Research Article

Study of $\bar{B}_{d,s}^{*0} \rightarrow D_{d,s}^+ M^-$ Decays with QCD Factorization Approach

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Motivated by the b -physics experiments at running LHC and upcoming SuperKEKB/Belle-II, the nonleptonic $\bar{B}_q^{*0} \rightarrow D_q^+ M^-$ ($q = d, s$ and $M = \pi, K, \rho, K^*$) weak decays are studied within QCD factorization framework. The observables of these decay modes are first predicted. It is found that the tree-dominated and CKM-favored $\bar{B}_q^{*0} \rightarrow D_q^+ \rho^-$ decays have the largest branching fractions $\sim \mathcal{O}(10^{-8})$ and thus are hopefully to be measured. The $\bar{B}_q^{*0} \rightarrow D_q^+ V^-$ decays are dominated by the longitudinal polarization states. In addition, associating with the relevant B meson decays, some interesting phenomena and relations are discussed in detail; for example, $R_{\mathcal{B}} \equiv \mathcal{B}(\bar{B}_q \rightarrow D_q^* P) / \mathcal{B}(\bar{B}_q \rightarrow D_q P) \approx 3\tau_B / \tau_{B^*}$, and $f_{L,\parallel}(\bar{B}_q \rightarrow D_q^* V) \approx f_{L,\parallel}(\bar{B}_q \rightarrow D_q V)$.

1. Introduction

Thanks to the fruitful running of BABAR, Belle, CDF, and D0 experiments in the past decade, most of the B meson decays with branching fractions $\geq \mathcal{O}(10^{-7})$ are measured precisely, which provide a very fertile testing ground for the Standard Model (SM) pictures of flavor physics, charge-parity (CP) asymmetry, and QCD mechanism. As the particle physics enter a new era of precision, more experimental information about b -physics will be explored at ongoing LHC and forthcoming SuperKEKB/Belle-II. In the $(b\bar{q})$ system, besides the bound state $\bar{B}_q$ meson, some excited states decays are also hopefully to be well measured in the future.

The $\bar{B}_q^*$ meson with quantum number of $n^{2s+1}L_J = 1^3S_1$ and $J^P = 1^-$ is the first excited state in the spectra of heavy-light $(b\bar{q})$ system [1–4]. In the past years, the $\bar{B}_q^*$ masses (or the differences $m_{B_q^*} - m_{B_q}$) are well measured by many collaborations [5–10]. However, the experimental information about the $\bar{B}_q^*$ meson decays is very limited due to the following facts: (i) At e^+e^- collider, $B_{u,d}^*$ and B_s^* mesons are produced mainly through $Y(5S)$ resonance decays, while the past b -physics experiments (BABAR and Belle) run mainly

around $Y(4S)$ resonance for producing $B_{u,d}$ mesons, and the collected data sample of e^+e^- collisions in the vicinity of $Y(5S)$ resonance is not sufficient enough to probe $\bar{B}^*$ rare decays; (ii) the decays of unstable $\bar{B}^*$ mesons are dominated by the electromagnetic transition $\bar{B}^* \rightarrow \bar{B}\gamma$, and thus the other decay modes are too rare to be detected soon. Fortunately, such situation will possibly be improved by the running LHC and upgrading Belle-II experiments in the near future.

For the Belle-II experiment at SuperKEKB, one of the main goals of the $Y(5S)$ physics program is to study the decays of B_s^* meson. Meanwhile, it also should be noted that a plenty of B^* samples would be produced simultaneously. With the target luminosity $8 \times 10^{35} \text{ cm}^{-2} \text{ s}^{-1}$, the annual integrated luminosity is expected to be 13 ab^{-1} /year after 2018 [11]. Using the cross section of $Y(5S)$ production $\sigma(e^+e^- \rightarrow Y(5S)) = 0.301 \text{ nb}$ [12] and the branching fractions of $Y(5S)$ decays related to B^* final states [13], we find that about $N(B_{u,d}^* + \bar{B}_{u,d}^*)/\text{year} \sim 4 \times 10^9$ and $N(B_s^* + \bar{B}_s^*)/\text{year} \sim 2 \times 10^9$ samples could be collected per year. As a result, the B^* decays with branching fractions $\geq \mathcal{O}(10^{-9})$ are hopefully to be observed by Belle-II. Moreover, because of the much larger beauty production cross section of pp collisions [14–16],

the LHC experiments may also provide a lot of experimental information for B^* decays. For instance, as analyzed in [17], even though $B_s^* \rightarrow l^+ l^-$ decay having branching fraction $\sim \mathcal{O}(10^{-11})$ is very rare and obviously out of the scope of Belle-II, it is still possible to be measured by LHC after high-luminosity upgrade (Run-III). So, with the rapid development of experiment, the theoretical studies of B^* weak decays are worthwhile then.

Recently, a few interesting theoretical studies of B^* decays have been performed. For instance, in [18–20], the theoretical estimates of the semileptonic B_c^* decays are made within QCD sum rules framework. In [17], the pure leptonic $B_s^* \rightarrow ll$ and $B_{u,c}^* \rightarrow l\nu$ decays are studied, and the detectability of LHC on these decays is analyzed in detail. Besides the (semi-)leptonic B^* decays, the study of nonleptonic B^* weak decays, which involve much more decay channels, is also essential. In this paper, we will focus on the $\bar{B}_{d,s}^{*0} \rightarrow D_{d,s}^+ M^-$ ($M = \pi, K, \rho$, and K^*) decays, which are tree-dominated and thus principally have relative large branching fractions. In the B meson system, in order to evaluate the QCD corrections, some calculation approaches, such as the QCD factorization (QCDF) [21, 22], the perturbative QCD [23, 24], and the soft-collinear effective theory [25–28], are presented. Then, the nonleptonic B^* decays provide another testing ground for these approaches. In our following calculation, the QCD factorization approach is employed.

Our paper is organized as follows. In Section 2, the basic theoretical framework and the amplitudes of $\bar{B}_{d,s}^{*0} \rightarrow D_{d,s}^+ M^-$ decays calculated through the QCDF approach are presented. Section 3 is devoted to the numerical results and discussion. Finally, a short summary is given.

2. Theoretical Framework

The low energy effective Hamiltonian responsible for the $\bar{B}_{d,s}^{*0} \rightarrow D_{d,s}^+ M^-$ decays could be written as [31, 32]

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{q=d,s} V_{cb} V_{uq}^* \{C_1(\mu) Q_1(\mu) + C_2(\mu) Q_2(\mu)\} + \text{h.c.}, \quad (1)$$

where G_F is the Fermi coupling and $V_{cb} V_{uq}^*$ is product of Cabibbo-Kobayashi-Maskawa (CKM) matrix elements. The Wilson coefficients $C_{1,2}(\mu)$ summarize the physical contributions above scale of μ and are calculable with the perturbation theory. Their values at scales of $\mu \sim \mathcal{O}(m_b/2, m_b, 2m_b)$ in naive dimensional regularization scheme are listed in Table 2. Q_1 and Q_2 are local tree four-quark operators and defined as

$$\begin{aligned} Q_1 &= [\bar{c}_i \gamma_\mu (1 - \gamma_5) b_i] [\bar{q}_j \gamma^\mu (1 - \gamma_5) u_j], \\ Q_2 &= [\bar{c}_i \gamma_\mu (1 - \gamma_5) b_j] [\bar{q}_j \gamma^\mu (1 - \gamma_5) u_i], \end{aligned} \quad (2)$$

where i and j are color indices and the sum over repeated indices is understood. One can refer to [31, 32] for details of this part. To obtain the decay amplitudes, the remaining

works are to accurately calculate the hadronic matrix elements of local operators.

