

## Research Article

# Properties of Quasi-Oscillator in Position-Dependent Mass Formalism

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Schrödinger equation is considered within position-dependent mass formalism with a quasi-oscillator interaction term. Wave functions and energy spectra have been obtained analytically. Thermodynamic properties, information entropy, and uncertainty in coordinate and momentum spaces are calculated. To provide a better physical insight into the solutions, some figures are included.

## 1. Introduction

The position-dependent mass (PDM) approach has a wide range of applications in various areas of science [1–8] and has motivated many recent researches [9–17]. A very notable application of the field is in the micro fabrication techniques such as molecular-beam epitaxy and nanolithography [18–20]. In quantum mechanics, the PDM Schrödinger equation has been already studied via path integral approach [21], supersymmetric quantum mechanics [22], Darboux transformation [23], de Broglie-Bohm technique [24], and Hamiltonian factorization [25]. On the other hand, in a search for possible alternatives ordinary entropy, some authors have investigated Shannon and Fisher information entropies. There are also attempts which connect the PDM with alternative entropies. Sun et al. [26] presented the Shannon entropy for the position and momentum eigenstates of the PDM Schrödinger equation for a particle with a nonuniform solitonic mass density in the case of a hyperbolic-type potential. Falaye et al. [27] considered a particle in PDM formalism with a nonuniform solitonic mass density and a special squared hyperbolic cosecant interaction. Amir and Iqbal [28] quantized the classical oscillator by considering the symmetric ordering of operator equivalents of momentum and PDM, respectively, and obtained a quantum Hamiltonian which was manifestly Hermitian in the configuration space. With inspiration of above points, we consider the Schrödinger

equation within PDM formalism with a quasi-harmonic interaction term. We investigate this system by obtaining the wave function, energy spectra, Shannon information entropy, and corresponding thermodynamics properties.

We have organized this article as follows. We first introduce the PDM Schrödinger equation and consider it with a quasi-oscillator in detail in Section 2. We next obtain the corresponding exact analytical solutions. Having calculated the energy spectra and the wave functions, in Section 3, we obtain the thermodynamic properties of the system. In Section 4, some discussions regarding the associated information entropy are included. In Section 5, some expectation values are reported and the related uncertainty principle is investigated. The results are discussed via various figures throughout the text.

## 2. One-Dimensional Schrödinger Equation with Position-Dependent Mass

To obtain Schrödinger equation, we first define the Lagrangian density as in [29]

$$\begin{aligned}\mathcal{L} = & \frac{i\hbar}{2} \Phi(x, t) \partial_t \Psi(x, t) \\ & + \frac{\hbar^2}{8} \left[ \frac{d}{dx} \left( \frac{1}{m(x)} \right) \right] \Phi(x, t) \partial_x \Psi(x, t)\end{aligned}$$

$$\begin{aligned}
& + \frac{\hbar^2}{4m(x)} \Phi(x, t) \partial_x^2 \Psi(x, t) \\
& - \frac{1}{2} V(x) \Phi(x, t) \Psi(x, t) \\
& - \frac{i\hbar}{2} \Phi^*(x, t) \partial_t \Psi^*(x, t) \\
& + \frac{\hbar^2}{8} \left[ \frac{d}{dx} \left( \frac{1}{m(x)} \right) \right] \Phi^*(x, t) \partial_x \Psi^*(x, t) \\
& + \frac{\hbar^2}{4m(x)} \Phi^*(x, t) \partial_x^2 \Psi^*(x, t) \\
& - \frac{1}{2} V(x) \Phi^*(x, t) \Psi^*(x, t),
\end{aligned} \tag{1}$$

where  $\Psi(x, t)$  and  $\Phi(x, t)$  are fields with  $\Psi^*(x, t)$  and  $\Phi^*(x, t)$  being their conjugates,  $\hbar$  is the Planck constant, and  $m(x)$  denotes a position-dependent mass. On the other hand, the Euler-Lagrange equation can be written as

$$\frac{\partial \mathcal{L}}{\partial \Phi} - \frac{\partial}{\partial x} \left( \frac{\partial \mathcal{L}}{\partial (\partial_x \Phi)} \right) - \frac{\partial}{\partial t} \left( \frac{\partial \mathcal{L}}{\partial (\partial_t \Phi)} \right) = 0 \tag{2}$$

or in generalized form

$$\begin{aligned}
& \frac{\partial \mathcal{L}}{\partial \Phi} - \frac{\partial}{\partial x} \left( \frac{\partial \mathcal{L}}{\partial (\partial_x \Phi)} \right) - \frac{\partial}{\partial t} \left( \frac{\partial \mathcal{L}}{\partial (\partial_t \Phi)} \right) \\
& + \frac{\partial^2}{\partial x^2} \left( \frac{\partial \mathcal{L}}{\partial (\partial_x^2 \Phi)} \right) = 0.
\end{aligned} \tag{3}$$

