Singlet Fermion Assisted Dominant Seesaw with Lepton Flavor and Number Violations and Leptogenesis

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Embedding type I seesaw in GUTs, left-right gauge theories, or even in extensions of the SM requires large right-handed neutrino masses making the neutrino mass generation mechanism inaccessible for direct experimental tests. This has been circumvented by introducing additional textures or high degree of fine-tuning in the Dirac neutrino or right-handed neutrino mass matrices. In this work we review another new mechanism that renders type I seesaw vanishing but other seesaw mechanisms dominant. Such mechanisms include extended seesaw, type II, linear, or double seesaw. The linear seesaw, double seesaw, and extended seesaw are directly verifiable at TeV scale. New observable predictions for lepton flavor and lepton number violations by ongoing searches are noted. Type II embedding in \( SO(10) \) also predicts these phenomena in addition to new mechanism for leptogenesis and displaced vertices mediated by gauge singlet fermions.

1. Introduction

Although the standard model (SM) of strong, weak, and electromagnetic interactions has enjoyed tremendous success through numerous experimental tests, it has outstanding failures in three notable areas: neutrino masses, baryon asymmetry of the universe, and dark matter. Two of the other conceptual and theoretical difficulties are that the SM cannot explain disparate values of its gauge couplings nor can it account for the monopoly of parity violation in weak interaction leaving out strong, electromagnetic, and gravitational interactions. In hitherto unverified extensions of the SM in the scalar, fermion, or gauge sectors, however, there are different theories for neutrino masses [1–11], which mainly exploit various seesaw mechanisms [9, 11–48].

The most popular method of neutrino mass generation has been through type I or canonical seesaw mechanism [12–17] which was noted to apply in the simplest extension of the SM through right-handed (RH) neutrinos encompassing family mixings [17]. Most of the problems of the SM have potentially satisfactory solutions in the minimal left-right symmetric (LRS) [21, 49–52] grand unified theory based on \( SO(10) \) [53–70]. Although the neutrino masses measured by the oscillation data [71–74] are most simply accommodated in \( SO(10) \) if both the left-handed (LH) and the right-handed (RH) neutrinos are Majorana fermions, alternative interpretations in favour of Dirac neutrino masses within the standard model paradigm have been also advanced [11, 75–78]. Currently a number of experiments [79–84] on neutrinoless double beta (0\( \beta\beta \)) decay are in progress to resolve the issue on the Dirac or Majorana nature of neutrinos [21–23].

Another set of physical processes under active experimental investigation are charged lepton flavor violating (LFV) decays, \( \tau \rightarrow e\gamma \), \( \tau \rightarrow \mu\gamma \), \( \mu \rightarrow e\gamma \), and \( \mu \rightarrow eee \), where the minimally extended SM embracing small neutrino masses and GIM mechanism predict branching ratios many orders smaller than their current experimental limits [85–96]. However supersymmetric theories possess high potential to explain LFV decays closer to the current limits [97–104]. A special feature of left-right gauge theories [49–52] and \( SO(10) \) grand unified theory (GUT) [53–60] is that
the canonical seesaw formula [12–17] for Majorana neutrino masses is invariably accompanied by the type II seesaw formula [18–22]

\[ M_s = m^2 + m^1 = f v_L - M_D \frac{1}{M_N} M^T_D \]  

(1)

The parameters entering into this hybrid seesaw formula have fundamentally appealing interpretations in Pati-Salam model [49, 50] or SO(10) GUT. In (1) \( M_D(M_N) \) is Dirac (RH Majorana) neutrino mass, \( v_L \sim (\lambda v^a_W g_\nu^T m_\nu^2) \) is the induced vacuum expectation value (VEV) of the LH triplet \( \Delta_L, V_\nu = SU(2)_L \times U(1)_{B-L} \) breaking VEV of the RH triplet \( \Delta_R \), and \( f \) is the Yukawa coupling of the triplets \( \langle \frac{1}{\sqrt{2}} \rangle \) of SO(10). The same Yukawa coupling \( f \) also defines the RH neutrino mass \( M_N = f V_\nu \). Normally, because of the underlying quark-lepton symmetry in SO(10) or Pati-Salam model, \( M_D \) is of the same order as \( M_N \), the up quark mass matrix, that drives the canonical seesaw scale to be large, \( M_N \sim 10^{14} \text{GeV} \). In the LR theory based upon \( SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C \) \((\approx G_{2213})\), \( M_D \sim M_L \) is charged lepton mass matrix. The neutrino oscillation data then pushes this seesaw scale to \( M_N \sim 10^{18} \text{GeV} \). Similarly the type II seesaw scale is also around this mass. With such high seesaw scales in nonsupersymmetric (non-SUSY) SO(10) model or LRS theory, there is no possibility of direct experimental verification of the seesaw mechanism or the associated \( W_R \) boson mass in near future. Likewise, the predicted LFV decay rates are far below the experimental limits.

On the other hand, if experimental investigations at the Large Hadron Collider (LHC) [105, 106] are to confirm [107] TeV scale \( W_R \) [9, 108–111] or \( Z_R \) boson production [112, 113] accompanied by RH Majorana neutrinos in the like-sign dilepton channel with jets, \( pp \rightarrow l^+ l^- jjX \), this should be also consistent with the neutrino oscillation data [71–74] through seesaw mechanisms. This is possible if the relevant seesaw scales are brought down to \( \sim (1–10) \text{TeV} \) (in the RH neutrino extension of the SM, explanation of neutrino oscillation data with baryon asymmetry of the universe has been addressed for \( \mathcal{O}(1–10) \text{GeV} \) type I seesaw scale [114–116]; interesting connections with double beta decay and dilepton production with displaced vertices at LHC have been also discussed with such low canonical seesaw scales [117] and the models need fine-tuning of the \( v \sim N - \phi \) Yukawa coupling \( y \sim 10^{-8} \) which is about 3 orders smaller than the electron Yukawa coupling; neutrino mass generation with standard model paradigm and their interesting applications have been discussed in a recent review [111]). Observable displaced vertices within SM extension have been also discussed in [118, 119] and references therein.

The scope and applications of type I seesaw to TeV scale \( W_R \) boson models have been discussed in the recent interesting review [9]. In such models D-Parity is at first broken at high scale that makes the left-handed triplet much heavier than the \( W_R \)-mass but keeps \( G_{2213}(g_{2L} \neq g_{2R}) \) unbroken down to much lower scale [55–57]. This causes the type II seesaw contribution of the hybrid seesaw formula of (1) to be severely damped out in the LHC scale \( W_R \) models where type I seesaw dominates. But because \( M_N \) is also at the TeV scale, the predicted type I seesaw contribution to light neutrino mass turns out to be \( 10^6–10^{11} \) times larger than the experimental values unless it is adequately suppressed while maintaining its dominance over type II seesaw. Such suppressions have been made possible in two ways: (i) using fine-tuned values of the Dirac neutrino mass matrices \( M_D \) [9, 117, 120–122] and (ii) introducing specific textures to the fermion mass matrices \( M_D \) and/or \( M_N \) [98, 108, 123–136].

Possible presence of specific textures of constituent matrices in the context of inverse, linear, or type I seesaw models has been also explored [137–145]. More recently the hybrid seesaw ansatz for matter parity conserving SO(10) has been applied to explain neutrino masses, dark matter, and baryon asymmetry of the universe without invoking any texture or intermediate scale in the nonsupersymmetric SO(10) framework [146].

Even without going beyond the SM paradigm and treating the added RH neutrinos in type I seesaw as gauge singlet fermions at \( \sim \text{GeV} \) scale, rich structure of new physics has been predicted including neutrino masses, dark matter, and baryon asymmetry of the universe. The fine-tuned value of the associated Dirac neutrino Yukawa coupling in these models is \( y \sim 10^{-7} \) [114–116].

There are physical situations where type II seesaw dominance, rather than type I seesaw or inverse seesaw, is desirable [17–23, 112, 147–162].

In the minimal case, being a mechanism driven by intermediate scale mass of LH triplet, type II seesaw may not be directly verifiable; nevertheless it can be clearly applicable to TeV scale \( Z_R \) models in non-SUSY SO(10) to account for neutrino masses [112] provided type I contribution is adequately suppressed. However, as in the fine-tuning of Dirac neutrino mass in the type I seesaw case in LR models, the induced VEV needed for type II seesaw can also be fine-tuned using more than one electroweak bidoublets reducing the triplet mass to lower scales accessible to accelerator tests. Looking to (1) and the structure of the induced VEV \( v_L \), the most convenient method of suppressing type I seesaw with respect to type II seesaw is to make the type I seesaw scale \( M_N = f V_\nu \) larger and the triplet mass much smaller, \( M_{\Delta_R} \ll M_N \). This requires the \( SU(2)_L \times U(1)_{B-L} \) breaking scale or \( M_{\Delta_R} \gg M_{\Delta_L} \) SUSY and non-SUSY SO(10) models have been constructed with this possibility and also in the case of split-SUSY [157, 160] where \( M_{\Delta_R} = 10^{17} \text{GeV} \). Obviously such models have no relevance in the context of TeV scale \( W_R \) or \( Z_R \) bosons accessible to LHC searches.

