATS Institute of Information Technology, Lahore 54000, Pakistan

Correspondence should be addressed to Abdul Jawad; jawadab181@yahoo.com

Received 31 July 2015; Revised 30 September 2015; Accepted 4 October 2015

Academic Editor: Elias C. Vagenas

Copyright © 2015 Surajit Chattopadhyay et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited. The publication of this article was funded by SCOAP³.

The present paper reports a study on the cosmological consequences arising from reconstructing $f(T)$ gravity through new holographic polytrropic dark energy. We assume two approaches, namely, a particular form of Hubble parameter H and a solution for $f(T)$. We obtain the deceleration parameter and effective equation of state, as well as torsion equation of state parameters from total density and pressure in both cases. It is interesting to mention here that the deceleration and torsion equation of state represent transition from deceleration to acceleration phase. We study the statefinder parameters under both approaches which result in the fact that statefinder trajectories are found to attain Λ CDM point. The comparison with observational data represents consistent results. Also, we discuss the stability of reconstructed models through squared speed of sound which represents stability in late times.

1. Introduction

The accelerated expansion of the universe is strongly manifested after the discovery of unexpected reduction in the detected energy fluxes coming from SNe Ia [1, 2]. Other observational data like CMBR, LSS, and galaxy redshift surveys [3–5] also provide evidences in this favor. These observations propose a mysterious form of force, referred to as dark energy (DE), reviewed in [6–9], which takes part in the expansion phenomenon and dominates overall energy density of the universe. This has two remarkable features: its pressure must be negative in order to cause the cosmic acceleration and it does not cluster at large scales. In spite of solid favor about the presence of DE from the observations, its unknown nature is the biggest puzzle in astronomy. In the last nineties, this expansion was detected, but the evidence for DE has been developed during the past decade.

Physical origin of DE is one of the largest mysteries not only in cosmology but also in fundamental physics [6, 10–13]. The dynamical nature of DE can be originated from different models such as cosmological constant, scalar field models, holographic DE (HDE), Chaplygin gas, polytrropic gas, and modified gravity theories. Various DE models are

discussed in [14–21]. The modified theories of gravity are the generalized models which came into being by modifying gravitational part in general relativity (GR) action while matter part remains unchanged. At large distances, these modified theories modify the dynamics of the universe. The $f(R)$ theory is the modification of GR which modifies the Ricci (curvature) scalar R to a general differentiable function. The gravitational interaction is established through curvature with the help of Levi-Civita connection. There is another theory which is the result of unification of gravitation and electromagnetism. It is based on mathematical structure of absolute or distant parallelism, also referred to as teleparallelism which led to teleparallel gravity. In this gravity, torsion is used as the gravitational field via Wittenböck connection. The modification of teleparallel gravity in the similar fashion of $f(R)$ gravity gives generalized teleparallel gravity $f(T)$, where f is general differentiable function of torsion scalar.

The search for a viable DE model (representing accelerated expansion of the universe) is the basic key leading to the reconstruction phenomenon, particularly in modified theories of gravity [22–25]. This reconstruction scheme works on the idea of comparison of corresponding energy densities to obtain the modified function in the underlying gravity.

Daouda et al. [26] developed the reconstruction scheme via HDE model in $f(T)$ gravity and found that the reconstructed model may cross the phantom divide line in future era. Setare and Darabi [27] assumed the scale factors in power-law form and obtained well defined solutions. Farooq et al. [28] reconstructed $f(T)$ model by taking (m, n) type HDE model and discussed its viability as well as cosmography. They showed that this model is viable, compatible with solar system test, and ghost-free and has positive gravitational constant. Karami and Abdolmaleki [29] obtained equation of state (EoS) parameter for the reconstructed $f(T)$ models by taking HDE, new agegraphic DE as well as their entropy-corrected versions and found transition from nonphantom to phantom phase only in entropy-corrected versions showing compatibility with the recent observations. Sharif and Rani [30, 31] explored this theory via some scalar fields, nonlinear electrodynamics, and entropy-corrected HDE models and analyze the accelerated expansion of the universe.

Holographic DE models are widely used for explaining the present day DE scenario and evolution of the universe. These are based on the holographic principle which naively asks that the combination of quantum mechanics and quantum gravity requires three-dimensional world to be an image of data that can be stored on a two-dimensional projection much like a holographic image [32, 33]. It is useful to reveal the entropy bounds of black holes (BHs) which lead to the formulation of the holographic principle. It is well established that the area of a BH event horizon never decreases with time, the so-called area theorem. If a matter undergoes gravitational collapse and converts into a BH, the entropy associated with the original system seems to disappear since the final state is unique. This process clearly violates the second law of thermodynamics. In order to avoid this problem, Bekenstein [34] proposed generalized second law of thermodynamics on the basis of area theorem which is stated as follows: BH carries an entropy proportional to its horizon area and the total entropy of ordinary matter system and BH never decreases. Mathematically, it can be written as

$$\frac{dS_{\text{tot}}}{dt} \geq 0. \quad (1)$$

Here, $S_{\text{tot}} = S + S_{\text{BH}}$, S represents the entropy of matter (body) outside a BH, and S_{BH} is the entropy of BH.

In the construction of HDE model, the relation between ultraviolet (UV) (Λ) and infrared (IR) (L) cutoffs as proposed by Cohen et al. [35] plays a key role. It is suggested that, for an effective field theory in a box of size L with Λ , the entropy S scales extensively; that is, $S \sim L^3 \Lambda^3$. However, the maximum entropy in a box possessing volume L^3 (growing with the area of the box) behaves nonextensively in the framework of BH thermodynamics, the so-called Bekenstein entropy bound. For any Λ (containing sufficiently large volume), the entropy of effective field theory will exceed the Bekenstein limit which can be satisfied if we limit the volume of the system as follows:

$$S = L^3 \Lambda^3 \lesssim S_{\text{BH}} \equiv \pi L^2 M_p^2, \quad (2)$$

where S_{BH} has radius L .

It can be seen from the above inequality that IR cutoff L (scales as Λ^{-3}) is directly associated with UV cutoff and cannot be chosen independently from it. Moreover, there occur some problems in saturating the above inequality because Schwarzschild radius is much larger than the box size and hence produces incompatibility problem with effective field theory. To avoid this problem, Cohen et al. [35] proposed a strong constraint on the IR cutoff which excludes all states that lie within the Schwarzschild radius; that is,

$$L^3 \Lambda^4 = L^3 \rho_\Lambda \lesssim L M_p^2. \quad (3)$$

Here, left and right hand sides correspond to the total energy of the system (since the maximum energy density in the effective field theory is Λ^4) and mass of the Schwarzschild BH, respectively. Also, IR cutoff L is being scaled as Λ^{-2} which is more restrictive limit than (2). The above relation indicates that the maximum entropy of the system will be $S_{\text{BH}}^{3/4}$. Li [36] developed the energy density for DE model by saturating the above inequality as follows:

$$\rho_\Lambda = 3\zeta^2 M_p^2 L^{-2}, \quad (4)$$

where ζ is the dimensionless HDE constant parameter. The interesting feature of this density is that it provides a relation between UV (bound of vacuum energy density) and IR (size of the universe) cutoffs. However, a controversy about the selection of IR cutoff of HDE has been raised since its birth. As a result, different proposals of IR cutoffs for HDE and its entropy-corrected versions [37, 38] have been developed.

