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Research Article

Study of the $\psi(1S, 2S)$ and $\eta_c(1S, 2S)$ Weak Decays into DM

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Inspired by the recent measurements on the $J/\psi(1S) \to D_s\rho$, D_uK^* weak decays at BESIII and the potential prospects of charmonium at high-luminosity heavy-flavor experiments, we study $\psi(1S, 2S)$ and $\eta_c(1S, 2S)$ weak decays into final states including one charmed meson plus one light meson, considering QCD corrections to hadronic matrix elements with QCD factorization approach. It is found that the Cabibbo-favored $\psi(1S, 2S) \to D_s^-\rho^+, D_s^-\pi^+, \overline{D}_u^0\overline{K}^{*0}$ decays have large branching ratios $\gtrsim 10^{-10}$, which might be accessible at future experiments.

1. Introduction

More than forty years after the discovery of the $J/\psi(1S)$ meson, the properties of charmonium (bound state of $c\bar{c}$) continue to be the subject of intensive theoretical and experimental study. It is believed that charmonium, resembling bottomonium (bound state of $b\bar{b}$), plays the same role in exploring hadronic dynamics as positronium and/or the hydrogen atom in understanding the atomic physics. Charmonium and bottomonium are good objects to test the basic ideas of QCD [1]. There is a renewed interest in charmonium due to the plentiful dedicated investigation from BES, CLEO-c, LHCb, and the studies via decays of the B mesons at B factories.

The $\psi(1S, 2S)$ and $\eta_c(1S, 2S)$ mesons are S-wave charmonium states below open-charm kinematic threshold and have the well-established quantum numbers of $I^GJ^{PC} = 0^+1^{--}$ and 0^+0^{-+}, respectively. They decay mainly through the strong and electromagnetic interactions. Because the G-parity conserving hadronic decays $\psi(2S) \to \pi\pi J/\psi(1S)$, $\eta J/\psi(1S)$ and $\eta_c(2S) \to \pi\pi\eta_c(1S)$ are suppressed by the compact phase space of final states, and because the decays into light hadrons are suppressed by the phenomenological Okubo-Zweig-Iizuka (OZI) rules [2–4], the total widths of $\psi(1S, 2S)$ and $\eta_c(1S, 2S)$ are narrow (see Table 1), which might render the charmonium weak decay as a necessary supplement. Here, we will concentrate on the $\psi(1S, 2S)$ and $\eta_c(1S, 2S)$ weak decays into DM final states, where M denotes the low-lying $SU(3)$ pseudoscalar and vector meson nonet. Our motivation is listed as follows.

From the experimental point of view, (1) some 10^9 $\psi(1S, 2S)$ data samples have been collected by BESIII since 2009 [5]. It is inspiringly expected to have about 10 billion $J/\psi(1S)$ and 3 billion $\psi(2S)$ events at BESIII experiment per year of data taking with the designed luminosity [6]: over 10^{10} $J/\psi(1S)$ at LHCb [7], ATLAS [8], and CMS [9] per fb^{-1} data in pp collisions. A large amount of data sample offers a realistic possibility to explore experimentally the charmonium weak decays. Correspondingly, theoretical study is very necessary to provide a ready reference. (2) Identification of the single D meson would provide an unambiguous signature of the charmonium weak decay into DM states. With the improvements of experimental instrumentation and particle identification techniques, accurate measurements on the nonleptonic charmonium weak decay might be feasible. Recently, a search for the $J/\psi(1S) \to D_s\rho$, D_uK^* decays has been performed at BESIII, although signals are unseen for the moment [10]. Of course, the branching ratios for the inclusive charmonium weak decay are tiny within the standard model,

Table 1: The properties of $\psi(1S, 2S)$ and $\eta_c(1S, 2S)$ mesons [12].

Meson	$I^G J^{PC}$	Mass (MeV)	Width
$\psi(1S)$	0^+1^{--}	3096.916 ± 0.011	92.9 ± 2.8 keV
$\psi(2S)$	0^+1^{--}	$3686.109^{+0.012}_{-0.014}$	299 ± 8 keV
$\eta_c(1S)$	0^+0^{-+}	2983.6 ± 0.7	32.2 ± 0.9 MeV
$\eta_c(2S)$	0^+0^{-+}	3639.4 ± 1.3	$11.3^{+3.2}_{-2.9}$ MeV

about $2/(\tau_D \Gamma_\psi) \sim 10^{-8}$ and $2/(\tau_D \Gamma_{\eta_c}) \sim 10^{-10}$, where D denotes the neutral charmed meson [11] and Γ_ψ and Γ_{η_c} stand for the total widths of $\psi(1S, 2S)$ and $\eta_c(1S, 2S)$ resonances, respectively. Observation of an abnormally large production rate of single charmed mesons in the final state would be a hint of new physics beyond the standard model [11].

From the theoretical point of view, (1) the charm quark weak decay is more favorable than the bottom quark weak decay, because the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements obey $|V_{cb}| \ll |V_{cs}|$ [12]. Penguin and annihilation contributions to nonleptonic charm quark weak decay, being proportional to the CKM factor $|V_{cb}V_{ub}| \sim \mathcal{O}(\lambda^5)$ with the Wolfenstein parameter $\lambda \simeq 0.22$ [12], are highly suppressed and hence negligible relative to tree contributions. Both c and $\bar{c}$ quarks in charmonium can decay individually, which provides a good place to investigate the dynamical mechanism of heavy-flavor weak decay and crosscheck model parameters obtained from the charmed hadron weak decays. (2) There are few works devoted to nonleptonic $J/\psi(1S)$ weak decays in the past, such as [13] with the covariant light-cone quark model, [14] with QCD sum rules, and [15–17] with the Wirbel-Stech-Bauer (WSB) model [18]. Moreover, previous works of [13–17] concern mainly the weak transition form factors between the $J/\psi(1S)$ and charmed mesons. Fewer papers have been devoted to nonleptonic $\psi(2S)$ and $\eta_c(1S, 2S)$ weak decays until now even though a rough estimate of branching ratios is unavailable. In this paper, we will estimate the branching ratios for nonleptonic two-body charmonium weak decay, taking the nonfactorizable contributions to hadronic matrix elements into account with the attractive QCD factorization (QCDF) approach [19].

This paper is organized as follows. In Section 2, we will present the theoretical framework and the amplitudes for $\psi(1S, 2S)$, $\eta_c(1S, 2S) \to DM$ decays. Section 3 is devoted to numerical results and discussion. Finally, Section 4 is our summation.

2. Theoretical Framework

2.1. The Effective Hamiltonian. Phenomenologically, the effective Hamiltonian responsible for charmonium weak decay into DM final states can be written as follows [25]:

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{q_1, q_2} V^*_{cq_1} V_{uq_2} \{C_1(\mu) Q_1(\mu) + C_2(\mu) Q_2(\mu)\} + \text{H.c.}, \quad (1)$$

where $G_F = 1.166 \times 10^{-5}\,\text{GeV}^{-2}$ [12] is the Fermi coupling constant; $V^*_{cq_1} V_{uq_2}$ is the CKM factor with $q_{1,2} = d, s$; the Wilson coefficients $C_{1,2}(\mu)$, which are independent of one particular process, summarize the physical contributions above the scale of μ. The expressions of the local tree four-quark operators are

$$Q_1 = \left[\bar{q}_{1,\alpha}\gamma_\mu (1-\gamma_5) c_\alpha\right]\left[\bar{u}_\beta \gamma^\mu (1-\gamma_5) q_{2,\beta}\right],$$
$$Q_2 = \left[\bar{q}_{1,\alpha}\gamma_\mu (1-\gamma_5) c_\beta\right]\left[\bar{u}_\beta \gamma^\mu (1-\gamma_5) q_{2,\alpha}\right], \quad (2)$$

where α and β are color indices.

It is well known that the Wilson coefficients C_i could be systematically calculated with perturbation theory and have properly been evaluated to the next-to-leading order (NLO). Their values at the scale of $\mu \sim \mathcal{O}(m_c)$ can be evaluated with the renormalization group (RG) equation [25]:

$$C_{1,2}(\mu) = U_4(\mu, m_b) U_5(m_b, m_W) C_{1,2}(m_W), \quad (3)$$

where $U_f(\mu_f, \mu_i)$ is the RG evolution matrix which transforms the Wilson coefficients from scale of μ_i to μ_f. The expression for $U_f(\mu_f, \mu_i)$ can be found in [25]. The numerical values of the leading-order (LO) and NLO $C_{1,2}$ in the naive dimensional regularization scheme are listed in Table 2. The values of coefficients $C_{1,2}$ in Table 2 agree well with those obtained with "effective" number of active flavors $f = 4.15$ [25] rather than formula (3).

