Research Article

Spin and Pseudospin Symmetry in Generalized Manning-Rosen Potential

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Spin and pseudospin symmetric Dirac spinors and energy relations are obtained by solving the Dirac equation with centrifugal term for a new suggested generalized Manning-Rosen potential which includes the potentials describing the nuclear and molecular structures. To solve the Dirac equation the Nikiforov-Uvarov method is used and also applied the Pekeris approximation to the centrifugal term. Energy eigenvalues for bound states are found numerically in the case of spin and pseudospin symmetry. Besides, the data attained in the present study are compared with the results obtained in the previous studies and it is seen that our data are consistent with the earlier ones.

1. Introduction

Spin and pseudospin symmetries are symmetries of the Dirac Hamiltonian. Spin symmetry leads to degeneracy between two states with quantum numbers $(n, l, j = l - s)$ and $(n, l, j = l + s)$. These two states are considered as a spin doublet with $(n, l, j = l \mp s)$. n , l , j , and s are radial, orbital angular momentum, total angular momentum, and spin quantum numbers, respectively. The spin doublet or spin symmetry is used to explain the spectrum of antinucleon in a nucleus [1–4] and small spin-orbit splitting in hadrons [5]. Pseudospin symmetry causes degeneracy between two states with quantum numbers $(n, l, j = l + 1/2)$ and $(n - 1, l + 2, j = l + 3/2)$. These two states are regarded as a pseudospin doublet with quantum numbers $(\tilde{n} = n - 1, \tilde{l} = l + 1, \tilde{j} = \tilde{l} \mp \tilde{s})$, where $\tilde{l}$ and $\tilde{s} = 1/2$ are pseudoorbital angular momentum and pseudospin quantum numbers, respectively [6, 7]. The pseudospin doublet or pseudospin symmetry is used to explain deformation [8], identical bands [9–11], magnetic moment [12–14], and effective shell-model [15] in the nuclear structures.

Pseudospin and pseudoorbital angular momentum were introduced for the first time to explain experimental observation of quasi-degeneracy between $(n, l, j = l + 1/2)$ and

$(n - 1, l + 2, j = l + 3/2)$ states of single-nucleon in nuclei [6, 7]. Pseudospin symmetry was discussed firstly in nonrelativistic framework [6]. However, in the 1990s, Blokhin et al. found a connection between pseudospin symmetry and relativistic mean field theory [16–18]. Then Ginocchio recognized that the pseudoorbital angular momentum $\tilde{l}$ is the orbital angular momentum of lower component of the Dirac spinor [19]. He showed that the pseudospin symmetry occurs in nuclei when the sum of the scalar potential $S(r)$ and vector potential $V(r)$ is approximately equal to zero ($V(r) + S(r) \sim 0$) [1, 2, 19–22]. Then, Meng et al. proved that the exact pseudospin symmetry exists when $d[V(r) + S(r)]/dr = 0$ or $V(r) + S(r) = \text{constant}$ in the Dirac equation [23, 24]. Also they introduced that the exact spin symmetry occurs when $d[V(r) - S(r)]/dr = 0$ or $V(r) - S(r) = \text{constant}$ in the Dirac equation [3]. For two symmetries, the Dirac Hamiltonian is invariant under the SU(2) algebra [25]. After these studies, in recent years, spin and pseudospin symmetric solutions of the Dirac equation have been obtained and investigated by using different methods for various external potentials [26–66]. In [26], Wei and Dong have studied the Manning-Rosen potential under the spin symmetry limit. Also they have examined pseudospin symmetric solutions and energy eigenvalues for the Manning-Rosen potential [27]. Moreover

the relativistic symmetries of the Manning-Rosen potential have been investigated by considering different approximation scheme for the centrifugal term [28, 29, 32, 33].

In this paper we have considered the new suggested generalized Manning-Rosen potential [67]. The new suggested generalized Manning-Rosen potential is defined in the following form:

$$V(r) = \left[\frac{Ae^{-2b(r-L)}}{(q + pe^{-b(r-L)})^2} + \frac{Be^{-b(r-L)}}{q + pe^{-b(r-L)}} \right]. \quad (1)$$

Generalized Manning-Rosen potential includes some molecular potentials which are Manning-Rosen [68–73], Morse [74–77], hyperbolic Pöschl-Teller [78], q -parameter hyperbolic Pöschl-Teller [79], and Kratzer-Fues [80–82] potentials. Also it can be reduced to Woods-Saxon [35–39, 83] and Hulthen [40–45, 84] potentials which are used in the explanation of the nuclear structures and to the Yukawa potential that represents the interactions between nucleons in nuclei [55–59, 85].

Therefore, the generalized Manning-Rosen potential is important for being able to explain both nuclear and molecular structures. If the generalized Manning-Rosen potential is considered as a nuclear potential, the spin and pseudospin symmetric solutions should be examined. Also spin and pseudospin symmetry can be investigated for diatomic molecular potentials by defining the reduced mass $\mu = m_1 m_2 / (m_1 + m_2)$, where m_1 and m_2 are two nuclei masses of a diatomic molecule. In this context, it is aimed at investigating spin and pseudospin symmetric solutions for the generalized Manning-Rosen potential and at finding the bound state energy eigenvalues of the considered potential under the spin and pseudospin symmetry limits. Another aim is to examine the special cases of the generalized Manning-Rosen potential and to compare the obtained results for special cases with the results that are obtained in the previous studies and also to check up whether these results are compatible.

The scheme of paper is as follows. In Section 2, the Nikiforov-Uvarov method that we use to solve the Dirac equation is introduced briefly. In Section 3, the second-order differential equations are obtained from Dirac equation for the spin and pseudospin symmetry. In Sections 4 and 5, the spin and pseudospin symmetric solutions and bound state energy eigenvalues are found for the generalized Manning-Rosen potential, respectively. In Section 6, the effects of potential parameters on the energy eigenvalues are investigated. In Section 7, special cases of the generalized Manning-Rosen potential are introduced and energy eigenvalues are calculated for each special case in the spin and pseudospin symmetric limits. Also in the same section our energy eigenvalues are compared with the previous ones. Finally, an appraisal of the obtained results is made in Section 8.

2. Nikiforov-Uvarov Method

This section has been prepared by using [86]. The Nikiforov-Uvarov method is used to solve the second-order differential equations which must be in the following form:

$$\frac{d^2 U(y)}{dy^2} + \frac{\bar{\varepsilon}(y)}{\xi(y)} \frac{dU(y)}{dy} + \frac{\bar{\xi}(y)}{\xi^2(y)} U(y) = 0, \quad (2)$$

where $\bar{\varepsilon}(y)$ can be first-degree polynomial maximally. $\xi(y)$ and $\bar{\xi}(y)$ can be polynomials no more than second degree. The function $U(y)$ is defined as

$$U(y) = H(y) W(y) \quad (3)$$

and the form of (2) is required to be covariant. In this case the following equation is obtained:

$$\frac{d^2 W(y)}{dy^2} + \frac{\varepsilon(y)}{\xi(y)} \frac{dW(y)}{dy} + \frac{\bar{\xi}(y)}{\xi^2(y)} W(y) = 0, \quad (4)$$

where $\varepsilon(y)$ which can be first-degree polynomials maximally and $\bar{\xi}(y)$ that can be second-degree polynomials maximally are found as follows, respectively:

$$\varepsilon(y) = \left[2 \frac{dH(y)/dy}{H(y)} + \frac{\bar{\varepsilon}(y)}{\xi(y)} \right] \xi(y) \quad (5)$$

$$\Rightarrow \frac{dH(y)/dy}{H(y)} = \frac{\pi(y)}{\xi(y)},$$

$$\begin{aligned} \bar{\xi}(y) &= \pi^2(y) + \bar{\xi}(y) + \pi(y) \left[\bar{\varepsilon}(y) - \frac{d\xi(y)}{dy} \right] \\ &+ \xi(y) \frac{d\pi(y)}{dy} \end{aligned} \quad (6)$$

by defining the polynomial $\pi(y)$ as

$$\pi(y) = \frac{1}{2} [\varepsilon(y) - \bar{\varepsilon}(y)]. \quad (7)$$

The convenient $\pi(y)$ must be found to obtain solutions of (4) or (2) and to study properties of solutions. Therefore the following relation is established between $\bar{\xi}(y)$ and $\xi(y)$:

$$\bar{\xi}(y) = \lambda \xi(y), \quad (8)$$

where λ is a constant. In this case (4) reduces to following equation:

$$\xi(y) \frac{d^2 W(y)}{dy^2} + \varepsilon(y) \frac{dW(y)}{dy} + \lambda W(y) = 0. \quad (9)$$

This is a hypergeometric type equation and solution of this equation is functions of hypergeometric type. By using (8), the polynomial $\pi(y)$ is found:

$$\begin{aligned} \pi(y) &= \frac{d\xi(y)/dy - \bar{\varepsilon}(y)}{2} \\ &\pm \sqrt{\left(\frac{d\xi(y)/dy - \bar{\varepsilon}(y)}{2} \right)^2 - \bar{\xi}(y) + k\xi(y)}, \end{aligned} \quad (10)$$

where

$$k = \lambda - \frac{d\pi(y)}{dy}. \quad (11)$$

$\pi(y)$ is a first-degree polynomial. Therefore the expression in the square root in (10) must be written a square of a polynomial. Considering this property, the values are found for k . Derivatives of the hypergeometric functions are also the hypergeometric functions. Accordingly, n th derivative of $W(y)$ satisfies the hypergeometric type equation. By using this property, a relation for λ is obtained:

$$\lambda = -n \frac{d\varepsilon(y)}{dy} - \frac{n(n-1)}{2} \frac{d^2\xi(y)}{dy^2}. \quad (12)$$

When the quantum mechanical problems are discussed, an energy relation is obtained by equating (11) to (12).