Plan of the paper is as follows. In Section 2, we provide information briefly about holographic polytropic DE model and some cosmological parameters. Also, we assume a particular form of Hubble parameter, subsequently considering a correspondence between new HDE and polytropic gas model of DE derived a new form of polytropic gas dark energy that was further assumed to be an effective description of dark energy in $f(T)$ gravity to study the cosmological consequences. In this section, we also assume a particular solution for $f(T)$ and derive solution for H in the backdrop of a correspondence between new HDE and polytropic DE. This reconstructed H has been utilized to get reconstructed effective torsion EoS and statefinder parameters. Also, we compare the obtained results with observational data in this section. In Section 3, we check the stability of reconstructed models in all cases. We conclude the results in Section 4.

2. New Holographic Polytropic DE in $f(T)$ Gravity

Holographic reconstruction of modified gravity model is a very active area of research in cosmology. Unfortunately, nature of DE is still not known and probably that has motivated theoretical physicists towards development of various candidates of DE and recently geometric DE or modified gravity has been proposed as a second approach to account for the late time acceleration of the universe. In literature, mostly reconstructed work has been done with polytropic EoS, family of holographic DE models, family of Chaplygin

gas, and scalar field models in general relativity, as well as modified theories of gravity (in framework of $f(T)$ gravity; see [26–31, 39–41]). However, we do holographic reconstruction of polytropic DE and based on that we experiment the cosmological implications of $f(T)$ gravity.

The polytropic gas model can explain the EoS of degenerate white dwarfs, neutron stars, and also the EoS of main sequence stars. Polytropic gas EoS is given by [42]

$$p_\Lambda = K\rho_\Lambda^{1+1/n}, \quad (5)$$

where K is a positive constant and n is the polytropic index. The important role played by polytropic EoS in astrophysics has been emphasized in [42, 43]. It is a simple example which is nevertheless not too dissimilar from realistic models [42]. Moreover, there are cases where a polytropic EoS is a good approximation to reality [42]. From continuity equation

$$\rho_\Lambda = \left(\frac{1}{Ba^{3/n} - K} \right)^n. \quad (6)$$

In the present work, we are considering a correspondence between polytropic DE and new HDE with an IR cutoff proposed by [44] with the density given by

$$\rho_D = 3(\mu\dot{H} + \nu H^2). \quad (7)$$

Statefinder and Cosmographic Parameters. Some cosmological parameters are very important for describing the geometry of the universe which include EoS, parameter, deceleration parameter, and statefinder. The physical state of a homogenous substance can be described by EoS. This state is associated with the matter including pressure, temperature, volume, and internal energy. It can be defined in the form $p = p(\rho, \hat{T})$, where ρ , p and $\hat{T}$ are the mass density, isotropic pressure, and absolute temperature, respectively. In cosmological context, EoS is the relation between energy density and pressure such as $p = p(\rho)$ and is given by

$$p = \omega\rho, \quad (8)$$

where ω represents the dimensionless EoS parameter which helps to classify different phases of the universe.

In order to differentiate different DE models on behalf of their role in explaining the current status of the universe, Sahni et al. [45] proposed statefinder parameters. These are denoted by (r, s) and are defined in terms of Hubble as well as deceleration parameters. The deceleration parameter is defined as

$$q = -\frac{\ddot{a}}{aH^2} = -\left(1 + \frac{\dot{H}}{H^2}\right). \quad (9)$$

The negative value of this parameter represents the accelerated expansion of the universe due to the term $\ddot{a} > 0$ (indicating expansion with acceleration). The statefinder parameters are given by

$$r = \frac{\ddot{a}}{aH^3}, \quad (10)$$

$$s = \frac{r-1}{3(q-1/2)}.$$

These parameters possess geometrical diagnostic because of their total dependence on the expansion factor. The most remarkable feature of (r, s) plane is that we can find the distance of a given DE model from Λ CDM limit. This depicts the well-known regions given as follows:

- (i) $(r, s) = (1, 0)$ shows Λ CDM limit;
- (ii) $(r, s) = (1, 1)$ describes CDM limit;
- (iii) $r < 1$ and $s > 0$ constitute quintessence and phantom DE regions.

Moreover, r can be expressed in terms of deceleration parameter as

$$r = 2q^2 + q - \frac{\dot{q}}{H}. \quad (11)$$

Both H and $\{r, s\}$ are categorized as cosmographic parameters. The cosmographic parameters, being dependent on the only stringent assumption of homogeneous and isotropic universe, marginally depend on the choice of a given cosmological model. Secondly, cosmography alleviates degeneracy, because it bounds only cosmological quantities which do not strictly depend on a model. The cosmographic set of parameters arising out of Taylor series expansion of $a(t)$ around the present epoch can be summarized as [46, 47]

$$H = \frac{1}{a} \frac{da}{dt},$$

$$q = -\frac{1}{aH^2} \frac{d^2a}{dt^2}, \quad (12)$$

$$j = \frac{1}{aH^3} \frac{d^3a}{dt^3},$$

$$s = \frac{1}{aH^4} \frac{d^4a}{dt^4}.$$

Differentiating Friedman equation with respect to t and using (12), one can write

$$\dot{H} = -H^2(1+q), \quad (13)$$

$$\ddot{H} = H^3(j+3q+2), \quad (14)$$

$$\ddot{\ddot{H}} = H^4(s-4j-3q(q+4)-6). \quad (15)$$

In the context of cosmological reconstruction problem, some notable contributions are [48–50]. It may be noted that the present work is motivated by Karami and Abdolmaleki [29].

2.1. With a Specific Choice of H . We consider that the Hubble rate H is given by [51]

$$H = H_0 + \frac{H_1}{t}, \quad (16)$$

leading to

$$a(t) = C_1 e^{H_0 t} t^{H_1}. \quad (17)$$

Due to this choice of Hubble parameter, the EoS takes the form

$$w_\Lambda = -1 + \frac{B(C_1 e^{H_0 t} t^{H_1})^{1/n}}{-K + B(C_1 e^{H_0 t} t^{H_1})^{1/n}}, \quad (18)$$

and subsequently NHDE density becomes

$$\rho_D = 3 \left(-\frac{H_1 \mu}{t^2} + \left(H_0 + \frac{H_1}{t} \right)^2 \nu \right). \quad (19)$$

From continuity equation, we have

$$w_D = -1 - \frac{2H_1 \mu / t^3 - 2H_1 (H_0 + H_1/t) \nu / t^2}{3(H_0 + H_1/t) (-H_1 \mu / t^2 + (H_0 + H_1/t)^2 \nu)}. \quad (20)$$