Therefore, the position-dependent mass Schrödinger equation appears as

$$\begin{aligned}
& -\frac{\hbar^2}{2m(x)} \frac{\partial^2 \Psi(x, t)}{\partial x^2} - \frac{3\hbar^2}{4} \left[ \frac{d}{dx} \left( \frac{1}{m(x)} \right) \right] \frac{\partial \Psi(x, t)}{\partial x} \\
& + \left( V(x) - \frac{\hbar^2}{4} \left[ \frac{d^2}{dx^2} \left( \frac{1}{m(x)} \right) \right] \right) \Psi(x, t) \\
& = -i\hbar \frac{d}{dt} \Psi(x, t),
\end{aligned} \tag{4}$$

where

$$\begin{aligned}
\widehat{H} = & -\frac{\hbar^2}{2m(x)} \frac{\partial^2}{\partial x^2} - \frac{3\hbar^2}{4} \left[ \frac{d}{dx} \left( \frac{1}{m(x)} \right) \right] \frac{\partial}{\partial x} \\
& - \frac{\hbar^2}{4} \left[ \frac{d^2}{dx^2} \left( \frac{1}{m(x)} \right) \right] + V(x)
\end{aligned} \tag{5}$$

with the atomic units  $\hbar = 1$  and  $c = 1$  being used. Performing the following transformations in (4)

$$\Psi(x) = m(x)^{1/2} \phi(y(x)), \tag{6a}$$

$$\frac{dy}{dx} = m(x)^{1/2}, \tag{6b}$$

$$m(x) = \frac{m_0}{(1 + \gamma x^2)}, \tag{6c}$$

we have [30–32]

$$y(x) = \frac{\sqrt{m_0}}{\sqrt{\gamma}} \arctan(x\sqrt{\gamma}). \tag{7}$$

Thus, we have to deal with

$$-\frac{1}{2} \frac{d^2 \phi(y)}{dy^2} + V(y) \phi(y) = E \phi(y). \tag{8}$$

We now consider

$$V(x) = V_0 (\arctan(x\sqrt{\gamma}))^2. \tag{9}$$

The Taylor expansion of the potential is

$$V(x) = V_0 x^2 \gamma - \frac{2}{3} V_0 x^4 \gamma^2 + \frac{25}{45} V_0 x^6 \gamma^3 + \dots; \tag{10}$$

when  $\gamma$  tends to zero, the higher order terms can be neglected and we may write

$$V(x) = V_0 \gamma x^2, \tag{11}$$

or, in terms of  $y$ ,

$$V(y) = V_0 \frac{\gamma}{m_0} y^2. \tag{12}$$

By considering  $\omega = \sqrt{2V_0\gamma}/m_0$ , the potential is more familiarly written as

$$V(y) = \frac{1}{2} m_0 \omega^2 y^2, \tag{13}$$

where  $\gamma$  is the mass deformation parameter and  $V_0$  denotes the potential parameter. In Figure 1, we have plotted the potential versus  $x$  for some different  $\gamma$  values.

### 3. The Wave Function and Energy for Oscillator Term

Substituting (13) into (8), we have Weber's differential equation as

$$\frac{d^2 \phi(y)}{dy^2} + (2E - \omega^2 y^2) \phi(y) = 0. \tag{14}$$

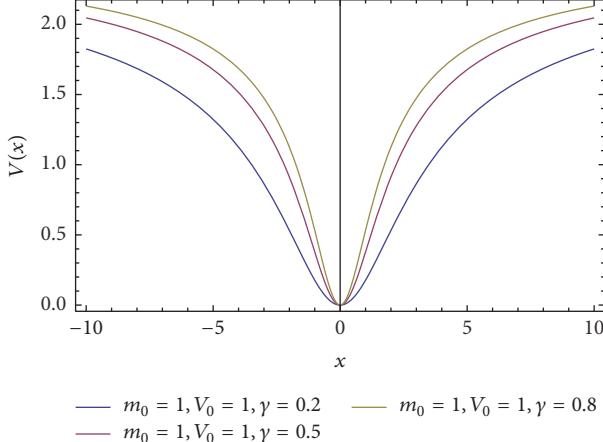
The change of variable  $s = \omega y^2$  brings (14) into the form

$$s \frac{d^2 \phi(s)}{ds^2} + \frac{1}{2} \frac{d\phi(s)}{ds} + \left( \frac{E}{2\omega} - \frac{s}{4} \right) \phi = 0. \tag{15}$$