To obtain the decay amplitudes and branching ratios, the remaining works are to evaluate accurately the hadronic matrix elements (HME) where the local operators are sandwiched between the charmonium and final states, which is also the most intricate work in dealing with the weak decay of heavy hadrons by now.

2.2. Hadronic Matrix Elements. Analogous to the exclusive processes with perturbative QCD theory proposed by Lepage and Brodsky [26], the QCDF approach is developed by Beneke et al. [19] to deal with HME based on the collinear factorization approximation and power counting rules in the heavy quark limit and has been extensively used for B meson decays. Using the QCDF master formula, HME of nonleptonic decays could be written as the convolution integrals of the process-dependent hard scattering kernels and universal light-cone distribution amplitudes (LCDA) of participating hadrons.

The spectator quark is the heavy-flavor charm quark for charmonium weak decays into DM final states. It is commonly assumed that the virtuality of the gluon connecting to the heavy spectator is of order Λ^2_{QCD}, where Λ_{QCD} is the characteristic QCD scale. Hence, the transition form factors between charmonium and D mesons are assumed to be dominated by the soft and nonperturbative contributions, and the amplitudes of the spectator rescattering subprocess are power-suppressed [19]. Taking $\eta_c \to DM$ decays, for example, HME can be written as

$$\langle DM | Q_{1,2} | \eta_c \rangle = \sum_i F_i^{\eta_c \to D} f_M \int H_i(x) \Phi_M(x)\, dx, \quad (4)$$

Table 2: Numerical values of the Wilson coefficients $C_{1,2}$ and parameters $a_{1,2}$ for $\eta_c \to D\pi$ decay with $m_c = 1.275$ GeV [12], where $a_{1,2}$ in [20] is used in the D meson weak decay.

μ	LO		NLO		QCDF		Previous works		
	C_1	C_2	C_1	C_2	a_1	a_2	Ref.	a_1	a_2
$0.8m_c$	1.335	−0.589	1.275	−0.504	$1.275e^{+i4^\circ}$	$0.503e^{-i154^\circ}$	[14, 16, 17]	1.26	−0.51
m_c	1.276	−0.505	1.222	−0.425	$1.219e^{+i3^\circ}$	$0.402e^{-i154^\circ}$	[15]	1.3 ± 0.1	−0.55 ± 0.10
$1.2m_c$	1.240	−0.450	1.190	−0.374	$1.186e^{+i3^\circ}$	$0.342e^{-i154^\circ}$	[20]	1.274	−0.529

where $F_i^{\eta_c \to D}$ is the weak transition form factor and f_M and $\Phi_M(x)$ are the decay constant and LCDA of the meson M, respectively. The leading twist LCDA for the pseudoscalar and longitudinally polarized vector mesons can be expressed in terms of Gegenbauer polynomials [23, 24]:

$$\Phi_M(x) = 6x\overline{x}\sum_{n=0}^{\infty} a_n^M C_n^{3/2}(x-\overline{x}), \tag{5}$$

where $\overline{x} = 1 - x$; $C_n^{3/2}(z)$ is the Gegenbauer polynomial,

$$\begin{aligned} C_0^{3/2}(z) &= 1, \\ C_1^{3/2}(z) &= 3z, \\ C_2^{3/2}(z) &= \frac{3}{2}\left(5z^2-1\right), \\ &\vdots \end{aligned} \tag{6}$$

a_n^M is the Gegenbauer moment corresponding to the Gegenbauer polynomials $C_n^{3/2}(z)$; $a_0^M \equiv 1$ for the asymptotic form; and $a_n = 0$ for $n = 1, 3, 5, \ldots$ because of the G-parity invariance of the π, $\eta^{(\prime)}$, ρ, ω, ϕ meson distribution amplitudes. In this paper, to give a rough estimation, the contributions from higher-order $n \geq 3$ Gegenbauer polynomials are not considered for the moment.

Hard scattering function $H_i(x)$ in (4) is, in principle, calculable order by order with the perturbative QCD theory. At the order of α_s^0, $H_i(x) = 1$. This is the simplest scenario, and one goes back to the naive factorization where there is no information about the strong phases and the renormalization scale hidden in the HME. At the order of α_s and higher orders, the renormalization scale dependence of hadronic matrix elements could be recuperated to partly cancel the μ-dependence of the Wilson coefficients. In addition, part of the strong phases could be reproduced from nonfactorizable contributions.

Within the QCDF framework, amplitudes for $\eta_c \to DM$ decays can be expressed as

$$\begin{aligned} \mathscr{A}(\eta_c \longrightarrow DM) &= \langle DM | \mathscr{H}_{\text{eff}} | \eta_c \rangle \\ &= \frac{G_F}{\sqrt{2}} V_{cq_1}^* V_{uq_2} a_i \langle M | J^\mu | 0 \rangle \langle D | J_\mu | \eta_c \rangle. \end{aligned} \tag{7}$$

In addition, the HME for the $\psi(1S, 2S) \to DV$ decays are conventionally expressed as the helicity amplitudes with the decomposition [27, 28],

$$\begin{aligned} \mathscr{H}_\lambda = \langle V | J^\mu | 0 \rangle \langle D | J_\mu | \psi \rangle = \epsilon_V^{*\mu} \epsilon_\psi^\nu \Bigg\{ & a g_{\mu\nu} \\ & + \frac{b}{m_\psi m_V} \left(p_\psi + p_D\right)^\mu p_V^\nu \\ & + \frac{ic}{m_\psi m_V} \epsilon_{\mu\nu\alpha\beta} p_V^\alpha \left(p_\psi + p_D\right)^\beta \Bigg\}. \end{aligned} \tag{8}$$

The relations among helicity amplitudes and invariant amplitudes a, b, c are

$$\begin{aligned} \mathscr{H}_0 &= -ax - 2b\left(x^2-1\right), \\ \mathscr{H}_\pm &= a \pm 2c\sqrt{x^2-1}, \\ x &= \frac{p_\psi \cdot p_V}{m_\psi m_V} = \frac{m_\psi^2 - m_D^2 + m_V^2}{2m_\psi m_V}, \end{aligned} \tag{9}$$

where three scalar amplitudes a, b, c describe s, d, p wave contributions, respectively.

The effective coefficient a_i at the order of α_s can be expressed as [19]

$$\begin{aligned} a_1 &= C_1^{\text{NLO}} + \frac{1}{N_c} C_2^{\text{NLO}} + \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} C_2^{\text{LO}} \mathscr{V}, \\ a_2 &= C_2^{\text{NLO}} + \frac{1}{N_c} C_1^{\text{NLO}} + \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} C_1^{\text{LO}} \mathscr{V}, \end{aligned} \tag{10}$$

where the color factor $C_F = 4/3$; the color number $N_c = 3$. For the transversely polarized light vector meson, the factor $\mathscr{V} = 0$ in the helicity $\mathscr{H}_\pm$ amplitudes beyond the leading twist contributions. With the leading twist LCDA for the pseudoscalar and longitudinally polarized vector mesons, the factor $\mathscr{V}$ is written as [19]

$$\begin{aligned} \mathscr{V} = 6\log\left(\frac{m_c^2}{\mu^2}\right) - 18 - \left(\frac{1}{2} + i3\pi\right) \\ + \left(\frac{11}{2} - i3\pi\right) a_1^M - \frac{21}{20} a_2^M + \cdots. \end{aligned} \tag{11}$$

From the numbers in Table 2, it is found that (1) the values of coefficients $a_{1,2}$ agree generally with those used in previous works [14–17, 20], (2) the strong phases appear by taking

nonfactorizable corrections into account, which is necessary for CP violation, and (3) the strong phase of a_1 is small due to the suppression of α_s and $1/N_c$. The strong phase of a_2 is large due to the enhancement from the large Wilson coefficients C_1.