Considering a correspondence between polytropic DE and new HDE, that is, $\rho_\Lambda = \rho_D$ and $w_\Lambda = w_D$, we express B and K in terms of a in the following arrangement:

$$B = \frac{23^{-(1+n)/n} H_1 (C_1 e^{H_0 t} t^{H_1})^{-1/n} (-\mu + (H_1 + H_0 t) \nu) (t^2 / (-H_1 \mu + (H_1 + H_0 t)^2 \nu))^{1+1/n}}{t^2 (H_1 + H_0 t)}, \quad (21)$$

$$K = 3^{-(1+n)/n} \left(\frac{t^2}{-H_1 \mu + (H_1 + H_0 t)^2 \nu} \right)^{1/n} \left(-3 + \frac{2}{H_1 + H_0 t} - \frac{2H_0 t \nu}{-H_1 \mu + (H_1 + H_0 t)^2 \nu} \right).$$

It may be noted that B and K , being integration constants, are not functions of a . Rather it is a new arrangement arising out of the consideration of a correspondence between new holographic dark energy and polytropic gas dark energy. Using (21) in (6), we get the new holographic polytropic gas density as

$$\rho_\Lambda = 3 \left(\frac{t^2}{-H_1 \mu + (H_1 + H_0 t)^2 \nu} \right)^{-1}. \quad (22)$$

The modified Friedmann equations in the case of $f(T)$ gravity for the spatially flat FRW universe are given by

$$H^2 = \frac{1}{3} (\rho_m + \rho_T), \quad (23)$$

$$2\dot{H} + 3H^2 = -(\rho_m + p_T), \quad (24)$$

where

$$\rho_T = \frac{1}{2} (2Tf_T - f - T), \quad (25)$$

$$p_T = -\frac{1}{2} [-8\dot{H}f_{TT} + (2T - 4\dot{H})f_T - f + 4\dot{H} - T], \quad (26)$$

$$T = -6H^2. \quad (27)$$

Here, ρ_m and p_m are the energy density and pressure of matter inside the universe, respectively. Also ρ_T and p_T are the torsion contributions to the energy density and pressure. The energy conservation laws are given by

$$\dot{\rho}_T + 3H(\rho_T + p_T) = 0, \quad (28)$$

$$\rho_m + 3H(\rho_m + p_m) = 0.$$

Using (25) and (26), the the effective torsion EoS parameter comes out to be

$$w_T = -1 + \frac{4\dot{H}(2Tf_{TT} + f_T - 1)}{2Tf_T - f - T}. \quad (29)$$

Using (23), (25), and (27), one can get

$$\rho_m = \frac{1}{2} (f - 2Tf_T). \quad (30)$$

The deceleration parameter is

$$q_T = 2 \left(\frac{f_T - Tf_{TT} - 3f/4T}{f_T + 2Tf_{TT}} \right). \quad (31)$$

The dark torsion contribution in $f(T)$ gravity can justify the observed acceleration of the universe without resorting to DE. This motivates us to reconstruct an $f(T)$ gravity model according to the new holographic polytropic DE. Considering $\rho_T = \rho_\Lambda$, that is, equating (22) and (25), we have the following differential equation:

$$6 \left(H_0 + \frac{H_1}{t} \right)^2 - f - \frac{t^2}{H_1} \left(H_0 + \frac{H_1}{t} \right) \frac{df}{dt} = 6 \left(\frac{t^2}{-\mu H_1 + (H_0 t + H_1)^2 \nu} \right)^{-1}. \quad (32)$$

Solving (32), we obtain reconstructed f in terms of cosmic time t

$$f(t) = \frac{1}{H_1 t^2} \left[H_1 \{ H_0 t (C_2 t - 12\mu) + H_1 (C_2 t - 6\mu + 6H_0 t (-1 + \nu)) + 6H_1^2 (-1 + \nu) \} + 12H_0 t (H_1 + H_0 t) \mu \ln \left(\frac{H_1}{t} + H_0 \right) \right]. \quad (33)$$

Considering $H = (-T/6)^{1/2}$, we have

$$t = \frac{6H_1}{-6H_0 + \sqrt{-6T}}, \quad (34)$$

that lead us to reexpress f of (33) as a function of T as

$$f(T) = \frac{C_2 H_1 \sqrt{-6T} + 6(6H_0^2 \mu - H_0 H_1 \sqrt{-6T}(-1 + \nu) + T(H_1 + \mu - H_1 \nu)) + 6H_0 \sqrt{-6T} \mu \ln[-T/6]}{6H_1}. \quad (35)$$

Subsequently using (35) in (29) and (31), we get the effective torsion EoS and deceleration parameters as

$$w_T = -1 + \frac{(-6H_0 + \sqrt{-6T})^2 (6H_0 \mu + \sqrt{-6T}(-\mu + H_1 \nu))}{9\sqrt{-6T}H_1 (-6H_0^2 \mu + 2\sqrt{-6T}H_0 \mu + T(\mu - H_1 \nu))}, \quad (36)$$

$$q_T = \frac{-(-18H_0^2 + 4\sqrt{-6T}H_0 + T)\mu + H_1 T(-1 + \nu)}{-2(H_0 \sqrt{-6T} + T)\mu + 2H_1 T(-1 + \nu)}. \quad (37)$$

Using (35) in (30), density of the dark matter inside the universe becomes

$$\rho_m = \frac{(6H_0^2 - 2\sqrt{-6T}H_0 - T)\mu + H_1 T(-1 + \nu)}{2H_1}. \quad (38)$$

In the case of pressureless dust matter, $p_m = 0$, we obtain

$$\dot{H} = -\frac{1}{2} \left(\frac{\rho_m}{f_T + 2Tf_{TT}} \right). \quad (39)$$

Using (35) and (38) in (39), we get

$$\dot{H} = \frac{\sqrt{-3T/2} (2H_0 (-3H_0 + \sqrt{-6T})\mu + T(H_1(1 - \nu) + \mu))}{2(-6H_0 \mu + \sqrt{-6T}(H_1(1 - \nu) + \mu))}. \quad (40)$$

Defining effective energy density and pressure as $\rho_{\text{tot}} = \rho_T + \rho_m$ and $p_{\text{tot}} = p_T$ ($p_m = 0$), the effective EoS $w_{\text{tot}} = p_{\text{tot}}/\rho_{\text{tot}}$ becomes (using (40))

$$w_{\text{tot}} = -1 - \frac{2\dot{H}}{3H^2} = -1 - \frac{3\sqrt{6}(2H_0(-3H_0 + \sqrt{-6T})\mu + T(H_1(1 - \nu) + \mu))}{6H_0\sqrt{-T}\mu + \sqrt{6T}(H_1(1 - \nu) + \mu)}. \quad (41)$$

