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Research Article

An Alternative Approach to Solutions of the MGECSC Potential in Presence of External Electric Field

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For the first time the Schrödinger equation with more general exponential cosine screened Coulomb potential in the presence of external electric field is solved approximately and analytically by applying an ansatz to eigenfunction of corresponding Hamiltonian and then energy values and wave functions are obtained. Since this potential turns into four different potential cases when considering different cases of the parameters in the potential, energies and eigenfunctions for these four different potentials are already to be found by solving Schrödinger equation with MGECSC potential. Energy values and wave functions obtained by using different values of potential parameters for each of these four different potential are compared with the results of other studies. Since the obtained general solutions in this study have been found in the presence of external electric field, the external electric field effects on systems with the mentioned four different potentials are also easily investigated. One of advantages of the present results and method is that if external electric field is equal to zero, general mathematical structure of corresponding equations does not change and then electric field effect can be eliminated. The presence or absence of electric field does not prevent solving the Schrödinger equation analytically.

1. Introduction

Exponential Coulomb potentials with and without cosine term are used to investigate important interactions in various fields of physics such as plasma physics, nuclear physics, condensed matter physics, and atomic physics. Exponential Coulomb (EC) potential also called Screened Coulomb (SC) potential is expressed as

$$V_1(r) = -\frac{A}{r}e^{-\alpha_1 r}, \tag{1}$$

where α_1 is screening parameter and A is strength coupling constant. EC potential that is also known as Yukawa-type potential acts between two particles and this is well known in nuclear physics as dominant central part of nucleon-nucleon interaction [1]. However, this potential is considered as Thomas-Fermi potential in applications of condensed matter and atomic physics [2, 3]. In addition, EC (or SC) potential is the well-known Debye-Hückel potential for plasma physics applications and used to study Debye plasma [4–20].

For some specific cases of potential screening parameters, exponential cosine screened Coulomb potential (ECSC) (or generalized exponential cosine screened Coulomb potential) can be reduced Yukawa and Pure Coulomb (PC) potentials and it has the following form:

$$V_2(r) = -\frac{B}{r}e^{-\alpha_2 r}\,\mathrm{Cos}\left(\beta_2 r\right), \tag{2}$$

where α_2 is screening parameter and B is constant. This potential can be also used for some specific cases of potential screening parameters in studies of nuclear and atomic physics. But ECSC potential is considered to model quantum plasma rather than using in studies of nuclear and atomic physics. Investigation of relativistic and nonrelativistic systems including ECSC potential is very difficult analytically due to cosine term in the potential and used methods up to now in order to obtain corresponding solutions have been performed numerically. The Schrödinger equation with ECSC potential was solved numerically within framework

Ritz variation method in considering hydrogenic wave function as a trial wave function [21]. As an alternative method, the same problem has been examined numerically using asymptotic iteration method (AIM) and only bound states energies could be obtained. The numerical methods such as SUSY-perturbation, perturbation, variational, and hypervirial Pade scheme method have been applied on mentioned problem [22, 23]. The more general exponential cosine screened Coulomb (MGECSC) potential has been considered to model Debye and quantum plasma for the first time [24]. MGECSC potential has more complex form compared to EC, PC and ECSC potentials and to obtain its numerical solutions is more difficult than that of EC, PC and ECSC potentials. The MGECSC potential is expressed in the following form:

$$V_3(r) = -\frac{C}{r}(1+br)\,e^{-\alpha_3 r}\,\mathrm{Cos}\left(\beta_3 r\right), \tag{3}$$

where α_3, β_3, and b are screening parameters and C is a constant. For the first time in [24], this potential has been suggested to investigate plasma screening effects on hydrogen atom in Debye and quantum plasmas and bound states energies of hydrogen atom have been obtained numerically within framework AIM. However, in [24], it was not possible to find wave functions of hydrogen atom in plasmas due to the nature of using numerical AIM. There are important differences between all abovementioned potentials in terms of screening effects exhibited by potential parameters. To put it more clearly, ECSC potential exhibits a stronger screening effect than that of SC and PC potentials because it contains cosine term in the structure. It should be pointed out that SC potential can be reduced to PC potential under suitable conditions [25]. The MGECSC potential used in present study is more operable and important according to SC, PC, and ECSC potentials in both the physical and mathematical sense, one of the justifications in which is that MGECSC potential exhibits stronger screening effect than that of SC, PC, and ECSC potential. Note that mentioned stronger screening effect arises from both cosine term and b screening parameter in the MGECSC potential. In addition to this, it should be particularly noted that since it has a more complex form than other potentials serving the same purpose, more different and specific methods are needed in order to be able to investigate systems with MGECSC potential. One of the advantages of MGECSC potential is also that this can be reduced SC, PC, and ECSC potentials in consideration of different cases of screening parameters in the structure. By this means, when examining relativistic and nonrelativistic systems including MGECSC potential, solutions of other systems with SC, PC, and ECSC potential would also be obtained already. It is an important point to mention that since the Schrödinger equation including MGECSC potential in presence of external electric field has been considered in present study, solutions obtained in here include all results of studies with SC, PC, and ECSC potential in presence and absence of external electric field. The most crucial point of present study is that for the first time the Schrödinger equation with MGECSC potential in presence and absence of external electric field has been solved analytically by applying an ansatz to eigenfunction of corresponding Hamiltonian. Applying an ansatz to corresponding eigenfunction is operable method for relativistic and nonrelativistic quantum mechanical systems with certain central potentials [26–31].