2.3. Form Factors. The weak transition form factors between charmonium and a charmed meson are defined as follows [18]:

$$\begin{aligned}
&\left\langle D(p_2)\left|V_\mu - A_\mu\right|\eta_c(p_1)\right\rangle \\
&\quad = \left\{(p_1+p_2)_\mu - \frac{m_{\eta_c}^2 - m_D^2}{q^2} q_\mu\right\} F_1\left(q^2\right) \\
&\qquad + \frac{m_{\eta_c}^2 - m_D^2}{q^2} q_\mu F_0\left(q^2\right), \\
&\left\langle D(p_2)\left|V_\mu - A_\mu\right|\psi(p_1,\epsilon)\right\rangle \\
&\quad = -\epsilon_{\mu\nu\alpha\beta}\epsilon_\psi^\nu q^\alpha (p_1+p_2)^\beta \frac{V\left(q^2\right)}{m_\psi + m_D} \\
&\qquad - i\frac{2m_\psi \epsilon_\psi \cdot q}{q^2} q_\mu A_0\left(q^2\right) \\
&\qquad - i\epsilon_{\psi,\mu}\left(m_\psi + m_D\right) A_1\left(q^2\right) \\
&\qquad - i\frac{\epsilon_\psi \cdot q}{m_\psi + m_D}(p_1+p_2)_\mu A_2\left(q^2\right) \\
&\qquad + i\frac{2m_\psi \epsilon_\psi \cdot q}{q^2} q_\mu A_3\left(q^2\right),
\end{aligned} \tag{12}$$

where $q = p_1 - p_2$; ϵ_ψ denotes the ψ's polarization vector. The form factors $F_0(0) = F_1(0)$ and $A_0(0) = A_3(0)$ are required compulsorily to cancel singularities at the pole of $q^2 = 0$. There is a relation among these form factors:

$$\begin{aligned}
2m_\psi A_3\left(q^2\right) &= \left(m_\psi + m_D\right) A_1\left(q^2\right) \\
&\quad + \left(m_\psi - m_D\right) A_2\left(q^2\right).
\end{aligned} \tag{13}$$

There are four independent transition form factors, $F_0(0)$, $A_{0,1}(0)$, and $V(0)$, at the pole of $q^2 = 0$. They could be written as the overlap integrals of wave functions [18]:

$$\begin{aligned}
F_0(0) &= \int\int_0^1 \left\{\Phi_{\eta_c}\left(\vec{k}_\perp, x, 0, 0\right)\right. \\
&\quad \left.\cdot \Phi_D\left(\vec{k}_\perp, x, 0, 0\right)\right\} dx\, d\vec{k}_\perp, \\
A_0(0) &= \int\int_0^1 \left\{\Phi_\psi\left(\vec{k}_\perp, x, 1, 0\right)\right. \\
&\quad \left.\cdot \sigma_z \Phi_D\left(\vec{k}_\perp, x, 0, 0\right)\right\} dx\, d\vec{k}_\perp, \\
A_1(0) &= \frac{m_c + m_q}{m_\psi + m_D} I, \\
V(0) &= \frac{m_c - m_q}{m_\psi - m_D} I, \\
I &= \sqrt{2}\int\int_0^1 \left\{\Phi_\psi\left(\vec{k}_\perp, x, 1, -1\right) i\sigma_y \Phi_D\left(\vec{k}_\perp, x, 0, 0\right)\right\} \\
&\quad \cdot \frac{1}{x} dx\, d\vec{k}_\perp,
\end{aligned} \tag{14}$$

where $\sigma_{y,z}$ is the Pauli matrix acting on the spin indices of the decaying charm quark; x and $\vec{k}_\perp$ denote the fraction of the longitudinal momentum and the transverse momentum of the nonspectator quark, respectively.

With the separation of the spin and spatial variables, wave functions can be written as

$$\Phi\left(\vec{k}_\perp, x, j, j_z\right) = \phi\left(\vec{k}_\perp, x\right)\left|s, s_z, s_1, s_2\right\rangle, \tag{15}$$

where the total angular momentum $\vec{j} = \vec{L} + \vec{s}_1 + \vec{s}_2 = \vec{s}_1 + \vec{s}_2 = \vec{s}$ because the orbital angular momentum between the valence quarks in $\psi(1S, 2S)$, $\eta_c(1S, 2S)$, D mesons in question have $\vec{L} = 0$; $s_{1,2}$ denote the spins of valence quarks in meson; $s = 1$ and 0 for the ψ and η_c mesons, respectively.

The charm quark in the charmonium state is nearly nonrelativistic with an average velocity $v \ll 1$ based on arguments of nonrelativistic quantum chromodynamics (NRQCD) [29–31]. For the D meson, the valence quarks are also nonrelativistic due to $m_D \approx m_c + m_q$, where the light quark mass $m_u \approx m_d \approx 310\,\text{MeV}$ and $m_s \approx 510\,\text{MeV}$ [32]. Here, we will take the solution of the Schrödinger equation with a scalar harmonic oscillator potential as the wave functions of the charmonium and D mesons:

$$\begin{aligned}
\phi_{1S}\left(\vec{k}\right) &\sim e^{-\vec{k}^2/2\alpha^2}, \\
\phi_{2S}\left(\vec{k}\right) &\sim e^{-\vec{k}^2/2\alpha^2}\left(2\vec{k}^2 - 3\alpha^2\right),
\end{aligned} \tag{16}$$

where the parameter α determines the average transverse quark momentum, $\langle\phi_{1S}|\vec{k}_\perp^2|\phi_{1S}\rangle = \alpha^2$. With the NRQCD power counting rules [29], $|\vec{k}_\perp| \sim mv \sim m\alpha_s$ for heavy quarkonium. Hence, parameter α is approximately taken as $m\alpha_s$ in our calculation.

Using the substitution ansatz [33],

$$\vec{k}^2 \longrightarrow \frac{\vec{k}_\perp^2 + \bar{x}m_q^2 + xm_c^2}{4x\bar{x}}, \tag{17}$$

one can obtain

$$\begin{aligned}
\phi_{1S}\left(\vec{k}_\perp, x\right) &= A\exp\left\{\frac{\vec{k}_\perp^2 + \bar{x}m_q^2 + xm_c^2}{-8\alpha^2 x\bar{x}}\right\}, \\
\phi_{2S}\left(\vec{k}_\perp, x\right) &= B\phi_{1S}\left(\vec{k}_\perp, x\right)\left\{\frac{\vec{k}_\perp^2 + \bar{x}m_q^2 + xm_c^2}{6\alpha^2 x\bar{x}} - 1\right\},
\end{aligned} \tag{18}$$

Table 3: The numerical values of transition form factors at $q^2 = 0$, where uncertainties of this work come from the charm quark mass.

Transition	Reference	$F_0(0)$	$A_0(0)$	$A_1(0)$	$V(0)$
$\eta_c(1S), \psi(1S) \to D_{u,d}$	This work	0.85 ± 0.01	0.85 ± 0.01	0.72 ± 0.01	1.76 ± 0.03
	[13][a]	$\cdots$	0.68 ± 0.01	0.68 ± 0.01	1.6 ± 0.1
	[21][b]	$\cdots$	$0.27^{+0.02}_{-0.03}$	$0.27^{+0.03}_{-0.02}$	$0.81^{+0.12}_{-0.08}$
	[15][c]	$\cdots$	0.40 (0.61)	0.44 (0.68)	1.17 (1.82)
	[17][d]	$\cdots$	0.55 ± 0.02	$0.77^{+0.09}_{-0.07}$	$2.14^{+0.15}_{-0.11}$
	[17][e]	$\cdots$	0.54	0.80	2.21
$\eta_c(1S), \psi(1S) \to D_s$	This work	0.90 ± 0.01	0.90 ± 0.01	0.81 ± 0.01	1.55 ± 0.04
	[13][a]	$\cdots$	0.68 ± 0.01	0.68 ± 0.01	1.8
	[21][b]	$\cdots$	0.37 ± 0.02	$0.38^{+0.02}_{-0.01}$	$1.07^{+0.05}_{-0.02}$
	[15][c]	$\cdots$	0.47 (0.66)	0.55 (0.78)	1.25 (1.80)
	[17][d]	$\cdots$	$0.71^{+0.04}_{-0.02}$	0.94 ± 0.07	$2.30^{+0.09}_{-0.06}$
	[17][e]	$\cdots$	0.69	0.96	2.36
$\eta_c(2S), \psi(2S) \to D_{u,d}$	This work	0.62 ± 0.01	0.61 ± 0.01	0.54 ± 0.01	1.00 ± 0.04
$\eta_c(2S), \psi(2S) \to D_s$	This work	0.65 ± 0.01	0.64 ± 0.01	0.59 ± 0.02	0.83 ± 0.04

[a]The form factors are computed with the covariant light-front quark model, where uncertainties come from the decay constant of charmed meson.
[b]The form factors are computed with QCD sum rules, where uncertainties are from the Borel parameters.
[c]The form factors are computed with parameter $\omega = 0.4$ (0.5) GeV using the WSB model.
[d]The form factors are computed with flavor dependent parameter ω using the WSB model.
[e]The form factors are computed with parameter $\omega = m\alpha_s$ using the WSB model.

where the parameters A and B are the normalization coefficients satisfying the normalization condition,

$$\int \int_0^1 \left|\phi\left(\vec{k}_\perp, x\right)\right|^2 dx\, d\vec{k}_\perp = 1. \tag{19}$$

The numerical values of transition form factors at $q^2 = 0$ are listed in Table 3. It is found that (1) the model dependence of form factors is large; (2) isospin-breaking effects are negligible and flavor breaking effects are small; and (3) as stated in [18] $F_0 \simeq A_0$ holds within collinear symmetry.