The statefinder parameters are given by

$$r = q + 2q^2 + \frac{\dot{q}}{H}, \quad (42)$$

$$s = \frac{r - 1}{3(q - 1/2)}.$$

In the current framework, (42) take the form

$$r = -\frac{1}{H_1 T^2 (-6H_0 \mu + \sqrt{-6T}(H_1 + \mu - H_1 \nu))^2} \cdot 3(-3H_0(36\sqrt{6}H_0^4 \sqrt{-T} + 36\sqrt{6}H_0^2 (-T)^{3/2} + \sqrt{6}(-T)^{5/2} + 144H_0^3 T - 24H_0 T^2)\mu^2 - H_1^2 T^2 (-54H_0^2 + 13H_0 \sqrt{-6T} + 4T)\mu(-1 + \nu) + 2H_1^3 T^3 (-1 + \nu)^2 + H_1 T \mu (2T^2 \mu - 6H_0^2 T(29\mu + 12(-1 + \nu)) + 108H_0^4 (-2 + 3\mu + 2\nu) + H_0 \sqrt{-6T} T(-3 + 13\mu + 3\nu) - 18\sqrt{6}H_0^3 \sqrt{-T}(-5 + 9\mu + 5\nu))), \quad (43)$$

$$s = \frac{1}{3H_1 T (6H_0 \mu + \sqrt{6}\sqrt{-T}(-H_1 - \mu + H_1 \nu))} [36H_0^3 \mu + \sqrt{6}(-1 + 3H_1) \sqrt{-T} T(-H_1 - \mu + H_1 \nu) + 18H_0 T(-\mu + H_1(-1 + 2\mu + \nu)) + 6\sqrt{6}H_0^2 \sqrt{-T}(-3\mu + H_1(-2 + 3\mu + 2\nu))].$$

It may be noted that in the present and subsequent figures, red, green, and blue lines correspond to $n = 6, 8$, and 10 , respectively. Figure 1 shows that $f(T)$ is decreasing with the increase of $|T|$. Figure 2 shows the evolution of the effective torsion EoS parameter w_T as a function of t . In this case, $w_T > -1$ and it is running close to -1 , but it is not crossing -1 boundary. This indicates “quintessence” behavior. In later time, $-6H_0 + \sqrt{-6T} \rightarrow 0$ (see (36)) and as a consequence $w_T \rightarrow -1$. A clear transition from $q > 0$ to $q < 0$ is apparent at $t \approx 0.5$ in Figure 3. This indicates transition from decelerated to accelerated phase of the universe. In Figure 4, it is observed that w_{tot} behaves differently from w_{eff} . The w_{tot} transits from > -1 , that is, quintessence, to < -1 , that is, phantom at $t \approx 1$. Statefinder parameters as obtained in (43) are plotted in Figure 5, and it is observed that the fixed point $\{r = 1, s = 0\}_{\Lambda\text{CDM}}$ is attainable and the $\{r - s\}$ trajectory goes beyond the ΛCDM . It is palpable that, for finite r , we have $s \rightarrow -\infty$. This indicates that the holographic polytropic $f(T)$ gravity interpolates between dust and ΛCDM phase of

FIGURE 1: Reconstructed $f(T)$ (35) and we see that $f(T) \rightarrow 0$ as $T \rightarrow 0$.

FIGURE 2: Effective torsion EoS parameter as in (36).

the universe. In this framework, the cosmographic parameter j (jerk) comes out to be

$$j = -\frac{7}{2} + \frac{2H_1}{(H_1 + H_0 t)^3} + \frac{9H_0 t \mu}{2(H_1 + H_0 t)(H_1 + H_0 t + \mu - (H_1 + H_0 t)\nu)}. \quad (44)$$

FIGURE 3: Deceleration parameter as in (37).

FIGURE 4: Plot of w_{tot} as in (41).

2.2. With Specific Form of f and without Any Assumption about H

Power-Law Model of Bengochea and Ferraro. In this section, we are not assuming any form of H or a . Rather we assume f as the power-law model of Bengochea and Ferraro [52]

$$f(T) = \alpha (-T)^b, \quad (45)$$

where α and b are the two model parameters. Considering $\rho_\Lambda = \rho_D$, we have the following differential equation:

$$\frac{\mu a}{2} \left(\frac{dH^2}{da} \right) + \nu H^2 = \frac{1}{3} (Ba^{1/n} - K)^{-n} \quad (46)$$

solving which we get

$$H^2 = a^{-2\nu/\mu} C_1 + \frac{(1 - a^{1/n} B/K)^n (a^{1/n} B - K)^{-n} 2F_1 [2n\nu/\mu, n, 1 + 2n\nu/\mu, a^{1/n} B/K]}{3\nu} \quad (47)$$

that leads to

$$\dot{H} = \frac{-3a^{-2\nu/\mu} C_1 \nu + (a^{1/n} B - K)^{-n} (1 - (1 - a^{1/n} B/K)^n 2F_1 [2n\nu/\mu, n, 1 + 2n\nu/\mu, a^{1/n} B/K])}{3\mu}. \quad (48)$$

Therefore, using $T = -6H^2$ in (45) and thereafter using (30), we have the dark matter density of the universe as a function of a as

$$\rho_m = \frac{1}{2} (1 - 2b) \alpha \left(6a^{-2\nu/\mu} C_1 + \frac{2(1 - a^{1/n} B/K)^n (a^{1/n} B - K)^{-n} 2F_1 [2n\nu/\mu, n, 1 + 2n\nu/\mu, a^{1/n} B/K]}{\nu} \right)^b. \quad (49)$$

Using (49) in (39), we have for the present choice of $f(T)$

$$\dot{H} = - \frac{3a^{-2\nu/\mu} C_1 + (1 - a^{1/n} B/K)^n (a^{1/n} B - K)^{-n} 2F_1 [2n\nu/\mu, n, 1 + 2n\nu/\mu, a^{1/n} B/K] / \nu}{2b}. \quad (50)$$

As we are considering new holographic polytropic dark energy in $f(T)$ gravity, we can consider equality of (50) and

(48) from which we can express the integration constant C_1 as

$$C_1 = \frac{a^{2\nu/\mu} (a^{1/n} B - K)^{-n} (2b\nu + (1 - a^{1/n} B/K)^n (3\mu - 2b\nu) 2F_1 [2n\nu/\mu, n, 1 + 2n\nu/\mu, a^{1/n} B/K])}{3\nu(-3\mu + 2b\nu)}. \quad (51)$$

As (51) is used in (47), H^2 reduces to

and hence

$$H^2 = - \frac{2b(a^{1/n} B - K)^{-n}}{9\mu - 6b\nu} \quad (52)$$

$$\dot{H} = \frac{a^{1/n} b B (a^{1/n} B - K)^{-1-n}}{9\mu - 6b\nu}. \quad (53)$$