The paper is organized as follows. In Section 2, in the presence of external electric field, the Schrödinger equation with MGECSC potential is briefly outlined and applied ansatz to corresponding eigenfunction is introduced. Later, bound states and eigenfunctions are obtained. In Section 3, the obtained results are given and discussed. Section 4 is assigned to summary and conclusion of present study.

2. The Schrödinger Equation with MGECSC Potential in Presence of External Electric Field

If the MGECSC potential form is considered for hydrogen-like atoms, the MGECSC potential in this case has the following form:

$$V(r) = -\frac{Ze^2}{r}(1+br)\,e^{-r/\lambda}\,\mathrm{Cos}\left(\frac{cr}{\lambda}\right), \tag{4}$$

where b, c, and λ are the screening parameters of MGECSC potential. In order to use MGECSC potential in plasma physics, for example, if this potential will be used to model Debye plasma, λ is taken into consideration as Debye screening parameter. When applying external electric field on hydrogen atom in the MGECSC potential, the corresponding radial Schrödinger equation is established in the following form:

$$\frac{d^2R_\ell(r)}{dr^2} + \frac{2m}{\hbar^2}\left(E_\ell - \frac{\hbar^2\ell(\ell+1)}{2mr^2} + \frac{Ze^2}{r}(1+br)\,e^{-r/\lambda}\,\mathrm{Cos}\left(\frac{cr}{\lambda}\right) + eFr\right)R_\ell(r) = 0, \tag{5}$$

where $\ell(\ell+1)/r^2$ is centrifugal potential and F is external electric field and eFr is contribution of external electric field on system. Here, it should be noted that θ is angle between F and r and external electric field contribution is obtained as $Fr\,\mathrm{Cos}(\theta)$ by using $\vec{F}\cdot\vec{r}$. In (5), Fr means $\theta = 0$ case. When using atomic units $m = \hbar = e = 1$ as well as $Z = 1$, (5) is given by

$$\frac{d^2R_\ell(r)}{dr^2} + \left(2E_\ell - \frac{\ell(\ell+1)}{r^2} + \frac{2(1+br)}{r}e^{-r/\lambda}\,\mathrm{Cos}\left(\frac{cr}{\lambda}\right) + 2Fr\right)R_\ell(r) = 0. \tag{6}$$

2.1. Approximate Analytical Solutions. The part with cosine term of (6) can be expanded in the power series of r up to the second term. Therefore, the part with cosine term of (6) becomes

$$\begin{aligned}\frac{2\left(1+br\right)e^{-r/\lambda}}{r}\,\mathrm{Cos}\left(\frac{cr}{\lambda}\right)&\cong\left(2b-\frac{2}{\lambda}\right)+\frac{2}{r}+\left(\frac{1-c^2-2b\lambda}{\lambda^2}\right)r\\&+\left(\frac{-1+3c^2+3b\lambda-3bc^2\lambda}{3\lambda^3}\right)r^2.\end{aligned}\tag{7}$$

Inserting (7) into (6),

$$\frac{d^2R_\ell\left(r\right)}{dr^2}+\left(\varepsilon-\frac{\ell\left(\ell+1\right)}{r^2}+\frac{2}{r}+\alpha_1 r+\alpha_2 r^2\right)R_\ell\left(r\right)=0\tag{8}$$

is obtained, where ansatzs are in the following form:

$$\varepsilon=2E_\ell+2b-\frac{2}{\lambda},\tag{9}$$

$$\alpha_1=\frac{1-c^2-2b\lambda}{\lambda^2}+2F,\tag{10}$$

$$\alpha_2=\frac{-1+3c^2+3b\lambda-3bc^2\lambda}{3\lambda^3}.\tag{11}$$

To solve (8), applying an ansatz to the radial wave function $R_\ell(r)$ should be performed as follows:

$$R_\ell\left(r\right)=\exp\left[\beta r+\frac{\gamma}{2}r^2\right]\sum_{n=0}^{\infty}a_n r^{n+\delta},\tag{12}$$

where β, γ, and δ are constants that will be determined later. If (12) is inserted into (8), (8) becomes

$$\sum_{n=0}^{\infty}a_nA_nr^{n+\delta}+\sum_{n=0}^{\infty}a_nB_nr^{n+\delta-1}+\sum_{n=0}^{\infty}a_nC_nr^{n+\delta-2}=0,\tag{13}$$

where A_n, B_n, and C_n are given by

$$A_n=\gamma+\beta^2+2\gamma\left(n+\delta\right)+\varepsilon,\tag{14}$$

$$B_n=2\beta\left(n+\delta\right)+2,\tag{15}$$

$$C_n=\left(n+\delta\right)\left(n+\delta-1\right)-\ell\left(\ell+1\right),\tag{16}$$

$$2\beta\gamma=-\alpha_1,\tag{17}$$

$$\gamma^2=-\alpha_2.\tag{18}$$

Equations (17)-(18) are constraint conditions which are used to obtain (13). Using (13),

$$\begin{aligned}&\left(a_0B_0+a_1C_1\right)r^{\delta-1}+a_0C_0r^{\delta-2}\\&+\sum_{n=0}^{\infty}\left(a_nA_n+b_{n+1}B_{n+1}+a_{n+2}C_{n+2}\right)r^{n+\delta}=0\end{aligned}\tag{19}$$

is obtained. Considering applied ansatz to the radial wave function $R_\ell(r)$, it is clear that $a_0\neq0$; otherwise bound states do not consist in the quantum system. Therefore, C_0 must be equal to zero in accordance with

$$C_0=\delta\left(\delta-1\right)-\ell\left(\ell+1\right)=0.\tag{20}$$

Given (20) leads to attaining δ parameter as $\delta=\ell+1$ that is physically acceptable solution. Moreover, in order to be able to confirm (19), it should be implemented that

$$\begin{aligned}a_0B_0+a_1C_1&=0,\\a_nA_n+a_{n+1}B_{n+1}+a_{n+2}C_{n+2}&=0.\end{aligned}\tag{21}$$