For further convenience, we apply the gauge transformation  $\phi(s) = e^{-s/2} \chi(s)$  which leads to

$$s \frac{d^2 \chi(s)}{ds^2} + \left( \frac{1}{2} - s \right) \frac{d\chi(s)}{ds} + \left( \frac{E}{2\omega} - \frac{1}{4} \right) \chi = 0. \tag{16}$$

We identify the above equation as the Kummer differential equation (16) and its eigenfunctions may be expressed

FIGURE 1:  $V(x)$  versus  $x$ .

in terms of regular confluent hypergeometric functions  $M(a, c, s)$  as [33]

$$\chi(s) = \tilde{A}M\left(a, \frac{1}{2}, s\right) + \tilde{B}s^{1/2}M\left(a + \frac{1}{2}, \frac{3}{2}, s\right), \quad (17)$$

where  $\tilde{A}$  and  $\tilde{B}$  are arbitrary constants and

$$a = -\left(\frac{E}{2\omega} - \frac{1}{4}\right). \quad (18)$$

In terms of variable  $y$ , the wave functions can be written as

$$\begin{aligned} \phi(y) &= \tilde{A}e^{-(1/2)\omega y^2}M\left(a, \frac{1}{2}, \omega y^2\right) \\ &\quad + \tilde{B}(\omega y^2)^{1/2}M\left(a + \frac{1}{2}, \frac{3}{2}, \omega y^2\right). \end{aligned} \quad (19)$$

Recalling (6a), (6c), and (7),  $\Psi(x)$  becomes [33]

$$\begin{aligned} \Psi(x) &= \sqrt{\frac{m_0}{(1+\gamma x^2)^2}} \left( \tilde{A}e^{-(\omega m_0/2y)\arctan^2(x\sqrt{\gamma})}M\left(a, \right. \right. \\ &\quad \left. \left. \frac{1}{2}, \frac{\omega m_0}{\gamma} \arctan^2(x\sqrt{\gamma})\right) + \tilde{B}\frac{\sqrt{\omega m_0}}{\sqrt{\gamma}} \arctan(x\sqrt{\gamma}) \right. \\ &\quad \left. \cdot M\left(a + \frac{1}{2}, \frac{3}{2}, \frac{\omega m_0}{\gamma} \arctan^2(x\sqrt{\gamma})\right) \right). \end{aligned} \quad (20)$$

The confluent Hypergeometric function are related to the Hermit polynomials via

$$\begin{aligned} H_{n_{\text{even}}}(\xi) &= (-1)^n \frac{(2n)!}{n!} M\left(-n, \frac{1}{2}, \omega y^2\right), \\ H_{n_{\text{odd}}}(\xi) &= (-1)^n \frac{2(2n+1)!}{n!} M\left(-n, \frac{3}{2}, \omega y^2\right). \end{aligned} \quad (21)$$

In view of the above equations, the even and odd eigenfunctions may be, respectively, expressed as

$$\Psi_{\text{even}} = N_n \sqrt{\frac{m_0}{(1+\gamma x^2)^2}} e^{-\omega y^2/2} H_n(y\sqrt{\omega}), \quad (22a)$$

$$\Psi_{\text{odd}} = N_n \sqrt{\frac{m_0}{(1+\gamma x^2)^2}} e^{-\omega y^2/2} H_n(y\sqrt{\omega}), \quad (22b)$$

where  $N_n$  is the normalization constant. However, the even odd eigenfunctions may be combined and the stationary states of the relativistic oscillator are

$$\begin{aligned} \Psi_n(x) &= N_n \sqrt{\frac{m_0}{(1+\gamma x^2)^2}} e^{-(\omega m_0/2y)\arctan^2(x\sqrt{\gamma})} H_n\left(\frac{\sqrt{\omega m_0}}{\sqrt{\gamma}} \right. \\ &\quad \left. \cdot \arctan(x\sqrt{\gamma})\right). \end{aligned} \quad (23)$$