The simplest way to deal with the hadronic matrix elements is the naive factorization (NF) scheme [33, 34] based on the color transparency mechanism [35–37]. Within the NF scheme, the hadronic matrix element is approximated by the product of two current matrix elements, which are further parameterized by decay constants and transition form factors. Explicitly, for the $\bar{B}_q^{*0} \rightarrow D_q^+ M^-$ ($q = d, s$) decays, the hadronic matrix element could be written as

$$\begin{aligned} H_{DM} &= \langle MD | Q | \bar{B}^* \rangle \\ &= \langle M | \bar{q} \gamma^\mu (1 - \gamma_5) u | 0 \rangle \langle D | \bar{c} \gamma_\mu (1 - \gamma_5) b | \bar{B}^* \rangle. \end{aligned} \quad (3)$$

For the case that M is a light pseudoscalar (P), evaluating (3), we get

$$H_{DP} = i \frac{G_F}{\sqrt{2}} (-2m_{B^*}) (\eta \cdot p_2) A_0^{B^* \rightarrow D}(0) f_P, \quad (4)$$

where η is the polarization four-vector of $\bar{B}^*$ meson, p_2 is the momentum of P , and their production could be simplified by replacing

$$\begin{aligned} \eta \cdot p_2 &\longrightarrow p_c \\ &= \frac{\sqrt{[m_{B^*}^2 - (m_D + m_P)^2][m_{B^*}^2 - (m_D - m_P)^2]}}{2m_{B^*}}. \end{aligned} \quad (5)$$

For the case that M is a light vector (V), corresponding to the different helicity amplitudes, (3) could be written as

$$\begin{aligned} H_{DV}^0 &= -if_V \frac{m_{B^*}^2 - m_D^2}{2m_{B^*}} \left[(m_{B^*} + m_D) A_1^{B^* \rightarrow D}(0) \right. \\ &\quad \left. + (m_{B^*} - m_D) A_2^{B^* \rightarrow D}(0) \right] = -if_V (m_{B^*}^2 - m_D^2) \\ &\quad \cdot A_0^{B^* \rightarrow D}(0), \\ H_{DV}^\pm &= -if_V m_V \left[(m_{B^*} + m_D) A_1^{B^* \rightarrow D}(0) \right. \\ &\quad \left. \mp (m_{B^*} - m_D) V^{B^* \rightarrow D}(0) \right]. \end{aligned} \quad (6)$$

In the evaluations, the definition of decay constants

$$\begin{aligned} \langle P(p_2) | \bar{q}_1 \gamma^\mu \gamma_5 q_2 | 0 \rangle &= -if_P p_2^\mu, \\ \langle V(p_2, \varepsilon^*) | \bar{q}_1 \gamma^\mu q_2 | 0 \rangle &= -if_V m_V \varepsilon^{*\mu}, \end{aligned} \quad (7)$$

the form factors (the expression of parameterization of the hadronic matrix $\langle P | \bar{q} \Gamma q | V \rangle$, such as (8), could be obtained

through taking the Hermitian conjugate of $\langle V|\bar{q}\Gamma q|P\rangle$; for the latter of which, we take the same conventions as [38].)

$$\begin{aligned} & \langle D(p_1) | q_3 \gamma_\mu b | B^*(p, \eta) \rangle \\ &= -\frac{2iV(q^2)}{m_{B^*} + m_D} \varepsilon_{\mu\nu\rho\sigma} \eta^\nu p^\rho p_1^\sigma, \\ & \langle D(p_1) | q_3 \gamma_\mu \gamma_5 b | B^*(p) \rangle = 2m_{B^*} A_0(q^2) \frac{\eta \cdot q}{q^2} q^\mu \\ &+ (m_p + m_{B^*}) A_1(q^2) \left(\eta^\mu - \frac{\eta \cdot q}{q^2} q^\mu \right) \\ &+ A_2(q^2) \frac{\eta \cdot q}{m_D + m_{B^*}} \left[(p + p_1)^\mu - \frac{m_{B^*}^2 - m_D^2}{q^2} q^\mu \right], \end{aligned} \quad (8)$$

and the relation

$$\begin{aligned} A_0^{B^* \rightarrow D}(0) &= \frac{1}{2m_{B^*}} \left[(m_{B^*} + m_D) A_1^{B^* \rightarrow D}(0) \right. \\ &\left. + (m_{B^*} - m_D) A_2^{B^* \rightarrow D}(0) \right] \end{aligned} \quad (9)$$

are used. Then, one can easily get the amplitudes of $\bar{B}_q^{*0} \rightarrow D_q^+ M^-$ decays within NF, which are proportional to H_{DP} or $H_{DV}^{0,\pm}$.

However, in the NF framework, the amplitudes are renormalization scale dependence, and the nonfactorizable contributions dominated by the hard gluon exchange are lost. As a result, the amplitude is unphysical, and the strong phase, which is essential for evaluating CP asymmetry, cannot be calculated. In order to remedy these deficiencies and take into account the nonfactorizable contribution, the QCDF approach is proposed by BBNS [21, 22] and has been widely used to deal with the hadronic matrix elements (e.g., [39–49]). Within the framework of QCDF, in the heavy-quark limit ($m_b \gg \Lambda_{\text{QCD}}$), the hadronic matrix elements $\langle DM | Q_i | B^* \rangle$ are expressed by the factorization formula [21, 22]:

$$\langle MD | Q_i | B^* \rangle = \sum_j F_j^{B^* \rightarrow D} \int dx \mathcal{T}_{ij}(x) \Phi_M(x). \quad (10)$$

Here, $F_j^{B^* \rightarrow D}$ is $B^* \rightarrow D$ form factor; $\mathcal{T}_{ij}(x)$ is hard-scattering function, which is perturbatively calculable; $\Phi_M(x)$ is the light-cone distribution amplitude for the quark-antiquark Fock state of meson M . The leading-twist distribution amplitudes of pseudoscalars (π and K) and longitudinal polarized vectors (ρ and K^*) are conventionally expanded in Gegenbauer polynomials [29, 30, 50]:

$$\Phi_M(x) = 6x(1-x) \left[1 + \sum_{n=1}^{\infty} a_n^M C_n^{3/2}(2x-1) \right], \quad (11)$$

where a_n^M is the Gegenbauer moment. In factorization formula (10), the spectator scattering contribution does not appear due to the fact that it is not only α_s -suppressed but

also power-suppressed by the factor of Λ_{QCD}/m_b relative to the LO contribution for case of heavy-light final states, while the vertex correction included in (10) is only α_s -suppressed. In fact, after a detailed analysis for B decays, the authors of [22] have concluded that the spectator interaction does not contribute to heavy-light final states at leading power in the heavy-quark expansion [22].