Serial in (12) should be convergent due to fact that (12) must be physically acceptable solution. So, if it is considered that pth nonvanishing coefficient is $a_p\neq0$, it is clear that $a_{p+1}=a_{p+2}=\cdots=0$. Then, it is clear that $A_p=0$. Namely, it is

$$A_p=\gamma+\beta^2+2\gamma\left(p+\delta\right)+\varepsilon.\tag{22}$$

If the above form of A_p is considered, bound energy eigenvalues can be obtained. It is noted that A_n, B_n, and C_n should satisfy the following determinant relation for a nontrivial solution:

$$\begin{vmatrix}B_0 & C_1 & \cdots & \cdots & \cdots & 0\\A_0 & B_1 & C_2 & \cdots & \cdots & .\\\vdots & \vdots & \vdots & \ddots & \vdots & \vdots\\0 & 0 & 0 & 0 & A_{p-1} & B_p\end{vmatrix}=0.\tag{23}$$

In order to generalize this method and thus obtain bound states and wave functions of hydrogen atom in MGECSC potential in presence of external electric field, which is used as a model in the present study, the approximate analytical solutions are presented for the cases $p=0,1$ as follows.

If $p=0$, it is true according to (23) that $\det|B_0|=0$ and $B_0=0$. In that case,

$$B_0=2\beta\delta+2=0\Longrightarrow\beta=\frac{-1}{\ell+1}.\tag{24}$$

The corresponding energy values are calculated using (22), but it should not be ruled out that restrictions created in (14)–(18) on the parameters should be taken into account when calculating corresponding energy values.

The corresponding eigenfunction for $p=0$ is given by using (12) as

$$R_\ell^0\left(r\right)=a_0\exp\left[\beta r+\frac{\gamma}{2}r^2\right]r^\delta,\tag{25}$$

where a_0 is normalization constant.

When $p=1$, considering (23), it is obtained that $\det\left|\begin{smallmatrix}B_0 & C_1\\A_0 & B_1\end{smallmatrix}\right|=0$. Due to this determinant relation, a restriction on the parameters is obtained as

$$\left[2+\left(6+4\ell\right)\beta+\left(3+5\ell+2\ell^2\right)\left(\beta^2-\gamma\right)-\varepsilon\left(1+\ell\right)\right]=0.\tag{26}$$

If this and restrictions in (17) and (18) are considered together with (22), corresponding energy values are obtained. The corresponding eigenfunction for $p = 1$ is given by using (12) as

$$R_\ell^1(r) = \exp\left[\beta r + \frac{\gamma}{2}r^2\right](a_0 + a_1 r)\, r^\delta, \tag{27}$$

where a_1 can be calculated by using normalization condition together with obtained a_0 from (25). In order to obtain corresponding energy values for other $p = 2, 3, \ldots$, the same way applied above should be repeated. More clearly, if $a_p \neq 0$, $a_{p+1} = a_{p+2} = \cdots = 0$. In this case, the energy eigenvalues E_p are obtained by using (22). But, it should be pointed out that when calculating energy eigenvalues E_p, there are two issues to be considered: for corresponding parameters, restrictions created in (17)-(18) and constraints that would be obtained from (23). So, the corresponding eigenfunction for pth case is

$$R_\ell^p(r) = \exp\left[\beta r + \frac{\gamma}{2}r^2\right]\left(a_0 + a_1 r + \cdots + a_p r^p\right) r^\delta, \tag{28}$$

where a terms can be calculated using normalization condition.

3. Results and Discussions

3.1. Case of $b = c = 0$. Since approximation in (7) is used in order to bound states and eigenfunctions of quantum systems with MGECSC potential in presence of external electric field, new form of MGECSC potential obtained by using mentioned approximation is given by

$$\overline{V}(r) \cong \left(-b + \frac{1}{\lambda}\right) - \frac{1}{r} + \left(\frac{-1 + c^2 + 2b\lambda}{2\lambda^2}\right) r + \left(\frac{1 - 3c^2 - 3b\lambda + 3bc^2\lambda}{6\lambda^3}\right) r^2. \tag{29}$$

As mentioned previously, MGECSC potential reduces to SC potential when using $b = c = 0$ case, which is considered to model hydrogen atom in Debye plasma in plasma physics. The corresponding Schrödinger equation has been solved analytically by applying ansatz to eigenfunction and using approximation in (29), and the energy eigenvalues of $1s$ and $2p$ quantum states have been obtained for $\lambda = 70, 100, 150, 200$ in Rydberg units, as can be seen in Table 1. In $b = c = 0$ case, in Table 1 comparison of $1s$ and $2p$ energy eigenvalues in the present study with results of [21, 32] in Rydberg units is shown. The obtained results in Table 1 of [21, 32] were calculated numerically and when comparing the results of [21, 32] with the present results obtained by using analytical method, it is seen that there is a very good agreement between each of the three results. However, it should be pointed out that energy eigenvalues obtained in the present study with results of [21, 32] become more consistent in larger values of λ screening parameters as can be seen in Table 1, reason of which can be understood better when investigating Figures 1(a) and 1(b). When $\lambda = 10$ and $\lambda = 200$, in $b = c = 0$ case, comparison of MGECSC potential $V(r)$ and $\overline{V}(r)$ potential in (29) obtained by using corresponding approximation on MGECSC potential is seen in Figures 1(a) and 1(b), respectively. As can be also seen in Figures 1(a) and 1(b), while a very good agreement has been obtained between $\overline{V}(r)$ and $V(r)$ in case of $\lambda = 200$, the same agreement between potential profiles for case of $\lambda = 10$ can not be obtained, which is the cause of trivial difference between obtained energy values in the present study with results of [21, 32].