Whereas the pristine type I or type II seesaw are essentially high scale formulas inaccessible for direct verification and need fine-tuning or textures to bring them down to the TeV scale, the well known classic inverse seesaw mechanism [24] which has been also discussed by a number of authors [25–34] is essentially TeV scale seesaw. It has the high potential to be directly verifiable at accelerator energies and also by ongoing experiments on charged lepton flavor violations [85–96].

Even without taking recourse to string theories, in addition to the three RH neutrinos \( (N_i, i = 1, 2, 3) \), one more gauge singlet fermion per generation \( (S_i, i = 1, 2, 3) \) is added...
to the SM where the Lagrangian contains the $N$-$S$ mixing mass term $M$. The heavy Majorana mass term is absent for RH neutrinos which turn out to be heavy pseudo Dirac fermions. The introduction of the global lepton symmetry breaking term in the Lagrangian, $\mu_S S^2 S$, gives rise to the well known classic inverse seesaw formula [24]

$$m_\nu = \frac{M_D}{M} \mu_S \left(\frac{M_D}{M}\right)^T. \quad (2)$$

Naturally small value of $\mu_S$ in the 't Hooft sense [163] brings down the inverse seesaw mechanism to the TeV scale without having the need to fine tune the associated Dirac mass matrix or Yukawa couplings. The presence of texture zeros in the constituent matrices of different types of seesaw formulas has been investigated consistently with neutrino oscillation data [138–140, 142, 143].

Recently models have been discussed using TeV scale heavy pseudo Dirac neutrinos [109, 164–168] where dominant RH Majorana mass term $M_N N N$ is either absent in the Lagrangian or negligible. In a contrasting situation [37, 38, 43], when $\mu_S \sim M_{\text{Planck}}$, the seesaw scale $M \sim M_{\text{GUT}}$, and the model avoids the presence of any additional intermediate symmetry. While operating with the SM paradigm, it also dispenses with the larger Higgs representation $126_{H} \subset SO(10)$ in favour of much smaller one $16_{H}$ leading to the double seesaw. The heavy RH neutrinos with $M_{N} \sim 10^{15}$ GeV turn out to be Majorana fermions instead of being pseudo Dirac. While this is an attractive scenario in SUSY SO(10) [37, 38], the coupling unification is challenging in the non-SUSY SO(10).

As discussed above if type I seesaw is the neutrino mass mechanism at the TeV scale, it must be appropriately suppressed either by fine-tuning or by introducing textures to the relevant mass matrices [9]. On the other hand if type II seesaw dominance in LR models or SO(10) is to account for neutrino masses, $W_R, Z_R$ boson mass must be at the GUT-Planck scale in the prevailing dominance mechanisms [157, 160].

In view of this, it would be quite interesting to explore, especially in the context of nonsupersymmetric SO(10), possible new physics implications when the would-be dominant type I seesaw cancels out exactly and analytically from the light neutrino mass matrix even without needing any fine-tuning or fermion mass textures in $M_D$ and/or $M_N$. The complete cancellation of type I seesaw in the presence of heavy RH Majorana mass term $M_{\nu} N N$ was explicitly proved in [45, 46] in the context of SM extension when both $N_i$ and $S_i$ are present manifesting in heavy RH neutrinos and lighter singlet fermions. We call this as gauge singlet fermion assisted extended seesaw dominance mechanism. Since then the mechanism has been utilised in explaining baryon asymmetry of the universe via low scale leptogenesis [45, 46] and the phenomenon of dark matter (DM) [169] along with cosmic ray anomalies [170]. More recently this extended seesaw mechanism for neutrino masses in the SM extension has been exploited to explain the keV singlet fermion DM along with low scale leptogenesis [171].

In the context of LR intermediate scales in SUSY SO(10), this mechanism has been applied to study coupling unification and leptogenesis [172–174] under gravitino constraint. Application to non-SUSY LR theory originating from Pati-Salam model [175] and non-SUSY SO(10) with TeV scale $W_R, Z_R$ bosons have been made [176, 177] with the predictions of a number of experimentally testable physical phenomena by low-energy experiments and including the observed dilepton excess at LHC [110]. In these models the singlet fermion assisted type I seesaw cancellation mechanism operates and the extended seesaw (or inverse seesaw) formula dominates.

Following the standard lore in type II seesaw dominant models, the dominant double beta decay rate in the $W_L - W_L$ channel is expected to be dominated by the exchange of the LH Higgs triplet $\Delta_L$ carrying $|B - L| = 2$. As such the predicted decay rate tends to be negligible in the limit of larger Higgs triplet mass. But it has been shown quite recently [112] that the type II seesaw dominance can occur assisted by the gauge singlet fermion but with a phenomenal difference. Even for large LH triplet mass in such models that controls the type II seesaw formula for light neutrino masses and mixings, the double beta decay rate in the $W_L - W_L$ channel remains dominant as it is controlled by the light singlet fermion exchanges. Other attractive predictions are observable LFV decays, nonunitarity effects, and resonant leptogenesis mediated by TeV scale quasi-degenerate singlet fermions of the second and third generations. The model has been noted to have its origin in non-SUSY SO(10) [112]. All the three types of gauge singlet fermions in these models mentioned above are Majorana fermions on which we focus in this review.

This article is organised in the following manner. In Section 2 we explain how the Kang-Kim mechanism [45, 46] operates within the SM paradigm extended by singlet fermions. In Section 3 we show how a generalised neutral fermion mass matrix exists in the appropriate extensions of the SM, LR theory, or SO(10). In Section 4 we show emergence of the other dominant seesaw mechanism including the extended or inverse seesaw and type II seesaw and cancellation of type I seesaw. Predictions for LFV, CP violation, and nonunitarity effect are discussed in Section 5. Predictions on double beta decay mediated by light singlet fermions in the $W_L - W_L$ channel are discussed in Section 6 where we have given its mass limits from the existing experimental data. Applications to resonant leptogenesis mediated by TeV scale singlet fermions in MSSM and SUSY SO(10) are briefly discussed in Section 7. Singlet fermion assisted leptogenesis in non-SUSY SO(10) is discussed in Section 8. This work is summarized in Section 9 with conclusions.

### 2. Mechanism of Extended Seesaw Dominance in SM Extension

Using the explicit derivation of Kang and Kim [45, 46], here we discuss how the type I contribution completely cancels out paving the way for the dominance of extended seesaw mechanism. The SM is extended by introducing RH neutrinos $N_i$ ($i = 1, 2, 3$) and an additional set of fermion
singlets $S_i (i = 1, 2, 3)$, one for each generation. After electroweak symmetry breaking, the Yukawa Lagrangian in the charged lepton mass basis gives for the neutral fermions

$$\mathcal{L}_{\text{mass}} = \left( M_D \bar{N} + \frac{1}{2} M_N N^T N + M \bar{N} S + \text{h.c.} \right) + \mu_S S^T S, \tag{3}$$

where $M_D$ is the Dirac neutrino mass matrix which is equal to $Y (\phi), Y$ being the Yukawa matrix. This gives the $9 \times 9$ neutral fermion mass matrix on the $(\nu, N, S)$ basis as follows:

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & M_D & 0 \\ M_D^T & M_N & M^T \\ 0 & M & \mu_S \end{pmatrix}. \tag{4}$$

The type I seesaw cancellation leading to dominance of extended seesaw (or inverse seesaw) [24] proceeds in two steps: as $M_N \gg M \gg M_D, \mu_S$, it is legitimate to integrate out the RH $N_i$ fields at first leading to the corresponding effective Lagrangian

$$-\mathcal{L}_{\text{eff}} = \left( M_D \frac{1}{M_N} M_D^T \right) y_\alpha y_\beta + \left( M_L + M_D \frac{1}{M_N} M^T \right) S_m S_n + \mu_S S^T S. \tag{5}$$

Then diagonalisation of the $9 \times 9$ neutral fermion mass matrix including the result of $\mathcal{L}_{\text{eff}}$ gives conventional type I seesaw term and another of opposite sign leading to the cancellation. The light neutrino mass predicted is the same as in the inverse seesaw case given in (2).