The energy eigenvalues of spin-zero particles bound in this oscillator potential may be found using (14). Therefore, the energy for Even and odd states can be written as

$$E_n = \left(n + \frac{1}{2}\right)\omega = \sqrt{2V_0\gamma}\left(n + \frac{1}{2}\right). \quad (24)$$

#### 4. Thermodynamic Properties

In order to consider the thermodynamic properties of a neutral particle, we concentrate, at first, on the calculation of the partition function [34]

$$\begin{aligned} Q_1 &= \sum_{n=0}^{\infty} e^{-\beta E_n} = e^{-\beta\omega/2} + e^{-3\beta\omega/2} + \dots \\ &= \frac{1}{e^{\beta\omega/2} - e^{-\beta\omega/2}} = \left(2 \sinh \frac{\sqrt{2V_0\gamma}\beta}{2}\right)^{-1}, \end{aligned} \quad (25)$$

where  $\beta = 1/KT$ . The partition function for noninteracting  $N$ -body system is

$$Q_N = (Q_1)^N = \left(2 \sinh \frac{\sqrt{2V_0\gamma}\beta}{2}\right)^{-N}, \quad (26)$$

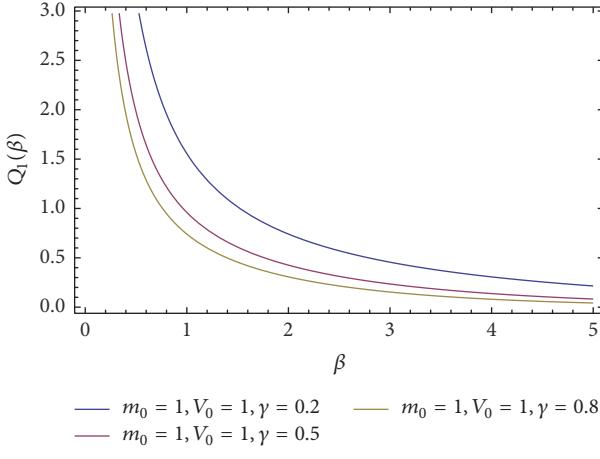
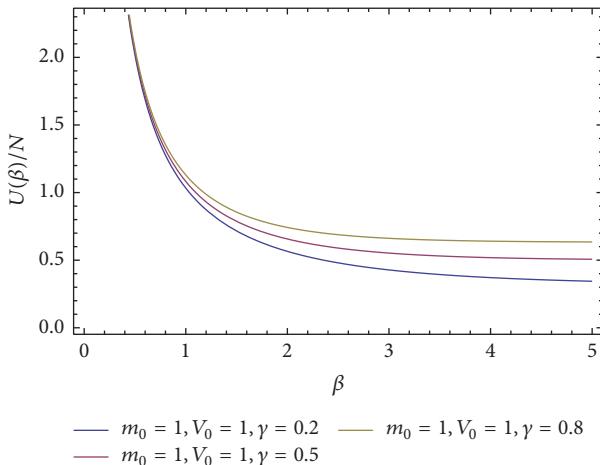
where

$$\ln Q_N = -N \ln \left(2 \sinh \frac{\sqrt{2V_0\gamma}\beta}{2}\right). \quad (27)$$

In Figure 2,  $Q_1(\beta)$  is plotted in terms of  $\beta$  by setting different constant values for  $m_0$ ,  $V_0$ , and  $\gamma$ . As it can be seen, by increasing  $\gamma$  and  $\beta$ ,  $Q_1(\beta)$  decreases as in  $a$ , somehow, the same manner.

Once the Helmholtz free energy is obtained, the other statistical quantities are obtained in a straightforward way. We therefore start from

$$A = -\frac{1}{\beta} \ln Q_N = \frac{N}{\beta} \ln \left(2 \sinh \frac{\sqrt{2V_0\gamma}\beta}{2}\right). \quad (28)$$

FIGURE 2:  $Q_1(\beta)$  versus  $\beta$ .FIGURE 3:  $U(\beta)/N$  versus  $\beta$ .