Table 1: When $b = c = 0$, comparison of $1s$ and $2p$ energy eigenvalues obtained in the present study with results of [21, 32] for different λ parameters in Rydberg units.

λ	[21]	[32]	Present results
		$1s$	
70	−0.971731	−0.971732	−0.971733
100	−0.980149	−0.980149	−0.980150
150	−0.986733	−0.986733	−0.986733
200	−0.990037	−0.990037	−0.990037
		$2p$	
70	−0.222421	−0.222421	−0.222444
100	−0.230490	−0.230490	−0.230498
150	−0.236886	−0.236886	−0.236888
200	−0.240123	−0.240124	−0.240125

3.2. Case of $b = 0, c = 1$. In $b = 0, c = 1$ case, MGECSC potential reduces to ECSC potential which is used to model hydrogen atom in quantum plasma in plasma physics. The corresponding Schrödinger equation has been solved analytically by applying ansatz to eigenfunction and using corresponding approximation, and the energy eigenvalues of $1s$ quantum state have been obtained for $1/\lambda = 0.01, 0.02, 0.03, 0.04$ and 0.05 in Rydberg units, as can be seen in Table 2. In $b = c = 0$ case, in Table 2 comparison of ground state energy eigenvalues obtained in the present study with results of [23, 24, 33] in atomic units has been shown. However, it should be pointed out that ground state energy eigenvalues obtained in the present study with results of [23, 24, 33] become more consistent in smaller values of $1/\lambda$ as can be seen in Table 2, reason of which can be understood better when investigating Figures 2(a) and 2(b). When $\lambda = 15$ and $\lambda = 100$, in $b = 0, c = 1$ case, comparison of MGECSC potential $V(r)$ and potential in (29) obtained by using corresponding approximation on MGECSC potential is seen in Figures 2(a) and 2(b), respectively. While a very good agreement has been obtained between $\overline{V}(r)$ and $V(r)$ in case of $\lambda = 100$, the same agreement between potential profiles for case of $\lambda = 15$ can not be obtained as can be also seen in Figures 2(a) and 2(b), which is the cause of trivial difference shown in Table 2 between energy values obtained in the present study with results of [23, 24, 33].

3.3. Case of $b \neq 0$, $c = 0$. When $b \neq 0$, $c = 0$ case in MGECSC potential is considered, it is reduced to the following form:

$$V(r) = -\frac{Ze^2}{r}(1 + br)\, e^{-r/\lambda}, \tag{30}$$

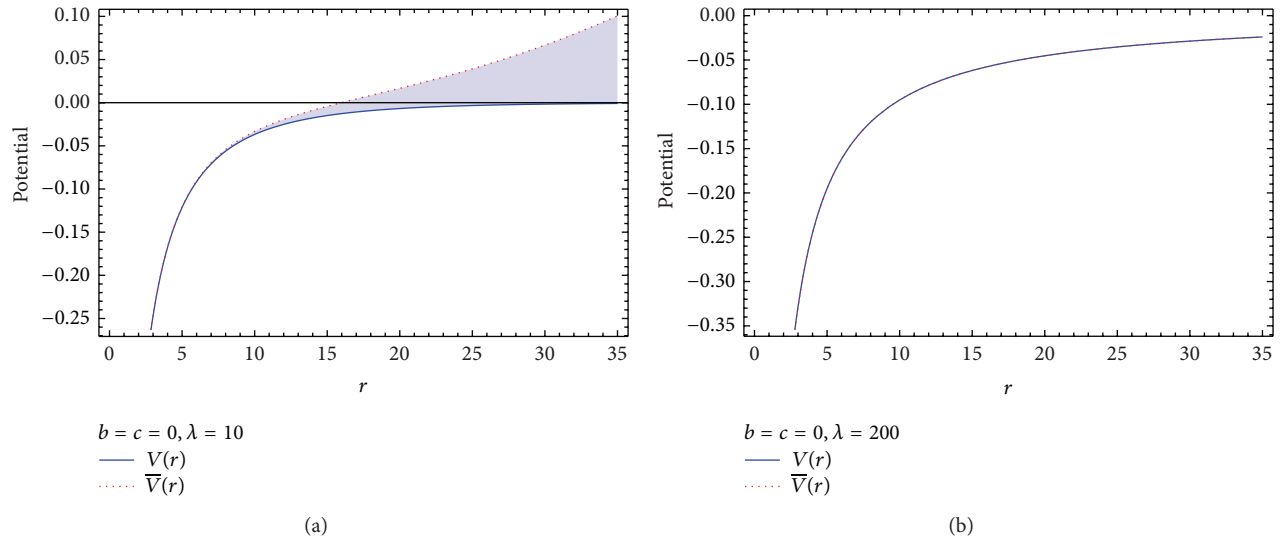

FIGURE 1: (a) Comparison of $\overline{V}(r)$ and $V(r)$ potentials in case of $b = c = 0$ and $\lambda = 10$ (in arbitrary units). (b) Comparison of $\overline{V}(r)$ and $V(r)$ potentials in case of $b = c = 0$ and $\lambda = 200$ (in arbitrary units).