The hypergeometric type function $W(y)$ is found by using the following Rodrigues relation for hypergeometric type equations:

$$W_n(y) = \frac{C_n}{\rho(y)} \frac{d^n}{dy^n} [\xi^n(y) \rho(y)], \quad (13)$$

where $\rho(y)$ is a weight function and satisfies the following equation:

$$\frac{d}{dy} [\xi(y) \rho(y)] = \varepsilon(y) \rho(y). \quad (14)$$

3. Dirac Equation

In the presence of an attractive scalar $S(r)$ and a repulsive vector $V(r)$ potentials, the Dirac equation for a nucleon with mass m is written as follows ($\hbar = c = 1$) [27, 46, 87]:

$$\begin{aligned} [\boldsymbol{\alpha} \cdot \mathbf{p} + \beta(m + S(r))] \Psi(r, \theta, \phi) \\ = [E - V(r)] \Psi(r, \theta, \phi), \end{aligned} \quad (15)$$

where $\boldsymbol{\alpha}$ and β are the 4×4 Dirac matrices, while $\boldsymbol{\alpha}$ is defined in terms of 2×2 Pauli spin matrices, and β is described in terms of 2×2 unit matrices [87]:

$$\begin{aligned} \boldsymbol{\alpha} &= \begin{pmatrix} 0 & \boldsymbol{\sigma} \\ \boldsymbol{\sigma} & 0 \end{pmatrix}, \\ \beta &= \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}. \end{aligned} \quad (16)$$

The spherically symmetric Dirac spinor wave function [1] is written as

$$\Psi(\mathbf{r}) = \begin{pmatrix} \frac{F_{n\kappa}(r)}{r} Y_{jm}^\ell(\theta, \phi) \\ i \frac{G_{n\kappa}(r)}{r} Y_{jm}^{\bar{\ell}}(\theta, \phi) \end{pmatrix}, \quad (17)$$

where $F_{n\kappa}(r)$ and $G_{n\kappa}(r)$ are the upper and lower radial wave functions, respectively. $Y_{jm}^\ell(\theta, \phi)$ and $Y_{jm}^{\bar{\ell}}(\theta, \phi)$ are the spin

and pseudospin spherical harmonics, respectively. κ is the eigenvalues of spin-orbit coupling operator $K = -\beta(\boldsymbol{\sigma} \cdot \mathbf{L} + 1)$. The values of κ are ℓ for unaligned spin ($j = \ell - 1/2$) and $-(\ell + 1)$ for aligned spin ($j = \ell + 1/2$). Substituting (9) and (10) into (8) and by using the following relations [88]:

$$\begin{aligned} (\boldsymbol{\sigma} \cdot \mathbf{A})(\boldsymbol{\sigma} \cdot \mathbf{B}) &= \mathbf{A} \cdot \mathbf{B} + i\boldsymbol{\sigma} \cdot (\mathbf{A} \times \mathbf{B}), \\ \boldsymbol{\sigma} \cdot \mathbf{p} &= \boldsymbol{\sigma} \cdot \hat{\mathbf{r}} \left(\hat{\mathbf{r}} \cdot \mathbf{p} + i \frac{\boldsymbol{\sigma} \cdot \mathbf{L}}{r} \right) \end{aligned} \quad (18)$$

together with the following properties [88]:

$$\begin{aligned} (\boldsymbol{\sigma} \cdot \mathbf{L}) Y_{jm}^{\bar{\ell}}(\theta, \phi) &= (\kappa - 1) Y_{jm}^{\bar{\ell}}(\theta, \phi), \\ (\boldsymbol{\sigma} \cdot \mathbf{L}) Y_{jm}^\ell(\theta, \phi) &= -(\kappa - 1) Y_{jm}^\ell(\theta, \phi), \\ (\boldsymbol{\sigma} \cdot \hat{\mathbf{r}}) Y_{jm}^\ell(\theta, \phi) &= -Y_{jm}^{\bar{\ell}}(\theta, \phi), \\ (\boldsymbol{\sigma} \cdot \hat{\mathbf{r}}) Y_{jm}^{\bar{\ell}}(\theta, \phi) &= -Y_{jm}^\ell(\theta, \phi), \end{aligned} \quad (19)$$

the Dirac equation reduces to two coupled differential equations as follows:

$$\left(\frac{d}{dr} + \frac{\kappa}{r} \right) F_{n\kappa}(r) = [m + E - \Delta(r)] G_{n\kappa}(r), \quad (20)$$

$$\left(\frac{d}{dr} - \frac{\kappa}{r} \right) G_{n\kappa}(r) = [m - E + \Sigma(r)] F_{n\kappa}(r), \quad (21)$$

where

$$\begin{aligned} \Delta(r) &= V(r) - S(r), \\ \Sigma(r) &= V(r) + S(r). \end{aligned} \quad (22)$$

From these two coupled differential equations, two second-order differential equations have been obtained for the upper and lower radial wave functions, respectively:

$$\begin{aligned} \left[\frac{d^2}{dr^2} - \frac{\kappa(\kappa+1)}{r^2} + \frac{(d\Delta(r)/dr)(d/dr + \kappa/r)}{m + E - \Delta(r)} \right] \\ \cdot F_{n\kappa}(r) + (E + m - \Delta(r))(E - m - \Sigma(r)) F_{n\kappa}(r) \\ = 0, \end{aligned} \quad (23)$$

$$\left[\frac{d^2}{dr^2} - \frac{\kappa(\kappa-1)}{r^2} - \frac{(d\Sigma(r)/dr)(d/dr - \kappa/r)}{m - E + \Sigma(r)} \right] \cdot G_{n\kappa}(r) + (E + m - \Delta(r))(E - m - \Sigma(r))G_{n\kappa}(r) = 0, \quad (24)$$

where $\kappa(\kappa+1) = l(l+1)$ and $\kappa(\kappa-1) = \tilde{l}(\tilde{l}+1)$.

4. Spin Symmetric Solution

In the spin symmetric case the sum of scalar potential with vector potential $\Sigma(r)$ and the difference between these potentials $\Delta(r)$ are written as follows:

$$\Delta(r) = S_s, \quad \Sigma(r) = \left[\frac{Ae^{-2b(r-L)}}{(q + pe^{-b(r-L)})^2} + \frac{Be^{-b(r-L)}}{q + pe^{-b(r-L)}} \right], \quad (25)$$

where S_s is a constant, L is the nuclear radius, $1/b$ is the thickness of surface layer, and A and B are related with the potential dept for nuclei. Also q and p are dimensionless parameters and they are used to determine shape of the potential. Substituting (25) into (23), (23) reduces to

$$\left\{ \frac{d^2}{dr^2} - \frac{\kappa(\kappa+1)}{r^2} - (m + E - S_s) \left[\frac{A}{(p + qe^{b(r-L)})^2} + \frac{B}{p + qe^{b(r-L)}} \right] \right\} \cdot F_{n\kappa}(r) - (m - E)(m + E - S_s)F_{n\kappa}(r) = 0. \quad (26)$$

To solve this equation analytically, the following approximation which is called the Pekeris approximation [89] is applied to the centrifugal term:

$$\frac{1}{r^2} \approx C_0 + \frac{C_1}{p + qe^{b(r-L)}} + \frac{C_2}{(p + qe^{b(r-L)})^2}, \quad (27)$$

where C_0 , C_1 , and C_2 are constants and their values are found as follows by expanding the terms in (27):

$$\begin{aligned} C_0 &= \frac{(3 + bL)p^2 - 2(-3 + bL)pq + (3 - 3bL + b^2L^2)q^2}{b^2q^2L^4}, \\ C_1 &= -\frac{2(p+q)^2[(3 + bL)p + (3 - 2bL)q]}{b^2q^2L^4}, \\ C_2 &= \frac{(p+q)^3[(3 + bL)p + (3 - bL)q]}{b^2q^2L^4}. \end{aligned} \quad (28)$$

The terms in (27) are expanded in a series about $r \approx L$ because the nuclear distance r does not fluctuate very far from the equilibrium position at L [35]. Therefore the approximation in (27) gives good result only for $r \approx L$. The equivalence

FIGURE 1: The equivalence of the Pekeris approximation to the centrifugal term.

of the Pekeris approximation to centrifugal term is shown in Figure 1. In this figure the blue curve shows the Pekeris approximation and the dotted curve indicates the centrifugal term.

Using (27) in (26) and defining variable like $z = p/(p + qe^{b(r-L)})$, the following equation is obtained:

$$\frac{d^2 F_{n\kappa}(z)}{dz^2} + \frac{(1-2z)}{z(1-z)} \frac{dF_{n\kappa}(z)}{dz} + \frac{(-\mu^2 - \nu z - \eta z^2)}{z^2(1-z)^2} F_{n\kappa}(z) = 0, \quad (29)$$

where

$$\begin{aligned} \mu^2 &= \frac{\kappa(\kappa+1)C_0 + (m-E)(m+E-S_s)}{b^2}, \\ \nu &= \frac{\kappa(\kappa+1)C_1 + (m+E-S_s)B}{pb^2}, \\ \eta &= \frac{\kappa(\kappa+1)C_2 + (m+E-S_s)A}{p^2b^2}. \end{aligned} \quad (30)$$

Equation (29) has the same form and properties with (2). Therefore the Nikiforov-Uvarov method can be used to solve (29). By comparing this equation with (2), it is seen that

$$\begin{aligned} \tilde{\varepsilon}(z) &= 1 - 2z, \\ \tilde{\xi}(z) &= -\mu^2 - \nu z - \eta z^2, \\ \xi(z) &= z(1-z). \end{aligned} \quad (31)$$

Using (10) and considering the condition of polynomial $\pi(z)$ to be first-degree polynomial, the following equation is found:

$$\pi(z) = \begin{cases} \pm \left[\mu - \left(\mu + \sqrt{\mu^2 + \nu + \eta} \right) z \right]; & k = -(\nu + 2\mu^2) - 2\mu\sqrt{\mu^2 + \nu + \eta} \\ \pm \left[\mu - \left(\mu - \sqrt{\mu^2 + \nu + \eta} \right) z \right]; & k = -(\nu + 2\mu^2) + 2\mu\sqrt{\mu^2 + \nu + \eta}. \end{cases} \quad (32)$$

The polynomial $\pi(z)$ has four values but only one of them satisfies the physical solution. By using (5) and considering the boundary conditions ($r \rightarrow \infty$, $F_{nk}(r) = 0$ and $r \rightarrow 0$, $F_{nk}(r) = 0$), the polynomial $\pi(z)$ which satisfies physical solution is found as follows:

$$\pi(z) = \mu - \left(\mu + \sqrt{\mu^2 + \nu + \eta} \right) z \quad (33)$$

for $k = -(\nu + 2\mu^2) - 2\mu\sqrt{\mu^2 + \nu + \eta}$.