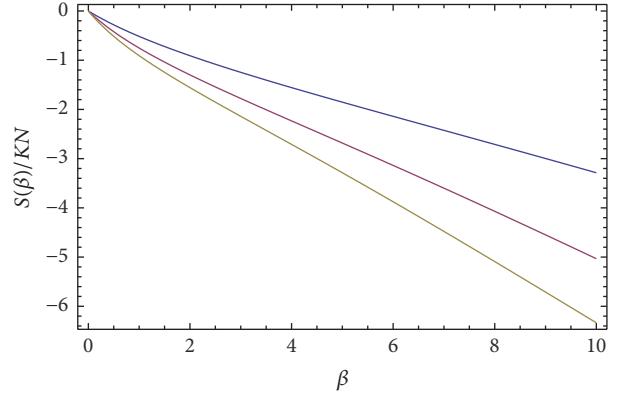
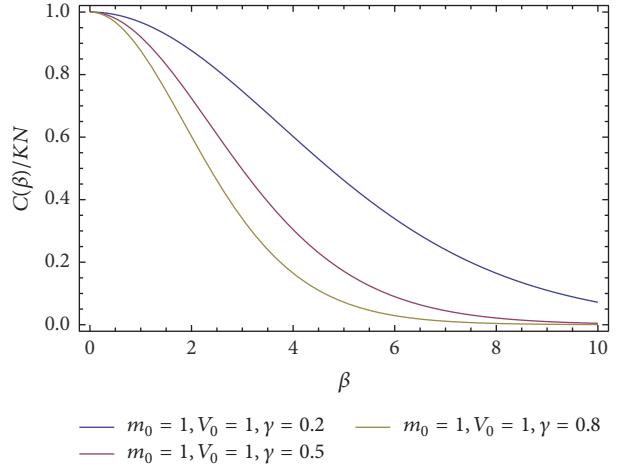
The chemical potential is obtained from (28) by a simple differentiation as

$$\mu = \frac{\partial A}{\partial N} = KT \ln \left( 2 \sinh \frac{\sqrt{2V_0\gamma}\beta}{2} \right). \quad (29)$$

The mean energy is obtained from

$$U = -\frac{\partial}{\partial \beta} \ln Q_N = \frac{\sqrt{2V_0\gamma}N}{2} \coth \frac{\sqrt{2V_0\gamma}\beta}{2}. \quad (30)$$

We have plotted  $U(\beta)/N$  in terms of  $\beta$ . In Figure 3, for small values of  $\beta$ , we face with convergences, but as  $\beta$  grows up, a small smooth divergence is observed.  $U(\beta)/N$ , as  $\gamma$  and  $\beta$  have higher values, gains higher values as well.

FIGURE 4:  $S(\beta)/KN$  versus  $\beta$ .FIGURE 5:  $C(\beta)/KN$  versus  $\beta$ .

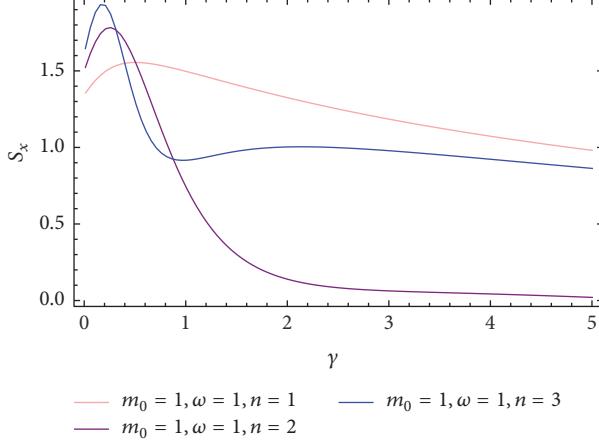
The main statistical quantity, that is, the entropy, is related to other quantities via

$$\begin{aligned} \frac{S}{K} &= \beta^2 \left( \frac{\partial A}{\partial \beta} \right) \\ &= -N \ln \left( 2 \sinh \frac{\sqrt{2V_0\gamma}\beta}{2} \right) \\ &\quad + \frac{\sqrt{2V_0\gamma}N}{2} \coth \frac{\sqrt{2V_0\gamma}\beta}{2}. \end{aligned} \quad (31)$$

In Figure 4,  $S(\beta)/KN$  is shown as  $\beta$  varies. Although a divergent behavior is seen at initial values of  $\beta$ , the curves tend to zero at large values. The specific heat capacity at constant volume is obtained from

$$\frac{C}{K} = -\beta^2 \left( \frac{\partial U}{\partial \beta} \right) = \frac{2V_0\gamma\beta^2 N}{4} \left( \operatorname{csch} \frac{\sqrt{2V_0\gamma}\beta}{2} \right)^2. \quad (32)$$

In Figure 5, we have plotted  $C(\beta)/KN$  versus  $\beta$ . In this figure, somehow, like the last figure, we have convergence but

FIGURE 6: Position Shannon information entropies  $S_x$  versus  $\gamma$ .

depending on the value of  $\gamma$ , values of  $C(\beta)/KN$  plunge into zero values with different slopes.