Subsequently, effective torsion EoS and deceleration parameters become

$$w_T = -1$$

$$+ \frac{a^{1/n} b B \left(4 + \alpha \left(b (a^{1/n} B - K)^{-n} / (-3\mu + 2b\nu) \right)^b \left(-2^{3+2b} b + (a^{1/n} B - K)^n (-34^b \mu + 2^{1+2b} b\nu) \right) \right)}{3(a^{1/n} B - K) \left(4b + (a^{1/n} B - K)^n \alpha \left(b (a^{1/n} B - K)^{-n} / (-3\mu + 2b\nu) \right)^b (34^b \mu + 2^{1+2b} b(-3\mu + (-1 + 2b)\nu)) \right)}, \quad (54)$$

$$q_T = \frac{4 - 3/b + 16b(a^{1/n} B - K)^{-n} / (-3\mu + 2b\nu)}{2(-1 + 2b)}, \quad (55)$$

$$w_{\text{tot}} = -1 + \frac{4a^{1/n} B + (a^{1/n} (34^b - 74^b b + 2^{1+2b} b^2) B - 34^b K + 32^{1+2b} b K) \alpha \left(b (a^{1/n} B - K)^{-n} / (-3\mu + 2b\nu) \right)^{-1+b}}{12(a^{1/n} B - K)}. \quad (56)$$

In this framework, (42) take the form

$$r = \frac{\left(4 - 3/b + 16b \left(a^{1/n}B - K\right)^{-n} / (-3\mu + 2b\nu)\right) \left(3 - 3/b + b \left(2 + 16 \left(a^{1/n}B - K\right)^{-n} / (-3\mu + 2b\nu)\right)\right)}{2(1 - 2b)^2}, \quad (57)$$

$$s = \frac{2 - 3/b + b \left(4 + 16 \left(a^{1/n}B - K\right)^{-n} / (-3\mu + 2b\nu)\right)}{3(-1 + 2b)}$$

and the other cosmographic parameter j (jerk parameter) (using (15)) takes the form

$$j = \frac{1}{2} \left(-4 + \frac{a^{1/n}B(K + a^{1/n}Bn)}{(-a^{1/n}B + K)^2 n} + \frac{3 \left(-4 + 3/b + 16b \left(a^{1/n}B - K\right)^{-n} / (3\mu - 2b\nu) \right)}{-1 + 2b} \right). \quad (58)$$

In Figure 6, $f(T)$ is plotted against T and it is observed that $f(T) \rightarrow -\infty$ as $T \rightarrow 0$. The effective torsion parameter is plotted in Figure 7 and it is palpable that $w_T < -1$ that behaves like phantom. The deceleration parameter plotted in Figure 8 shows an ever-accelerating universe; $w_{\text{tot}} < -1$ that

behaves like phantom as seen in Figure 9. The statefinders as obtained in (57) are plotted in Figure 10 and $\{r - s\}$ trajectory attains the Λ CDM point, that is, $\{r = 1, s = 0\}$. However, unlike the previous model, the dust phase is not apparently attained by the statefinder trajectory.

Exponential Model. We consider exponential $f(T)$ gravity [53] as

$$f(T) = \delta \exp(\xi T). \quad (59)$$

Subsequently, using $T = -6H^2$ in (59), where H^2 is as obtained in (47), and thereafter using (30), we have the dark matter density of the universe as a function of a for the present choice of f as follows:

$$\rho_m = \frac{1}{2\nu} a^{-2\nu/\mu} e^{(-6a^{-2\nu/\mu} C_1 \xi - 2(1 - a^{1/n}B/K)^n (a^{1/n}B - K)^{-n} \xi 2F_1[2n\nu/\mu, n, 1 + 2n\nu/\mu, a^{1/n}B/K]/\nu)} (a^{1/n}B - K)^{-n} \cdot \delta \left((a^{1/n}B - K)^n \nu (a^{2\nu/\mu} + 12C_1 \xi) + 4a^{2\nu/\mu} \left(1 - \frac{a^{1/n}B}{K} \right)^n \xi 2F_1 \left[\frac{2n\nu}{\mu}, n, 1 + \frac{2n\nu}{\mu}, \frac{a^{1/n}B}{K} \right] \right). \quad (60)$$

Using (60) in (39), we have for the present choice of $f(T)$

$$\dot{H} = \frac{(a^{1/n}B - K)^n \nu (a^{2\nu/\mu} + 12C_1 \xi) + 4a^{2\nu/\mu} (1 - a^{1/n}B/K)^n \xi 2F_1[2n\nu/\mu, n, 1 + 2n\nu/\mu, a^{1/n}B/K]}{4\xi \left(- (a^{1/n}B - K)^n \nu (a^{2\nu/\mu} - 12C_1 \xi) + 4a^{2\nu/\mu} (1 - a^{1/n}B/K)^n \xi 2F_1[2n\nu/\mu, n, 1 + 2n\nu/\mu, a^{1/n}B/K] \right)}. \quad (61)$$

Considering equality of (50) and (61), we can express C_1 as

$$C_1 = \frac{1}{24\nu^2 \xi^2} (a^{1/n}B - K)^{-2n} \left((a^{4\nu/\mu} (a^{1/n}B - K)^{2n} \cdot \nu^2 \xi^2 \left((a^{1/n}B - K)^{2n} (9\mu^2 - 18\mu\nu + \nu^2) - 8(a^{1/n}B - K)^n (3\mu + \nu) \xi + 16\xi^2 \right) \right)^{1/2}$$

$$+ a^{2\nu/\mu} (a^{1/n}B - K)^n \nu \xi \left((a^{1/n}B - K)^n (-3\mu + \nu) + 4\xi - 8 \left(1 - \frac{a^{1/n}B}{K} \right)^n \xi 2F_1 \left[\frac{2n\nu}{\mu}, n, 1 + \frac{2n\nu}{\mu}, \frac{a^{1/n}B}{K} \right] \right) \right)$$

(62)

that finally leads to

FIGURE 5: Statefinder parameters for the choice of $H = H_0 + H_1/t$. FIGURE 6: Plot of $f(T)$ based on reconstructed H .

$$\begin{aligned} \dot{H} = & \left(a^{1/n} B (a^{1/n} B - K)^{-1-n} \left(a^{2\nu/\mu} (a^{1/n} B - K)^n \nu \left((a^{1/n} B - K)^n (3\mu + \nu) - 4\xi \right) \xi \right. \right. \\ & \left. \left. - \sqrt{a^{4\nu/\mu} (a^{1/n} B - K)^{2n} \nu^2 \xi^2 \left((a^{1/n} B - K)^{2n} (9\mu^2 - 18\mu\nu + \nu^2) - 8(a^{1/n} B - K)^n (3\mu + \nu) \xi + 16\xi^2 \right)} \right) \right) \\ & \cdot \left(12\nu \sqrt{a^{4\nu/\mu} (a^{1/n} B - K)^{2n} \nu^2 \xi^2 \left((a^{1/n} B - K)^{2n} (9\mu^2 - 18\mu\nu + \nu^2) - 8(a^{1/n} B - K)^n (3\mu + \nu) \xi + 16\xi^2 \right)} \right)^{-1}. \end{aligned} \quad (63)$$

Figure 11 shows that $f(T)$ is decreasing with the increase in T . It is also observed that after certain stage $f(T)$ is behaving asymptotically. So, this behavior is contrary to what happened in the last two models. Effective torsion parameter w_T displayed in Figure 12 behaves like phantom and deceleration parameter displayed in Figure 13 makes an ever-accelerating universe apparent. For $n = 6$ (red line), w_{total} crosses phantom-divide line at $z \approx 3.8$ (Figure 14). However, for $n = 8$ and 10 , w_{total} stays below the phantom-divide line. The statefinder parameters $\{r, s\}$, when plotted in Figure 15, is found to reach $\{r = 1, s = 0\}_{\Lambda_{\text{CDM}}}$, but they can not effectively go beyond it.