It must be emphasized that the earlier realisations of the classic inverse seesaw formula [24] were possible [25–32, 34] with vanishing RH Majorana mass $M_N = 0$ in (12).

Under the similar condition in which the type I seesaw cancels out the Majorana mass $m_\nu$ of the sterile neutrino and its mixing angle $\theta_5$ with light neutrinos are governed by

$$m_\nu = \mu_S - M \frac{1}{M_N} M^T \sim - M \frac{1}{M_N} M^T, \tag{6}$$

$$\tan 2 \theta_5 = \frac{2 M_D}{M}. \tag{7}$$

As $\mu_S$ is naturally small, it is clear that type I seesaw now controls the gauge singlet fermion mass, although it has no role to play in determining the LH neutrino mass. These results have been shown to emerge [47, 109, 110, 112, 176, 177] from SO(10) with gauge fermion singlet extensions by following the explicit block diagonalisation procedure in two steps while safeguarding the hierarchy $M_N \gg M \gg M_D, \mu_S$ with the supplementary condition $\mu_S M_N < M^2$.

3. Generalised Neutral Fermion Mass Matrix

A left-right symmetric (LRS) gauge theory $G_{2213D} (g_{2L} = g_{2R})$ at higher scale ($\mu = M_{\nu}$) is known to lead to $\text{TeV}$ scale asymmetric LR gauge theory $G_{2213S} (g_{2L} \neq g_{2R})$ via D-Parity breaking [55–57]. This symmetry further breaks to the SM gauge symmetry by the VEV of the RH triplet $\Delta g(1, 3, -2, 1)$ leading to massive $W_R, Z_R$ bosons and RH neutrinos at the intermediate scale $M_R$. Instead of $G_{2213D} (g_{2L} = g_{2R})$ it is possible to start directly from SO(10) which has been discussed at length in a number of investigations that normally leads to the type I $\oplus$ type II hybrid seesaw formula. In the absence of additional sterile neutrinos, the neutral fermion mass is standard $6 \times 6$ form. Here we discuss how a generalised $9 \times 9$ neutral fermion mass matrix that emerges in the presence of additional singlet fermions contains the rudiments of various seesaw formulas. As noted in Section 1, the derivation of the minimal classic inverse seesaw mechanism [24] has been possible in theories with gauge singlet fermion extensions of the SM [25–34, 178]. Extensive applications of this mechanism have been discussed and reported in a number of recent reviews [11, 99–102, 179, 180]. Exploring possible effects on invisible Higgs decays [181–183], prediction of observable lepton flavor violation as a hallmark of the minimal classic inverse seesaw mechanism has attracted considerable attention earlier and during recent investigations [103, 184–189]. The effects of massive gauge singlet fermions have been found to be consistent with electroweak precision observables [190–192]. Earlier its impact on a class of left-right symmetric models has been examined [193–195]. Prospects of lepton flavor violation in the context of linear seesaw and dynamical left-right symmetric model have been also investigated earlier [178].

It is well known that 15 fermions of one generation plus a right-handed neutrino form the spinorial representation 16 of SO(10) grand unified theory [53, 54]. In addition to three generation of fermions 16, $(i = 1, 2, 3)$, we also include one SO(10)-singlet fermion per generation $S_i (i = 1, 2, 3)$. We note that such singlets under the LR gauge group or the SM can originate from the nonstandard fermion representations in SO(10) such as $45_c$ or $210_F$.

Under $G_{2213}$ symmetry the fermion and Higgs representations are as follows:

**Fermions**

$$Q_L = \begin{pmatrix} u \\ d \\ \eta \end{pmatrix}_L (2, 1, 1, 3),$$

$$Q_R = \begin{pmatrix} u \\ d \\ \eta \end{pmatrix}_R (1, 2, -1, 3^*),$$

$$L = \begin{pmatrix} l \\ \eta \end{pmatrix}_L (2, 1, -1, 1),$$

$$R = \begin{pmatrix} N_i \\ l \end{pmatrix}_R (1, 2, -1, 1),$$

$$S_i = (1, 1, -1, 1).$$
Higgs

\[ \phi = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^- \end{pmatrix} (2, 2, 0, 1), \]

\[ \Delta_L = \begin{pmatrix} \Delta_L^+ / \sqrt{2} & \Delta_L^{++} \\ \Delta_L^0 / \sqrt{2} & \Delta_L^{++} \end{pmatrix} (3, 1, -2, 1), \]

\[ \Delta_R = \begin{pmatrix} \Delta_R^+ / \sqrt{2} & \Delta_R^{++} \\ \Delta_R^0 / \sqrt{2} & \Delta_R^{++} \end{pmatrix} (1, 3, -2, 1), \]  

\[ \eta_0 = (1, 1, 1, 1), \]

where \( \eta_0 \) is a D-Parity odd singlet with transformation property \( \eta_0 \to -\eta_0 \) under \( L \to R \). When this singlet acquires VEV \( \langle \eta_0 \rangle \sim M_p \), D-Parity breaks along with the underlying left-right discrete symmetry but the asymmetric LR gauge theory proposed recently in the context of like-sign scales. The \( G \) acquires VEV \((1,1,0,1)\) under Pati-Salam symmetry.\( G \) can be broken further to the \( SU(2)_L \times SU(2)_R \times U(1) \) symmetry by the VEV of RH Higgs triplet \( \chi_\text{R} \).\( G \) can further break down to the SM directly by the VEV of RH Higgs triplet \( \Delta \subset SU(10) \). The D-Parity odd (even) singlets \( \eta_0 \) were found to occur naturally in \( SO(10) \) GUT theory [55–57].

Designating the quantum numbers of submultiplets under Pati-Salam symmetry \( SU(2)_L \times SU(2)_R \times SU(4)_C \) (=\( G_{223} \)), the submultiplet \( (1,1,1) \subset 210 \subset SO(10) \) is \( \eta_0 \) whereas the submultiplet \( (1,1,1) \subset 54 \subset SO(10) \) is \( \eta_0 \). Likewise the neutral component of the submultiplet \( (1,1,15) \subset 45 \subset SO(10) \) behaves as \( \eta_0 \), but that in \( (1,1,15) \subset 210 \subset SO(10) \) behaves as \( \eta_0 \). Thus the GUT scale symmetry breaking \( SO(10) \to G_{224D} \) can occur by the VEV of \( 54_\text{H} \) in the direction \( \eta_0 \sim M_{\text{GUT}} \), so \( SO(10) \to G_{224} \) can occur by the VEV of \( 210_\text{H} \) in the direction \( \eta_0 \sim M_{\text{GUT}} \). Likewise \( SO(10) \to G_{2213D} \) can occur by the VEV of the neutral component \( (1,1,0,0)_H \subset 210_\text{H} \), and \( SO(10) \to G_{2213} \) can occur by the VEV of the neutral component \( (1,1,15,0)_H \subset 210_\text{H} \). As an example, one minimal chain with TeV scale LR gauge theory proposed recently in the context of like-sign dilepton signals observed at LHC is

\[ \begin{align*}
&SO(10) \xrightarrow{M_0 = M_p} SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C \ [G_{2213}] \\
&\quad \xrightarrow{M_0} SU(2)_L \times U(1)_{Y} \times SU(3)_C \ [SU(5)] \\
&\quad \xrightarrow{M_\Sigma} SU(3)_C \times U(1)_Q.
\end{align*} \]  

In this symmetry breaking pattern all LH triplets and doublets are near the GUT scale, but RH triplets or doublets are near the \( G_{2213} \) breaking intermediate scale \( M_\Sigma \) which could be \( \sim \) (few–100) TeV. Out of two minimal models with GUT scale D-Parity breaking satisfying the desired decoupling criteria \( M_\Sigma \gg M \gg M_D, \mu_5 \) [110, 112], dominance of extended seesaw in the presence of gauge singlet fermions has been possible in [110] with single \( G_{2213} \) intermediate scale corresponding to TeV scale \( W_R \) and \( Z_R \) bosons. The extended seesaw dominance in the presence of fermion singlets in \( SO(10) \) has been also realised including additional intermediate symmetries \( G_{2214D} \) and \( G_{224} \) where observable proton decay, TeV scale \( Z_R \) boson and RH Majorana neutrinos, observable proton decay, \( n - \bar{n} \) oscillation, and rare kaon decay have been predicted. Interestingly the masses of \( W_R \) boson and leptoquark gauge bosons of \( SU(4)_C \) have been predicted at \( \sim 100 \) TeV which could be accessible to planned LHC at those energies where \( W_R \) boson scale \( \sim (100–1000) \) TeV matching with observable \( n - \bar{n} \) oscillation and rare kaon decay have been predicted. But the heavy RH neutrino and \( Z_R \) boson scales being near TeV scale have been predicted to be accessible to LHC and planned accelerators [176, 177]. That non-SUSY GUTs with two-intermediate scales permit a low mass \( Z_R \) boson was noted much earlier [196].