TABLE 2: When $b = 0$, $c = 1$, comparison of $1s$ energy eigenvalues obtained in the present study with results of [23, 24, 33] for different $1/\lambda$ in atomic units.

	$1s$			
$1/\lambda$	[23]	[24]	[33]	Present results
0.01	−0.490000	−0.490000	−0.490001	−0.490001
0.02	−0.480007	−0.480007	−0.480008	−0.480004
0.03	−0.470025	−0.470026	−0.470026	−0.470014
0.04	−0.460060	−0.460060	−0.460061	−0.460032
0.05	−0.450117	−0.450117	−0.450117	−0.450063

TABLE 3: When $c = 0$, $b \neq 0$, and $\lambda = 200$, comparison of $1s$ and $2p$ energy eigenvalues obtained in the present study with results of [24] for different b screening parameters in Rydberg units.

b	$1s$		$2p$	
	[24]	Present results	[24]	Present results
0.1	−1.188545	−1.188540	−0.435223	−0.435143
0.2	−1.387055	−1.387050	−0.630376	−0.630170
0.3	−1.585566	−1.585550	−0.825578	−0.825211
0.5	−1.982592	−1.982570	−1.216118	−1.215330
0.7	−2.379624	−2.379580	−1.606823	−1.605500

where λ is considered as Debye screening parameter if hydrogen atom will be investigated in Debye plasma. In $b \neq 0$, $c = 0$ case, in Table 3 effect of b screening parameter ($b = 0.1, 0.2, 0.3, 0.5$ and 0.7) on the energy eigenvalues for two different quantum states ($1s$ and $2p$) at $\lambda = 200$ is shown. There is also a very good agreement between the present results and numerical results obtained by using AIM in [24]. But it should be mentioned that smaller b screening parameter leads to getting a better consistency between numerical and analytical results as can be seen in Table 3. In order to analyze influence of b screening parameter on $\overline{V}(r)$ and $V(r)$, in other words, to examine trivial differences between numerical and analytical results, Figures 3(a) and 3(b) can be investigated. However, it is seen that although effect of b screening parameter on used approximation in the present study is less than that of λ screening parameter, this effect of b screening parameter changes analytically obtained results in the present study. More clearly, SC, ECSC, and MGECSC potentials and the potential in (30) are more effective in the shorter range. However, if Figures 3(a) and 3(b) are plotted in the larger range, the difference between profiles of $\overline{V}(r)$ and $V(r)$ can be clearly seen. In this manner, the energies of ground state are obtained more precisely compared to that of other states, due to the approximation in (7).

3.4. Case of $b \neq 0$, $c \neq 0$. The $b \neq 0$, $c \neq 0$ case is the MGECSC potential, which is also used to investigate interaction of hydrogen atom in quantum plasma in plasma physics. In $b \neq 0$, $c = 1$ case and $\lambda = 200$, energy eigenvalues of $1s$ and $2p$ quantum states for $0, 0.1, 0.2, 0.3$, and 0.4 values of b screening parameter have been calculated in Rydberg units and they have been shown in Table 4. In Table 4, comparison of energy values obtained in the present study with results of [24] for different b screening parameters in Rydberg units is seen. Moreover, the consistency between the results in the present study and results of [24] is better in smaller values of b screening parameter as can be seen in Table 4. However, energy values obtained in Table 4 have also a very good agreement with results of [24]. When $c = 1$, in order to examine screening effect of b parameter on $\overline{V}(r)$ and $V(r)$, Figures 4(a) and 4(b) can be investigated. This investigation enables examining trivial difference between numerical results of other studies and analytical results in the present study as previously emphasized. Smaller values of b screening parameter cause obtaining a better agreement