## 5. Information Entropy

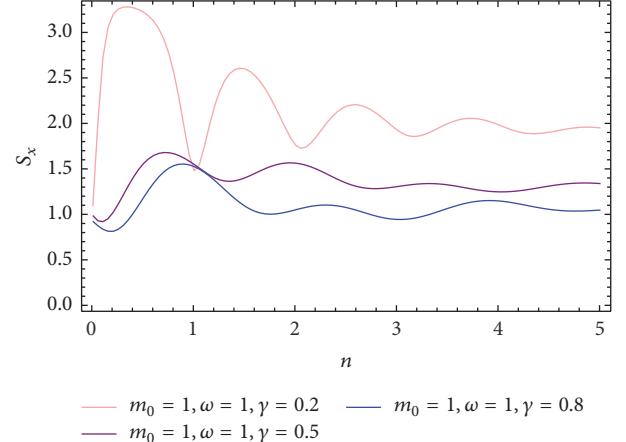
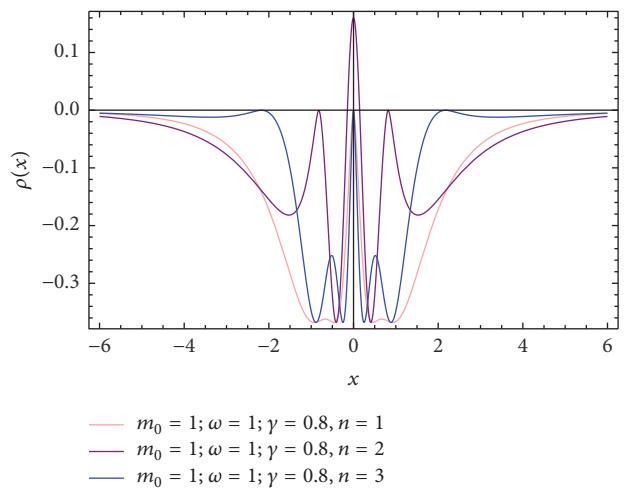
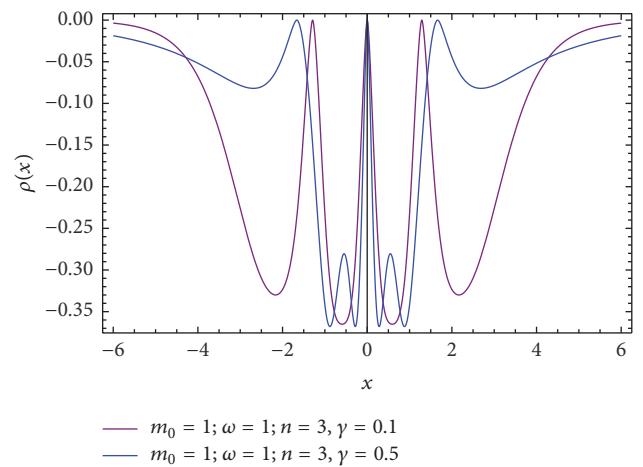
The position space Shannon and Fisher information entropies for a one-dimensional system can be calculated as  $S_x = -\int |\Psi_n(x)|^2 \ln |\Psi_n(x)|^2 dx$  and  $F_x = -\int |\Psi_n(x)|^2 [(d/dx) \ln |\Psi_n(x)|^2] dx$ . In general, explicit derivation of the Shannon information entropy is quite difficult. In particular the derivation of analytical expression for the  $S_x$  is almost impossible. We represent the position Shannon and Fisher information entropy densities, respectively, by  $\rho_{S(x)} = |\Psi_n(x)|^2 \ln(|\Psi_n(x)|^2)$  and  $\rho_{F(x)} = |\Psi_n(x)|^2 [(d/dx) \ln |\Psi_n(x)|^2]$  [35–40]. In our case, these quantities are obtained as