3. Numerical Results and Discussion

In the charmonium center-of-mass frame, the branching ratio for the charmonium weak decay can be written as

$$\begin{aligned} \mathscr{B}r\left(\eta_c \longrightarrow DM\right) &= \frac{p_{\mathrm{cm}}}{4\pi m_{\eta_c}^2 \Gamma_{\eta_c}} \left|\mathscr{A}\left(\eta_c \longrightarrow DM\right)\right|^2, \\ \mathscr{B}r\left(\psi \longrightarrow DM\right) &= \frac{p_{\mathrm{cm}}}{12\pi m_{\psi}^2 \Gamma_{\psi}} \left|\mathscr{A}\left(\psi \longrightarrow DM\right)\right|^2, \end{aligned} \tag{20}$$

where the common momentum of final states is

$$p_{\mathrm{cm}} = \frac{\sqrt{\left[m_{\eta_c,\psi}^2 - \left(m_D + m_M\right)^2\right]\left[m_{\eta_c,\psi}^2 - \left(m_D - m_M\right)^2\right]}}{2m_{\eta_c,\psi}}. \tag{21}$$

The decay amplitudes for $\mathscr{A}(\psi \to DM)$ and $\mathscr{A}(\eta_c \to DM)$ are collected in Appendices A and B, respectively.

In our calculation, we assume that the light vector mesons are ideally mixed; that is, $\omega = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $\phi = s\bar{s}$. For the mixing of pseudoscalar η and η' meson, we will adopt the quark-flavor basis description proposed in [22] and neglect the contributions from possible gluonium compositions; that is,

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix}, \tag{22}$$

where $\eta_q = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $\eta_s = s\bar{s}$; the mixing angle $\phi = (39.3 \pm 1.0)^\circ$ [22]. The mass relations are

$$\begin{aligned} m_{\eta_q}^2 &= m_\eta^2 \cos^2\phi + m_{\eta'}^2 \sin^2\phi \\ &\quad - \frac{\sqrt{2} f_{\eta_s}}{f_{\eta_q}} \left(m_{\eta'}^2 - m_\eta^2\right) \cos\phi \sin\phi, \\ m_{\eta_s}^2 &= m_\eta^2 \sin^2\phi + m_{\eta'}^2 \cos^2\phi \\ &\quad - \frac{f_{\eta_q}}{\sqrt{2} f_{\eta_s}} \left(m_{\eta'}^2 - m_\eta^2\right) \cos\phi \sin\phi. \end{aligned} \tag{23}$$

The input parameters, including the CKM Wolfenstein parameters, decay constants, and Gegenbauer moments, are collected in Table 4. If not specified explicitly, we will take their central values as the default inputs. Our numerical results on branching ratios for the nonleptonic two-body $\psi(1S, 2S)$, $\eta_c(1S, 2S) \to DM$ weak decays are displayed in Tables 5 and 6, where the uncertainties of this work come from the CKM parameters, the renormalization scale $\mu = (1 \pm 0.2)m_c$, and hadronic parameters including decay constants and Gegenbauer moments, respectively. For comparison, previous results on $J/\psi(1S)$ weak decays [14, 16, 17] with parameters $a_1 = 1.26$ and $a_2 = -0.51$ are also listed in Table 5. The following are some comments.

Table 4: Numerical values of input parameters.

$\lambda = 0.22537 \pm 0.00061$ [12]	$A = 0.814^{+0.023}_{-0.024}$ [12]
$\overline{\rho} = 0.117 \pm 0.021$ [12]	$\overline{\eta} = 0.353 \pm 0.013$ [12]
$m_c = 1.275 \pm 0.025$ GeV [12]	$m_{D_u} = 1864.84 \pm 0.07$ MeV [12]
$m_{D_d} = 1869.61 \pm 0.10$ MeV [12]	$m_{D_s} = 1968.30 \pm 0.11$ MeV [12]
$f_\pi = 130.41 \pm 0.20$ MeV [12]	$f_K = 156.2 \pm 0.7$ MeV [12]
$f_{\eta_q} = (1.07 \pm 0.02)\, f_\pi$ [22]	$f_{\eta_s} = (1.34 \pm 0.06)\, f_\pi$ [22]
$f_\rho = 216 \pm 3$ MeV [23]	$f_\omega = 187 \pm 5$ MeV [23]
$f_\phi = 215 \pm 5$ MeV [23]	$f_{K^*} = 220 \pm 5$ MeV [23]
$a_2^\pi = a_2^{\eta_q} = a_2^{\eta_s} = 0.25 \pm 0.15$ [24]	$a_2^\rho = a_2^\omega = 0.15 \pm 0.07$ [23]
$a_1^{\overline{K}} = -a_1^K = 0.06 \pm 0.03$ [24]	$a_2^K = a_2^{\overline{K}} = 0.25 \pm 0.15$ [24]
$a_1^{\overline{K}^*} = -a_1^{K^*} = 0.03 \pm 0.02$ [23]	$a_2^{K^*} = a_2^{\overline{K}^*} = 0.11 \pm 0.09$ [23]
$a_1^\pi = a_1^\rho = a_1^\omega = a_1^\phi = 0$	$a_2^\phi = 0.18 \pm 0.08$ [23]

Table 5: Branching ratios for the nonleptonic two-body $J/\psi(1S)$ weak decays, where the uncertainties of this work come from the CKM parameters, the renormalization scale $\mu = (1 \pm 0.2)m_c$, and hadronic parameters including decay constants and Gegenbauer moments, respectively. The results of [14, 16, 17] are calculated with $a_1 = 1.26$ and $a_2 = -0.51$. The results of [14] are based on QCD sum rules. The numbers in columns of "A," "B," "C," and "D" are based on the WSB model with flavor dependent ω, QCD inspired $\omega = \alpha_s m$, and universal $\omega = 0.4$ GeV and 0.5 GeV, respectively.

Final states	Case	Reference [14]	Reference [17]		Reference [16]		This work
			A	B	C	D	
$D_s^-\pi^+$	1-a	2.0×10^{-10}	7.41×10^{-10}	7.13×10^{-10}	3.32×10^{-10}	8.74×10^{-10}	$(1.09^{+0.01+0.10+0.01}_{-0.01-0.06-0.01}) \times 10^{-9}$
$D_s^-K^+$	1-b	1.6×10^{-11}	5.3×10^{-11}	5.2×10^{-11}	2.4×10^{-11}	5.5×10^{-11}	$(6.18^{+0.03+0.59+0.08}_{-0.03-0.33-0.08}) \times 10^{-11}$
$D_d^-\pi^+$	1-b	0.8×10^{-11}	2.9×10^{-11}	2.8×10^{-11}	1.5×10^{-11}	5.5×10^{-11}	$(6.37^{+0.03+0.60+0.03}_{-0.03-0.34-0.03}) \times 10^{-11}$
$D_d^-K^+$	1-c	$\cdots$	2.3×10^{-12}	2.2×10^{-12}	1.2×10^{-12}	$\cdots$	$(3.79^{+0.04+0.36+0.05}_{-0.04-0.20-0.05}) \times 10^{-12}$
$\overline{D}_u^0\pi^0$	2-b	$\cdots$	2.4×10^{-12}	2.3×10^{-12}	1.2×10^{-12}	5.5×10^{-12}	$(3.50^{+0.02+1.98+0.06}_{-0.02-0.97-0.06}) \times 10^{-12}$
$\overline{D}_u^0K^0$	2-c	$\cdots$	4.0×10^{-13}	4.0×10^{-13}	2.0×10^{-13}	$\cdots$	$(4.16^{+0.04+2.35+0.11}_{-0.04-1.15-0.10}) \times 10^{-13}$
$\overline{D}_u^0\overline{K}^0$	2-a	3.6×10^{-11}	1.39×10^{-10}	1.34×10^{-10}	7.2×10^{-11}	2.8×10^{-10}	$(1.44^{+0.01+0.81+0.03}_{-0.01-0.40-0.03}) \times 10^{-10}$
$\overline{D}_u^0\eta$		$\cdots$	7.0×10^{-12}	6.7×10^{-12}	3.6×10^{-12}	1.6×10^{-12}	$(1.03^{+0.01+0.58+0.10}_{-0.01-0.28-0.10}) \times 10^{-11}$
$\overline{D}_u^0\eta'$		$\cdots$	4.0×10^{-13}	4.0×10^{-13}	2.0×10^{-13}	3.0×10^{-13}	$(5.83^{+0.03+3.29+1.72}_{-0.03-1.61-1.50}) \times 10^{-13}$
$D_s^-\rho^+$	1-a	1.26×10^{-9}	5.11×10^{-9}	5.32×10^{-9}	1.77×10^{-9}	3.63×10^{-9}	$(3.82^{+0.01+0.36+0.11}_{-0.01-0.20-0.11}) \times 10^{-9}$
$D_s^-K^{*+}$	1-b	0.82×10^{-10}	2.82×10^{-10}	2.96×10^{-10}	0.97×10^{-10}	2.12×10^{-10}	$(2.00^{+0.01+0.19+0.10}_{-0.01-0.11-0.09}) \times 10^{-10}$
$D_d^-\rho^+$	1-b	0.42×10^{-10}	2.16×10^{-10}	2.28×10^{-10}	0.72×10^{-10}	2.20×10^{-10}	$(2.12^{+0.01+0.20+0.06}_{-0.01-0.11-0.06}) \times 10^{-10}$
$D_d^-K^{*+}$	1-c	$\cdots$	1.3×10^{-11}	1.3×10^{-11}	4.2×10^{-12}	$\cdots$	$(1.14^{+0.01+0.11+0.06}_{-0.01-0.06-0.05}) \times 10^{-11}$
$\overline{D}_u^0\rho^0$	2-b	$\cdots$	1.8×10^{-11}	1.9×10^{-11}	6.0×10^{-12}	2.2×10^{-11}	$(1.08^{+0.01+0.61+0.04}_{-0.01-0.30-0.04}) \times 10^{-11}$
$\overline{D}_u^0\omega$	2-b	$\cdots$	1.6×10^{-11}	1.7×10^{-11}	5.0×10^{-12}	1.8×10^{-11}	$(8.10^{+0.04+4.56+0.50}_{-0.04-2.25-0.48}) \times 10^{-12}$
$\overline{D}_u^0\phi$	2-b	$\cdots$	4.2×10^{-11}	4.4×10^{-11}	1.4×10^{-11}	6.5×10^{-11}	$(1.92^{+0.01+1.08+0.10}_{-0.01-0.53-0.10}) \times 10^{-11}$
$\overline{D}_u^0K^{*0}$	2-c	$\cdots$	2.1×10^{-12}	2.2×10^{-12}	7.0×10^{-13}	$\cdots$	$(1.19^{+0.01+0.67+0.07}_{-0.01-0.33-0.07}) \times 10^{-12}$
$\overline{D}_u^0\overline{K}^{*0}$	2-a	1.54×10^{-10}	7.61×10^{-10}	8.12×10^{-10}	2.51×10^{-10}	1.03×10^{-9}	$(4.09^{+0.01+2.30+0.24}_{-0.01-1.14-0.23}) \times 10^{-10}$