In (9), instead of breaking directly to SM, the \( G_{2213} \) breaking may occur in two steps \( G_{22131} \to G_{2213} \) where \( G_{22131} \) represents the gauge symmetry \( SU(2)_L \times SU(3)_C \times U(1)_{Y} \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C \ [G_{22111}] \). This promises the interesting possibility of TeV scale \( Z_R \) boson with the constraint \( M_{\Sigma} \gg M_Z \). Thus the model can be discriminated from the direct LR models if \( Z_R \) boson is detected at lower mass scales than the \( W_R \)-boson. There are currently ongoing accelerator searches for this extra heavy neutral gauge boson. This has been implemented recently with type II seesaw dominance in the presence of added fermion singlets [112]. As we will discuss below both these types of models predict light neutrinos capable of mediating double beta decay rates in the \( W_L - W_R \) channel saturating the current experimental limits. In addition resonant leptogenesis mediated by heavy sterile neutrinos has been realised in the model of [112].

The \( G_{2213} \) symmetric Yukawa Lagrangian descending from \( SO(10) \) symmetry can be written as

\[ \mathcal{L}_{\text{Yuk}} = \sum_{i=1,2} Y_i^f \bar{\psi}_L^f \psi_R^f \Phi_i + f (\bar{\psi}_L^f \psi_R^f \Delta_R + \bar{\psi}_L^f \psi_L^f \Delta_L) \]

\[ + y_h^f (\bar{\psi}_R^f \chi_R + \bar{\psi}_L^f \chi_L) + \text{h.c.}, \]

where \( \Phi_i \subset 1 \times 10_{H,H_s} \) are two doublets, \( (\Delta_L, \Delta_R) \subset 126_F \), and \( (\chi_L, \chi_R) \subset 16_{H_s} \).

Including the induced VEV contribution to \( \Delta_L \), the Yukawa mass term can be written as

\[ \mathcal{L}_{\text{mass}} = \left(M_D \bar{\nu} \nu + \frac{1}{2} M_{\Sigma} N^T N + M_{\Sigma} S + M_{\Sigma} \bar{\nu} S + \text{h.c.} \right) \]

\[ + m^n_{\nu} \nu^T \nu + \mu_6 S^T S. \]

Here the last term denotes the gauge invariant singlet mass term where naturalness criteria demand \( \mu_6 \) to be a very small
parameter. In the \((\nu, S, N^C)\) basis the generalised form of the 9\times9 neutrino mass matrix after electroweak symmetry breaking can be written as

\[
\mathcal{M}_\nu = \begin{pmatrix}
m_{\nu}^2 & M_L & M_D \\
M_L^T & \mu_S & M_T \\
M_D^T & M & M_N
\end{pmatrix},
\]

(12)

where \(M_D = Y(\Phi), M_N = f v_R, M = y_\nu \langle \chi_0 \rangle\), and \(M_L = y_\nu \langle \chi_L \rangle\). In this model the symmetry breaking mechanism and the VEVs are such that \(M_N > M \gg M_D\). The LH triplet scalar mass \(M_N\), and RH neutrino masses being at the heaviest mass scales in the Lagrangian, this triplet scalar field and the RH neutrinos are at first integrated out leading to the effective Lagrangian at lower scales \([45,46,112]\) as follows:

\[
-\mathcal{L}_{\text{eff}} = \left( m_{\nu}^2 + M_D M_N^{-1} M_D^T \right) v_\alpha v_\beta \\
+ \left( M_L + M_D M_N^{-1} M_T \right)_{am} (\bar{\nu}_a S_m + \bar{S}_m \nu_a) \\
+ \left( M \frac{1}{M_N} M_T \right)_{mn} S_m^T S_n + \mu_S S^T S
\]

(13)

4. Cancellation of Type I Seesaw and Dominance of Others

4.1. Cancellation of Type I Seesaw. Whereas the heaviest RH neutrino mass matrix \(M_N\) separates out trivially, the other two 3 \times 3 mass matrices \(\mathcal{M}_\nu\), and \(\mathcal{M}_S\) are extracted through various steps of block diagonalisation. The details of various steps are given in \([112,176,177]\).

\[
\mathcal{M}_\nu = m_{\nu}^2 + \left( M_D M_N^{-1} M_D^T \right) - \left( M_D M_N^{-1} M_D^T \right) \\
+ M_L \left( M_T M_N^{-1} M_L \right)^{-1} M_L^T \\
- M_L \left( M_T M_N^{-1} M_L \right)^{-1} \left( M_T M_N^{-1} M_D \right) \\
- \left( M_D M_N^{-1} M_L \right) \left( M_T M_N^{-1} M_L \right)^{-1} M_L^T \\
+ M_D M_N^{-1} M_L M_T^{-1} \\
\mathcal{M}_S = \mu_S - M_M^{-1} M_T + \cdots, \\
\mathcal{M}_N = M_N.
\]

(14)

From the first of the above three equations, it is clear that the type I seesaw term cancels out with an opposite sign in this generalised form of the light neutrino mass matrix turns out to be

\[
\mathcal{M}_\nu = f v_L + M_L M_N^{-1} M_N \left( M_T \right)^{-1} M_L^T \\
- \left[ M_L M_N^{-1} M_D^T + M_D \left( M_L M_T^{-1} \right)^T \right] \\
+ \frac{M_D}{M} \mu_S \left( \frac{M_D}{M} \right)^T.
\]

(15)

In different limiting cases this generalised light neutrino mass matrix reduces to the corresponding well-known neutrino mass formulas.

4.2. Linear Seesaw and Double Seesaw. With \(M_L = y_\nu v_L\) that induces \(\nu-S\) mixing, the second term in (15) is the double seesaw formula

\[
\mathcal{M}_\nu^{(\text{double})} = M_L M_N^{-1} M_N \left( M_T \right)^{-1} M_L^T.
\]

(16)

The third term in (15) represents the linear seesaw formula

\[
\mathcal{M}_\nu^{(\text{linear})} = - \left[ M_L M_N^{-1} M_L^T + M_D \left( M_L M_T^{-1} \right)^T \right].
\]

(17)

Similar formulas have been shown to emerge from single-step breaking of SUSY GUT models \([39,40,44]\) which require the presence of three gauge singlet fermions.

Using the D-Parity breaking mechanism of \([55,56]\), an interesting model of linear seesaw mechanism in the context of supersymmetric SO(10) with successful gauge coupling unification \([197]\) has been suggested in the presence of three gauge singlet fermions. A special feature of this linear seesaw, compared to others \([39,40,44]\), is that the neutrino mass formula is suppressed by the SUSY GUT scale but it is not decoupled from the low U(1)_B-L breaking scale. In addition to prediction of TeV scale superpartners, the model provides another important testing ground through manifestation of extra Z' boson at LHC or via low-energy neutrino scattering experiment \([198]\).

4.3. Type II Seesaw. When VEV \(\langle \chi_L \rangle = 0\) or becomes negligible and \(\mu_S = 0\) in (2), type II seesaw dominates leading to

\[
m_\nu = f v_L.
\]

(18)

As noted briefly in Section 1, in the conventional models \([157,160]\) of type II seesaw dominance in SO(10), the \(W_R, Z_R\) boson masses have to be at the GUT-Planck scale. As a phenomenal development, this singlet fermion assisted type II seesaw dominance permits \(U(1)_{B-L}\) breaking scale associated with \(G_{2213}\) or \(G_{2113}\) breaking (i.e., the \(W_R\) and \(Z_R\) boson masses) accessible to accelerator energies including LHC. At the same time the heavy \(N-S\) mixing mass terms \(M_i (i = 1, 2, 3)\) at the TeV scale are capable of mediating observable LFV decay rates closer to their current experimental values \([85-96]\) as discussed in Section 5. Consequences of this new type II seesaw dominance with TeV scale \(Z_R\) boson mass have been investigated in detail \([112]\) in which charged triplet mediated LFV decay rates are negligible but singlet fermion decay rates are observable. Also predictions of observable double beta decay rates close to their experimental limits are discussed below in Section 6. While the principle of such a dominance is clearly elucidated in this derivation, the details of the model with TeV scale \(G_{2213}\) symmetry will be reported elsewhere.