Applying the QCDF formula, the matrix elements of the effective weak Hamiltonian for $\bar{B}_q^{*0} \rightarrow D_q^+ M^-$ ($q = d, s$) decays could be written as

$$\begin{aligned} \mathcal{A}(\bar{B}_q^{*0} \rightarrow D_q^+ M^-) &= \langle M^- D_q^+ | \mathcal{H}_{\text{eff}} | \bar{B}_q^{*0} \rangle = \frac{G_F}{\sqrt{2}} \\ &\cdot V_{cb} V_{uq}^* \alpha_1 \langle M | \bar{q} \gamma^\mu (1 - \gamma_5) u | 0 \rangle \\ &\cdot \langle D | \bar{c} \gamma_\mu (1 - \gamma_5) b | \bar{B}^* \rangle, \end{aligned} \quad (12)$$

where the products of matrix elements of two current, $\langle M | \bar{q} \gamma^\mu (1 - \gamma_5) u | 0 \rangle \langle D | \bar{c} \gamma_\mu (1 - \gamma_5) b | \bar{B}^* \rangle$, have been given explicitly by (4) and (6). The effective coefficient α_1 in the amplitude, including nonfactorizable contributions from QCD radiative vertex corrections, is defined as [22, 51]

$$\alpha_1 = C_1^{\text{NLO}} + \frac{1}{N_c} C_2^{\text{NLO}} + \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} C_2^{\text{LO}} V_1. \quad (13)$$

Obviously, the NF result is recovered if the QCD-loop correction (the third term in (13)) is neglected. With the modified minimal subtraction ($\overline{\text{MS}}$) scheme, for the pseudoscalar M and the longitudinally polarized vector meson, the function V_1 is written as

$$\begin{aligned} V_1 &= 3 \log \left(\frac{m_b^2}{\mu^2} \right) + 3 \log \left(\frac{m_c^2}{\mu^2} \right) - 18 \\ &+ \int_0^1 dx T(x) \Phi_M(x), \end{aligned} \quad (14)$$

where the loop function $T(x)$ is

$$\begin{aligned} T(x) &= \frac{c_a}{1-c_a} \log(c_a) - \frac{c_b}{1-c_b} \log(c_b) \\ &+ \frac{c_d}{1-c_d} \log(c_d) - \frac{c_c}{1-c_c} \log(c_c) \\ &- r_c \left[\frac{c_a}{(1-c_a)^2} \log(c_a) + \frac{1}{1-c_a} \right] \\ &- r_c^{-1} \left[\frac{c_d}{(1-c_d)^2} \log(c_d) + \frac{1}{1-c_d} \right] + f(c_a) \\ &- f(c_b) - f(c_c) + f(c_d) \\ &+ 2 \log(r_c^2) [\log(c_a) - \log(c_b)], \end{aligned} \quad (15)$$

with the definitions $r_c = m_c/m_b$, $c_a = x(1-r_c^2)$, $c_b = \bar{x}(1-r_c^2)$, $c_c = -c_a/r_c^2$, and $c_d = -c_b/r_c^2$. For the transversely polarized

vector meson, the leading-twist contribution to V_1 is zero. The result of V_1 (see (14)) is exactly the same as the result in $B \rightarrow D^{(*)}L$ decays, where L is a light meson, given in [22].

With the amplitudes given in (12), we can evaluate the observables of $\bar{B}_q^{*0} \rightarrow D_q M$ decays. In the rest frame of $\bar{B}_q^{*0}$ meson, the spin-averaged branching fractions could be written as

$$\begin{aligned} \mathcal{B}(\bar{B}_q^{*0} \rightarrow D_q P) \\ = \frac{1}{3} \frac{1}{8\pi} \frac{p_c}{m_{\bar{B}_q^{*0}}^2 \Gamma_{\text{tot}}(\bar{B}_q^{*0})} \left| \mathcal{A}(\bar{B}_q^{*0} \rightarrow D_q P) \right|^2, \end{aligned} \quad (16)$$

$$\begin{aligned} \mathcal{B}(\bar{B}_q^{*0} \rightarrow D_q V) \\ = \frac{1}{3} \frac{1}{8\pi} \frac{p_c}{m_{\bar{B}_q^{*0}}^2 \Gamma_{\text{tot}}(\bar{B}_q^{*0})} \sum_{\lambda} \left| \mathcal{A}_{\lambda}(\bar{B}_q^{*0} \rightarrow D_q V) \right|^2, \end{aligned} \quad (17)$$

where $\Gamma_{\text{tot}}(\bar{B}_q^{*0})$ is the total decay width of $\bar{B}_q^{*0}$. Besides of the branching fraction, the polarization fractions of $\bar{B}_q^{*0} \rightarrow D_q V$ decays are also important observables. They are defined as

$$f_{L,\perp} = \frac{|\mathcal{A}_{0,\parallel,\perp}|^2}{|\mathcal{A}_0|^2 + |\mathcal{A}_{\parallel}|^2 + |\mathcal{A}_{\perp}|^2}, \quad (18)$$

where $\mathcal{A}_{\parallel}$ and $\mathcal{A}_{\perp}$ are parallel and perpendicular amplitudes and could be easily obtained through $\mathcal{A}_{\parallel,\perp} = (\mathcal{A}_{-} \pm \mathcal{A}_{+})/\sqrt{2}$.

3. Numerical Results and Discussions

With the theoretical framework given in Section 2, we then present our numerical results and discussions. Firstly, we would like to clarify the input parameters in our numerical evaluation. The input values of Wolfenstein parameters, masses of quarks, decay constants, and Gegenbauer moments are summarized in Table 1. Our numerical results of the Wilson coefficients C_1 and C_2 at different scales are listed in Table 2. Besides that, to evaluate the branching fractions of $\bar{B}_q^{*0} \rightarrow D_q M$ decays, the total decay widths $\Gamma_{\text{tot}}(B_q^*)$ are essential. Unfortunately, there are no available experimental or theoretical results until now. In our numerical evaluation, the approximation $\Gamma_{\text{tot}}(B_q^*) \simeq \Gamma(B_q^* \rightarrow B_q \gamma)$ is taken because of the known fact that the radiative process $B_q^* \rightarrow B_q \gamma$ dominates B_q^* meson decays. Theoretically, the predictions on $\Gamma(B^* \rightarrow B \gamma)$ have been widely evaluated in various models, such as relativistic quark model [52, 53], QCD sum rules [54], light-cone QCD sum rules [55], light front quark model [56], heavy-quark effective theory with vector meson dominance hypothesis [57], or covariant model [58]. In this paper, we employ the most recent results [56, 58]:

$$\begin{aligned} \Gamma(B^{*0} \rightarrow B^0 \gamma) &= (148 \pm 20) \text{ eV}, \\ \Gamma(B_s^{*0} \rightarrow B_s^0 \gamma) &= (68 \pm 17) \text{ eV}, \end{aligned} \quad (19)$$

which are consistent with the results in the other models.

TABLE 1: The values of input parameters.