(1) There are some differences among the estimates of branching ratios for $J/\psi(1S) \to DM$ weak decays (see the numbers in Table 5). These inconsistencies among previous works, although the same values of parameters $a_{1,2}$ are used, come principally from different values of form factors. Our results are generally in line with the numbers in columns "A" and "B" which are favored by [17].

(2) Branching ratios for $J/\psi(1S)$ weak decay are about two or more times as large as those for $\psi(2S)$ decay into the same final states, because the decay width of $\psi(2S)$ is about three times as large as that of $J/\psi(1S)$.

(3) Due to the relatively small decay width and relatively large space phases for $\eta_c(2S)$ decay, branching ratios for $\eta_c(2S)$ weak decay are some five (ten) or more times as large as those for $\eta_c(1S)$ weak decay into the same DP (DV) final states.

(4) Among $\psi(1S, 2S)$ and $\eta_c(1S, 2S)$ mesons, $\eta_c(1S)$ has a maximal decay width and a minimal mass resulting in a small phase space, while $J/\psi(1S)$ has a minimal decay width. These facts lead to the smallest [or the largest] branching ratio for $\eta_c(1S)$ [or $J/\psi(1S)$] weak decay among $\psi(1S, 2S)$, $\eta_c(1S, 2S)$ weak decays into the same final states.

Table 6: Branching ratios for the nonleptonic two-body $\psi(2S)$, $\eta_c(1S)$, and $\eta_c(2S)$ weak decays, where the uncertainties come from the CKM parameters, the renormalization scale $\mu = (1 \pm 0.2)m_c$, and hadronic parameters including decay constants and Gegenbauer moments, respectively.

Case	Final states	$\psi(2S)$ decay	$\eta_c(1S)$ decay	$\eta_c(2S)$ decay
1-a	$D_s^-\pi^+$	$(5.07^{+0.01+0.48+0.03}_{-0.01-0.27-0.02}) \times 10^{-10}$	$(7.35^{+0.01+0.69+0.04}_{-0.01-0.39-0.04}) \times 10^{-12}$	$(3.90^{+0.01+0.37+0.02}_{-0.01-0.21-0.02}) \times 10^{-11}$
1-b	$D_s^-K^+$	$(3.43^{+0.02+0.33+0.04}_{-0.02-0.18-0.04}) \times 10^{-11}$	$(4.97^{+0.03+0.48+0.06}_{-0.03-0.27-0.06}) \times 10^{-13}$	$(2.87^{+0.01+0.27+0.04}_{-0.01-0.15-0.04}) \times 10^{-12}$
1-b	$D_d^-\pi^+$	$(2.76^{+0.01+0.26+0.01}_{-0.01-0.15-0.01}) \times 10^{-11}$	$(4.39^{+0.02+0.41+0.02}_{-0.02-0.23-0.02}) \times 10^{-13}$	$(2.13^{+0.01+0.20+0.01}_{-0.01-0.11-0.01}) \times 10^{-12}$
1-c	$D_d^-K^+$	$(1.90^{+0.02+0.18+0.02}_{-0.02-0.10-0.02}) \times 10^{-12}$	$(3.04^{+0.03+0.29+0.04}_{-0.03-0.16-0.04}) \times 10^{-14}$	$(1.58^{+0.02+0.15+0.02}_{-0.02-0.08-0.02}) \times 10^{-13}$
2-b	$\overline{D}_u^0\pi^0$	$(1.51^{+0.01+0.85+0.02}_{-0.01-0.42-0.02}) \times 10^{-12}$	$(2.41^{+0.01+1.36+0.04}_{-0.01-0.67-0.04}) \times 10^{-14}$	$(1.16^{+0.01+0.66+0.02}_{-0.01-0.32-0.02}) \times 10^{-13}$
2-c	$\overline{D}_u^0K^0$	$(2.07^{+0.02+1.17+0.05}_{-0.02-0.57-0.05}) \times 10^{-13}$	$(3.35^{+0.04+1.89+0.09}_{-0.04-0.93-0.08}) \times 10^{-15}$	$(1.73^{+0.02+0.97+0.04}_{-0.02-0.48-0.04}) \times 10^{-14}$
2-a	$\overline{D}_u^0\overline{K}^0$	$(7.15^{+0.01+4.04+0.17}_{-0.01-1.98-0.16}) \times 10^{-11}$	$(1.16^{+0.01+0.65+0.03}_{-0.01-0.32-0.03}) \times 10^{-12}$	$(5.96^{+0.01+3.37+0.14}_{-0.01-1.65-0.14}) \times 10^{-12}$
	$\overline{D}_u^0\eta$	$(5.35^{+0.03+3.02+0.54}_{-0.03-1.48-0.50}) \times 10^{-12}$	$(8.66^{+0.04+4.89+0.88}_{-0.04-2.40-0.82}) \times 10^{-14}$	$(4.55^{+0.02+2.57+0.46}_{-0.02-1.26-0.43}) \times 10^{-13}$
	$\overline{D}_u^0\eta'$	$(5.63^{+0.03+3.18+1.68}_{-0.03-1.56-1.46}) \times 10^{-13}$	$(7.66^{+0.04+4.32+2.28}_{-0.04-2.12-1.98}) \times 10^{-15}$	$(6.02^{+0.03+3.40+1.79}_{-0.03-1.67-1.56}) \times 10^{-14}$
1-a	$D_s^-\rho^+$	$(1.67^{+0.01+0.15+0.05}_{-0.01-0.09-0.05}) \times 10^{-9}$	$(5.28^{+0.01+0.50+0.15}_{-0.01-0.28-0.15}) \times 10^{-12}$	$(7.24^{+0.01+0.68+0.21}_{-0.01-0.38-0.21}) \times 10^{-11}$
1-b	$D_s^-K^{*+}$	$(9.59^{+0.05+0.89+0.46}_{-0.05-0.50-0.45}) \times 10^{-11}$	$(1.18^{+0.01+0.11+0.06}_{-0.01-0.06-0.06}) \times 10^{-13}$	$(3.47^{+0.02+0.33+0.17}_{-0.02-0.18-0.16}) \times 10^{-12}$
1-b	$D_d^-\rho^+$	$(8.99^{+0.05+0.83+0.26}_{-0.05-0.47-0.26}) \times 10^{-11}$	$(4.32^{+0.02+0.41+0.12}_{-0.02-0.23-0.12}) \times 10^{-13}$	$(4.13^{+0.02+0.39+0.12}_{-0.02-0.22-0.12}) \times 10^{-12}$
1-c	$D_d^-K^{*+}$	$(5.15^{+0.06+0.48+0.25}_{-0.05-0.27-0.24}) \times 10^{-12}$	$(1.38^{+0.01+0.13+0.07}_{-0.01-0.07-0.07}) \times 10^{-14}$	$(2.02^{+0.02+0.19+0.10}_{-0.02-0.11-0.10}) \times 10^{-13}$
2-b	$\overline{D}_u^0\rho^0$	$(4.36^{+0.02+2.44+0.15}_{-0.02-1.21-0.15}) \times 10^{-12}$	$(2.38^{+0.01+1.35+0.08}_{-0.01-0.66-0.08}) \times 10^{-14}$	$(2.24^{+0.01+1.27+0.08}_{-0.01-0.62-0.08}) \times 10^{-13}$
2-b	$\overline{D}_u^0\omega$	$(3.28^{+0.02+1.84+0.20}_{-0.02-0.91-0.19}) \times 10^{-12}$	$(1.74^{+0.01+0.98+0.11}_{-0.01-0.48-0.10}) \times 10^{-14}$	$(1.67^{+0.01+0.94+0.10}_{-0.01-0.46-0.10}) \times 10^{-13}$
2-b	$\overline{D}_u^0\phi$	$(9.40^{+0.05+5.28+0.52}_{-0.05-2.61-0.50}) \times 10^{-12}$	$(8.57^{+0.04+4.84+0.47}_{-0.04-2.38-0.45}) \times 10^{-15}$	$(3.28^{+0.02+1.85+0.18}_{-0.02-0.91-0.17}) \times 10^{-13}$
2-c	$\overline{D}_u^0K^{*0}$	$(5.09^{+0.05+2.86+0.31}_{-0.05-1.42-0.30}) \times 10^{-13}$	$(1.50^{+0.02+0.85+0.08}_{-0.02-0.42-0.08}) \times 10^{-15}$	$(2.18^{+0.02+1.23+0.12}_{-0.02-0.60-0.12}) \times 10^{-14}$
2-a	$\overline{D}_u^0\overline{K}^{*0}$	$(1.74^{+0.01+0.98+0.11}_{-0.01-0.49-0.10}) \times 10^{-10}$	$(5.20^{+0.01+2.94+0.29}_{-0.01-1.44-0.28}) \times 10^{-13}$	$(7.57^{+0.01+4.27+0.42}_{-0.01-2.10-0.40}) \times 10^{-12}$