2.3. Comparison with Observational Schemes. By implying different combination of observational schemes at 95% confidence level, Ade et al. [54] (Planck data) provided the following constraints for EoS:

$$\begin{aligned} w_{\text{DE}} &= -1.13_{-0.25}^{+0.24} \quad (\text{Planck} + \text{WP} + \text{BAO}), \\ w_{\text{DE}} &= -1.09 \pm 0.17, \quad (\text{Planck} + \text{WP} + \text{Union 2.1}), \\ w_{\text{DE}} &= -1.13_{-0.14}^{+0.13}, \quad (\text{Planck} + \text{WP} + \text{SNLS}), \\ w_{\text{DE}} &= -1.24_{-0.19}^{+0.18}, \quad (\text{Planck} + \text{WP} + H_0). \end{aligned} \quad (64)$$

The trajectories of EoS parameter also favor the following nine-year WMAP observational data

$$\begin{aligned} w_{\text{DE}} &= -1.073_{-0.089}^{+0.090} \\ & \quad (\text{WMAP} + \text{eCMB} + \text{BAO} + H_0), \\ w_{\text{DE}} &= -1.084 \pm 0.063, \\ & \quad (\text{WMAP} + \text{eCMB} + \text{BAO} + H_0 + \text{SNe}). \end{aligned} \quad (65)$$

It is interesting to mention here that the ranges of EoS parameter for both cases lie within these observational constraints.

3. Stability

The stability analysis of underconsideration models in the present framework is being discussed in this section. For this purpose, we consider squared speed of sound which has the following expression:

$$v_s^2 = \frac{\dot{P}_T}{\dot{\rho}_T}. \quad (66)$$

FIGURE 7: Effective torsion EoS parameter for the Bengochea and Ferraro model.

FIGURE 9: Plot of w_{tot} as in (56).

FIGURE 8: Deceleration parameter q_T for the Bengochea and Ferraro model.

FIGURE 10: Statefinder parameters for the choice of $f(T) = \alpha(-T)^b$.

The sign of this parameter is very important in order to analyze the stability of the model. This depicts the stable behavior for positive v_s^2 while its negativity expresses instability of the underconsideration model. Inserting corresponding expressions and after some calculations, we can obtain squared speed of sound for all cases. We draw the graphs versus t for $n = 6$ in each case taking same values for the parameters to discuss the stability of the reconstructed $f(T)$ model. We provide a discussion about stability in each case in the following.

(i) *With a Specific Choice of H .* Figure 16 represents the behavior of v_s^2 versus t for the particular choice of H . The graph shows unstable behavior initially but for a period $t < 1.4$. After this interval of time, squared speed of sound parameter maintains increasing behavior and becomes positive expressing stability of the model.

(ii) *Without Any Choice of H .* In this case, squared speed of sound shows increasing and positive behavior which exhibits the stability of the reconstructed model. The corresponding plot is given in Figure 17.

FIGURE 11: Reconstructed $f(T)$ for the exponential model and we see that $f(T)$ becomes a decreasing function of T .

FIGURE 12: Effective torsion EoS parameter based on reconstructed $\dot{H}$ in (63).

(iii) *Exponential Model.* Taking into account the case of exponential model, we plot the squared speed of sound parameter versus t as shown in Figure 18. v_s^2 represents a positively decreasing behavior establishing stability of the reconstructed model in this case throughout the time interval.

4. Concluding Remarks

In the present work, we have new holographic reconstructed polytropic dark energy and these kinds of holographic reconstruction of other dark energy models are already reported

FIGURE 13: Deceleration parameter using (63) in (37).

FIGURE 14: Plot of w_{tot} using (63) in (41).

in [48–50, 55]. Viewing $f(T)$ as an effective description of the underlying theory of DE and considering the new holographic polytropic dark energy as point in the direction of the underlying theory of DE, we have studied how the modified gravity can describe the new holographic polytropic dark energy as effective theory of DE. This approach is largely motivated by [56, 57]. We have carried out this work through two approaches. In the first approach, we have chosen H as $H = H_0 + H_1/t$ and consequently generated reconstructed $f(T)$ that is found to tend to 0 with T tending to 0 and thereby satisfying one of the sufficient conditions for a realistic model [57]. The effective torsion EoS parameter

FIGURE 15: Statefinder parameters for the choice of $f(T) = \delta \exp(\xi T)$.

FIGURE 16: Plot of squared speed of sound with specific form of H .

coming out of this reconstructed $f(T)$ is found to stay above -1 in contradiction to w_{tot} showing a clear transition from quintessence to phantom, that is, quintom. The deceleration parameter exhibits transition from decelerated to accelerated phase. The statefinder parameters $\{r, s\}$ could attain ΛCDM $\{r = 1, s = 0\}$ and could go beyond it. More particularly, it has been apparent from the statefinder plot that for finite r we have $s \rightarrow -\infty$ that indicates dust phase. Hence, this reconstructed $f(T)$ model interpolates between dust and ΛCDM phase of the universe.

In the second approach instead of considering any particular form of the scale factor, we have assumed a power-law and exponential solutions for $f(T)$ as proposed in [52]

FIGURE 17: Plot of squared speed of sound for power-law form of $f(T)$.

FIGURE 18: Plot of squared speed of sound for exponential form of $f(T)$.

and [53], respectively. Under power-law solution, we derived expressions for $\dot{H}$ in terms of a . Thereafter, we derived effective torsion EoS and deceleration parameters and also the statefinder parameters. For this reconstructed H , the $f(T)$ has been found to behave like the earlier approach that is tending to 0 as T tends to 0. As plotted against redshift z , the effective torsion EoS and w_{tot} are found to exhibit phantom-like behavior. The deceleration parameter is found to stay negative, that is, exhibited accelerated expansion. Although the statefinder plot could attain ΛCDM , no clear attainment of dust phase is apparent. Under exponential solution of $f(T)$, we derived expressions for $\dot{H}$ in terms of a and subsequently reconstructed f does not tend to 0 as T tends to 0 and hence

FIGURE 19: Jerk parameter plot corresponding to (44).

FIGURE 20: Jerk parameter plot against redshift z corresponding to (58).

it does not satisfy the sufficient condition for realistic model. The effective torsion equation of state parameter derived this way exhibited phantom-like behavior. However, w_{total} exhibits a transition from > -1 to < -1 for $n = 6$. We have discussed the stability of the model through squared speed of sound in all cases. We have obtained large intervals where the models behave like stable models. Cosmographic parameter j , based on (44) and (58) plotted against t and z in Figures 19 and 20, respectively, shows that for both reconstruction models with and without any choice of H the jerk parameter j is increasing gradually with evolution of the universe and

remains positive throughout. This observation is somewhat consistent with the work of [58].