The Wolfenstein parameters	
$\lambda = 0.22548_{-0.00034}^{+0.00068}$ [13]	$A = 0.810_{-0.024}^{+0.018}$ [13]
Masses of quarks	
$m_c = 1.67 \pm 0.07 \text{ GeV}$ [13]	$m_b = 4.78 \pm 0.06 \text{ GeV}$ [13]
Decay constants	
$f_{\pi} = 130.41 \pm 0.20 \text{ MeV}$ [13]	$f_K = 156.2 \pm 0.7 \text{ MeV}$ [13]
$f_{\rho} = 216 \pm 3 \text{ MeV}$ [29]	$f_{K^*} = 220 \pm 5 \text{ MeV}$ [29]
The Gegenbauer moments at the scale $\mu = 1 \text{ GeV}$	
$a_1^{\rho} = 0$ [29]	$a_2^{\rho} = 0.15 \pm 0.07$ [29]
$a_1^{K^*} = 0.03 \pm 0.02$ [29]	$a_2^{K^*} = 0.11 \pm 0.09$ [29]
$a_1^{\pi} = 0$ [30]	$a_2^{\pi} = 0.25 \pm 0.15$ [30]
$a_1^K = 0.06 \pm 0.03$ [30]	$a_2^K = 0.25 \pm 0.15$ [30]

In addition, the values of $B_q^* \rightarrow D_q$ transition form factors are also unknown. In this paper, the Bauer-Stech-Wirbel (BSW) model [59] is employed to evaluate the values of $A_0(0)$, $A_1(0)$, and $V(0)$, which could be written as the overlap integrals of wave functions of mesons [59]. With the meson wave function $\varphi_M(\vec{p}_{\perp}, x)$ as solution of a relativistic scalar harmonic oscillator potential and $\omega = 0.4 \text{ GeV}$ which determines the average transverse quark momentum, we get

$$\begin{aligned} A_0^{B_d^* \rightarrow D_d}(0) &= 0.71, \\ A_1^{B_d^* \rightarrow D_d}(0) &= 0.75, \\ V^{B_d^* \rightarrow D_d}(0) &= 0.76, \\ A_0^{B_s^* \rightarrow D_s}(0) &= 0.66, \\ A_1^{B_s^* \rightarrow D_s}(0) &= 0.69, \\ V^{B_s^* \rightarrow D_s}(0) &= 0.72. \end{aligned} \quad (20)$$

In our numerical evaluation, these numbers and 15% of them are treated as default inputs and uncertainties, respectively.

Using the given values of input parameters and the theoretical formula, we then present QCDF predictions of the CP-averaged branching ratios of $\bar{B}_q^{*0} \rightarrow D_q^+ M^-$ ($M = \pi, K, \rho, K^*$) decays in Table 3, in which the three theoretical uncertainties are induced by the CKM parameters, hadronic parameters (decay constants and form factors), and total decay widths, respectively. In comparison, the NF results are also listed in Table 3. The followings are some analyses and discussions:

- (1) In Table 2, the values of effective coefficient α_1 within NF and QCDF are summarized. It could be found that information of strong phases is obtained by considering gluon radiative corrections to vertex, which plays an important role in exploring the direct CP violation. However, due to lack of interference, the direct CP asymmetries of $\bar{B}_q^{*0} \rightarrow D_q^+ M^-$ ($M = \pi, K, \rho, K^*$) decays are zero.

TABLE 2: The values of Wilson's coefficients $C_{1,2}(\mu)$ and effective coefficient α_1 .

μ	C_1^{LO}	C_2^{LO}	C_1^{NLO}	C_2^{NLO}	α_1 (NF)	α_1 (QCDF)
$m_b/2$	1.166	-0.335	1.126	-0.266	1.037	$1.075 - 0.027i$
m_b	1.11	-0.236	1.076	-0.173	1.018	$1.054 - 0.016i$
$2m_b$	1.07	-0.16	1.041	-0.1	1.008	$1.037 - 0.009i$

TABLE 3: The results of branching fractions.

Decay modes	CKM	NF		QCDF	
		$\mu = m_b$	$\mu = m_b/2$	$\mu = m_b$	$\mu = 2m_b$
$\bar{B}^{*0} \rightarrow D^+ K^- [10^{-10}]$	λ^3	$3.3^{+0.2+1.1+0.5}_{-0.2-1.0-0.4}$	$3.7^{+0.2+1.2+0.6}_{-0.2-1.1-0.4}$	$3.5^{+0.2+1.2+0.6}_{-0.2-1.0-0.4}$	$3.4^{+0.2+1.1+0.5}_{-0.2-1.0-0.4}$
$\bar{B}_s^{*0} \rightarrow D_s^+ K^- [10^{-10}]$	λ^3	$6.3^{+0.3+2.1+2.1}_{-0.4-1.8-1.3}$	$7.1^{+0.3+2.3+0.6}_{-0.4-2.0-1.4}$	$6.8^{+0.3+2.2+2.3}_{-0.4-1.9-1.4}$	$6.5^{+0.3+2.1+2.2}_{-0.4-1.8-1.3}$
$\bar{B}^{*0} \rightarrow D^+ \pi^- [10^{-9}]$	λ^2	$4.4^{+0.2+1.5+0.7}_{-0.3-1.3-0.5}$	$4.9^{+0.2+1.6+0.8}_{-0.3-1.4-0.6}$	$4.7^{+0.2+1.6+0.7}_{-0.3-1.4-0.6}$	$4.6^{+0.2+1.5+0.7}_{-0.3-1.3-0.5}$
$\bar{B}_s^{*0} \rightarrow D_s^+ \pi^- [10^{-9}]$	λ^2	$8.5^{+0.4+2.8+2.8}_{-0.5-2.4-1.7}$	$9.5^{+0.4+3.1+3.2}_{-0.6-2.7-1.9}$	$9.1^{+0.4+3.0+3.0}_{-0.5-2.6-1.8}$	$8.8^{+0.4+2.9+2.9}_{-0.5-2.5-1.8}$
$\bar{B}^{*0} \rightarrow D^+ K^{*-} [10^{-10}]$	λ^3	$7.6^{+0.4+1.9+1.2}_{-0.4-1.7-0.9}$	$8.4^{+0.4+2.1+1.3}_{-0.5-1.9-1.0}$	$8.1^{+0.4+2.0+1.3}_{-0.5-1.8-1.0}$	$7.8^{+0.4+1.9+1.2}_{-0.5-1.7-0.9}$
$\bar{B}_s^{*0} \rightarrow D_s^+ K^{*-} [10^{-9}]$	λ^3	$1.5^{+0.1+0.4+0.5}_{-0.1-0.3-0.3}$	$1.6^{+0.1+0.4+0.5}_{-0.1-0.4-0.3}$	$1.5^{+0.1+0.4+0.5}_{-0.1-0.3-0.3}$	$1.5^{+0.1+0.4+0.5}_{-0.1-0.3-0.3}$
$\bar{B}^{*0} \rightarrow D^+ \rho^- [10^{-8}]$	λ^2	$1.3^{+0.1+0.3+0.2}_{-0.1-0.3-0.2}$	$1.5^{+0.1+0.4+0.2}_{-0.1-0.3-0.2}$	$1.4^{+0.1+0.3+0.2}_{-0.1-0.3-0.2}$	$1.4^{+0.1+0.3+0.2}_{-0.1-0.3-0.2}$
$\bar{B}_s^{*0} \rightarrow D_s^+ \rho^- [10^{-8}]$	λ^2	$2.6^{+0.1+0.6+0.9}_{-0.1-0.6-0.5}$	$2.8^{+0.1+0.7+0.9}_{-0.2-0.6-0.6}$	$2.7^{+0.1+0.7+0.9}_{-0.2-0.6-0.5}$	$2.6^{+0.1+0.6+0.9}_{-0.2-0.6-0.5}$