4.4. Extended Seesaw. It is quite clear that the classic inverse seesaw formula \([24]\) of (2) for light neutrino mass emerges
when the LH triplet mass is large and the VEV $\langle \chi_L \rangle = 0$ which is possible in a large class of non-SUSY models with left-right, Pati-Salam, and SO(10) gauge groups with D-Parity broken at high scales [55–57] with $M_N = 0$ leading to RH neutrinos as heavy pseudo Dirac fermions. Particularly in SO(10) some non-SUSY examples are [109, 166] and SUSY examples are [164, 165, 199]. The mechanism operates without supersymmetry provided we reconcile with gauge hierarchy problem and some non-SUSY models are [109, 110, 175, 176].

As noted in Section 1, the derivation of classic inverse seesaw mechanism [24–34] has $M_N = 0$ in (12). More recent applications in LRS and GUTs have been discussed with relevant reference to earlier works in [126, 166, 167, 200–204].

In this section we have discussed that, in spite of the presence of the heavy Majorana mass term of RH neutrino, each of the three seesaw mechanisms, (i) extended seesaw, (ii) Type II seesaw, and (iii) linear seesaw or double seesaw, can dominate as light neutrino mass ansatz when the respective limiting conditions are satisfied. Also the seesaw can operate in the presence of TeV scale $G_{213}$ or $G_{313}$ gauge symmetry originating from non-SUSY SO(10) [112, 176, 177]. As the TeV scale theory spontaneously breaks to low-energy theory $U(1)_{em} \times SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)' \times U(1)_{B-L}$ through the electroweak symmetry breaking of the standard model, these seesaw mechanisms are valid in the SM extensions with suitable Higgs scalars and three generations of $N_i$ and $S_i$. For example without taking recourse to LR gauge theory type II seesaw can be embedded into the SM extension by inclusion of LH Higgs triplet $\Delta_3 (3, -2, 1)$ with $Y = -2$. The induced VEV can be generated by the trilinear term $\Delta M_{
u_{ij}} \Delta i^j \phi i \phi j$ [205, 206]. The origin of such induced VEV in the direct breaking of SO(10) → SM is well known.

4.5. Hybrid Seesaw. In the minimal SO(10), without extra fermion singlets, one example of hybrid seesaw with type I ⊕ type II is given in (I). There are a number of investigations where this hybrid seesaw has been successful in parametrising small neutrino masses with large mixing angles along with $\theta_{13} \sim 8^\circ$ in SUSY SO(10) [148–153] and LR models. But the present mechanism of type I seesaw cancellation suggests a possible new hybrid seesaw formula as a combination of type II ⊕ Linear ⊕ Extended seesaw as revealed from (15). Neutrino physics phenomenology may yield interesting new results with this new combination with additional degrees of freedom to deal with neutrino oscillation data and lepton matter covering coupling unification in SO(10) which has a very rich structure for dark matter.

Using the D-Parity breaking mechanism of SO(10), an interesting model of linear seesaw mechanism in the context of supersymmetric SO(10) with successful gauge coupling unification [197] has been suggested in the presence of three gauge singlet fermions. A special feature of this linear seesaw, compared to others [39, 40], is that the neutrino mass formula is suppressed by the SUSY GUT scale but decoupled from the low $U(1)_{B-L}$ breaking scale which can be even at few TeV. This serves as a testing ground through manifestation of extra $Z'$ boson at LHC or via low-energy neutrino scattering experiments [198]. Being a SUSY model it also predicts TeV scale superpartners expected to be visible at LHC.

4.6. Common Mass Formula for Sterile Neutrinos. In spite of different types of seesaw formulas in the corresponding limiting cases the formula for sterile neutrino mass remains the same as in (6) which does not emerge from the classic inverse seesaw approach with $M_N = 0$.

We conclude this section by noting that the classic inverse seesaw mechanism was gauged at the TeV scale through its embedding in non-SUSY SO(10) with the prediction of experimentally accessible $Z'$ boson, LFV decays, and nonuniversality effects [166]. The possibility of gauged and extended inverse seesaw mechanism with dominant contributions to both lepton flavor and lepton number nonconservation was at first noted in the context Pati-Salam model in [175] and in the context of non-SUSY SO(10) in [176, 177] with type I seesaw cancellation. The generalised form of hybrid seesaw of (15) in non-SUSY SO(10) with type I cancellation was realised in [112]. As a special case of this model, the experimentally verifiable phenomena like extra $Z'$ boson, resonant leptogenesis, LFV decays, and double beta decay rates closer to the current search limits were decoupled from the intermediate scale type II seesaw dominated neutrino mass generation mechanism. Proton lifetime prediction for $p \to e + \nu^\circ$ mode also turns out to be within the accessible range.

5. Predictions for LFV Decays, CP Violation, and Nonuniversality

The presence of nonvanishing neutrino masses with generational mixing evidenced from the oscillation data [71–74], in principle, induces charged lepton flavor violating (LFV) decays. For a recent review see [99] and references therein. The observed neutrino mixings through weak charged currents lead to nonconservation of lepton flavor numbers $L_e$, $L_\mu$, and $L_\tau$ resulting in the predictions of $\mu \to e + \gamma$, $\tau \to e + \gamma$, $\tau \to \mu + \gamma$, $\mu \to e + \nu$, and a host of others [85–96]. If the non-SUSY SM is minimally extended to embrace tiny neutrino masses and mixings through GIM mechanism as the only underlying source of charged lepton flavor violation, the loop mediated branching ratio is

$$\text{Br}(\ell_a \to \ell_i + \gamma) = \frac{\alpha}{32\pi} \sum_{j=2,3} U_{ai} U^*_{ij} \frac{\Delta m^2_{ji}}{M_W^2}.$$

These branching ratios turn out to be $\lesssim 10^{-53}$ ruling out any possibility of experimental observation of the decay rates. In high intermediate scale non-SUSY SO(10) models with Dirac neutrino mass matrix $M_D$ similar to the up quark mass matrix $M_u$, the type I seesaw ansatz for neutrino masses constrain the heavy RH neutrino masses $M_N \geq 10^{12}$ GeV resulting in branching ratio values $\lesssim 10^{-40}$ which are far below the current experimental limits. Another drawback of the model is that the underlying neutrino mass generation mechanism and the predicted $W_\nu$ boson mass cannot be verified directly.

On the other hand SUSY GUTs are well known to provide profound predictions of CP-violations and LFV decay branching ratios closer to the current experimental limits in spite of their high scale seesaw mechanisms for neutrino masses. Some of the extensively available reviews on this...
subject are [97, 99, 100, 203, 207]. The superpartner masses near 100–1000 GeV are necessary for such predictions.

As profound applications of the classic inverse seesaw mechanism it has been noted that the presence of heavy pseudo Dirac fermions would manifest through LFV decays [33, 184–188] and also in lepton number violation [208, 209]. They are also likely to contribute to the modifications of the electroweak observables [190–192] keeping them within their allowed limits. It has been also emphasized that the SUSY inverse seesaw mechanism for neutrino masses further enhances the LFV decay rates [184]. As a direct test of the seesaw mechanism, these heavy particles with masses near the TeV scale can be produced at high energy colliders including LHC [109, 110, 112, 164, 165, 167]. Other interesting signatures have been reviewed in [11].

More interestingly a linear seesaw formula has been predicted from supersymmetric SO(10) with an extra Z’ boson mass accessible to LHC [42]. The TeV scale classic inverse seesaw mechanism and W_R gauge boson masses have been embedded in SUSY SO(10) with rich structure for leptonic CP violation, nonunitarity effects, and LFV decay branching ratios accessible to ongoing experiments [164, 165]. The impact of such a model with TeV scale pseudo Dirac type RH neutrinos has been investigated on proton lifetime predictions and leptogenesis [199].

As supersymmetry has not been experimentally observed so far, an interesting conceptual and practical issue is to confront LFV decay rates accessible to ongoing experimental searches along with the observed tiny values of light neutrino masses. In this section we summarise how, in the absence of SUSY, the classic inverse seesaw and the extended seesaw could still serve as powerful mechanisms to confront neutrino mass, observable lepton flavor violation [109, 166], and, in addition, dominant lepton number violation [110, 175–177] in non-SUSY SO(10). The possibilities of detecting TeV scale W_R bosons have been also explored recently in non-SUSY SO(10) [110, 111, 175–177]. Besides earlier works in left-right-symmetric model [210] and those cited in Sections I–4, more recent works include [61–66, 78, 211–217] and references cited in these papers.