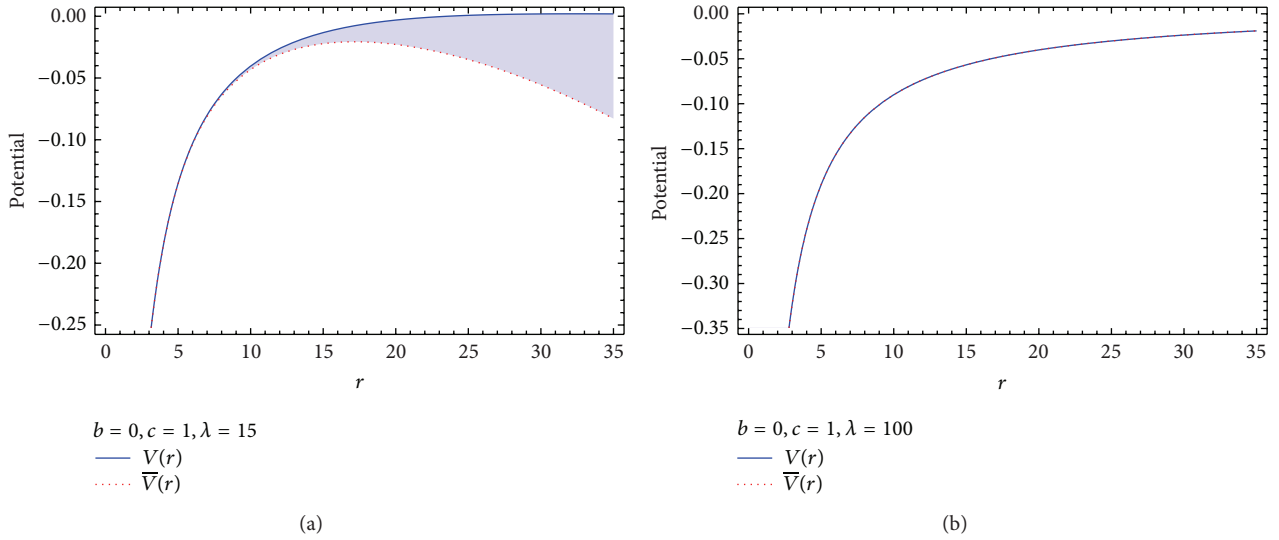

FIGURE 2: (a) Comparison of $\overline{V}(r)$ and $V(r)$ potentials in case of $b = 0$, $c = 1$, and $\lambda = 15$ (in arbitrary units). (b) Comparison of $\overline{V}(r)$ and $V(r)$ potentials in case of $b = 0$, $c = 1$, and $\lambda = 100$ (in arbitrary units).

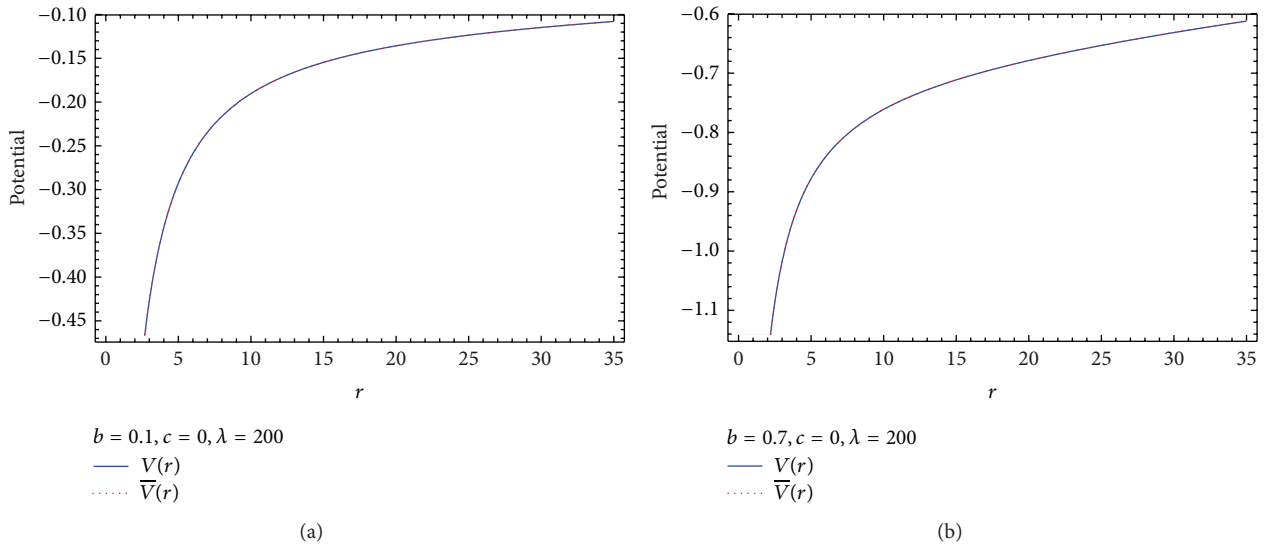

FIGURE 3: (a) Comparison of $\overline{V}(r)$ and $V(r)$ potentials in case of $b = 0.1$, $c = 0$, and $\lambda = 200$ (in arbitrary units). (b) Comparison of $\overline{V}(r)$ and $V(r)$ potentials in case of $b = 0.7$, $c = 0$, and $\lambda = 200$ (in arbitrary units).

between exact and approximate results obtained by using numerical or analytical methods. In Table 5, in $b = 0.1$ and $c = 1$ case, comparison of $1s$ and $2p$ energy eigenvalues obtained in the present study with results of [24] for $\lambda = 100, 150, 200, 250, 500$ in Rydberg units has been shown. If the used approximation in the present study is considered, it is clear that larger λ parameter provides obtaining a better approximation for solution, which is confirmed by examining of Figures 1(a), 1(b), 2(a), 2(b), 5(a), and 5(b).

3.5. Case of $b \neq 0$, $c \neq 0$ in Presence of External Electric Field. In $c = 0.5$, $b = 0.3$, and $\lambda = 300$ case, energy eigenvalues

TABLE 4: When $c = 1$, $b \neq 0$, and $\lambda = 200$, comparison of $1s$ and $2p$ energy eigenvalues obtained in the present study with results of [24] for different b screening parameters in Rydberg units.

b	$1s$		$2p$	
	[24]	Present results	[24]	Present results
0	−0.990000	−0.990000	−0.240002	−0.240001
0.1	−1.188501	−0.188501	−0.435033	−0.435003
0.2	−1.387003	−1.387003	−0.630119	−0.630022
0.3	−1.585507	−1.585506	−0.825258	−0.825050
0.4	−1.784012	−1.784011	−1.020445	−1.020090