In this case from (5), we obtain $H(z)$ which is a part of the upper radial wave function $F_{nk}(z)$:

$$H(z) = z^\mu (1-z)^{\sqrt{\mu^2 + \nu + \eta}}. \quad (34)$$

By using (11) and (33), the following equation is found:

$$\lambda = -\nu - 2\mu^2 - \mu - (1 + 2\mu)\sqrt{\mu^2 + \nu + \eta}. \quad (35)$$

Also using (7) and (12), the following equation is attained for λ :

$$\lambda = 2n \left[1 + \left(\mu + \sqrt{\mu^2 + \nu + \eta} \right) \right] + n(n-1). \quad (36)$$

Equating (35) with (36), we obtain an equation that gives the bound energy values:

$$\begin{aligned} E^2 - m^2 - S_s(E-m) - \kappa(\kappa+1)C_0 - \frac{\kappa(\kappa+1)C_1 + (E+m-S_s)B}{2p} - \frac{n(n+1)b^2}{2} - \frac{(2n+1)b^2}{2} \\ \cdot \sqrt{\frac{\kappa(\kappa+1)C_0 + (m-E)(m+E-S_s)}{b^2}} - \frac{b^2}{2} \left(1 + 2n + 2\sqrt{\frac{\kappa(\kappa+1)C_0 + (m-E)(m+E-S_s)}{b^2}} \right) \\ \cdot \sqrt{\frac{\kappa(\kappa+1)(C_0p^2 + C_1p + C_2) + (m+E-S_s)[A+Bp + (m-E)p^2]}{b^2p^2}} = 0. \end{aligned} \quad (37)$$

It is not possible to solve this equation analytically so we use the software program to obtain energy values for bound states depending on n , κ and the parameters of potential. Some energy eigenvalues are calculated for $q = 0.8$, $p = 0.65$, $b = 2 \text{ fm}^{-1}$, $L = 7 \text{ fm}$, $A = 8 \text{ fm}^{-1}$, and $B = -12.5 \text{ fm}^{-1}$ and they are given in Table 1.

The weight function $\rho(z)$ and another part of the upper radial wavefunction $F_{nk}(z)$ which is $W(z)$ are found as follows by using (14) and (13), respectively:

$$\begin{aligned} \rho(z) &= z^{2\mu} (1-z)^{2\sqrt{\mu^2 + \nu + \eta}}, \\ W(z) &= C_n z^{-2\mu} (1-z)^{-2\sqrt{\mu^2 + \nu + \eta}} \\ &\cdot \frac{d^n}{dz^n} \left[z^{n+2\mu} (1-z)^{n+2\sqrt{\mu^2 + \nu + \eta}} \right] \\ &\equiv P_n^{(2\mu, 2\sqrt{\mu^2 + \nu + \eta})} (1-2z), \end{aligned} \quad (38)$$

where $P_n^{(2\mu, 2\sqrt{\mu^2 + \nu + \eta})} (1-2z)$ is the jacobi polynomial. The upper radial wave function $F_{nk}(z)$ is written as follows:

$$\begin{aligned} F_{nk}(z) &= H(z) W(z) \\ &= N_{nk} z^\mu (1-z)^{\sqrt{\mu^2 + \nu + \eta}} P_n^{(2\mu, 2\sqrt{\mu^2 + \nu + \eta})} (1-2z), \end{aligned} \quad (39)$$

where N_{nk} is the normalization constant. The lower radial wave function $G_{nk}(z)$ can be obtained by using (20).

5. Pseudospin Symmetric Solution

The sum of scalar potential with vector potential $\Sigma(r)$ and the difference between these potentials $\Delta(r)$ for the pseudospin symmetric case become as follows:

$$\begin{aligned} \Sigma(r) &= S_{ps}, \\ \Delta(r) &= \left[\frac{Ae^{-2b(r-L)}}{(q + pe^{-b(r-L)})^2} + \frac{Be^{-b(r-L)}}{q + pe^{-b(r-L)}} \right], \end{aligned} \quad (40)$$

where S_{ps} is a constant. By using (40), (24) reduces to following equation:

TABLE 1: The bound state energy eigenvalues for the generalized Manning-Rosen potential in the spin symmetric case for $m = 4.76 \text{ fm}^{-1}$ and $S_s = 3.9 \text{ fm}^{-1}$.

l	n	$\kappa < 0$	(l, j)	$E_{n\kappa} (\text{fm}^{-1})$	n	$\kappa > 0$	(l, j)	$E_{n\kappa} (\text{fm}^{-1})$
1	0	-2	$0p_{3/2}$	1.19206	0	1	$0p_{1/2}$	1.19206
1	1	-2	$1p_{3/2}$	2.67722	1	1	$1p_{1/2}$	2.67722
1	2	-2	$2p_{3/2}$	3.60544	2	1	$2p_{1/2}$	3.60544
1	3	-2	$3p_{3/2}$	4.20757	3	1	$3p_{1/2}$	4.20757
2	0	-3	$0d_{5/2}$	1.22410	0	2	$0d_{3/2}$	1.22410
2	1	-3	$1d_{5/2}$	2.69576	1	2	$1d_{3/2}$	2.69576
2	2	-3	$2d_{5/2}$	3.62088	2	2	$2d_{3/2}$	3.62088
2	3	-3	$3d_{5/2}$	4.22255	3	2	$3d_{3/2}$	4.22255
3	0	-4	$0f_{7/2}$	1.27064	0	3	$0f_{5/2}$	1.27064
3	1	-4	$1f_{7/2}$	2.72324	1	3	$1f_{5/2}$	2.72324
3	2	-4	$2f_{7/2}$	3.64383	2	3	$2f_{5/2}$	3.64383
3	3	-4	$3f_{7/2}$	4.24478	3	3	$3f_{5/2}$	4.24478

$$\left\{ \frac{d^2}{dr^2} - \frac{\kappa(\kappa-1)}{r^2} - (E-m-S_{ps}) \right. \\ \cdot \left[\frac{A}{(p+qe^{b(r-L)})^2} + \frac{B}{p+qe^{b(r-L)}} \right] \Big\} G_{n\kappa}(r) + (E+m)(E-m-S_{ps})G_{n\kappa}(r) = 0. \quad (41)$$

Considering (27) instead of the centrifugal term and then changing variable like $z = p/(p+qe^{b(r-L)})$, the following equation is found:

$$\frac{d^2 G_{n\kappa}(z)}{dz^2} + \frac{(1-2z)}{z(1-z)} \frac{dG_{n\kappa}(z)}{dz} + \frac{(-\tilde{\mu}^2 - \tilde{\nu}z - \tilde{\eta}z^2)}{z^2(1-z)^2} G_{n\kappa}(z) = 0, \quad (42)$$

where

$$\tilde{\mu}^2 = \frac{\kappa(\kappa-1)C_0 - (E+m)(E-m-S_{ps})}{b^2}, \\ \tilde{\nu} = \frac{\kappa(\kappa-1)C_1 + (E-m-S_{ps})B}{pb^2}, \\ \tilde{\eta} = \frac{\kappa(\kappa-1)C_2 + (E-m-S_{ps})A}{p^2b^2}. \quad (43)$$

Comparing (42) with (29), it is seen that replacing μ, ν, η , and $F_{n\kappa}(z)$ with $\tilde{\mu}, \tilde{\nu}, \tilde{\eta}$, and $G_{n\kappa}(z)$ in (29), respectively, (42) can be obtained. Therefore the bound state energy relation and the lower radial wave function for pseudospin symmetric limit are found writing $\tilde{\mu}, \tilde{\nu}, \tilde{\eta}$, and $G_{n\kappa}(z)$ instead of μ, ν, η , and $F_{n\kappa}(z)$ in the solutions obtained in the spin symmetry limit. Then, the pseudospin symmetric bound state energy relation is directly written as follows:

$$E^2 - m^2 - S_{ps}(E+m) - \kappa(\kappa-1)C_0 - \frac{\kappa(\kappa-1)C_1 + (E-m-S_{ps})B}{2p} - \frac{n(n+1)b^2}{2} - \frac{(2n+1)b^2}{2} \\ \cdot \sqrt{\frac{\kappa(\kappa-1)C_0 - (E+m)(E-m-S_{ps})}{b^2}} - \frac{b^2}{2} \left(1 + 2n + 2 \sqrt{\frac{\kappa(\kappa-1)C_0 - (E+m)(E-m-S_{ps})}{b^2}} \right) \\ \cdot \sqrt{\frac{\kappa(\kappa-1)(C_0p^2 + C_1p + C_2) + (E-m-S_{ps})[A+Bp - (E+m)p^2]}{b^2p^2}} = 0. \quad (44)$$

The lower radial wave function is obtained in the following form:

$$G_{n\kappa}(z) = \tilde{N}_{n\kappa} z^{\tilde{\mu}} (1-z)^{\sqrt{\tilde{\mu}^2 + \tilde{\nu} + \tilde{\eta}}} P_n^{(2\tilde{\mu}, 2\sqrt{\tilde{\mu}^2 + \tilde{\nu} + \tilde{\eta}})}(1-2z), \quad (45)$$

where $\tilde{N}_{n\kappa}$ is the normalization constant. The upper radial wave function $F_{n\kappa}(z)$ can be found by using (21).