In view of the above, although both of the approaches are found to be somewhat consistent with the expected cosmological consequences, the first approach could be stated to be more acceptable as it could show a transition from decelerated to accelerated expansion and could interpolate between dust and Λ CDM phases of the universe. Secondly, in the first approach, w_{total} transited from quintessence to phantom that is found to be consistent with the outcomes of [53], where cosmological evolutions of the equation of state for DE in $f(T)$ gravity were seen to have a transition of similar nature. However, one major difference between [53] and the present work lies in the fact that in [53] the equation of state parameter behaved like quintom irrespective of exponential power-law or combined $f(T)$ gravity. Contrarily, in our present work, the equation of state parameter of holographic polytropic gas DE does not necessarily exhibit quintom behavior.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

Constructive suggestions from the reviewers are thankfully acknowledged by the authors. Visiting Associateship of IUCAA, Pune, India, and financial support from DST, Government of India under project Grant no. SR/FTP/PS-167/2011 are acknowledged by Surajit Chattopadhyay.

References

- [1] A. G. Riess, A. V. Filippenko, P. Challis et al., "Observational evidence from supernovae for an accelerating universe and a cosmological constant," *The Astronomical Journal*, vol. 116, no. 3, pp. 1009–1038, 1998.
- [2] S. Perlmutter, G. Aldering, G. Goldhaber et al., "Measurements of Ω and Λ from 42 high-redshift supernovae," *The Astrophysical Journal*, vol. 517, no. 2, pp. 565–586, 1999.
- [3] R. R. Caldwell and M. Doran, "Cosmic microwave background and supernova constraints on quintessence: concordance regions and target models," *Physical Review D*, vol. 69, no. 10, Article ID 103517, 2004.
- [4] T. Koivisto and D. F. Mota, "Dark energy anisotropic stress and large scale structure formation," *Physical Review D*, vol. 73, no. 8, Article ID 083502, 2006.
- [5] C. Fedeli, L. Moscardini, and M. Bartelmann, "Observing the clustering properties of galaxy clusters in dynamical dark-energy cosmologies," *Astronomy and Astrophysics*, vol. 500, no. 2, pp. 667–679, 2009.
- [6] E. J. Copeland, M. Sami, and S. Tsujikawa, "Dynamics of dark energy," *International Journal of Modern Physics D*, vol. 15, no. 11, pp. 1753–1936, 2006.
- [7] K. Bamba, S. Capozziello, S. Nojiri, and S. D. Odintsov, "Dark energy cosmology: the equivalent description via different theoretical models and cosmography tests," *Astrophysics and Space Science*, vol. 342, no. 1, pp. 155–228, 2012.