In contrast to negligible LFV decay rates and branching ratios predicted in the non-SUSY SM modified by GIM mechanism, we discuss in Section 5.2 how the non-SUSY SO(10) predicts the branching ratios in the range 10^{-13}–10^{-16} consistent with small neutrino masses dictated by classic inverse seesaw, extended inverse seesaw, or type II seesaw in the presence of added fermion singlets.

5.1. Neutrino Mixing and Nonunitarity Matrix. The light neutrino flavor state is now a mixture of three mass eigenstates  \( \bar{\nu}_i \),  \( \bar{S}_i \), and  \( \bar{N}_i \) as follows:

\[
\nu_a = U_{a \beta} \bar{\nu}_\beta + \mathcal{V}_{a \beta} \bar{S}_\beta + \mathcal{N}_{a \beta} \bar{N}_\beta,
\]

where in the diagonal bases of  \( \bar{S}_i \) and  \( \bar{N}_i \)  \( \mathcal{V}_{a \beta} = (M_D/M_N)_{a \beta} \) and  \( \mathcal{N}_{a \beta} = (M_D/M_N)_{a \beta} \). In cases where the matrices  \( M_D \) and  \( M_N \) are nondiagonal, the corresponding flavor mixing matrices are taken as additional factors to define  \( \nu-S \) or  \( \nu-N \) mixing matrices. For the sake of simplicity, treating the  \( N-S \) mixing mass matrix  \( M \) as diagonal,

\[
M = \text{diag}(M_1, M_2, M_3),
\]

and, under the assumed hierarchy  \( M_N \gg M \), the formula for the nonunitarity deviation matrix element  \( \eta_{\alpha \beta} \) has been defined in the respective cases [110, 112, 175–177, 218–231] as follows:

\[
\eta = \frac{1}{2} X \cdot X^\dagger = M_D^2 M_R^2 M_D^\dagger,
\]

\[
\eta_{\alpha \beta} = \frac{1}{2} \sum_{k=1,2,3} M_{D \alpha k} M_{D \beta k}^* M_k^2.
\]

The Dirac neutrino mass matrix  \( M_D \) needed for the fit to neutrino oscillation data through extended seesaw formula and prediction of nonunitarity effects has been derived from the GUT scale fit of charged fermion masses in the case of non-SUSY SO(10) [110, 112, 166, 176, 177]. For this purpose, the available data at the electroweak scale on charged fermion masses and mixings are extrapolated to the GUT scale [232]. The fitting is done following the method of [233] by suitably adding additional contributions due to VEVs of additional bidoublets or higher dimensional operators, wherever necessary. In the inverse seesaw case with almost degenerate heavy pseudo Dirac neutrinos,  \( M_D \) has been derived in the case of SUSY SO(10) with TeV scale  \( G_{2213} \) symmetry [164, 165] and in non-SUSY SO(10) with TeV scale  \( G_{2113} \) symmetry [166]. In the case of extended seesaw dominance in non-SUSY SO(10) it has been derived in [110, 175–177] whereas for type II seesaw dominance it has been derived in [112]. The value of  \( M_D \), thus derived at the GUT scale is extrapolated to the TeV scale following the top-down approach. It turns out that such values are approximately equal to the one shown in the following section in (32).

For the general nondegenerate case of the  \( M \) matrix, ignoring the heavier RH neutrino contributions and saturating the upper bound  \( |\eta_{\alpha \beta}| < 2.7 \times 10^{-3} \) gives

\[
\eta_{\tau \tau} = \frac{1}{2} \left[ \frac{0.1026}{M_1^2} + \frac{7.0756}{M_2^2} + \frac{6762.4}{M_3^2} \right] = 2.7 \times 10^{-3}.
\]

By inspection this equation gives the lower bounds  \( M_1 > 4.35 (\text{GeV}), M_2 > 36.2 (\text{GeV}), \) and  \( M_3 > 1120 (\text{GeV}) \). And for the degenerate case  \( M_{\text{Deg}} = 1213 \) GeV,  \( M_{(1213, 1213, 1212)} \) GeV. For the partial degenerate case of  \( M_1 = M_2 \neq M_3 \) the solutions can be similarly derived as in [112, 176] and one example is  \( M(100, 100, 1319.67) \) GeV.

Experimentally constrained lower bounds of the nonunitarity matrix elements are

\[
|\eta_{\tau \tau}| \leq 2.7 \times 10^{-3},
\]

\[
|\eta_{\nu \mu}| \leq 8.0 \times 10^{-4},
\]

\[
|\eta_{\nu \tau}| \leq 2.0 \times 10^{-3},
\]

\[
|\eta_{\nu \mu}| \leq 3.5 \times 10^{-5},
\]

\[
|\eta_{\nu \tau}| \leq 8.0 \times 10^{-3},
\]

\[
|\eta_{\tau \tau}| \leq 5.1 \times 10^{-3}.
\]
Table 1: Predictions of moduli and phases of nonunitarity matrix $\eta_{a\beta}$ as a function of allowed values of masses $M_1$, $M_2$, and $M_3$. The Dirac neutrino matrix is the same as in [112].

| $m_{R_1}$ (GeV) | $m_{R_2}$ (GeV) | $|\eta_{\mu\mu}|$ | $\delta_{\mu\mu}$ | $|\eta_{\mu\tau}|$ | $\delta_{\mu\tau}$ | $|\eta_{\mu\nu}|$ | $\delta_{\mu\nu}$ |
|-----------------|-----------------|----------------|-----------------|----------------|----------------|----------------|----------------|
| 1213.11         | 1213.11         | 2.737 $\times$ 10^{-4} | 1.920           | 4.543 $\times$ 10^{-7} | 1.78           | 2.318 $\times$ 10^{-5} | 2.391 $\times$ 10^{-7} |
| 500             | 1280            | 8.132 $\times$ 10^{-7} | 1.326           | 3.746 $\times$ 10^{-7} | 1.456          | 2.426 $\times$ 10^{-5} | 2.723 $\times$ 10^{-5} |
| 1119.67         | 500             | 6.543 $\times$ 10^{-6} | 0.728           | 4.834 $\times$ 10^{-6} | 0.974          | 2.975 $\times$ 10^{-5} | 8.932 $\times$ 10^{-4} |
| 50              | 1545.31         | 7.652 $\times$ 10^{-6} | 0.203           | 9.754 $\times$ 10^{-6} | 0.342          | 3.424 $\times$ 10^{-5} | 2.813 $\times$ 10^{-3} |

Out of several estimations of the elements of $\eta$-matrix [110, 112, 166, 175–177] carried out in non-SUSY $SO(10)$, here we give one example of [112]. Using the Dirac neutrino mass matrix from [112] and allowed solutions of $M_i$, the values of the $\eta_{a\beta}$ parameters and their phases as functions of $M_i$ are determined using (22). These results are presented in Table 1.

5.2. LFV Decay Branching Ratios versus Neutrino Mass. The most important outcome of nonunitarity effect is expected to manifest through ongoing experimental searches for LFV decays such as $\tau \rightarrow e\gamma$, $\tau \rightarrow \mu\gamma$, and $\mu \rightarrow e\gamma$ [85–96, 103]. In left-right symmetric models the LFV decay contribution due to $W_\alpha - N_i$ ($i = 1, 2$) mediation has been computed as early as 1981 assuming a GIM like mechanism in the RH sector [104]. Some of the current experimental bounds on the branching ratios are $Br(\mu^+ \rightarrow e^+\gamma) < 5.7 \times 10^{-13}$, $Br(\mu^+ \rightarrow e^+\gamma) < 5.7 \times 10^{-13}$, $Br(\mu^+ \rightarrow e^+\gamma) < 5.7 \times 10^{-13}$, and $Br(\mu^+ \rightarrow e^+\gamma) < 5.7 \times 10^{-13}$. An up-to-date list of experimental results on various LFV processes and their future projections have been summarised in [103]. In these models contribution to the branching ratios due to the heavier RH neutrinos is subdominant compared to the lighter singlet fermions

$$Br(\ell_\alpha \rightarrow \ell_\beta + \gamma) = \frac{\alpha^2 \lambda^2}{256 \pi^2} \frac{m^2_{\ell_\alpha}}{M_{W_1}} \left| S_{a\beta}^N + S_{a\beta}^S \right|^2,$$

(25)

where

$$S_{a\beta}^N = \sum_k (\gamma^{\alpha N})_{ak} (\gamma^{\beta N})_{bk} \mathcal{J} \left( \frac{m^2_{N_k}}{M_{W_1}} \right),$$

(26)

$$S_{a\beta}^S = \sum_j (\gamma^{\alpha S})_{aj} (\gamma^{\beta S})_{bj} \mathcal{J} \left( \frac{m^2_{\nu_j}}{M_{W_1}} \right),$$

$$\mathcal{J}(x) = \frac{-2x^3 + 5x^2 - x}{4(1-x)^3} - \frac{3x^3 \ln x}{2(1-x)^4}.$$ 

Because of the condition $M_N \gg M$, the RH neutrino exchange contribution is however damped out compared to the sterile fermion singlet contributions. Using allowed solutions, for $M_1$, $M_2$, and $M_3$, our estimations in the partial degenerate case are given in Table 2.