From (44), bound state energy eigenvalues can be calculated for pseudospin symmetry case. We calculate some energy eigenvalues which are given in Table 2 for $q = 0.8$, $p = 0.3$, $b = 2 \text{ fm}^{-1}$, $L = 7 \text{ fm}$, $A = -0.17 \text{ fm}^{-1}$, and $B = -0.9 \text{ fm}^{-1}$ depending on quantum numbers n, κ .

6. Effects of Parameters on Energy Eigenvalues

In this section, we investigate dependence of energy eigenvalues to parameters graphically for $2p_{3/2}$, $2d_{5/2}$, and $2f_{7/2}$ states in the spin symmetric limit and for $2s_{1/2}$, $2p_{3/2}$, and $2d_{5/2}$ states in the pseudospin symmetric limit.

The effects of b, p, L, q, m, S_s , and S_{ps} parameters on energy eigenvalues is given in Figures 2, 3, 4, 5, 6, and 7, respectively. It is seen from these figures that the energy eigenvalues increase as b, m , and S_s increase and decrease as p, L , and q increase in the spin symmetric case. On the other hand in the pseudospin symmetric case, energy eigenvalues increase as q, m , and S_{ps} increase and decrease as b, p , and L increase. Decrease of the energy eigenvalues means that the bound states become less bounded and increase in energy

TABLE 2: The bound state energy eigenvalues for the generalized Manning-Rosen potential in the pseudospin symmetric case for $m = 4.76 \text{ fm}^{-1}$ and $S_{ps} = -0.1 \text{ fm}^{-1}$.

$\tilde{l}$	n	$\kappa < 0$	(l, j)	$E_{m\kappa}$ (fm^{-1})	$n-1$	$\kappa > 0$	$(l+2, j+1)$	$E_{m\kappa}$ (fm^{-1})
1	1	-1	$1s_{1/2}$	-4.73362	0	2	$0d_{3/2}$	-4.73362
1	2	-1	$2s_{1/2}$	-4.19017	1	2	$1d_{3/2}$	-4.19017
1	3	-1	$3s_{1/2}$	-2.89434	2	2	$2d_{3/2}$	-2.89434
2	1	-2	$1p_{3/2}$	-4.74120	0	3	$0f_{5/2}$	-4.74120
2	2	-2	$2p_{3/2}$	-4.20134	1	3	$1f_{5/2}$	-4.20134
2	3	-2	$3p_{3/2}$	-2.91352	2	3	$2f_{5/2}$	-2.91352
3	1	-3	$1d_{5/2}$	-4.75252	0	4	$0g_{7/2}$	-4.75252
3	2	-3	$2d_{5/2}$	-4.21801	1	4	$1g_{7/2}$	-4.21801
3	3	-3	$3d_{5/2}$	-2.94202	2	4	$2g_{7/2}$	-2.94202

eigenvalues indicates that the bound states become more tightly bounded.

In Figure 4(a), the bound states do not occur in $2d_{5/2}$ and $2f_{7/2}$ for $L = 1$ because the energy values of $2d_{5/2}$ and $2f_{7/2}$ are greater than the mass of nucleon. In order to obtain a bound state the binding energies of nuclear states must be smaller than the mass of nucleon ($m \approx 4.76 \text{ fm}^{-1} \approx 939 \text{ MeV}$). Moreover, it is seen from Figure 4 that the binding energies of nuclear states are getting closer to each other as L increases.

7. Special Cases

In certain limits and depending on values of the potential parameters, the generalized Manning-Rosen potential can be reduced to various potentials that are used to explain different physical processes. The bound states energy eigenvalues for these potentials are investigated and compared with the previous studies.

7.1. Manning-Rosen Potential. If parameters of the generalized Manning-Rosen potential are chosen as $p = -1$, $b = 1/\beta$, $q = e^{bL}$, $A = \alpha(\alpha-1)/2m\beta^2$, and $B = -\tilde{A}/2m\beta^2$, the following potential is found:

$$V(r) = \frac{1}{2m\beta^2} \left[\frac{\alpha(\alpha-1)e^{-2r/\beta}}{(1-e^{-r/\beta})^2} - \frac{\tilde{A}e^{-r/\beta}}{1-e^{-r/\beta}} \right]. \quad (46)$$

This potential is known as the Manning-Rosen potential [68]. From (44) and (37), the bound energy eigenvalues of the Manning-Rosen potential can be obtained for pseudospin symmetric and spin symmetric cases. For $L = 17.5$, $p = -1$, $m = 1$, $S_{ps} = -6$, $S_s = 6$, $\alpha = 1.5$, $\beta = 20$, and $\tilde{A} = 30.52$, the bound energy eigenvalues for pseudospin and spin symmetric cases are calculated and these are given in Tables 3 and 4, respectively. In Table 3 our results are compared with those presented in [27]. Also if $L = 0.05$ and the values of parameters which are given in [26] are used for spin symmetric case, it can be seen that our results are consistent with the results of [26].

7.2. Kratzer-Fues Potential. By taking $p = -1$, $q = e^{bL}$, $A = D_e r_e^2 b^2$, $B = -2D_e r_e b$, and $\lim_{b \rightarrow 0}$ in (1), the generalized Manning-Rosen potential reduces to Kratzer-Fues potential [80, 81]:

$$V_{KF}(r) = \lim_{b \rightarrow 0} \left[\frac{D_e r_e^2 b^2 e^{-2br}}{(1-e^{-br})^2} - \frac{2D_e r_e b e^{-br}}{1-e^{-br}} \right] \quad (47)$$

$$= -D_e \left(\frac{2r_e}{r} - \frac{r_e^2}{r^2} \right),$$

where D_e and r_e are the dissociation energy and equilibrium internuclear length, respectively.

For $L = 4.9$, $p = -1$, $m = 5 \text{ fm}^{-1}$, $S_{ps} = 0$, $S_s = 0$, $b = 0.000002$, $D_e = 1.25 \text{ fm}^{-1}$, and $r_e = 0.35 \text{ fm}$, the bound state energy eigenvalues for Kratzer-Fues potential are found in the pseudospin and spin symmetric cases by using (44) and (37), respectively, and these are given in Tables 5 and 6. it can be seen from these tables that our energy eigenvalues are consistent with those obtained in [46] both for pseudospin and for spin symmetric cases.

7.3. Morse Potential. Considering $q = 1$, $A = D_e$, $B = -2D_e$, $b = a$, and $L = r_e$ and taking $\lim_{p \rightarrow 0}$, the generalized Manning-Rosen potential reduces to the Morse potential [74]:

$$V_M(r) = \lim_{p \rightarrow 0} \left[\frac{D_e e^{-2a(r-r_e)}}{(1 + p e^{-a(r-r_e)})^2} - \frac{2D_e e^{-a(r-r_e)}}{1 + p e^{-a(r-r_e)}} \right] \quad (48)$$

$$\approx D_e [e^{-2a(r-r_e)} - 2e^{-a(r-r_e)}],$$

where r_e is the equilibrium distance and a is related to the range of the potential well. We can not take $p = 0$ because the solutions of (44) and (37) go to infinity.

The bound state energy eigenvalues for the Morse potential in the pseudospin and spin symmetric cases are given in Tables 7 and 8, respectively, for $q = 1$, $m = 10 \text{ fm}^{-1}$, $S_{ps} = -10 \text{ fm}^{-1}$, $S_s = 10 \text{ fm}^{-1}$, $b = 0.988879$, $D_e = 5 \text{ fm}^{-1}$, $L = 2.40873$, and $p = 0.000000000013$. Equations (44) and (37) are used to calculate the bound state energy eigenvalues. These tables show that the energy eigenvalues obtained for pseudospin and spin symmetric cases are compatible with the energy eigenvalues of [49, 50] when they are compared.

If the parameters are chosen as $q = e^{bL}$, $A = 0$, $B = -V_0$, and $\lim_{p \rightarrow 0}$ in (1), the generalized Manning-Rosen potential reduces to the cusp potential [90]. The energy eigenvalues for this potential also can be calculated by using (44) in the pseudospin symmetry limit and (37) in the spin symmetry limit.

7.4. Hulthen Potential. Hulthen potential [84] can be obtained as follows by taking $p = -1$, $q = e^{bL}$, $b = 2\alpha$, $A = 0$, and $B = -V_0$ in (1):

$$V_H(r) = -V_0 \frac{e^{-2\alpha r}}{1 - e^{-2\alpha r}}, \quad (49)$$

FIGURE 2: (a) Energy eigenvalues versus b in spin symmetric limit for $A = 8 \text{ fm}^{-1}$, $B = -12.5 \text{ fm}^{-1}$, $S_s = 3.9 \text{ fm}^{-1}$, $L = 7 \text{ fm}$, $m = 4.76 \text{ fm}^{-1}$, $q = 0.8$, and $p = 0.65$. (b) Energy eigenvalues versus b in pseudospin symmetric limit for $A = -0.17 \text{ fm}^{-1}$, $B = -0.9 \text{ fm}^{-1}$, $S_{ps} = -0.1 \text{ fm}^{-1}$, $L = 7 \text{ fm}$, $m = 4.76 \text{ fm}^{-1}$, $q = 0.8$, and $p = 0.3$.