Table 7: Classification of the nonleptonic charmonium weak decays.

Case	Parameter	CKM factor
1-a	a_1	$\lvert V_{ud}V_{cs}^*\rvert \sim 1$
1-b	a_1	$\lvert V_{ud}V_{cd}^*\rvert, \lvert V_{us}V_{cs}^*\rvert \sim \lambda$
1-c	a_1	$\lvert V_{us}V_{cd}^*\rvert \sim \lambda^2$
2-a	a_2	$\lvert V_{ud}V_{cs}^*\rvert \sim 1$
2-b	a_2	$\lvert V_{ud}V_{cd}^*\rvert, \lvert V_{us}V_{cs}^*\rvert \sim \lambda$
2-c	a_2	$\lvert V_{us}V_{cd}^*\rvert \sim \lambda^2$

(5) Compared with $\psi(1S, 2S) \to DV$ decays, the corresponding $\psi(1S, 2S) \to DP$ decays, where P and V have the same flavor structures, are suppressed by the orbital angular momentum and so have relatively small branching ratios. There are some approximative relations $\mathscr{B}r(J/\psi(1S) \to DV) \approx 3\mathscr{B}r(J/\psi(1S) \to DP)$ and $\mathscr{B}r(\psi(2S) \to DV) \approx 3\mathscr{B}r(\psi(2S) \to DP)$.

(6) According to the CKM factors and parameters $a_{1,2}$, nonleptonic charmonium weak decays could be subdivided into six cases (see Table 7). Case "i-a" is the Cabibbo-favored one, so it generally has large branching ratios relative to cases "i-b" and "i-c." The a_2-dominated charmonium weak decays are suppressed by a color factor relative to a_1-dominated ones. Hence, the charmonium weak decays into $D_s\rho$ and $D_s\pi$ final states belonging to case "1-a" usually have relatively large branching ratios; the charmonium weak decays into the $\overline{D}_u^0K^{*0}$ final states belonging to case "2-c" usually have relatively small branching ratios. In addition, the branching ratio of case "2-a" (or "2-b") is usually larger than that of case "1-b" (or "1-c") due to $|a_2/a_1| \geq \lambda$.

(7) Branching ratios for the Cabibbo-favored $\psi(1S, 2S) \to D_s^-\rho^+$, $D_s^-\pi^+$, $\overline{D}_u^0\overline{K}^{*0}$ decays can reach up to 10^{-10}, which might be measurable in the forthcoming days. For example, $J/\psi(1S)$ production cross section can reach up to a few μb with the LHCb and ALICE detectors at LHC [7, 8]. Therefore, over 10^{12} $J/\psi(1S)$ samples are in principle available per 100 fb^{-1} data collected by LHCb and ALICE, corresponding to a few tens of $J/\psi(1S) \to D_s^-\rho^+$, $D_s^-\pi^+$, $\overline{D}_u^0\overline{K}^{*0}$ events for about 10% reconstruction efficiency.

(8) There is a large cancellation between the CKM factors $V_{ud}V_{cd}^*$ and $V_{us}V_{cs}^*$, which results in a very small branching ratio for charmonium weak decays into $D_u\eta'$ state.

(9) There are many uncertainties in our results. The first uncertainty from the CKM factors is small due to high precision on the Wolfenstein parameter λ with only 0.3% relative errors now [12]. The second uncertainty from the renormalization scale μ could, in principle, be reduced by the inclusion of higher order α_s corrections. For example, it has been shown [34] that tree amplitudes incorporating with the NNLO corrections are relatively less sensitive to the renormalization scale than the NLO amplitudes. The third uncertainty comes from hadronic parameters, which is expected to be cancelled or reduced with the relative ratio of branching ratios.

(10) The numbers in Tables 5 and 6 just provide an order of magnitude estimate. Many other factors, such as the final state interactions and q^2 dependence of form factors, which are not considered here, deserve many dedicated studies.

4. Summary

With the anticipation of abundant data samples on charmonium at high-luminosity heavy-flavor experiments, we studied the nonleptonic two-body $\psi(1S,2S)$ and $\eta_c(1S,2S)$ weak decays into one ground-state charmed meson plus one ground-state light meson based on the low energy effective Hamiltonian. By considering QCD radiative corrections to hadronic matrix elements of tree operators, we got the effective coefficients $a_{1,2}$ containing partial information of strong phases. The magnitude of $a_{1,2}$ agrees comfortably with those used in previous works [14–17]. The transition form factors between the charmonium and charmed meson are calculated by using the nonrelativistic wave functions with isotropic harmonic oscillator potential. Branching ratios for $\psi(1S,2S)$, $\eta_c(1S,2S) \to DM$ decays are estimated roughly. It is found that the Cabibbo-favored $\psi(1S,2S) \to D_s^-\rho^+, D_s^-\pi^+, \overline{D}_u^0\overline{K}^{*0}$ decays have large branching ratios $\gtrsim 10^{-10}$, which are promisingly detected in the forthcoming years.