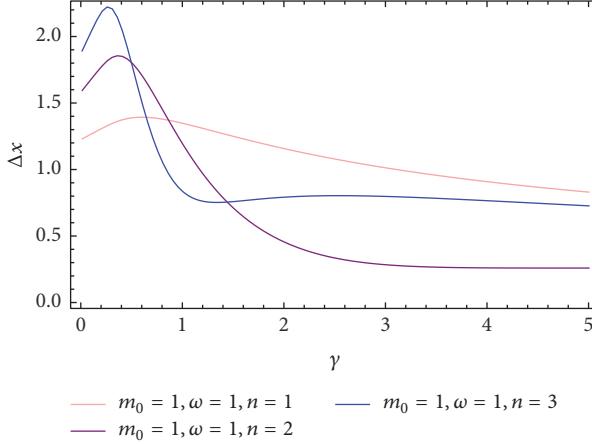
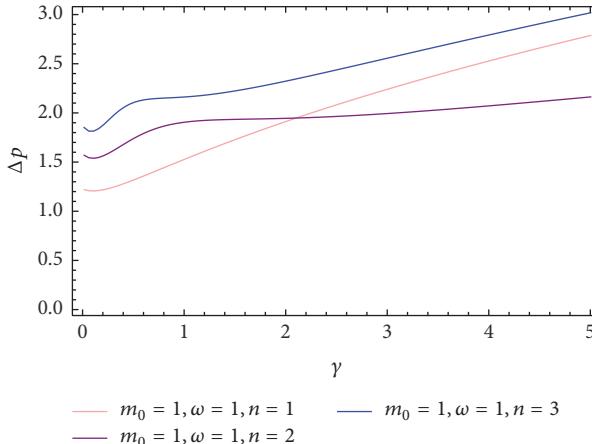
$$\begin{aligned} \rho_{S(x)} &= \frac{m_0 e^{-\omega y^2} H_n(y\sqrt{\omega})^2 \ln \left( m_0 e^{-\omega y^2} H_n(y\sqrt{\omega})^2 / (1 + \gamma x^2)^2 \right)}{(1 + \gamma x^2)^2}, \quad (33) \\ \rho_{F(x)} &= -\frac{4m_0 e^{-\omega y^2} x \gamma H_n(y\sqrt{\omega})^2}{(1 + \gamma x^2)^3}. \end{aligned}$$

To understand the essential features of the entropies, some related figures are included. At first case we plotted the position space Shannon information entropies  $S_x$  considering  $\gamma$  varying for  $m_0 = 1, \omega = 1$ . In Figures 6 and 7, we depicted the behavior of the position space Shannon information entropies  $S_x$  as function of  $n$  and  $\gamma$  by considering  $m_0 = 1, \omega = 1$ . The ascending nature can be seen easily. In Figures 8 and 9 we have plotted the position Shannon information entropy densities versus  $x$ .

## 6. Some Expectation Values and the Uncertainty Principle

The asymmetry caused by the parameter  $\gamma$  can be adequately quantified in terms of the average position calculated

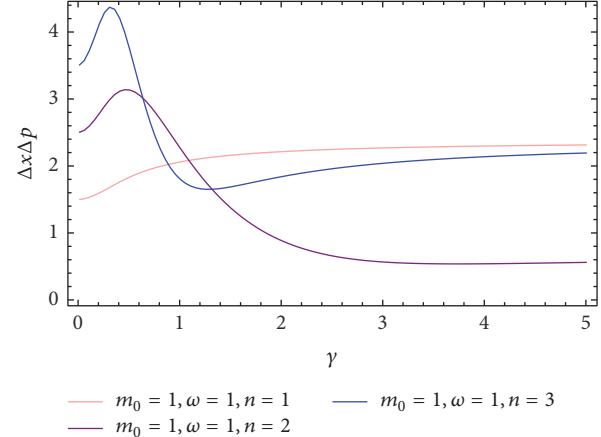
FIGURE 7: Position Shannon information entropy  $S_x$  versus  $n$ .FIGURE 8: Position Shannon information entropy densities  $\rho(x)$  versus  $x$ .FIGURE 9: Position Shannon information entropy density  $\rho(x)$  versus  $x$ .