FIGURE 3: (a) Energy eigenvalues versus p in spin symmetric limit for $A = 8 \text{ fm}^{-1}$, $B = -12.5 \text{ fm}^{-1}$, $S_s = 3.9 \text{ fm}^{-1}$, $L = 7 \text{ fm}$, $m = 4.76 \text{ fm}^{-1}$, $b = 2 \text{ fm}^{-1}$, and $q = 0.8$. (b) Energy eigenvalues versus p in pseudospin symmetric limit for $A = -0.17 \text{ fm}^{-1}$, $B = -0.9 \text{ fm}^{-1}$, $S_{ps} = -0.1 \text{ fm}^{-1}$, $L = 7 \text{ fm}$, $m = 4.76 \text{ fm}^{-1}$, $b = 2 \text{ fm}^{-1}$, and $q = 0.8$.

where V_0 is the dept of the potential and α is the screening parameter of the potential.

By using (44) and (37), bound state energy eigenvalues are calculated for Hulthen potential in the pseudospin and spin symmetric cases for $b = 0.1 \text{ fm}$, $L = 15$, $B = -4 \text{ fm}^{-1}$,

$S_{ps} = -5 \text{ fm}^{-1}$, $S_s = 5 \text{ fm}^{-1}$, and $m = 5 \text{ fm}^{-1}$. These energy eigenvalues are compared with the energy eigenvalues of [40] in Tables 9 and 10. Here our energy eigenvalues are little different from results of [40]. The reason of this is approximation that is applied to the centrifugal term. The authors of [40]

FIGURE 4: (a) Energy eigenvalues versus L in spin symmetric limit for $A = 8 \text{ fm}^{-1}$, $B = -12.5 \text{ fm}^{-1}$, $S_s = 3.9 \text{ fm}^{-1}$, $m = 4.76 \text{ fm}^{-1}$, $b = 2 \text{ fm}^{-1}$, $p = 0.65$, and $q = 0.8$. (b) Energy eigenvalues versus L in pseudospin symmetric limit for $A = -0.17 \text{ fm}^{-1}$, $B = -0.9 \text{ fm}^{-1}$, $S_{ps} = -0.1 \text{ fm}^{-1}$, $m = 4.76 \text{ fm}^{-1}$, $b = 2 \text{ fm}^{-1}$, $p = 0.3$, and $q = 0.8$.

FIGURE 5: (a) Energy eigenvalues versus q in spin symmetric limit for $A = 8 \text{ fm}^{-1}$, $B = -12.5 \text{ fm}^{-1}$, $S_s = 3.9 \text{ fm}^{-1}$, $m = 4.76 \text{ fm}^{-1}$, $L = 7 \text{ fm}$, $b = 2 \text{ fm}^{-1}$, and $p = 0.65$. (b) Energy eigenvalues versus q in pseudospin symmetric limit for $A = -0.17 \text{ fm}^{-1}$, $B = -0.9 \text{ fm}^{-1}$, $S_{ps} = -0.1 \text{ fm}^{-1}$, $m = 4.76 \text{ fm}^{-1}$, $L = 7 \text{ fm}$, $b = 2 \text{ fm}^{-1}$, and $p = 0.3$.

have used the Greene-Aldrich [91] approximation but we use the Pekeris [89] approximation instead of centrifugal term.

7.5. Yukawa Potential. Yukawa potential [85] can be obtained by considering $q = e^{bL}$, $b = 2\alpha$, $B = -2\alpha V_0$, $A = 0$, and $p = -1$ in (1):

$$V_Y(r) = -V_0 e^{-\alpha r} \left(\frac{2\alpha e^{-\alpha r}}{1 - e^{-2\alpha r}} \right) \approx -V_0 \frac{e^{-\alpha r}}{r}, \quad (50)$$

where $1/r \approx (2\alpha e^{-\alpha r} / (1 - e^{-2\alpha r}))$ and α is the range of the nucleon force in the meson theory [55, 85].

FIGURE 6: (a) Energy eigenvalues versus m in spin symmetric limit for $A = 8 \text{ fm}^{-1}$, $B = -12.5 \text{ fm}^{-1}$, $S_s = 3.9 \text{ fm}^{-1}$, $L = 7 \text{ fm}$, $b = 2 \text{ fm}^{-1}$, $q = 0.8$, and $p = 0.65$. (b) Energy eigenvalues versus m in pseudospin symmetric limit for $A = -0.17 \text{ fm}^{-1}$, $B = -0.9 \text{ fm}^{-1}$, $S_{ps} = -0.1 \text{ fm}^{-1}$, $L = 7 \text{ fm}$, $b = 2 \text{ fm}^{-1}$, $q = 0.8$, and $p = 0.3$.

FIGURE 7: (a) Energy eigenvalues versus S_s in spin symmetric limit for $A = 8 \text{ fm}^{-1}$, $B = -12.5 \text{ fm}^{-1}$, $m = 4.76 \text{ fm}^{-1}$, $L = 7 \text{ fm}$, $b = 2 \text{ fm}^{-1}$, $q = 0.8$, and $p = 0.65$. (b) Energy eigenvalues versus S_{ps} in pseudospin symmetric limit for $A = -0.17 \text{ fm}^{-1}$, $B = -0.9 \text{ fm}^{-1}$, $m = 4.76 \text{ fm}^{-1}$, $L = 7 \text{ fm}$, $b = 2 \text{ fm}^{-1}$, $q = 0.8$, and $p = 0.3$.

The energy eigenvalues for this potential are found in the pseudospin and spin symmetric cases by using (44) and (37), respectively, for $b = 0.2 \text{ fm}$, $L = 0.0001$, $V_0 = 1 \text{ fm}^{-1}$, $S_{ps} = -5 \text{ fm}^{-1}$, $S_s = 5 \text{ fm}^{-1}$, and $m = 5 \text{ fm}^{-1}$. These energy eigenvalues for pseudospin symmetry are compared with the

results of [55, 56] in Table 11 and spin symmetric energy eigenvalues are compared with the results of [56] in Table 12. As shown in Tables 11 and 12, pseudospin and spin doublets have not occurred in results of [55, 56]. Besides, in $1s_{1/2}$ and $2p_{3/2}$ for pseudospin symmetry and in $1p_{3/2}$ for spin

TABLE 3: The bound state energy eigenvalues for the Manning-Rosen potential in the pseudospin symmetric case for $m = 1$ and $S_{ps} = -6$. Here first states column is according to (l, j) and second states column is according to $(l + 2, j + 1)$.

$ n, \tilde{l}, \kappa\rangle$	State	$E_{n\kappa}$	$E_{n\kappa}$ [27]	$ n - 1, \tilde{l}, \kappa\rangle$	State	$E_{n\kappa}$	$E_{n\kappa}$ [27]
$ 1, 1, -1\rangle$	$1s_{1/2}$	-4.99867	-4.99866	$ 0, 1, 2\rangle$	$0d_{3/2}$	-4.99867	-4.99866
$ 1, 2, -2\rangle$	$1p_{3/2}$	-4.99774	-4.99772	$ 0, 2, 3\rangle$	$0f_{5/2}$	-4.99774	-4.99772
$ 1, 3, -3\rangle$	$1d_{5/2}$	-4.99657	-4.99656	$ 0, 3, 4\rangle$	$0g_{7/2}$	-4.99657	-4.99656
$ 1, 4, -4\rangle$	$1f_{7/2}$	-4.99517	-4.99517	$ 0, 4, 5\rangle$	$0h_{9/2}$	-4.99517	-4.99517

TABLE 4: The bound state energy eigenvalues for the Manning-Rosen potential in the spin symmetric case for $m = 1$ and $S_s = 6$. Here first and second states columns are according to (l, j) .

$ n, l, \kappa\rangle$	State	$E_{n\kappa}$	$ n, l, \kappa\rangle$	State	$E_{n\kappa}$
$ 0, 1, -2\rangle$	$0p_{3/2}$	4.99945	$ 0, 1, 1\rangle$	$0p_{1/2}$	4.99945
$ 0, 2, -3\rangle$	$0d_{5/2}$	4.99882	$ 0, 2, 2\rangle$	$0d_{3/2}$	4.99882
$ 0, 3, -4\rangle$	$0f_{7/2}$	4.99795	$ 0, 3, 3\rangle$	$0f_{5/2}$	4.99795
$ 0, 4, -5\rangle$	$0g_{9/2}$	4.99685	$ 0, 4, 4\rangle$	$0g_{7/2}$	4.99685

TABLE 5: The bound state energy eigenvalues for the Kratzer-Fues potential in the pseudospin symmetric case for $m = 5 \text{ fm}^{-1}$ and $S_{ps} = 0 \text{ fm}^{-1}$. Here first states column is according to (l, j) and second states column is according to $(l + 2, j + 1)$.