Appendices

A. The Amplitudes for $\psi \longrightarrow DM$ Decays

Consider

$$\mathscr{A}\left(\psi \longrightarrow D_s^-\pi^+\right) = \sqrt{2}G_F m_\psi \left(\epsilon_\psi \cdot p_\pi\right) \cdot f_\pi A_0^{\psi\to D_s} V_{cs}^* V_{ud} a_1,$$

$$\mathscr{A}\left(\psi \longrightarrow D_s^-K^+\right) = \sqrt{2}G_F m_\psi \left(\epsilon_\psi \cdot p_K\right) \cdot f_K A_0^{\psi\to D_s} V_{cs}^* V_{us} a_1,$$

$$\mathscr{A}\left(\psi \longrightarrow D_d^-\pi^+\right) = \sqrt{2}G_F m_\psi \left(\epsilon_\psi \cdot p_\pi\right) \cdot f_\pi A_0^{\psi\to D_d} V_{cd}^* V_{ud} a_1,$$

$$\mathscr{A}\left(\psi \longrightarrow D_d^-K^+\right) = \sqrt{2}G_F m_\psi \left(\epsilon_\psi \cdot p_K\right) \cdot f_K A_0^{\psi\to D_d} V_{cd}^* V_{us} a_1,$$

$$\mathscr{A}\left(\psi \longrightarrow \overline{D}_u^0\pi^0\right) = -G_F m_\psi \left(\epsilon_\psi \cdot p_\pi\right) \cdot f_\pi A_0^{\psi\to D_u} V_{cd}^* V_{ud} a_2,$$

$$\mathscr{A}\left(\psi \longrightarrow \overline{D}_u^0K^0\right) = \sqrt{2}G_F m_\psi \left(\epsilon_\psi \cdot p_K\right) \cdot f_K A_0^{\psi\to D_u} V_{cd}^* V_{us} a_2,$$

$$\mathscr{A}\left(\psi \longrightarrow \overline{D}_u^0\overline{K}^0\right) = \sqrt{2}G_F m_\psi \left(\epsilon_\psi \cdot p_K\right) \cdot f_K A_0^{\psi\to D_u} V_{cs}^* V_{ud} a_2,$$

$$\mathscr{A}\left(\psi \longrightarrow \overline{D}_u^0\eta_q\right) = G_F m_\psi \left(\epsilon_\psi \cdot p_{\eta_q}\right) \cdot f_{\eta_q} A_0^{\psi\to D_u} V_{cd}^* V_{ud} a_2,$$

$$\mathscr{A}\left(\psi \longrightarrow \overline{D}_u^0\eta_s\right) = \sqrt{2}G_F m_\psi \left(\epsilon_\psi \cdot p_{\eta_s}\right) \cdot f_{\eta_s} A_0^{\psi\to D_u} V_{cs}^* V_{us} a_2,$$

$$\mathscr{A}\left(\psi \longrightarrow \overline{D}_u^0\eta\right) = \cos\phi\,\mathscr{A}\left(\psi \longrightarrow \overline{D}_u^0\eta_q\right) - \sin\phi \cdot \mathscr{A}\left(\psi \longrightarrow \overline{D}_u^0\eta_s\right),$$

$$\mathscr{A}\left(\psi \longrightarrow \overline{D}_u^0\eta'\right) = \sin\phi\,\mathscr{A}\left(\psi \longrightarrow \overline{D}_u^0\eta_q\right) + \cos\phi \cdot \mathscr{A}\left(\psi \longrightarrow \overline{D}_u^0\eta_s\right),$$

$$\mathscr{A}\left(\psi \longrightarrow D_s^-\rho^+\right) = -i\frac{G_F}{\sqrt{2}} f_\rho m_\rho V_{cs}^* V_{ud} a_1 \left\{\left(\epsilon_\rho^* \cdot \epsilon_\psi\right) \cdot \left(m_\psi + m_{D_s}\right) A_1^{\psi\to D_s} + \left(\epsilon_\rho^* \cdot p_\psi\right)\left(\epsilon_\psi \cdot p_\rho\right) \cdot \frac{2A_2^{\psi\to D_s}}{m_\psi + m_{D_s}} - i\epsilon_{\mu\nu\alpha\beta}\epsilon_\rho^{*\mu}\epsilon_\psi^\nu p_\rho^\alpha p_\psi^\beta \frac{2V^{\psi\to D_s}}{m_\psi + m_{D_s}}\right\},$$

$$\mathscr{A}\left(\psi \longrightarrow D_s^-K^{*+}\right) = -i\frac{G_F}{\sqrt{2}} \cdot f_{K^*} m_{K^*} V_{cs}^* V_{us} a_1 \left\{\left(\epsilon_{K^*}^* \cdot \epsilon_\psi\right)\left(m_\psi + m_{D_s}\right) A_1^{\psi\to D_s} + \left(\epsilon_{K^*}^* \cdot p_\psi\right)\left(\epsilon_\psi \cdot p_{K^*}\right) \frac{2A_2^{\psi\to D_s}}{m_\psi + m_{D_s}} - i\epsilon_{\mu\nu\alpha\beta}\epsilon_{K^*}^{*\mu}\epsilon_\psi^\nu p_{K^*}^\alpha p_\psi^\beta \frac{2V^{\psi\to D_s}}{m_\psi + m_{D_s}}\right\},$$

$$\mathscr{A}\left(\psi \longrightarrow D_d^-\rho^+\right) = -i\frac{G_F}{\sqrt{2}} f_\rho m_\rho V_{cd}^* V_{ud} a_1 \left\{\left(\epsilon_\rho^* \cdot \epsilon_\psi\right) \cdot \left(m_\psi + m_{D_d}\right) A_1^{\psi\to D_d} + \left(\epsilon_\rho^* \cdot p_\psi\right)\left(\epsilon_\psi \cdot p_\rho\right) \cdot \frac{2A_2^{\psi\to D_d}}{m_\psi + m_{D_d}} - i\epsilon_{\mu\nu\alpha\beta}\epsilon_\rho^{*\mu}\epsilon_\psi^\nu p_\rho^\alpha p_\psi^\beta \frac{2V^{\psi\to D_d}}{m_\psi + m_{D_d}}\right\},$$

$$\mathscr{A}\left(\psi \longrightarrow D_d^-K^{*+}\right) = -i\frac{G_F}{\sqrt{2}} \cdot f_{K^*} m_{K^*} V_{cd}^* V_{us} a_1 \left\{\left(\epsilon_{K^*}^* \cdot \epsilon_\psi\right)\left(m_\psi + m_{D_d}\right) A_1^{\psi\to D_d} + \left(\epsilon_{K^*}^* \cdot p_\psi\right)\left(\epsilon_\psi \cdot p_{K^*}\right) \frac{2A_2^{\psi\to D_d}}{m_\psi + m_{D_d}} - i\epsilon_{\mu\nu\alpha\beta}\epsilon_{K^*}^{*\mu}\epsilon_\psi^\nu p_{K^*}^\alpha p_\psi^\beta \frac{2V^{\psi\to D_d}}{m_\psi + m_{D_d}}\right\},$$

$$\mathscr{A}\left(\psi \longrightarrow \overline{D}_u^0 \rho^0\right) = +i\frac{G_F}{2} f_\rho m_\rho V_{cd}^* V_{ud} a_2 \left\{\left(\epsilon_\rho^* \cdot \epsilon_\psi\right) \cdot \left(m_\psi + m_{D_u}\right) A_1^{\psi\to D_u} + \left(\epsilon_\rho^* \cdot p_\psi\right)\left(\epsilon_\psi \cdot p_\rho\right) \cdot \frac{2A_2^{\psi\to D_u}}{m_\psi + m_{D_u}} - i\epsilon_{\mu\nu\alpha\beta}\epsilon_\rho^{*\mu}\epsilon_\psi^\nu p_\rho^\alpha p_\psi^\beta \frac{2V^{\psi\to D_u}}{m_\psi + m_{D_u}}\right\},$$

$$\mathscr{A}\left(\psi \longrightarrow \overline{D}_u^0 \omega\right) = -i\frac{G_F}{2} f_\omega m_\omega V_{cd}^* V_{ud} a_2 \left\{\left(\epsilon_\omega^* \cdot \epsilon_\psi\right) \cdot \left(m_\psi + m_{D_u}\right) A_1^{\psi\to D_u} + \left(\epsilon_\omega^* \cdot p_\psi\right)\left(\epsilon_\psi \cdot p_\omega\right) \cdot \frac{2A_2^{\psi\to D_u}}{m_\psi + m_{D_u}} - i\epsilon_{\mu\nu\alpha\beta}\epsilon_\omega^{*\mu}\epsilon_\psi^\nu p_\omega^\alpha p_\psi^\beta \frac{2V^{\psi\to D_u}}{m_\psi + m_{D_u}}\right\},$$

$$\mathscr{A}\left(\psi \longrightarrow \overline{D}_u^0 \phi\right) = -i\frac{G_F}{\sqrt{2}} f_\phi m_\phi V_{cs}^* V_{us} a_2 \left\{\left(\epsilon_\phi^* \cdot \epsilon_\psi\right) \cdot \left(m_\psi + m_{D_u}\right) A_1^{\psi\to D_u} + \left(\epsilon_\phi^* \cdot p_\psi\right)\left(\epsilon_\psi \cdot p_\phi\right) \cdot \frac{2A_2^{\psi\to D_u}}{m_\psi + m_{D_u}} - i\epsilon_{\mu\nu\alpha\beta}\epsilon_\phi^{*\mu}\epsilon_\psi^\nu p_\phi^\alpha p_\psi^\beta \frac{2V^{\psi\to D_u}}{m_\psi + m_{D_u}}\right\},$$