As a demonstration of solutions to the conceptual and practical issue of predicting experimentally accessible LFV decay branching ratios consistent with tiny neutrino masses in the dominant seesaw mechanisms (in the event of type I seesaw cancellation) we present results in two specific non-SUSY examples in $SO(10)$: (i) type II dominance [112] and (ii) extended inverse seesaw dominance [110, 175–177]. We have used the Dirac neutrino mass matrices and the fixed value of the matrix $\mu_3$ from the respective references. Our predictions for branching ratios as a function of the lightest neutrino mass are shown in in Figure 1 for the type II dominance case. In this figure we have also shown variation of the LH triplet mass as expected from the type II seesaw formula. But in spite of the large value of the triplet mass that normally predicts negligible LFV branching ratios, our model gives experimentally accessible values.

The corresponding results for the extended inverse seesaw case is shown in Figure 2 for the degenerate values of the $N$-$S$ mixing mass term $M = M_1 = M_2 = M_3 = 1.2$ TeV.

We note that the predictions of LFV decay branching with classic inverse seesaw [24] embedding in non-SUSY $SO(10)$ as investigated in [166] with TeV scale $Z'$ boson would be also similar to Figure 2.

5.3. CP Violation due to Nonunitarity. The standard contribution to the CP violation is determined by the rephasing invariant $J_{CP}$ associated with the Dirac phase $\delta_{CP}$ and matrix elements of the PMNS matrix

$$J_{CP} \equiv \text{Im} \left( U_{\alpha\alpha} U_{\beta\beta} U_{\alpha\beta} \right)$$

$$= \cos \theta_{12} \cos^2 \theta_{13} \cos \theta_{23} \sin \theta_{12} \sin \theta_{13} \sin \theta_{23} \sin \delta_{CP}.$$

(27)

Because of the presence of nonunitarity effects, the leptonic CP violation can be written as

$$\Delta f_{\alpha\beta} \equiv \text{Im} \left( V_{\alpha\alpha}^* V_{\beta\beta}^* V_{\alpha\beta} \right) = J_{CP} + \Delta f_{\alpha\beta},$$

(28)
where \([110, 112, 120-122, 164-166, 176, 218-231]\)

\[
\Delta f_{ij}^{\alpha \beta} = - \sum_{p=e,\mu,\tau} \text{Im} \left[ \eta_{\alpha p} U_{\rho i} U_{\rho j}^* U_{aj}^* U_{\beta i} \right] 
+ \eta_{\beta p} U_{\alpha i} U_{\rho j}^* U_{\alpha j}^* U_{\rho i} + \eta_{\alpha p}^* U_{\alpha i} U_{\beta j} U_{\rho j}^* U_{\rho i}^* U_{\beta i} 
+ \eta_{\beta p}^* U_{\alpha i} U_{\beta j} U_{\alpha j}^* U_{\rho i}^* U_{\rho j} \right].
\] (29)

Model predictions for deviations from rephasing invariant matrix defined in (29) are presented in Table 3 for different allowed values of \(M_i \equiv m_{\nu_i}\). In order to visualise how far the predicted LFV decay branching ratios are in agreement with neutrino mass values derived from the new seesaw mechanisms which survive after type I cancellation, we have examined two cases as examples: (i) Type II seesaw dominance [112] and (ii) extended seesaw dominance [175–177]. We have used the values of the Dirac neutrino mass matrix and the value of \(\mu_3\) from these references.

### 6. Neutrinoless Double Beta Decay

#### 6.1. Double Beta Decay Predictions in the \(W_L-W_R\) Channel

The ongoing experiments on double beta decay without any conclusive results have led to a surge of investigations through different theoretical predictions including LR gauge theories [214, 215, 234–236]. While the standard light neutrino exchange amplitude is \(\propto G_F^2 \sum U_{ei}^2 m_i\) in the \(W_L-W_L\) channel, the RH neutrino exchange amplitude in the \(W_R-W_R\) channel is \([237–240] \propto G_F^2 |M_W/M_W|^4 (1/M_N)\). Thus the major suppression factor \(|M_W/M_W|^4 < 10^{-4}\) apart from the inverse proportionality factor \(M_N^{-1}\). The RH triplet exchange contribution is also suppressed [241, 242] and the available
Table 3: Allowed deviations of the rephasing invariant matrix $J_{CP}$ as a result of weak leptonic CP-violation and nonunitarity effects defined in (29).

<table>
<thead>
<tr>
<th>$M_{12}$ (GeV)</th>
<th>$M_{1}$ (GeV)</th>
<th>$\Delta J_{ep}^{12}$</th>
<th>$\Delta J_{ep}^{23}$</th>
<th>$\Delta J_{ep}^{23}$</th>
<th>$\Delta J_{ep}^{31}$</th>
<th>$\Delta J_{ep}^{12}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1213.11</td>
<td>1213.11</td>
<td>$-2.1 \times 10^{-6}$</td>
<td>$-2.4 \times 10^{-6}$</td>
<td>$1.4 \times 10^{-4}$</td>
<td>$1.2 \times 10^{-4}$</td>
<td>$1.1 \times 10^{-4}$</td>
</tr>
<tr>
<td>500</td>
<td>1280</td>
<td>$-2.1 \times 10^{-6}$</td>
<td>$-2.4 \times 10^{-6}$</td>
<td>$1.4 \times 10^{-4}$</td>
<td>$1.2 \times 10^{-4}$</td>
<td>$1.1 \times 10^{-4}$</td>
</tr>
<tr>
<td>100</td>
<td>1319.67</td>
<td>$-2.0 \times 10^{-6}$</td>
<td>$-3.0 \times 10^{-6}$</td>
<td>$1.3 \times 10^{-4}$</td>
<td>$1.8 \times 10^{-4}$</td>
<td>$3.4 \times 10^{-5}$</td>
</tr>
<tr>
<td>50</td>
<td>1545.31</td>
<td>$-1.7 \times 10^{-6}$</td>
<td>$-1.98 \times 10^{-6}$</td>
<td>$1.1 \times 10^{-4}$</td>
<td>$1.8 \times 10^{-4}$</td>
<td>$8.4 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

Experimental limit on the double beta decay rates has led to the lower bound on the doubly charged Higgs mass [234, 235]

$$M_{AA^*} \geq 500 \left( \frac{3.5 \text{ TeV}}{M_{W^*}} \right)^2 \times \left( \frac{M_{N}^*}{3 \text{ TeV}} \right)^{1/2}. \quad (30)$$

Possible dominant contribution [175] in the $W_L^*W_R^*$ channel has been also suggested due to what are known as $\eta$ and $\lambda$ diagrams [214, 215]. It has been concluded [9] that the lack of observation of double beta decay cannot rule out TeV scale LR models.

6.2. Double Beta Decay Predictions in the $W_L^*W_L^*$ Channel. In [176] it was noted in the context of extended gauged inverse seesaw mechanism in non-SUSY SO(10) that a new and dominant contribution to $0\nu2\beta$ decay exists through left-handed weak charged currents in the $W_L^*W_L^*$ channel via exchanges of fermion singlets $S_i$. That the light sterile neutrinos may have a dominant effect was also noted in the context of extensions of SM or LR models [48] where type I contributions to neutrino mass were also included. As such this model [48] did not use cancellation mechanism for the type I seesaw contributions. The sterile fermions mix quite prominently through Dirac neutrino mass matrix which is well known to be of order of up quark mass matrix. As already noted the sterile neutrinos acquire Majorana masses $m_S = \mu_S - M(1/M_N)M^T = -M(1/M_N)M^T$. The Feynman diagram for double beta decay due to light sterile neutrino exchange is shown in Figure 3.