$ n, \tilde{l}, \kappa\rangle$	State	$E_{n\kappa}$	$E_{n\kappa}$ [46]	$ n - 1, \tilde{l}, \kappa\rangle$	State	$E_{n\kappa}$	$E_{n\kappa}$ [46]
$ 1, 1, -1\rangle$	$1s_{1/2}$	-4.671859	-4.672305	$ 0, 1, 2\rangle$	$0d_{3/2}$	-4.671859	-4.672305
$ 1, 2, -2\rangle$	$1p_{3/2}$	-4.860367	-4.860421	$ 0, 2, 3\rangle$	$0f_{5/2}$	-4.860367	-4.860421
$ 1, 3, -3\rangle$	$1d_{5/2}$	-4.916765	-4.916782	$ 0, 3, 4\rangle$	$0g_{7/2}$	-4.916765	-4.916782
$ 1, 4, -4\rangle$	$1f_{7/2}$	-4.943945	-4.943953	$ 0, 4, 5\rangle$	$0h_{9/2}$	-4.943945	-4.943953
$ 2, 1, -1\rangle$	$2s_{1/2}$	-4.833423	-4.833547	$ 1, 1, 2\rangle$	$1d_{3/2}$	-4.833423	-4.833547
$ 2, 2, -2\rangle$	$2p_{3/2}$	-4.913175	-4.913195	$ 1, 2, 3\rangle$	$1f_{5/2}$	-4.913175	-4.913195
$ 2, 3, -3\rangle$	$2d_{5/2}$	-4.942933	-4.942941	$ 1, 3, 4\rangle$	$1g_{7/2}$	-4.942933	-4.942941
$ 2, 4, -4\rangle$	$2f_{7/2}$	-4.959099	-4.959104	$ 1, 4, 5\rangle$	$1h_{9/2}$	-4.959099	-4.959104

TABLE 6: The bound state energy eigenvalues for the Kratzer-Fues potential in the spin symmetric case for $m = 5 \text{ fm}^{-1}$ and $S_s = 0 \text{ fm}^{-1}$. Here first and second states columns are according to (l, j) .

$ n, l, \kappa\rangle$	State	$E_{n\kappa}$	$E_{n\kappa}$ [46]	$ n, l, \kappa\rangle$	State	$E_{n\kappa}$	$E_{n\kappa}$ [46]
$ 0, 1, -2\rangle$	$0p_{3/2}$	4.686684	4.686570	$ 0, 1, 1\rangle$	$0p_{1/2}$	4.686684	4.686570
$ 0, 2, -3\rangle$	$0d_{5/2}$	4.825721	4.825688	$ 0, 2, 2\rangle$	$0d_{3/2}$	4.825721	4.825688
$ 0, 3, -4\rangle$	$0f_{7/2}$	4.893179	4.893167	$ 0, 3, 3\rangle$	$0f_{5/2}$	4.893179	4.893167
$ 0, 4, -5\rangle$	$0g_{9/2}$	4.928792	4.928786	$ 0, 4, 4\rangle$	$0g_{7/2}$	4.928792	4.928786
$ 1, 1, -2\rangle$	$1p_{3/2}$	4.840692	4.840668	$ 1, 1, 1\rangle$	$1p_{1/2}$	4.840692	4.840668
$ 1, 2, -3\rangle$	$1d_{5/2}$	4.896919	4.896810	$ 1, 2, 2\rangle$	$1d_{3/2}$	4.896919	4.896810
$ 1, 3, -4\rangle$	$1f_{7/2}$	4.930006	4.930002	$ 1, 3, 3\rangle$	$1f_{5/2}$	4.930006	4.930002
$ 1, 4, -5\rangle$	$1g_{9/2}$	4.949914	4.949911	$ 1, 4, 4\rangle$	$1g_{7/2}$	4.949914	4.949911

TABLE 7: The bound state energy eigenvalues for the Morse potential in the pseudospin symmetric case for $m = 10 \text{ fm}^{-1}$ and $S_{ps} = -10 \text{ fm}^{-1}$. Here first states column is according to (l, j) and second states column is according to $(l + 2, j + 1)$.

$ n, \tilde{l}, \kappa\rangle$	State	$E_{n\kappa}$	$E_{n\kappa}$ [49, 50]	$ n - 1, \tilde{l}, \kappa\rangle$	State	$E_{n\kappa}$	$E_{n\kappa}$ [49, 50]
$ 1, 1, -1\rangle$	$1s_{1/2}$	-0.0064125	-0.0064123	$ 0, 1, 2\rangle$	$0d_{3/2}$	-0.0064125	-0.0064123
$ 1, 2, -2\rangle$	$1p_{3/2}$	-0.0155780	-0.0155771	$ 0, 2, 3\rangle$	$0f_{5/2}$	-0.0155780	-0.0155771
$ 1, 3, -3\rangle$	$1d_{5/2}$	-0.0243675	-0.0243659	$ 0, 3, 4\rangle$	$0g_{7/2}$	-0.0243675	-0.0243659
$ 1, 4, -4\rangle$	$1f_{7/2}$	-0.0305324	-0.0305297	$ 0, 4, 5\rangle$	$0h_{9/2}$	-0.0305324	-0.0305297
$ 2, 1, -1\rangle$	$2s_{1/2}$	-0.0070207	-0.0070204	$ 1, 1, 2\rangle$	$1d_{3/2}$	-0.0070207	-0.0070204
$ 2, 2, -2\rangle$	$2p_{3/2}$	-0.0190450	-0.0190441	$ 1, 2, 3\rangle$	$1f_{5/2}$	-0.0190450	-0.0190441
$ 2, 3, -3\rangle$	$2d_{5/2}$	-0.0337736	-0.0337719	$ 1, 3, 4\rangle$	$1g_{7/2}$	-0.0337736	-0.0337719
$ 2, 4, -4\rangle$	$2f_{7/2}$	-0.0492178	-0.0492150	$ 1, 4, 5\rangle$	$1h_{9/2}$	-0.0492178	-0.0492150

TABLE 8: The bound state energy eigenvalues for the Morse potential in the spin symmetric case for $m = 10 \text{ fm}^{-1}$ and $S_s = 10 \text{ fm}^{-1}$. Here first and second states columns are according to (l, j) .

$ n, l, \kappa\rangle$	State	$E_{n\kappa}$	$E_{n\kappa}$ [49]	$ n, l, \kappa\rangle$	State	$E_{n\kappa}$	$E_{n\kappa}$ [49]
$ 0, 1, -2\rangle$	$0p_{3/2}$	0.0188480	0.0188481	$ 0, 1, 1\rangle$	$0p_{1/2}$	0.0188480	0.0188481
$ 0, 2, -3\rangle$	$0d_{5/2}$	0.0336560	0.0336562	$ 0, 2, 2\rangle$	$0d_{3/2}$	0.0336560	0.0336562
$ 0, 3, -4\rangle$	$0f_{7/2}$	0.0525267	0.0525273	$ 0, 3, 3\rangle$	$0f_{5/2}$	0.0525267	0.0525273
$ 0, 4, -5\rangle$	$0g_{9/2}$	0.0754340	0.0754350	$ 0, 4, 4\rangle$	$0g_{7/2}$	0.0754340	0.0754350
$ 1, 1, -2\rangle$	$1p_{3/2}$	0.0899995	0.0899995	$ 1, 1, 1\rangle$	$1p_{1/2}$	0.0899995	0.0899995
$ 1, 2, -3\rangle$	$1d_{5/2}$	0.1136725	0.1136725	$ 1, 2, 2\rangle$	$1d_{3/2}$	0.1136725	0.1136725
$ 1, 3, -4\rangle$	$1f_{7/2}$	0.1438029	0.1438031	$ 1, 3, 3\rangle$	$1f_{5/2}$	0.1438029	0.1438031
$ 1, 4, -5\rangle$	$1g_{9/2}$	0.1791421	0.1791425	$ 1, 4, 4\rangle$	$1g_{7/2}$	0.1791421	0.1791425

TABLE 9: The bound state energy eigenvalues for the Hulthen potential in the pseudospin symmetric case for $m = 5 \text{ fm}^{-1}$ and $S_{ps} = -5 \text{ fm}^{-1}$. Here first states column is according to (l, j) and second states column is according to $(l + 2, j + 1)$.

$ n, \tilde{l}, \kappa\rangle$	State	$E_{n\kappa}$	$E_{n\kappa}$ [40]	$ n - 1, \tilde{l}, \kappa\rangle$	State	$E_{n\kappa}$	$E_{n\kappa}$ [40]
$ 1, 1, -1\rangle$	$1s_{1/2}$	-0.00875024	-0.00862755	$ 0, 1, 2\rangle$	$0d_{3/2}$	-0.00875024	-0.00862755
$ 1, 2, -2\rangle$	$1p_{3/2}$	-0.01566113	-0.01538462	$ 0, 2, 3\rangle$	$0f_{5/2}$	-0.01566113	-0.01538462
$ 1, 3, -3\rangle$	$1d_{5/2}$	-0.02467877	-0.02413384	$ 0, 3, 4\rangle$	$0g_{7/2}$	-0.02467877	-0.02413384
$ 1, 4, -4\rangle$	$1f_{7/2}$	-0.03708404	-0.03492388	$ 0, 4, 5\rangle$	$0h_{9/2}$	-0.03708404	-0.03492388
$ 2, 1, -1\rangle$	$2s_{1/2}$	-0.01554783	-0.01538462	$ 1, 1, 2\rangle$	$1d_{3/2}$	-0.01554783	-0.01538462
$ 2, 2, -2\rangle$	$2p_{3/2}$	-0.02452379	-0.02413384	$ 1, 2, 3\rangle$	$1f_{5/2}$	-0.02452379	-0.02413384
$ 2, 3, -3\rangle$	$2d_{5/2}$	-0.03574590	-0.03492388	$ 1, 3, 4\rangle$	$1g_{7/2}$	-0.03574590	-0.03492388
$ 2, 4, -4\rangle$	$2f_{7/2}$	-0.04888397	-0.04781723	$ 1, 4, 5\rangle$	$1h_{9/2}$	-0.04888397	-0.04781723

TABLE 10: The bound state energy eigenvalues for the Hulthen potential in the spin symmetric case for $m = 5 \text{ fm}^{-1}$ and $S_s = 5 \text{ fm}^{-1}$. Here first and second states columns are according to (l, j) .