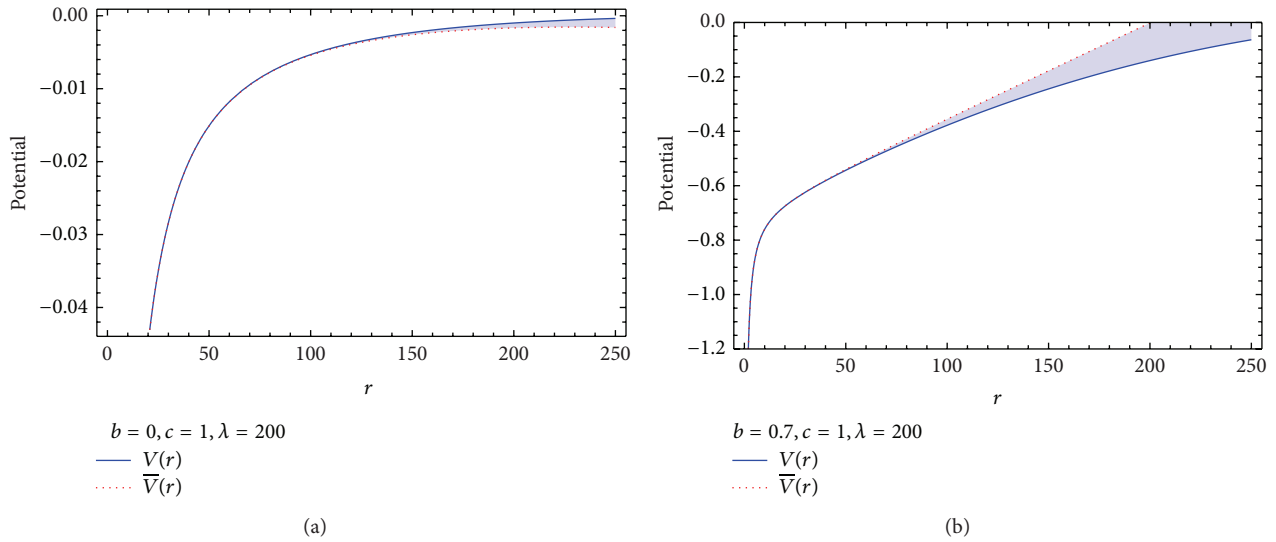

FIGURE 4: (a) Comparison of $\overline{V}(r)$ and $V(r)$ potentials in case of $b = 0$, $c = 1$, and $\lambda = 200$ (in arbitrary units). (b) Comparison of $\overline{V}(r)$ and $V(r)$ potentials in case of $b = 0.7$, $c = 1$, and $\lambda = 200$ (in arbitrary units).

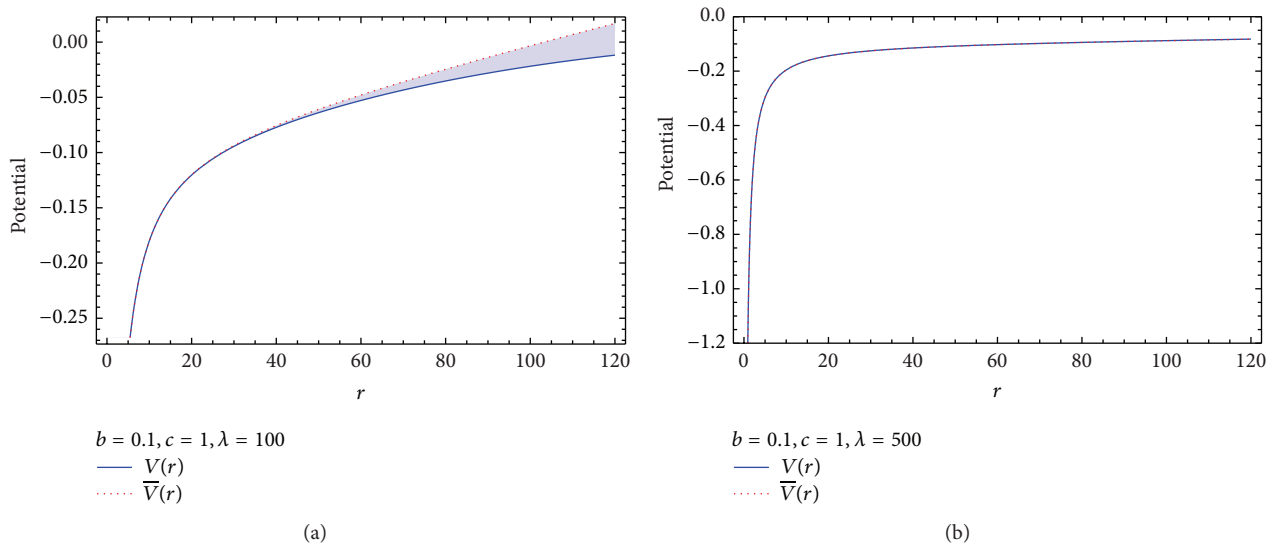

FIGURE 5: (a) Comparison of $\overline{V}(r)$ and $V(r)$ potentials in case of $b = 0.1$, $c = 1$, and $\lambda = 100$ (in arbitrary units). (b) Comparison of $\overline{V}(r)$ and $V(r)$ potentials in case of $b = 0.1$, $c = 1$, and $\lambda = 500$ (in arbitrary units).

TABLE 5: When $b = 0.1$, $c = 1$, comparison of $1s$ and $2p$ energy eigenvalues obtained in the present study with results of [24] for different λ screening parameters in Rydberg units.