In Figure 1, the dependence of tree coefficient α_1 on the renormalization scale μ is shown. As Figure 1(b) shows, the imaginary part $\text{Im}(\alpha_1)$, which is zero at LO (NF result), arises after taking into account the NLO corrections. For the real part $\text{Re}(\alpha_1)$, as Figure 1(a) shows, the scale dependence has been reduced partly at low scales when the NLO corrections are taken into account. To further clarify such partial reduction, we define the quantity $k(\mu) = |d\text{Re}[\alpha_1(\mu)]/d\mu|$, which is equal to zero if $\text{Re}[\alpha_1(\mu)]$ is totally scale-independent. It is found that the value of k at NLO is a little bit smaller than the one at LO; for instance, $k(2 \text{ GeV}) \times 10^3 = 21$ (LO), 14 (NLO), and $k(3 \text{ GeV}) \times 10^3 = 9.9$ (LO), 9.7 (NLO), as found from Figure 1(a). However, one also should note that the reduction of scale dependence is not very obvious as one expected, which could be attributed to the fact that NLO correction is color-suppressed [60], while the scale dependence reduction effect becomes very significant when the NNLO correction, which is no longer color-suppressed, is taken into account as found in [60].

- (2) From Table 3, one may find a clear hierarchy of branching fractions, $\mathcal{B}(\bar{B}_q^{*0} \rightarrow D_q^+ \rho^-) > \mathcal{B}(\bar{B}_q^{*0} \rightarrow D_q^+ \pi^-) > \mathcal{B}(\bar{B}_q^{*0} \rightarrow D_q^+ K^{*-}) > \mathcal{B}(\bar{B}_q^{*0} \rightarrow D_q^+ K^-)$. It is mainly induced by the following two reasons: (i) The CKM element $V_{cb}V_{us}^*$ responsible for $\bar{B}_q^{*0} \rightarrow D_q^+ K^{(*)-}$ decays is suppressed by factor of λ compared with the one $V_{cb}V_{ud}^*$ for $\bar{B}_q^{*0} \rightarrow D_q^+ \pi^- (\rho^-)$ decays and (ii) the $\bar{B}_q^{*0} \rightarrow D_q^+ P$ decays are suppressed relatively by the orbital angular momentum compared with the corresponding $\bar{B}_q^{*0} \rightarrow D_q^+ V$ decays.

In addition, one also may find that the $\bar{B}_s^{*0}$ decay is always about two times larger than the corresponding

$\bar{B}_d^{*0}$ decay; for instance, $\mathcal{B}(\bar{B}_s^{*0} \rightarrow D_s^+ K^-) \approx 2\mathcal{B}(\bar{B}^{*0} \rightarrow D^+ K^-)$. It is mainly induced by the theoretical prediction $\Gamma(\bar{B}^{*0} \rightarrow B^0 \gamma)/\Gamma(\bar{B}_s^{*0} \rightarrow B_s^0 \gamma) \approx 2$ (see (19)) and the assumption $\Gamma_{\text{tot}}(B_q^*) \approx \Gamma(B_q^* \rightarrow B_q \gamma)$. Explicitly, such relation could be expressed as

$$R_{sd} \equiv \frac{\mathcal{B}(\bar{B}_s^{*0} \rightarrow D_s^+ M)}{\mathcal{B}(\bar{B}_d^{*0} \rightarrow D_d^+ M)} \simeq \frac{\Gamma(\bar{B}_d^{*0} \rightarrow B_d^0 \gamma)}{\Gamma(\bar{B}_s^{*0} \rightarrow B_s^0 \gamma)} \stackrel{\text{theo.}}{\simeq} 2, \quad (21)$$

which is a useful observable for measuring $\tau_{\bar{B}_d^{*0}}/\tau_{\bar{B}_s^{*0}}$ experimentally and further testing the theoretical predictions of $\Gamma(B_q^* \rightarrow B_q \gamma)$.

From Table 3, it could be found that $\bar{B}_q^{*0} \rightarrow D_q^+ \rho^-$ decays have the largest branching fractions, about $\mathcal{O}(10^{-8})$, and thus are hopefully to be well measured by Belle-II experiment in the near future. In addition, the processes that $\bar{B}_q^{*0}$ decays into two light mesons, such as $\pi\pi$ and πK final states, generally have much more interesting phenomena. However, they are generally CKM- and/or loop-suppressed and therefore hard to be observed soon.

- (3) Besides the branching ratio, the polarization fractions are also important observables. In the $\bar{B} \rightarrow VV$ decays, the hierarchy pattern of helicity amplitudes

$$\mathcal{A}_0 : \mathcal{A}_- : \mathcal{A}_+ = 1 : \frac{\Lambda_{\text{QCD}}}{m_b} : \left(\frac{\Lambda_{\text{QCD}}}{m_b} \right)^2 \quad (22)$$

is expected [61–63], especially for the tree-dominated decays. For the $\bar{B}_q^{*0} \rightarrow D_q^+ V$ decays, such hierarchical relation is also naively expected due to the following:

FIGURE 1: Dependence of the tree coefficient $\alpha_1(D\pi)$ on the renormalization scale μ , with asymptotic light-cone distribution amplitudes $\phi_\pi(x) = 6x\bar{x}$. The solid red and dotted blue lines denote $\alpha_1(D\pi)$ within QCDF and NF frameworks, respectively.

taking $\bar{B}^{*0} \rightarrow D^+ \rho^-$ decay ($b \rightarrow c\bar{u}d$ transition) as an example for convenience of discussion, in the longitudinal transition, the quark and antiquark in each meson have opposite helicities, in which the case $(h_q, h_{\bar{q}}) = (-1/2, 1/2)$ is favored by $(V - A)$ interaction. Relative to $\mathcal{A}_0$, for $\mathcal{A}_-$ to occur the $\bar{u}$ quark has to flip its helicity, which results in the so-called “helicity-flip” suppression. For $\mathcal{A}_+$, in addition to the “helicity-flip” suppression, a further chirality suppression appears since the c quark in the $(V - A)$ interaction has a “wrong” helicity at this moment.