$ n, l, \kappa\rangle$	State	$E_{n\kappa}$	$E_{n\kappa}$ [40]	$ n, l, \kappa\rangle$	State	$E_{n\kappa}$	$E_{n\kappa}$ [40]
$ 0, 1, -2\rangle$	$0p_{3/2}$	0.06582980	0.06784763	$ 0, 1, 1\rangle$	$0p_{1/2}$	0.06582980	0.06784763
$ 0, 2, -3\rangle$	$0d_{5/2}$	0.14592114	0.15074992	$ 0, 2, 2\rangle$	$0d_{3/2}$	0.14592114	0.15074992
$ 0, 3, -4\rangle$	$0f_{7/2}$	0.25504870	0.26338976	$ 0, 3, 3\rangle$	$0f_{5/2}$	0.25504870	0.26338976
$ 0, 4, -5\rangle$	$0g_{9/2}$	0.39036621	0.40263365	$ 0, 4, 4\rangle$	$0g_{7/2}$	0.39036621	0.40263365
$ 1, 1, -2\rangle$	$1p_{3/2}$	0.14708189	0.15074992	$ 1, 1, 1\rangle$	$1p_{1/2}$	0.14708189	0.15074992
$ 1, 2, -3\rangle$	$1d_{5/2}$	0.25580818	0.26338976	$ 1, 2, 2\rangle$	$1d_{3/2}$	0.25580818	0.26338976
$ 1, 3, -4\rangle$	$1f_{7/2}$	0.39073879	0.40263365	$ 1, 3, 3\rangle$	$1f_{5/2}$	0.39073879	0.40263365
$ 1, 4, -5\rangle$	$1g_{9/2}$	0.54849660	0.56482502	$ 1, 4, 4\rangle$	$1g_{7/2}$	0.54849660	0.56482502

TABLE 11: The bound state energy eigenvalues for the Yukawa potential in the pseudospin symmetric case for $m = 5 \text{ fm}^{-1}$ and $S_{ps} = -5 \text{ fm}^{-1}$. Here first states column is according to (l, j) and second states column is according to $(l + 2, j + 1)$.

$ n, \tilde{l}, \kappa\rangle$	State	$E_{n\kappa}$	$E_{n\kappa}$ [55, 56]	$ n - 1, \tilde{l}, \kappa\rangle$	State	$E_{n\kappa}$	$E_{n\kappa}$ [55, 56]
$ 1, 1, -1\rangle$	$1s_{1/2}$	-0.0184387	—	$ 0, 1, 2\rangle$	$0d_{3/2}$	-0.0184387	-0.0184387
$ 1, 2, -2\rangle$	$1p_{3/2}$	-0.0327981	-0.0020419	$ 0, 2, 3\rangle$	$0f_{5/2}$	-0.0327981	-0.0328771
$ 1, 3, -3\rangle$	$1d_{5/2}$	-0.0515637	-0.0081778	$ 0, 3, 4\rangle$	$0g_{7/2}$	-0.0515637	-0.0515686
$ 1, 4, -4\rangle$	$1f_{7/2}$	-0.0746132	-0.0184387	$ 0, 4, 5\rangle$	$0h_{9/2}$	-0.0746132	-0.0746134
$ 2, 1, -1\rangle$	$2s_{1/2}$	-0.0327981	-0.0020419	$ 1, 1, 2\rangle$	$1d_{3/2}$	-0.0327981	-0.0328771
$ 2, 2, -2\rangle$	$2p_{3/2}$	-0.0515686	—	$ 1, 2, 3\rangle$	$1f_{5/2}$	-0.0515686	-0.0515686
$ 2, 3, -3\rangle$	$2d_{5/2}$	-0.0744240	-0.0020419	$ 1, 3, 4\rangle$	$1g_{7/2}$	-0.0744240	-0.0746134
$ 2, 4, -4\rangle$	$2f_{7/2}$	-0.1021320	-0.0081778	$ 1, 4, 5\rangle$	$1h_{9/2}$	-0.1021320	-0.1021400

TABLE 12: The bound state energy eigenvalues for the Yukawa potential in the spin symmetric case for $m = 5 \text{ fm}^{-1}$ and $S_s = 5 \text{ fm}^{-1}$. Here first and second states columns are according to (l, j) .

$ n, l, \kappa\rangle$	State	$E_{n\kappa}$	$E_{n\kappa}$ [56]	$ n, l, \kappa\rangle$	State	$E_{n\kappa}$	$E_{n\kappa}$ [56]
$ 0, 1, -2\rangle$	$0p_{3/2}$	0.0078560	0.0019617	$ 0, 1, 1\rangle$	$0p_{1/2}$	0.0078560	0.0078560
$ 0, 2, -3\rangle$	$0d_{5/2}$	0.0177103	0.0078560	$ 0, 2, 2\rangle$	$0d_{3/2}$	0.0177103	0.0177103
$ 0, 3, -4\rangle$	$0f_{7/2}$	0.0315710	0.0177103	$ 0, 3, 3\rangle$	$0f_{5/2}$	0.0315710	0.0315710
$ 0, 4, -5\rangle$	$0g_{9/2}$	0.0495050	0.0315710	$ 0, 4, 4\rangle$	$0g_{7/2}$	0.0495050	0.0495050
$ 1, 1, -2\rangle$	$1p_{3/2}$	0.0177103	—	$ 1, 1, 1\rangle$	$1p_{1/2}$	0.0177103	0.0177103
$ 1, 2, -3\rangle$	$1d_{5/2}$	0.0314949	0.0019617	$ 1, 2, 2\rangle$	$1d_{3/2}$	0.0314949	0.0315710
$ 1, 3, -4\rangle$	$1f_{7/2}$	0.0495002	0.0078560	$ 1, 3, 3\rangle$	$1f_{5/2}$	0.0495002	0.0495050
$ 1, 4, -5\rangle$	$1g_{9/2}$	0.0716001	0.0177103	$ 1, 4, 4\rangle$	$1g_{7/2}$	0.0716001	0.0716004

TABLE 13: The bound state energy eigenvalues for the Woods-Saxon potential in the pseudospin symmetric case for $m = 5 \text{ fm}^{-1}$ and $S_{ps} = 0 \text{ fm}^{-1}$. Here first states column is according to (l, j) and second states column is according to $(l + 2, j + 1)$.

$ n, \tilde{l}, \kappa\rangle$	State	$E_{n\kappa}$	$ n, \tilde{l}, \kappa\rangle$	State	$E_{n\kappa}$
$ 1, 1, -1\rangle$	$1s_{1/2}$	-4.97891	$ 0, 1, 2\rangle$	$0d_{3/2}$	-4.97891
$ 1, 2, -2\rangle$	$1p_{3/2}$	-4.98562	$ 0, 2, 3\rangle$	$0f_{5/2}$	-4.98562
$ 1, 3, -3\rangle$	$1d_{5/2}$	-4.99564	$ 0, 3, 4\rangle$	$0g_{7/2}$	-4.99564
$ 1, 4, -4\rangle$	$1f_{7/2}$	-5.00894	$ 0, 4, 5\rangle$	$0h_{9/2}$	-5.00894
$ 2, 1, -1\rangle$	$2s_{1/2}$	-4.81296	$ 1, 1, 2\rangle$	$1d_{3/2}$	-4.81296
$ 2, 2, -2\rangle$	$2p_{3/2}$	-4.81873	$ 1, 2, 3\rangle$	$1f_{5/2}$	-4.81873
$ 2, 3, -3\rangle$	$2d_{5/2}$	-4.82742	$ 1, 3, 4\rangle$	$1g_{7/2}$	-4.82742
$ 2, 4, -4\rangle$	$2f_{7/2}$	-4.83904	$ 1, 4, 5\rangle$	$1h_{9/2}$	-4.83904

symmetry, bound energy eigenvalues have not achieved. However, we find bound energy eigenvalues for $1s_{1/2}$, $1p_{3/2}$, and $2p_{3/2}$ states and also we obtain pseudospin and spin doublets. The reason of this is the approximation. Unlike [55, 56] we use the Pekeris approximation to the centrifugal term.

7.6. Woods-Saxon Potential. If the potential parameters are taken $A = 0$, $q = p = 1$, $B = -V_0$, $b = 1/a$, and $L = R$, the generalized Manning-Rosen potential reduces to the Woods-Saxon potential [83]:

$$V_{\text{WS}}(r) = -\frac{V_0}{1 + e^{(r-R)/a}}, \quad (51)$$

where a is thickness of surface and R is nuclear radius.

For this potential, the bound energy eigenvalues are found in pseudospin and spin symmetric cases by using (44) and (37) for $b = 2 \text{ fm}^{-1}$, $L = 7 \text{ fm}$, $m = 5 \text{ fm}^{-1}$, $B = -2 \text{ fm}^{-1}$, and $S_s = S_{ps} = 0$. These energy eigenvalues are presented in Tables 13 and 14. Also for the other set of parameters like $m = 10 \text{ fm}^{-1}$, $B = -10 \text{ fm}^{-1}$, $b = 2 \text{ fm}^{-1}$, $L = 7 \text{ fm}$, and $S_{ps} = -10.1 \text{ fm}^{-1}$, our energy eigenvalues for pseudospin symmetry which can be found by using (44) are same as energy eigenvalues of [35] that is obtained in the absence of tensor potential.

7.7. q -Parameter Hyperbolic Pöschl-Teller Potential. q -parameter hyperbolic Pöschl-Teller potential [79] is obtained by

TABLE 14: The bound state energy eigenvalues for the Woods-Saxon potential in the spin symmetric case for $m = 5 \text{ fm}^{-1}$ and $S_s = 0 \text{ fm}^{-1}$. Here first and second states columns are according to (l, j) .