FIGURE 10: Uncertainty in position versus  $\gamma$ .FIGURE 11: Uncertainty in momentum versus  $\gamma$ .

as  $\langle x \rangle = \int_{-\infty}^{\infty} \Psi_n^*(x) x \Psi_n(x) dx$  and average for  $x^2$  is  $\langle x^2 \rangle = \int_{-\infty}^{\infty} \Psi_n^*(x) x^2 \Psi_n(x) dx$ . From (23), the average of the modified momentum is  $\langle p \rangle = 0$  and average for  $p^2$  is  $\langle p^2 \rangle = -\hbar^2 \int_{-\infty}^{\infty} \Psi_n^*(x) (d^2/dx^2) \Psi_n(x) dx$  [41, 42]. We have plotted the uncertainty for  $x$ ,  $p$  and the uncertainty principle. Figure 10 shows how uncertainty for  $x$  can change. Also treatments of uncertainty in momentum are shown in Figure 11 for  $m_0 = 1$  and  $\omega = 1$  for different  $n$  in terms of  $\gamma$ . Figure 12, assuming  $m_0 = 1$  and  $\omega = 1$ , illustrates the uncertainty principle versus  $\gamma$ .

## 7. Conclusion

We studied the physical characteristics of a nonrelativistic quasi-oscillator interaction within position-dependent mass formalism. We first obtained the wave functions and the energy spectra of the system in an exact analytical manner. Next, the thermodynamic properties, information entropy, some expectation values, and some uncertainty principles were evaluated. In addition, we included some figures to

FIGURE 12: Combined uncertainty versus  $\gamma$ .

illustrate the physical characteristics and asymptotic behavior of the results.

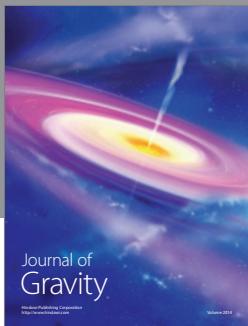
## Competing Interests

The authors declare that they have no competing interests.

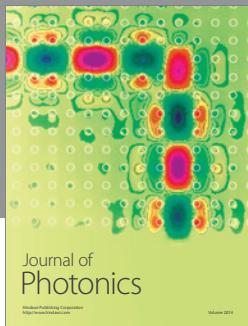
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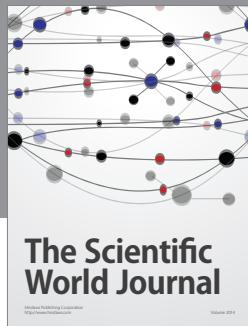
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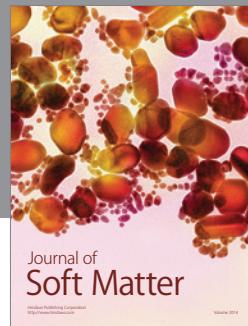
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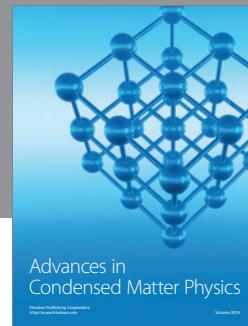
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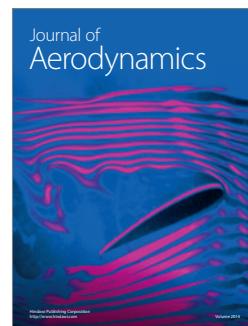
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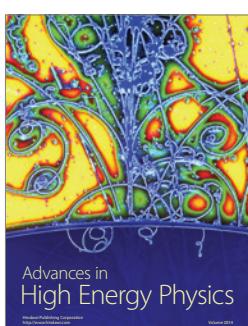
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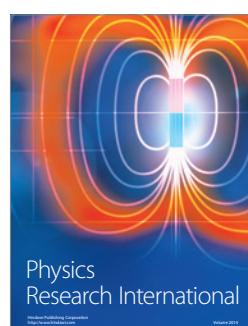
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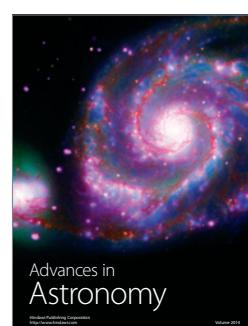
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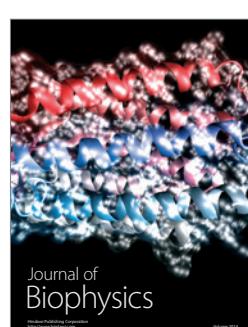
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