Exactly, from (6), it could be found that the transverse amplitudes $H_{DV}^\pm$ are suppressed by a factor $2m_{B^*}m_V/(m_{B^*}^2 - m_D^2) \sim \Lambda_{\text{QCD}}/m_b$ relative to H_{DV}^0 . In addition, the axial-vector and vector contribution to $H_{DV}^\pm$ cancel in the heavy-quark limit. As a result, the hierarchy pattern of helicity amplitudes (see (22)) is still fulfilled by the $\bar{B}_q^{*0} \rightarrow D_q V$ decays within NF framework. Further, considering that the QCD NLO correction in α_1 (see (13)) is much smaller than the LO one, the very large longitudinal polarization fractions of $\bar{B}_q^{*0} \rightarrow D_q V$ decays are generally expected in both NF and QCDF frameworks. Numerically, within the QCDF, using the default values of input parameters and taking $\mu = m_b$, we get

$$f_L(D^+ K^{*-}, D_s^+ K^{*-}, D^+ \rho^-, D_s^+ \rho^-) = (85, 86, 89, 89)\%, \quad (23)$$

$$f_\parallel(D^+ K^{*-}, D_s^+ K^{*-}, D^+ \rho^-, D_s^+ \rho^-) = (12, 12, 9, 9)\%.$$

- (4) In order to explore the relation between $\bar{B}_q \rightarrow D_q^* P$ and $\bar{B}_q^* \rightarrow D_q P$ decays, we define the ratio

$$R_{\mathcal{A}} \equiv \frac{|\mathcal{A}(\bar{B}_q \rightarrow D_q^* P)|}{|\mathcal{A}(\bar{B}_q^* \rightarrow D_q P)|} = \frac{(m_B^2 - m_{D^*}^2) A_0^{B \rightarrow D^*}}{(m_{B^*}^2 - m_D^2) A_0^{B^* \rightarrow D}}, \quad (24)$$

which is independent of the decay constants f_P and the coefficient α_1 and close to 1. Further, evaluating the branching fractions, we get

$$\begin{aligned} R_{\mathcal{B}} &\equiv \frac{\mathcal{B}(\bar{B}_q \rightarrow D_q^* P)}{\mathcal{B}(\bar{B}_q^* \rightarrow D_q P)} \\ &= 3 \frac{\tau_B}{\tau_{B^*}} \frac{(m_B^2 - m_{D^*}^2)^2 m_{B^*}^3}{(m_{B^*}^2 - m_D^2)^2 m_B^3} \frac{A_0^{B \rightarrow D^*}}{A_0^{B^* \rightarrow D}}, \end{aligned} \quad (25)$$

in which the prefactor of 3 corresponds to the factor 1/3 in (16) caused by averaging over the initial B^* spin. With the values of masses given by PDG [13], the ratio of masses in (25) is equal to 0.94 for $q = d$ and s . Moreover, the ratio of form factors in (25) is generally close to 1; for instance, $0.63/0.71(0.58/0.66) \approx 0.89(0.88)$ for $q = d(s)$ within WSB model. So, the relation $R_{\mathcal{B}} \approx 3\tau_B/\tau_{B^*}$ is expected. Numerically, with the assumption $\Gamma_{\text{tot}}(B_q^*) \approx \Gamma(B_q^* \rightarrow B_q \gamma)$ and the values of $\Gamma(B_q^* \rightarrow B_q \gamma)$ given by (19), we get $R_{\mathcal{B}} \approx 1.02 \times 10^6$ for $q = d$ and 0.47×10^6 for $q = s$, which could be tested experimentally.

For the $\bar{B}_q \rightarrow D_q^* V$ and $\bar{B}_q^* \rightarrow D_q V$ decays, the relation between their polarization fractions is much interesting. It could be found that the relation

$$f_{L,\parallel}(\bar{B}_q \rightarrow D_q^* V) \approx f_{L,\parallel}(\bar{B}_q^* \rightarrow D_q V) \quad (26)$$

is generally expected, because (i) the expressions of their helicity amplitudes are very similar to each other, except for the replacements $\bar{B}^* \leftrightarrow \bar{B}$ and $D \leftrightarrow D^*$ everywhere in (6), and (ii) different from branching ratio (see (26)) the polarization fraction is sensitive to the relative strengths of form factors rather than the absolute ones. In order to test the relation, we take $\bar{B}^{*0} \rightarrow D^+ \rho^-$ and $\bar{B}^0 \rightarrow D^{*+} \rho^-$ decays for example. One may find that our prediction

$f_L(\bar{B}^{*0} \rightarrow D^+\rho^-) = (89 \pm 1)\%$ numerically agrees well with $f_L(\bar{B}^0 \rightarrow D^{*+}\rho^-) = 87\%$ predicted in [64], which is consistent with experimental results $\sim 88.5\%$ [65].

4. Summary

In this paper, detailed analyses of the $\bar{B}_{d,s}^{*0} \rightarrow D_{d,s}^+ M^-$ ($M = \pi, K, \rho, K^*$) weak decays are performed within QCD factorization framework. The theoretical predictions for the branching fractions and polarization fractions are presented in Table 3 and (23), respectively. Some interesting phenomena and relations are discussed. It is found that (i) there is a clear hierarchy of branching fractions in $\bar{B}_{d,s}^{*0} \rightarrow D_{d,s}^+ M^-$ decays, in which the $\bar{B}_q^{*0} \rightarrow D_q^+ \rho^-$ decays have the largest branching fractions, $\sim \mathcal{O}(10^{-8})$, and thus are very hopefully to be observed by Belle-II experiment in the near future. (ii) The $\bar{B}_{d,s}^{*0} \rightarrow D_{d,s}^+ V^-$ decays are dominated by the longitudinal polarization states, numerically $f_L \sim [80, 90]\%$. (iii) Some interesting and useful correlations between $\bar{B}_q^{*0} \rightarrow D_q M$ and its corresponding $\bar{B}_q \rightarrow D_q^* M$ decays are presented. For instance, $R_{\mathcal{B}} \equiv \mathcal{B}(\bar{B}_q \rightarrow D_q^* P)/\mathcal{B}(\bar{B}_q^{*0} \rightarrow D_q P) \simeq 3\tau_B/\tau_{B^*}$ and $f_{L\parallel}(\bar{B}_q \rightarrow D_q^* V) \simeq f_{L\parallel}(\bar{B}_q^{*0} \rightarrow D_q V)$ are generally expected. All of above findings are waiting for the experimental test at LHC and SuperKEKB/Belle-II.

Competing Interests

The authors declare that they have no competing interests.