λ	$1s$		$2p$	
	[24]	Present results	[24]	Present results
100	−1.177005	−1.177000	−0.420145	−0.420025
150	−1.184668	−1.184669	−0.430061	−0.430010
200	−1.188501	−1.188500	−0.435033	−0.435006
250	−1.190800	−1.190800	−0.438020	−0.438004
500	−1.195400	−1.195400	−0.444005	−0.444001

of $1s$ and $2p$ quantum states for 0.0001, 0.001, 0.01, 0.5, and 2 values of external electric field strengths are calculated in atomic units and corresponding results have been shown in Table 6.

3.6. Plot of Wave Functions for $1s$ and $2p$ States. The wave functions of $1s$ and $2p$ states for different cases of b and c screening parameters are seen in Figures 6 and 7. As can be seen in Figures 6 and 7, plot of $1s$ and $2p$ wave functions and that in [32] are the same, which is an expected result. However, it should be pointed out that radial eigenfunction

Table 6: When $b = 0.3$, $c = 0.5$, and $\lambda = 300$, $1s$ and $2p$ energy eigenvalues for different external electric field strengths in atomic units.

F	$1s$	$2p$
0.0001	−0.795325	−0.417202
0.001	−0.796675	−0.421694
0.01	−0.810236	—
0.5	−2.22977	—
2	—	—

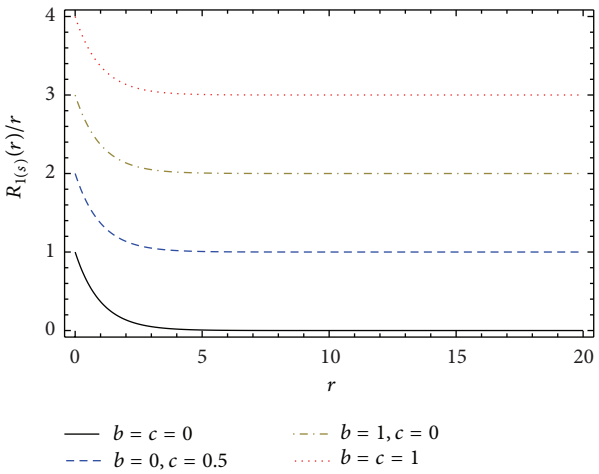

Figure 6: When $\lambda = 500$, plot of wave functions of $1s$ state for different cases of b and c screening parameters (in arbitrary units).

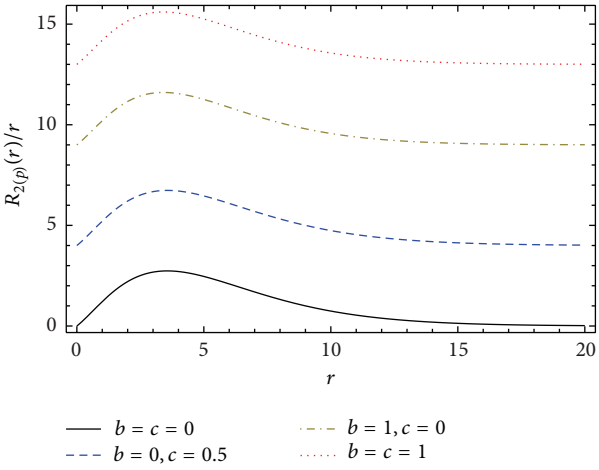

Figure 7: When $\lambda = 500$, plot of wave functions of $2p$ state for different cases of b and c screening parameters (in arbitrary units).

of corresponding Hamiltonian in [32] was taken as $P_{n\ell}(r) = rR_{n\ell}(r)$ in order to solve Schrödinger equation.

4. Conclusion

In this study, the Schrödinger equation with MGECSC potential has been analytically solved approximately by applying an ansatz to radial eigenfunction of corresponding Hamiltonian, and the energy values of bound states and eigenfunctions have been obtained, to the best of author's knowledge, which has been carried out in this study for the first time. The MGECSC potential is more complex and important in the physical sense compared to SC, ECSC, and PC potentials, because if the different cases of b and c screening parameters are considered, the bound states and wave functions for SC and ECSC potentials are obtained, as performed in the present study. The studied potentials in Section 3 play important role to model some interactions in plasma physics. The analytically obtained results in this study were compared with the numerically obtained results in corresponding references and it was observed that all the results have a very good agreement. The importance of this study can be outlined in four main points. Firstly, since for the first time the Schrödinger equation with MGECSC, ECSC, SC, and PC potentials has been analytically solved in the present study, the importance of obtained analytical solutions in this study is better understood if applications of each of these potentials are considered for plasma and nuclear physics. Secondly, to obtain analytical solution is very difficult for relativistic quantum systems with MGECSC and ECSC potential. For this reason, numerical methods such as perturbation, DPT, and variational calculus are used in order to investigate relativistic systems with MGECSC, ECSC, or SC potentials [34]. However, the ground state solutions need to be able to make perturbative calculus. Since the energy values of bound states and corresponding wave functions in analytical form have been obtained for quantum systems with MGECSC and ECSC potentials, results in this study make it possible to investigate relativistic quantum systems with MGECSC and ECSC potentials. This mentioned investigation was not possible because analytical solutions of nonrelativistic systems with MGECSC and ECSC potential have not been made up to now. Thirdly, the external electric field effect on MGECSC potential has been also considered in the present study. It should be noted that external electric field effect on energy values of bound states can be removed or taken into account on request as can be seen in (10). Finally, in this study, since the external electric field can be included in calculations, relativistic systems with MGECSC and ECSC potential under external electric field can be studied by using perturbative calculus.

Conflict of Interests

The author declares clearly that there is no conflict of interests regarding the publication of this paper.

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