$ n, l, \kappa\rangle$	State	$E_{n\kappa}$	$ n, l, \kappa\rangle$	State	$E_{n\kappa}$
$ 0, 1, -2\rangle$	$0p_{3/2}$	2.10339	$ 0, 1, 1\rangle$	$0p_{1/2}$	2.10339
$ 0, 2, -3\rangle$	$0d_{5/2}$	2.13554	$ 0, 2, 2\rangle$	$0d_{3/2}$	2.13554
$ 0, 3, -4\rangle$	$0f_{7/2}$	2.18283	$ 0, 3, 3\rangle$	$0f_{5/2}$	2.18283
$ 0, 4, -5\rangle$	$0g_{9/2}$	2.24422	$ 0, 4, 4\rangle$	$0g_{7/2}$	2.24422
$ 1, 1, -2\rangle$	$1p_{3/2}$	3.00686	$ 1, 1, 1\rangle$	$1p_{1/2}$	3.00686
$ 1, 2, -3\rangle$	$1d_{5/2}$	3.02052	$ 1, 2, 2\rangle$	$1d_{3/2}$	3.02052
$ 1, 3, -4\rangle$	$1f_{7/2}$	3.04086	$ 1, 3, 3\rangle$	$1f_{5/2}$	3.04086
$ 1, 4, -5\rangle$	$1g_{9/2}$	3.06769	$ 1, 4, 4\rangle$	$1g_{7/2}$	3.06769

taking the parameters as $p = \tilde{q}$, $A = 4V_0\tilde{q}$, $B = -4V_0$, $b = 2\alpha$, and $q = e^{2\alpha L}$ in (1):

$$V_{q\text{HPT}}(r) = -V_0 \frac{4}{(e^{\alpha r} + \tilde{q}e^{-\alpha r})^2} = -\frac{V_0}{\cosh_q^2(\alpha r)}. \quad (52)$$

For this potential, we find energy eigenvalues from (44) in pseudospin symmetric limit and from (37) in spin symmetric limit by considering $V_0 = 2$, $b = 0.2$, $L = 7$, $m = 5$, $S_s = 5$, and $S_{ps} = -5$. These energy eigenvalues are given in Tables 15 and 16 for pseudospin and spin symmetric limits, respectively.

8. Conclusion

In the present study, spin and pseudospin symmetric Dirac spinors have been obtained for a new suggested generalized Manning-Rosen potential by solving the Dirac equation with centrifugal term. In order to solve the Dirac equation, the Nikiforov-Uvarov method has been used and the Pekeris approximation has been applied to the centrifugal term. Besides, we have obtained energy eigenvalues equations for potential considered in this study in both spin and pseudospin symmetry cases and by using these energy equations, binding energies of some states are calculated for a special set of parameters. The obtained results are given in Tables 1 and 2. It is seen from these tables that the binding energies are taken negative values for pseudospin symmetry case and positive values for spin symmetry limit. In addition we have investigated the effects of potential parameters b , q , L , and p

TABLE 15: The bound state energy eigenvalues for the q -parameter hyperbolic Pöschl-Teller potential in the pseudospin symmetric case for $m = 5 \text{ fm}^{-1}$ and $S_{ps} = -5 \text{ fm}^{-1}$. Here first states column is according to (l, j) and second states column is according to $(l + 2, j + 1)$.

$ n, \tilde{l}, \kappa\rangle$	State	$(\tilde{q} = 1)$ $E_{n\kappa}$	$(\tilde{q} = 2)$ $E_{n\kappa}$	$ n, \tilde{l}, \kappa\rangle$	State	$(\tilde{q} = 1)$ $E_{n\kappa}$	$(\tilde{q} = 2)$ $E_{n\kappa}$
$ 1, 1, -1\rangle$	$1s_{1/2}$	-0.216038	-0.270379	$ 0, 1, 2\rangle$	$0d_{3/2}$	-0.216038	-0.270379
$ 1, 2, -2\rangle$	$1p_{3/2}$	-0.608149	-0.803145	$ 0, 2, 3\rangle$	$0f_{5/2}$	-0.608149	-0.803145
$ 1, 3, -3\rangle$	$1d_{5/2}$	-1.160371	-1.577490	$ 0, 3, 4\rangle$	$0g_{7/2}$	-1.160371	-1.577490
$ 1, 4, -4\rangle$	$1f_{7/2}$	-1.843543	-2.566032	$ 0, 4, 5\rangle$	$0h_{9/2}$	-1.843543	-2.566032
$ 2, 1, -1\rangle$	$2s_{1/2}$	-0.223653	-0.255494	$ 1, 1, 2\rangle$	$1d_{3/2}$	-0.223653	-0.255494
$ 2, 2, -2\rangle$	$2p_{3/2}$	-0.639701	-0.807669	$ 1, 2, 3\rangle$	$1f_{5/2}$	-0.639701	-0.807669
$ 2, 3, -3\rangle$	$2d_{5/2}$	-1.218299	-1.606114	$ 1, 3, 4\rangle$	$1g_{7/2}$	-1.218299	-1.606114
$ 2, 4, -4\rangle$	$2f_{7/2}$	-1.929906	-2.626000	$ 1, 4, 5\rangle$	$1h_{9/2}$	-1.929906	-2.626000

TABLE 16: The bound state energy eigenvalues for the q -parameter hyperbolic Pöschl-Teller potential in the spin symmetric case for $m = 5 \text{ fm}^{-1}$ and $S_s = 5 \text{ fm}^{-1}$. Here first and second states columns are according to (l, j) .

$ n, l, \kappa\rangle$	State	$(\tilde{q} = 1)$ $E_{n\kappa}$	$(\tilde{q} = 2)$ $E_{n\kappa}$	$ n, l, \kappa\rangle$	State	$(\tilde{q} = 1)$ $E_{n\kappa}$	$(\tilde{q} = 2)$ $E_{n\kappa}$
$ 0, 1, -2\rangle$	$0p_{3/2}$	3.173391	4.077032	$ 0, 1, 1\rangle$	$0p_{1/2}$	3.173391	4.077032
$ 0, 2, -3\rangle$	$0d_{5/2}$	3.321749	4.125918	$ 0, 2, 2\rangle$	$0d_{3/2}$	3.321749	4.125918
$ 0, 3, -4\rangle$	$0f_{7/2}$	3.484972	4.185197	$ 0, 3, 3\rangle$	$0f_{5/2}$	3.484972	4.185197
$ 0, 4, -5\rangle$	$0g_{9/2}$	3.642339	4.248060	$ 0, 4, 4\rangle$	$0g_{7/2}$	3.642339	4.248060
$ 1, 1, -2\rangle$	$1p_{3/2}$	3.321462	4.171962	$ 1, 1, 1\rangle$	$1p_{1/2}$	3.321462	4.171962
$ 1, 2, -3\rangle$	$1d_{5/2}$	3.468183	4.224145	$ 1, 2, 2\rangle$	$1d_{3/2}$	3.468183	4.224145
$ 1, 3, -4\rangle$	$1f_{7/2}$	3.630026	4.287317	$ 1, 3, 3\rangle$	$1f_{5/2}$	3.630026	4.287317
$ 1, 4, -5\rangle$	$1g_{9/2}$	3.786248	4.354272	$ 1, 4, 4\rangle$	$1g_{7/2}$	3.786248	4.354272

and parameters m , S_s , and S_{ps} on the binding energies of some states both for spin and pseudospin symmetry in Figures 2, 3, 4, 5, 6, and 7. The figures show that the binding energies increase as m , S_s for spin symmetry and m , S_{ps} for pseudospin symmetry increase and decrease as p and L increase in both symmetry cases. However unlike parameter q , the binding energies increase for spin symmetry and decrease for pseudospin symmetry as parameter b increases. Moreover, special cases of the generalized Manning-Rosen potential which are Manning-Rosen, Hulthen, Woods-Saxon, Morse, Kratzer-Fues, Yukawa, and q -parameter hyperbolic Pöschl-Teller potentials have been examined. Energy eigenvalues for these potentials have been found in spin and pseudospin symmetry cases. Also these energy eigenvalues have been compared with energy eigenvalues of the previous studies and it is seen that our results are consistent with the previous ones.

The other important results of this study have been summarized in the following.

(i) A new suggested generalized Manning-Rosen potential is considered as a nuclear potential because this potential can be reduced to Woods-Saxon, Hulthen, and Yukawa potentials which are nuclear potentials. At the same time the suggested potential in this study is regarded as a molecular potential too because it can be reduced to Manning-Rosen, Morse, Kratzer-Fues, and hyperbolic Pöschl-Teller potentials which are used to explain molecular structure.

(ii) The energy eigenvalues that are obtained for Hulthen potentials have been compared with the results of [40] in

Tables 9 and 10 and it is seen that our results are little different from the results of [40]. The reason of the difference is form of the approximation which is applied to the centrifugal term. The authors of [40] have used the Greene-Aldrich approximation while we have used the Pekeris approximation. Also the obtained results for Yukawa potential have been compared with the results of [55, 56] in Tables 11 and 12. The results of [55, 56] in which the Greene-Aldrich approximation is used show that bound energy eigenvalues do not exist for some states and the members of spin and pseudospin doublets do not have the same energies but our results indicate that bound energy eigenvalues exist in all states and members of spin and pseudospin doublets have the same energies. Explanation of this situation is that our Pekeris approximation is more convenient according to the Greene-Aldrich approximation for the centrifugal term.

(iii) We have used the Nikiforov-Uvarov method to solve Dirac equation, while in [49] standard method, in [27] SUSYQM, and in [46] asymptotic iteration method have been used. Nevertheless, our results are consistent with [27, 46, 49] as seen in Tables 3, 5, 6, 7, and 8. Therefore, it can be concluded that the methods which are used to solve Dirac equation do not affect the results.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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