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References

- [1] N. Isgur and M. B. Wise, "Spectroscopy with heavy-quark symmetry," *Physical Review Letters*, vol. 66, no. 9, pp. 1130–1133, 1991.
- [2] S. Godfrey and R. Kokoski, "Properties of P-wave mesons with one heavy quark," *Physical Review D*, vol. 43, no. 5, pp. 1679–1687, 1991.
- [3] E. J. Eichten, C. T. Hill, and C. Quigg, "Properties of orbitally excited heavy-light (Qq^-) mesons," *Physical Review Letters*, vol. 71, pp. 4116–4119, 1993.
- [4] D. Ebert, V. O. Galkin, and R. N. Faustov, "Mass spectrum of orbitally and radially excited heavy-light mesons in the relativistic quark model," *Physical Review D*, vol. 57, no. 9, pp. 5663–5669, 1998, Erratum in: *Physical Review D*, vol. 59, Article ID 019902, 1998.
- [5] R. Aaij, C. Abellan Beteta, A. Adametz et al., "First observation of the decay $B_{s2}^{*0} \rightarrow B^+ + K^-$ and studies of excited B_s^0 mesons," *Physical Review Letters*, vol. 110, no. 15, Article ID 151803, 2013.
- [6] R. Louvot, J. Wicht, O. Schneider et al., "Measurement of the decay $B_s^0 \rightarrow D_s^- \pi^+$ and evidence for $B_s^0 \rightarrow D_s^- K^+$ in e^+e^- annihilation at $\sqrt{s} \approx 10.87$ GeV," *Physical Review Letters*, vol. 102, no. 2, Article ID 021801, 2009.
- [7] O. Aquines, Z. Li, A. Lopez et al., "Measurements of the exclusive decays of the $Y(5S)$ to B meson final states and improved B_s^+ mass measurement," *Physical Review Letters*, vol. 96, Article ID 152001, 2006.
- [8] K. Ackerstaff, G. Alexander, J. Allison et al., " B^* production in Z^0 decays," *Zeitschrift für Physik C: Particles and Fields*, vol. 74, no. 3, pp. 413–423, 1997.
- [9] D. Buskulic, D. Casper, I. De Bonis et al., "Production of excited beauty states in Z decays," *Zeitschrift für Physik C Particles and Fields*, vol. 96, no. 3, pp. 393–404, 1996.
- [10] P. Abreu, W. Adam, T. Abye et al., " B^* production in Z decays," *Zeitschrift für Physik C Particles and Fields*, vol. 68, no. 3, pp. 353–362, 1995.
- [11] T. Abe, I. Adachi, K. Adamczyk et al., "Belle II technical designreport," <http://arxiv.org/abs/1011.0352>.
- [12] G. S. Huang, D. H. Miller, V. Pavlunin et al., "Measurement of $B(Y(5S)) \rightarrow B_s^{(*)} \bar{B}_s^{(*)}$ using ϕ mesons," <http://arxiv.org/abs/hep-ex/0607080v1>.
- [13] K. A. Olive, K. Agashe, C. Amsler et al., "Review of particle physics," *Chinese Physics C*, vol. 38, no. 9, Article ID 090001, 2014.
- [14] A. Bharucha, I. I. Bigi, C. Bobeth et al., "Implications of LHCb measurements and future prospects," *The European Physical Journal C*, vol. 73, article 2373, 2013.
- [15] R. Aaij, C. Abellan Beteta, B. Adeva et al., "Measurement of $\sigma(pp \rightarrow b\bar{b}X)$ at $\sqrt{s} = 7$ TeV in the forward region," *Physics Letters B*, vol. 694, pp. 209–216, 2010.
- [16] R. Aaij, B. Adeva, M. Adinolfi et al., "LHCb detector performance," *International Journal of Modern Physics A*, vol. 30, no. 7, Article ID 1530022, 2015.
- [17] B. Grinstein and J. M. Camalich, "Weak decays of unstable b -mesons," <http://arxiv.org/abs/1509.05049>.
- [18] Z.-G. Wang, "Semileptonic decays $B_c^{*+} \rightarrow \eta_c l^+ \bar{\nu}_l$ with QCD sum rules," *Communications in Theoretical Physics*, vol. 61, no. 1, article 81, 2014.
- [19] K. Zeynali, V. Bashiry, and F. Zolfagharpour, "Form factors and decay rate of $B_c^{*+} \rightarrow D_s^+ l^+ \bar{\nu}_l$ decays in the QCD sum rules," *The European Physical Journal A*, vol. 50, article 127, 2014.
- [20] V. Bashiry, "Investigation of the rare exclusive $B_c^{*+} \rightarrow D_s^+ \nu \bar{\nu}$ decays in the framework of the QCD sum rules," *Advances in High Energy Physics*, vol. 2014, Article ID 503049, 10 pages, 2014.
- [21] M. Beneke, G. Buchalla, M. Neubert, and C. Sachrajda, "QCD factorization for $B \rightarrow \pi\pi$ decays: strong phases and CP violation in the heavy quark limit," *Physical Review Letters*, vol. 83, no. 10, pp. 1914–1917, 1999.
- [22] M. Beneke, G. Buchalla, M. Neubert, and C. T. Sachrajda, "QCD factorization for exclusive non-leptonic B-meson decays: general arguments and the case of heavy-light final states," *Nuclear Physics B*, vol. 591, no. 1-2, pp. 313–418, 2000.