$$\mathscr{A}\left(\psi \longrightarrow \overline{D}_u^0 K^{*0}\right) = -i\frac{G_F}{\sqrt{2}} \cdot f_{K^*} m_{K^*} V_{cd}^* V_{us} a_2 \left\{\left(\epsilon_{K^*}^* \cdot \epsilon_\psi\right)\left(m_\psi + m_{D_u}\right) A_1^{\psi\to D_u} + \left(\epsilon_{K^*}^* \cdot p_\psi\right)\left(\epsilon_\psi \cdot p_{K^*}\right) \frac{2A_2^{\psi\to D_u}}{m_\psi + m_{D_u}} - i\epsilon_{\mu\nu\alpha\beta}\epsilon_{K^*}^{*\mu}\epsilon_\psi^\nu p_{K^*}^\alpha p_\psi^\beta \frac{2V^{\psi\to D_u}}{m_\psi + m_{D_u}}\right\},$$

$$\mathscr{A}\left(\psi \longrightarrow \overline{D}_u^0 \overline{K}^{*0}\right) = -i\frac{G_F}{\sqrt{2}} \cdot f_{K^*} m_{K^*} V_{cs}^* V_{ud} a_2 \left\{\left(\epsilon_{K^*}^* \cdot \epsilon_\psi\right)\left(m_\psi + m_{D_u}\right) A_1^{\psi\to D_u} + \left(\epsilon_{K^*}^* \cdot p_\psi\right)\left(\epsilon_\psi \cdot p_{K^*}\right) \frac{2A_2^{\psi\to D_u}}{m_\psi + m_{D_u}} - i\epsilon_{\mu\nu\alpha\beta}\epsilon_{K^*}^{*\mu}\epsilon_\psi^\nu p_{K^*}^\alpha p_\psi^\beta \frac{2V^{\psi\to D_u}}{m_\psi + m_{D_u}}\right\}. \tag{A.1}$$

B. The Amplitudes for the $\eta_c \longrightarrow DM$ Decays

Consider

$$\mathscr{A}\left(\eta_c \longrightarrow D_s^- \pi^+\right) = i\frac{G_F}{\sqrt{2}}\left(m_{\eta_c}^2 - m_{D_s}^2\right) f_\pi F_0^{\eta_c\to D_s} V_{ud} V_{cs}^* a_1,$$

$$\mathscr{A}\left(\eta_c \longrightarrow D_s^- K^+\right) = i\frac{G_F}{\sqrt{2}}\left(m_{\eta_c}^2 - m_{D_s}^2\right) f_K F_0^{\eta_c\to D_s} V_{us} V_{cs}^* a_1,$$

$$\mathscr{A}\left(\eta_c \longrightarrow D_d^- \pi^+\right) = i\frac{G_F}{\sqrt{2}}\left(m_{\eta_c}^2 - m_{D_d}^2\right) f_\pi F_0^{\eta_c\to D_d} V_{ud} V_{cd}^* a_1,$$

$$\mathscr{A}\left(\eta_c \longrightarrow D_d^- K^+\right) = i\frac{G_F}{\sqrt{2}}\left(m_{\eta_c}^2 - m_{D_d}^2\right) f_K F_0^{\eta_c\to D_d} V_{us} V_{cd}^* a_1,$$

$$\mathscr{A}\left(\eta_c \longrightarrow \overline{D}_u^0 \pi^0\right) = -i\frac{G_F}{2}\left(m_{\eta_c}^2 - m_{D_u}^2\right) f_\pi F_0^{\eta_c\to D_u} V_{ud} V_{cd}^* a_2,$$

$$\mathscr{A}\left(\eta_c \longrightarrow \overline{D}_u^0 K^0\right) = i\frac{G_F}{\sqrt{2}}\left(m_{\eta_c}^2 - m_{D_u}^2\right) f_K F_0^{\eta_c\to D_u} V_{us} V_{cd}^* a_2,$$

$$\mathscr{A}\left(\eta_c \longrightarrow \overline{D}_u^0 \overline{K}^0\right) = i\frac{G_F}{\sqrt{2}}\left(m_{\eta_c}^2 - m_{D_u}^2\right) f_K F_0^{\eta_c\to D_u} V_{ud} V_{cs}^* a_2,$$

$$\mathscr{A}\left(\eta_c \longrightarrow \overline{D}_u^0 \eta_q\right) = i\frac{G_F}{2}\left(m_{\eta_c}^2 - m_{D_u}^2\right) f_{\eta_q} F_0^{\eta_c\to D_u} V_{ud} V_{cd}^* a_2,$$

$$\mathscr{A}\left(\eta_c \longrightarrow \overline{D}_u^0 \eta_s\right) = i\frac{G_F}{\sqrt{2}}\left(m_{\eta_c}^2 - m_{D_u}^2\right) f_{\eta_s} F_0^{\eta_c\to D_u} V_{us} V_{cs}^* a_2,$$

$$\mathscr{A}\left(\eta_c \longrightarrow \overline{D}_u^0 \eta\right) = \cos\phi\,\mathscr{A}\left(\eta_c \longrightarrow \overline{D}_u^0 \eta_q\right) - \sin\phi\,\mathscr{A}\left(\eta_c \longrightarrow \overline{D}_u^0 \eta_s\right),$$

$$\mathscr{A}\left(\eta_c \longrightarrow \overline{D}_u^0 \eta'\right) = \sin\phi\,\mathscr{A}\left(\eta_c \longrightarrow \overline{D}_u^0 \eta_q\right) + \cos\phi\,\mathscr{A}\left(\eta_c \longrightarrow \overline{D}_u^0 \eta_s\right),$$

$$\mathscr{A}\left(\eta_c \longrightarrow D_s^- \rho^+\right) = \sqrt{2} G_F m_\rho \left(\epsilon_\rho^* \cdot p_{\eta_c}\right) f_\rho F_1^{\eta_c\to D_s} V_{ud} V_{cs}^* a_1,$$

$$\mathscr{A}\left(\eta_c \longrightarrow D_s^- K^{*+}\right) = \sqrt{2} G_F m_{K^*} \left(\epsilon_{K^*}^* \cdot p_{\eta_c}\right) f_{K^*} F_1^{\eta_c\to D_s} V_{us} V_{cs}^* a_1,$$

$$\begin{aligned}
&\mathscr{A}\left(\eta_c \longrightarrow D_d^- \rho^+\right) \\
&\quad = \sqrt{2} G_F m_\rho \left(\epsilon_\rho^* \cdot p_{\eta_c}\right) f_\rho F_1^{\eta_c \to D_d} V_{ud} V_{cd}^* a_1, \\
&\mathscr{A}\left(\eta_c \longrightarrow D_d^- K^{*+}\right) \\
&\quad = \sqrt{2} G_F m_{K^*} \left(\epsilon_{K^*}^* \cdot p_{\eta_c}\right) f_{K^*} F_1^{\eta_c \to D_d} V_{us} V_{cd}^* a_1, \\
&\mathscr{A}\left(\eta_c \longrightarrow \overline{D}_u^0 \rho^0\right) \\
&\quad = -G_F m_\rho \left(\epsilon_\rho^* \cdot p_{\eta_c}\right) f_\rho F_1^{\eta_c \to D_u} V_{ud} V_{cd}^* a_2, \\
&\mathscr{A}\left(\eta_c \longrightarrow \overline{D}_u^0 \omega\right) \\
&\quad = G_F m_\omega \left(\epsilon_\omega^* \cdot p_{\eta_c}\right) f_\omega F_1^{\eta_c \to D_u} V_{ud} V_{cd}^* a_2, \\
&\mathscr{A}\left(\eta_c \longrightarrow \overline{D}_u^0 \phi\right) \\
&\quad = \sqrt{2} G_F m_\phi \left(\epsilon_\phi^* \cdot p_{\eta_c}\right) f_\phi F_1^{\eta_c \to D_u} V_{us} V_{cs}^* a_2, \\
&\mathscr{A}\left(\eta_c \longrightarrow \overline{D}_u^0 K^{*0}\right) \\
&\quad = \sqrt{2} G_F m_{K^*} \left(\epsilon_{K^*}^* \cdot p_{\eta_c}\right) f_{K^*} F_1^{\eta_c \to D_u} V_{us} V_{cd}^* a_2, \\
&\mathscr{A}\left(\eta_c \longrightarrow \overline{D}_u^0 \overline{K}^{*0}\right) \\
&\quad = \sqrt{2} G_F m_{K^*} \left(\epsilon_{K^*}^* \cdot p_{\eta_c}\right) f_{K^*} F_1^{\eta_c \to D_u} V_{ud} V_{cs}^* a_2.
\end{aligned} \tag{B.1}$$

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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