- [8] R. R. Caldwell and M. Kamionkowski, "The physics of cosmic acceleration," *Annual Review of Nuclear and Particle Science*, vol. 59, no. 1, pp. 397–429, 2009.
- [9] S. Nojiri and S. D. Odintsov, "Unified cosmic history in modified gravity: from $f(R)$ theory to Lorentz non-invariant models," *Physics Reports*, vol. 505, no. 2–4, pp. 59–144, 2011.
- [10] V. Sahni and A. Starobinsky, "The case for a positive cosmological λ -term," *International Journal of Modern Physics D*, vol. 9, no. 4, pp. 373–443, 2000.
- [11] V. Sahni and A. A. Starobinsky, "Reconstructing dark energy," *International Journal of Modern Physics D*, vol. 15, no. 12, article 2105, 2006.
- [12] H. Motohashi, A. A. Starobinsky, and J. Yokoyama, "Phantom boundary crossing and anomalous growth index of fluctuations in viable $f(R)$ models of cosmic acceleration," *Progress of Theoretical Physics*, vol. 123, no. 5, pp. 887–902, 2010.
- [13] P. J. E. Peebles and B. Ratra, "The cosmological constant and dark energy," *Reviews of Modern Physics*, vol. 75, no. 2, pp. 559–606, 2003.
- [14] S. Nojiri, S. D. Odintsov, and O. G. Gorbunova, "Dark energy problem: from phantom theory to modified Gauss-Bonnet gravity," *Journal of Physics A: Mathematical and General*, vol. 39, no. 21, pp. 6627–6633, 2006.
- [15] A. V. Astashenok, S. Nojiri, S. D. Odintsov, and R. J. Scherrer, "Scalar dark energy models mimicking Λ CDM with arbitrary future evolution," *Physics Letters B*, vol. 713, no. 3, pp. 145–153, 2012.
- [16] B. Gumjudpai, T. Naskar, M. Sami, and S. Tsujikawa, "Coupled dark energy: towards a general description of the dynamics," *Journal of Cosmology and Astroparticle Physics*, vol. 2005, no. 6, pp. 7–7, 2005.
- [17] E. Elizalde, S. Nojiri, and S. D. Odintsov, "Late-time cosmology in a (phantom) scalar-tensor theory: dark energy and the cosmic speed-up," *Physical Review D*, vol. 70, no. 4, Article ID 043539, 2004.
- [18] S. Nojiri, S. D. Odintsov, and S. Tsujikawa, "Properties of singularities in the (phantom) dark energy universe," *Physical Review D*, vol. 71, no. 6, Article ID 063004, 2005.
- [19] H. Zhang and Z.-H. Zhu, "Interacting chaplygin gas," *Physical Review D*, vol. 73, no. 4, Article ID 043518, 2006.
- [20] K. Bamba, J. Matsumoto, and S. Nojiri, "Cosmological perturbations in the k -essence model," *Physical Review D*, vol. 85, Article ID 084026, 2012.
- [21] M. Forte, "Linking phantom quintessences and tachyons," *Physical Review D*, vol. 90, no. 2, Article ID 027302, 3 pages, 2014.
- [22] S. Nojiri and S. D. Odintsov, "Introduction to modified gravity and gravitational alternative for dark energy," *International Journal of Geometric Methods in Modern Physics*, vol. 4, no. 1, pp. 115–145, 2007.
- [23] S. Nojiri and S. D. Odintsov, "Unified cosmic history in modified gravity: from $f(R)$ theory to Lorentz non-invariant models," *Physics Reports*, vol. 505, no. 2–4, pp. 59–144, 2011.
- [24] S. Nojiri and S. D. Odintsov, "Unifying phantom inflation with late-time acceleration: scalar phantom-non-phantom transition model and generalized holographic dark energy," *General Relativity and Gravitation*, vol. 38, no. 8, pp. 1285–1304, 2006.
- [25] S. Nojiri and S. D. Odintsov, "Modified Gauss-Bonnet theory as gravitational alternative for dark energy," *Physics Letters B*, vol. 631, no. 1–2, pp. 1–6, 2005.
- [26] M. H. Daouda, M. E. Rodrigues, and M. J. S. Houndjo, "Reconstruction of $f(T)$ gravity according to holographic dark energy," *The European Physical Journal C*, vol. 72, article 1893, 2012.
- [27] M. R. Setare and F. Darabi, "Power-law solutions in $f(T)$ gravity," *General Relativity and Gravitation*, vol. 44, no. 10, pp. 2521–2527, 2012.
- [28] M. U. Farooq, M. Jamil, D. Momeni, and R. Myrzakulov, "Reconstruction of $f(T)$ and $f(R)$ gravity according to (m, n) -type holographic dark energy," *Canadian Journal of Physics*, vol. 91, no. 9, pp. 703–708, 2013.
- [29] K. Karami and A. Abdolmaleki, " $f(T)$ modified teleparallel gravity as an alternative for holographic and new agegraphic dark energy models," *Research in Astronomy and Astrophysics*, vol. 13, no. 7, pp. 757–771, 2013.
- [30] M. Sharif and S. Rani, "Generalized teleparallel gravity via some scalar field dark energy models," *Astrophysics and Space Science*, vol. 345, no. 1, pp. 217–223, 2013.
- [31] M. Sharif and S. Rani, "Entropy corrected holographic dark energy $f(T)$ gravity model," *Modern Physics Letters A*, vol. 29, no. 2, Article ID 1450015, 2014.
- [32] G. 't Hooft, "Dimensional reduction in quantum gravity," Report no. THU-93/26, <http://arxiv.org/abs/gr-qc/9310026>.
- [33] L. Susskind, "The world as a hologram," *Journal of Mathematical Physics*, vol. 36, no. 11, pp. 6377–6399, 1995.
- [34] J. D. Bekenstein, "Black holes and entropy," *Physical Review D*, vol. 7, no. 8, pp. 2333–2346, 1973.
- [35] A. G. Cohen, D. B. Kaplan, and A. E. Nelson, "Effective field theory, black holes, and the cosmological constant," *Physical Review Letters*, vol. 82, no. 25, pp. 4971–4974, 1999.
- [36] M. Li, "A model of holographic dark energy," *Physics Letters B*, vol. 603, no. 1–2, pp. 1–5, 2004.
- [37] W. Hao, "Entropy-corrected holographic dark energy," *Communications in Theoretical Physics*, vol. 52, no. 4, pp. 743–749, 2009.
- [38] A. Sheykhi and M. Jamil, "Power-Law entropy corrected holographic dark energy model," *General Relativity and Gravitation*, vol. 43, no. 10, pp. 2661–2672, 2011.
- [39] K. Karami and A. Abdolmaleki, "Polytropic and Chaplygin $f(T)$ -gravity models," *Journal of Physics: Conference Series*, vol. 375, no. 3, Article ID 032009, 2012.
- [40] M. Sharif and S. Rani, "Nonlinear electrodynamics in $f(T)$ gravity and generalized second law of thermodynamics," *Astrophysics and Space Science*, vol. 346, no. 2, pp. 573–582, 2013.
- [41] M. Sharif and S. Rani, "Pilgrim dark energy in $f(T)$ gravity," *Journal of Experimental and Theoretical Physics*, vol. 119, no. 1, pp. 75–82, 2014.
- [42] K. Karami and S. Ghaffari, "The generalized second law of thermodynamics for the interacting polytropic dark energy in non-flat FRW universe enclosed by the apparent horizon," *Physics Letters B*, vol. 688, no. 2–3, pp. 125–128, 2010.
- [43] K. Karami, S. Ghaffari, and J. Fehri, "Interacting polytropic gas model of phantom dark energy in non-flat universe," *The European Physical Journal C*, vol. 64, no. 1, pp. 85–88, 2009.
- [44] L. N. Granda and A. Oliveros, "Infrared cut-off proposal for the holographic density," *Physics Letters B*, vol. 669, no. 5, pp. 275–277, 2008.
- [45] V. Sahni, T. D. Saini, A. A. Starobinsky, and U. Alam, "Statefinder—a new geometrical diagnostic of dark energy," *Journal of Experimental and Theoretical Physics Letters*, vol. 77, no. 5, pp. 201–206, 2003.

- [46] A. Aviles, A. Bravetti, S. Capozziello, and O. Luongo, "Cosmographic reconstruction of $f(T)$ cosmology," *Physical Review D*, vol. 87, no. 6, Article ID 064025, 2013.
- [47] S. Capozziello, O. Luongo, and E. N. Saridakis, "Transition redshift in $f(T)$ cosmology and observational constraints," *Physical Review D*, vol. 91, no. 12, Article ID 124037, 2015.
- [48] K. Karami and J. Fehri, "New holographic scalar field models of dark energy in non-flat universe," *Physics Letters B*, vol. 684, no. 2-3, pp. 61-68, 2010.
- [49] U. Debnath, "Reconstructions of scalar field dark energy models from new holographic dark energy in Galileon universe," *The European Physical Journal Plus*, vol. 129, article 272, 2014.
- [50] A. Sheykhi, "Holographic scalar field models of dark energy," *Physical Review D*, vol. 84, no. 10, Article ID 107302, 2011.
- [51] S. Nojiri and S. D. Odintsov, "Modified $f(R)$ gravity consistent with realistic cosmology: from a matter dominated epoch to a dark energy universe," *Physical Review D*, vol. 74, no. 8, Article ID 086005, 2006.
- [52] G. R. Bengochea and R. Ferraro, "Dark torsion as the cosmic speed-up," *Physical Review D*, vol. 79, no. 12, Article ID 124019, 2009.
- [53] K. Bamba, C.-Q. Geng, C.-C. Lee, and L.-W. Luo, "Equation of state for dark energy in $f(T)$ gravity," *Journal of Cosmology and Astroparticle Physics*, vol. 2011, no. 1, article 021, 2011.
- [54] P. A. R. Ade, N. Aghanim, C. Armitage-Caplan et al., "Planck 2013 results. XVI. Cosmological parameters," *Astronomy & Astrophysics*, vol. 571, article A16, 66 pages, 2014.
- [55] S. Chattopadhyay, A. Pasqua, and M. Khurshudyan, "New holographic reconstruction of scalar-field dark-energy models in the framework of chameleon Brans-Dicke cosmology," *The European Physical Journal C*, vol. 74, no. 9, article 3080, 2014.
- [56] W.-Q. Yang, Y.-B. Wu, L.-M. Song et al., "Reconstruction of new holographic scalar field models of dark energy in Brans-Dicke universe," *Modern Physics Letters A*, vol. 26, no. 3, pp. 191-204, 2011.
- [57] M. R. Setare, "Holographic modified gravity," *International Journal of Modern Physics D*, vol. 17, no. 12, article 2219, 2008.
- [58] S. Pan and S. Chakraborty, "A cosmographic analysis of holographic dark energy models," *International Journal of Modern Physics D*, vol. 23, no. 11, Article ID 1450092, 2014.