- [23] Y. Y. Keum, H. N. Li, and A. I. Sanda, "Fat penguins and imaginary penguins in perturbative QCD," *Physics Letters B*, vol. 504, no. 1-2, pp. 6-14, 2001.
- [24] Y.-Y. Keum, H.-N. Li, and A. I. Sanda, "Penguin enhancement and $\bar{B}K\pi$ decays in perturbative QCD," *Physical Review D*, vol. 63, no. 5, Article ID 054008, 2001.
- [25] C. W. Bauer, S. Fleming, and M. Luke, "Summing Sudakov logarithms in $\bar{B}X_s\gamma$ in effective field theory," *Physical Review D*, vol. 63, no. 1, Article ID 014006, 2000.
- [26] C. W. Bauer, S. Fleming, D. Pirjol, and I. W. Stewart, "An effective field theory for collinear and soft gluons: heavy to light decays," *Physical Review D*, vol. 63, Article ID 114020, 2001.
- [27] C. W. Bauer and I. W. Stewart, "Invariant operators in collinear effective theory," *Physics Letters B*, vol. 516, no. 1-2, pp. 134-142, 2001.
- [28] C. W. Bauer, D. Pirjol, and I. W. Stewart, "Soft-collinear factorization in effective field theory," *Physical Review D*, vol. 65, no. 5, Article ID 054022, 2002.
- [29] P. Ball, G. W. Jones, and R. Zwicky, " $B \rightarrow V_\gamma$ beyond QCD factorization," *Physical Review D*, vol. 75, no. 5, Article ID 054004, 2007.
- [30] P. Ball, V. M. Braun, and A. Lenz, "Higher-twist distribution amplitudes of the K meson in QCD," *Journal of High Energy Physics*, vol. 5, article 4, 2006.
- [31] G. Buchalla, A. J. Buras, and M. E. Lautenbacher, "Weak decays beyond leading logarithms," *Reviews of Modern Physics*, vol. 68, no. 4, pp. 1125-1244, 1996.
- [32] A. J. Buras, "Weak hamiltonian, CPviolation and rare decays," <http://arxiv.org/abs/hep-ph/9806471>.
- [33] D. Fakirov and B. Stech, "F- and D-decays," *Nuclear Physics B*, vol. 133, no. 2, pp. 315-326, 1978.
- [34] N. Cabibbo and L. Maiani, "Two-body decays of charmed mesons," *Physics Letters B*, vol. 73, no. 4-5, pp. 418-422, 1978, Erratum in: *Physics Letters B*, vol. 76, p. 663, 1978.
- [35] J. D. Bjorken, "Topics in B-physics," *Nuclear Physics B—Proceedings Supplements*, vol. 11, pp. 325-341, 1989.
- [36] M. J. Dugan and B. Grinstein, "QCD basis for factorization in decays of heavy mesons," *Physics Letters B*, vol. 255, no. 4, pp. 583-588, 1991.
- [37] P. Jain, B. Pire, and J. P. Ralston, "Quantum color transparency and nuclear filtering," *Physics Report*, vol. 271, no. 2-3, pp. 67-179, 1996.
- [38] M. Beneke and T. Feldmann, "Symmetry-breaking corrections to heavy-to-light B meson form factors at large recoil," *Nuclear Physics B*, vol. 592, no. 1-2, pp. 3-34, 2001.
- [39] M. Beneke and M. Neubert, "Flavor-singlet B-decay amplitudes in QCD factorization," *Nuclear Physics B*, vol. 651, no. 3, pp. 225-248, 2003.
- [40] M. Beneke and M. Neubert, "QCD factorization for $B \rightarrow PP$ and $B \rightarrow PV$ decays," *Nuclear Physics B*, vol. 675, no. 1-2, pp. 333-415, 2003.
- [41] M. Beneke, J. Rohrer, and D. Yang, "Branching fractions, polarisation and asymmetries of $B \rightarrow VV$ decays," *Nuclear Physics B*, vol. 774, no. 1-3, pp. 64-101, 2007.
- [42] D. S. Du, H. J. Gong, J. F. Sun, D. S. Yang, and G. H. Zhu, "Phenomenological analysis of $B \rightarrow PP$ decays with QCD factorization," *Physical Review D*, vol. 65, no. 7, Article ID 074001, 2002.
- [43] J. F. Sun, G. H. Zhu, and D. S. Du, "Phenomenological analysis of charmless decays $B_s \rightarrow PP, PV$ with QCD factorization," *Physical Review D*, vol. 68, no. 5, Article ID 054003, 2003.
- [44] J. Sun, L. Chen, N. Wang, Q. Chang, J. Huang, and Y. Yang, "Study on the $\Upsilon(1S) \rightarrow B_c M$ weak decays," *Advances in High Energy Physics*, vol. 2015, Article ID 691261, 8 pages, 2015.
- [45] J. Sun, N. Wang, Q. Chang, and Y. Yang, " $B_c \rightarrow BP, BV$ decays with the QCD factorization approach," *Advances in High Energy Physics*, vol. 2015, Article ID 104378, 10 pages, 2015.
- [46] H.-Y. Cheng and C.-K. Chua, "Revisiting charmless hadronic $B_{u,d}$ decays in QCD factorization," *Physical Review D*, vol. 80, no. 11, Article ID 114008, 2009.
- [47] H.-Y. Cheng and C.-K. Chua, "QCD factorization for charmless hadronic B_s decays revisited," *Physical Review D*, vol. 80, no. 11, Article ID 114026, 2009.
- [48] Q. Chang, J. Sun, Y. Yang, and X. Li, "A combined fit on the annihilation corrections in $B_{u,d,s} \rightarrow PP$ decays within QCDF," *Physics Letters B*, vol. 740, pp. 56-60, 2015.
- [49] J. Sun, Q. Chang, X. Hu, and Y. Yang, "Constraints on hard spectator scattering and annihilation corrections in $B_{u,d} \rightarrow PV$ decays within QCD factorization," *Physics Letters B*, vol. 743, pp. 444-450, 2015.
- [50] P. Ball, "Theoretical update of pseudoscalar meson distribution amplitudes of higher twist: the nonsinglet case," *Journal of High Energy Physics*, vol. 1999, no. 1, article 010, 1999.
- [51] J. Sun, G. Xue, Y. Yang, G. Lu, and D. Du, "Study of $B_c^- \rightarrow J/\psi\pi^-, \eta_c\pi^-$ decays with QCD factorization," *Physical Review D*, vol. 77, no. 7, Article ID 074013, 9 pages, 2008.
- [52] J. L. Goity and W. Roberts, "Radiative transitions in heavy mesons in a relativistic quark model," *Physical Review D*, vol. 64, no. 9, Article ID 094007, 2001.
- [53] D. Ebert, R. N. Faustov, and V. O. Galkin, "Radiative M1-decays of heavy-light mesons in the relativistic quark model," *Physics Letters B*, vol. 537, no. 3-4, pp. 241-248, 2002.
- [54] S. L. Zhu, W. Y. P. Hwang, and Z. S. Yang, " $D^* \rightarrow D\gamma$ and $B^* \rightarrow B\gamma$ as derived from QCD Sum rules," *Modern Physics Letters A*, vol. 12, no. 39, pp. 3027-3035, 1997.
- [55] T. M. Aliev, D. A. Demir, E. Iltan, and N. K. Pak, "Radiative $B^* \rightarrow B\gamma$ and $D^* \rightarrow D\gamma$ decays in light-cone QCD sum rules," *Physical Review D*, vol. 54, no. 1, pp. 857-862, 1996.
- [56] H.-M. Choi, "Decay constants and radiative decays of heavy mesons in light-front quark model," *Physical Review D*, vol. 75, no. 7, Article ID 073016, 2007.
- [57] P. Colangelo, F. De Fazio, and G. Nardulli, "Radiative heavy meson transitions," *Physics Letters B*, vol. 316, no. 4, pp. 555-560, 1993.
- [58] C.-Y. Cheung and C.-W. Hwang, "Strong and radiative decays of heavy mesons in a covariant model," *Journal of High Energy Physics*, vol. 2014, article 177, 2014.
- [59] M. Wirbel, B. Stech, and M. Bauer, "Exclusive semileptonic decays of heavy mesons," *Zeitschrift für Physik C Particles and Fields*, vol. 29, no. 4, pp. 637-642, 1985.
- [60] M. Beneke, T. Huber, and X.-Q. Li, "NNLO vertex corrections to non-leptonic B decays: tree amplitudes," *Nuclear Physics B*, vol. 832, no. 1-2, pp. 109-151, 2010.
- [61] A. Ali, J. G. Körner, G. Kramer, and J. Willrodt, "Nonleptonic weak decays of bottom mesons," *Zeitschrift für Physik C: Particles and Fields*, vol. 1, no. 3, pp. 269-277, 1979.
- [62] J. G. Körner and G. R. Goldstein, "Quark and particle helicities in hadronic charmed particle decays," *Physics Letters B*, vol. 89, no. 1, pp. 105-110, 1979.
- [63] A. L. Kagan, "Polarization in $B \rightarrow VV$ decays," *Physics Letters B*, vol. 601, no. 3-4, pp. 151-163, 2004.

- [64] G. Kramer, T. Mannel, and W. F. Palmer, “Angular correlations in the decays $B \rightarrow VV$ using heavy quark symmetry,” *Zeitschrift für Physik C*, vol. 55, no. 3, pp. 497–501, 1992.
- [65] S. E. Csorna, I. Danko, G. Bonvicini et al., “Measurements of the branching fractions and helicity amplitudes in $\bar{B}D^*\rho$ decays,” *Physical Review D*, vol. 67, no. 11, Article ID 112002, 2003.