The $(ei)$ element of the $\nu-S$ mixing matrix is

$$\mathcal{V}^{neS}_{ei} = \left( \frac{M_D}{M} \right)_{ei}, \quad (31)$$

where the Dirac neutrino mass matrix $M_D$ has been given in different SO(10) models from fits to the charged fermion masses at the GUT scale and then running it down to the TeV scale in non-SUSY SO(10) models [110, 112, 166, 176, 177]. For example the Dirac neutrino mass matrix $M_D$ in the model [112] is

$$M_D \text{ (GeV)} = \begin{pmatrix} 0.014 & 0.04 - 0.01i & 0.109 - 0.3i \\ 0.04 + 0.01i & 0.35 & 2.6 + 0.0007i \\ 0.1 + 0.3i & 2.6 - 0.0007i & 79.20 \end{pmatrix} \quad (32)$$

The elements of the matrix $M$ treated as diagonal are determined from constraints on LFV decays and the diagonal elements are estimated using the nonunitarity equation as discussed in the previous section. We derive the relevant elements of the mixing matrix $\mathcal{V}^{neS}$ using the structures of the Dirac neutrino mass matrix $M_D$ given in (32) and values of the diagonal elements of $M = (M_1, M_2, M_3)$ satisfying the nonunitarity constraint in (23). The eigenvalues of the $S$-fermion mass matrix are estimated for different cases using the structures of the RH Majorana neutrino mass matrices [112, 175–177] and allowed values of $M_i$ satisfying the nonunitarity constraints through the formula $M_S = -M(1/M_N)M^T$.

Different particle exchange contributions for $0\nu2\beta$ decay are changed by the chirality of the hadronic currents involved in the nuclear mass matrix element. In this model only the LH currents are significant or dominant while the RH current effects are negligible. The inverse half-life has been estimated with proper normalization factors by taking into account the effects of nuclear matrix elements [214, 215, 237, 243–253]

$$[\tau_{1/2}^{0\nu}]^{-1} = C_{01} \left( \frac{\mathcal{M}_{0\nu}^2}{m_e} \right) \left( |m_S^e + m_S^\nu + m_S^{n}\nu| \right)^2 \quad (33)$$

The larger value of RH neutrino masses makes negligible contribution to the $0\nu2\beta$ decay amplitude. In the above equation $C_{01} = 0.686 \times 10^{-14} \text{ yrs}^{-1}$, $\mathcal{M}_{0\nu}^2 = 2.58 - 6.64.$
\[
K_{\nu\nu} = 1.57 \times 10^{-25} \text{ yrs}^{-1} \text{ eV}^{-2},
\]
and the two effective mass parameters are
\[
m_{\nu}^e = \sum_i (\nu_{ei}^\nu)^2 m_{\nu_i},
\]
\[
m_{S}^e = \sum_i (\nu_{ei}^S)^2 \frac{|p_i|^2}{m_{S_i}},
\]
\[
m_{N}^e = \sum_i (\nu_{ei}^N)^2 \frac{|p_i|^2}{m_{N_i}}.
\] (34)

Here \(m_{S_i}\) is the eigenvalue of the S-fermion mass
\[
M_S = -M_{\nu}^{-1} M_N^T.
\] (35)

6.3. Singlet Fermion Exchange Dominated Half-Life. The non-
standard contributions to half-life of double beta decay as
a function of sterile neutrino mass has been discussed in
[110, 112, 166, 176, 177]. The models predict \(\nu-S\) mixing and
sterile neutrino masses which have been used to predict the
half-life in the case of different hierarchies of light neutrino
masses: NH, IH, and QD. Using the estimations of [177]
which are also applicable to models of [110, 112, 175, 176],
scattered plots for the predicted half-life are shown in Figures
4, 5, and 6 for the three types of neutrino mass hierarchies.

In Figure 7 we show predictions in the QD case excluding
and including the CP phases of Majorana type sterile
neutrinos. The cancellations between light neutrino exchange
amplitude and the sterile neutrino exchange amplitude are
shown by the two peaks. When CP phases associated with
the Majorana type sterile neutrino mass eigenvalue(s) are
included the peaks are smoothened as shown by dotted lines
[177].

In Figure 8 estimations on the lightest sterile neutrino
masses are predicted which saturate the current experimental
limit on the observed double beta decay half-life using Ge-
76 and Xe-136 nuclei for three different light neutrino mass
hierarchies in each case. The uncertainties in the predicted
masses correspond to the existing uncertainty in the neutrino
virtuality momentum \(|p| = 120-200 \text{ MeV}\). The green hori-
zontal line represents the average value
\[
\bar{M}_{S_i} = 18 \pm 4 \text{ GeV}
\] (36)
of the lightest sterile neutrino mass determined from double
beta decay experimental bound [177]. Lower values of this
mass have been obtained using light neutrino assisted type
II seesaw dominance [112].

These predictions suggest that sterile neutrino exchange
contribution dominates the double beta decay rate even
when the light neutrino masses have NH or IH type of
mass hierarchies. To predict double beta decay saturating the
current experimental bounds, it is not necessary that light
neutrinos should be quasi-degenerate in mass. On the other
hand if double beta decay is not found with half-life close
to the current limits, then the solutions with light sterile
neutrino masses in the range \(\sim(2-15) \text{ GeV}\) are ruled out, but
the model with larger mass eigenvalues easily survives.

7. Leptogenesis in Extended MSSM and
SUSY SO(10)

In the conventional type I seesaw based leptogenesis models
where heavy RH neutrino decays give rise to the desired
lepton asymmetry [254], the Davidson-Ibarra bound [255,
256] imposes a lower limit on the scale of leptogenesis,
\(M_{N_1} > 4.5 \times 10^8 \text{ GeV}\) [255, 256]. This also suggests the
lower bound for the reheating temperature after inflation,
\(T_{\text{RH}} \geq 10^9 \text{ GeV}\), that would lead to overproduction
of gravitino severely affecting the relic abundance of light
nuclei since the acceptable limit has been set as \(T_{\text{RH}} \leq 10^7 \text{ GeV}\) [257]. Several attempts have been made to evade the
gravitino constraint on leptogenesis where sterile neutrino
assisted results are interesting. Obviously gravitino constraint
is satisfied in models with TeV scale resonant leptogenesis
[258]. Also in the singlet fermion extended SUSY SO(10)
Figure 5: Same as Figure 4 but for inverted hierarchy (IH) of light neutrino masses.

Figure 6: Same as Figure 4 but for quasi-degenerate (QD) type light neutrino masses.

Figure 7: Prediction of half-life for double beta decay as a function of sterile neutrino mass in the case of QD type mass hierarchy with the common mass parameter $m_0 = 0.23$ eV. The peaks correspond to cancellation between light neutrino and sterile neutrino exchange amplitudes when Majorana CP phases of sterile neutrino are ignored. The dotted line shows the absence of peaks when CP phases are included [177].

where RH neutrinos are heavy pseudo Dirac neutrinos and neutrino mass formula is through inverse seesaw [24], there is no problem due to gravitino constraint [164, 165, 199]. We discuss the cases where all the three types of neutrinos are Majorana fermions.

7.1. Leptogenesis with Extended Seesaw Dominance. With extended seesaw realisation of leptogenesis two types of SUSY models have been investigated under gravitino constraint: (i) MSSM extension with fermion singlets [45, 46, 169–171] and (ii) Singlet extension of SO(10) with intermediate scale $G_{2213}$ gauge symmetry [172, 173]. We discuss their salient features.

7.1.1. MSSM Extension with Fermion Singlets. The Dirac neutrino mass matrix is identified with the charged lepton mass matrix in this model where MSSM is extended with the addition of heavy RH neutrinos $N_i$ as well as additional singlets $S_i$ [45, 46], one for each generation. As already explained in the limit $M_{N_i} > M \gg M_{D}, \mu_9$ extended seesaw formula, which is the same as the inverse seesaw formula
for active neutrino mass. In this case resonant leptogenesis is implemented via quasi-degenerate RH neutrino decays at the TeV scale. It is well known that such resonant leptogenesis scenario with $M_{N_1} \sim M_{N_2} \sim 1$ TeV implemented through canonical seesaw needs a very small mass splitting between the RH neutrinos $(M_{N_1} - M_{N_2})/(M_{N_1} + M_{N_2}) \sim 10^{-6}$. With the tension arising out of fitting the neutrino oscillation data being transferred from type I seesaw to extended seesaw in the presence of additional sterile fermions, it is not unimaginable that this fine-tuning associated with very tiny RH neutrino mass splitting could be adequately alleviated. The fermion singlets $S_i$ give rise to a new self-energy contribution and using this successful resonant leptogenesis has been found to be possible for a much larger mass ratio $M_{N_1}/M_{N_2} \sim 10$. Possibilities of $\sim 100$ MeV to $\sim 10$ GeV mass range for light sterile neutrinos have been pointed out. In a separate analysis the possibility of singlet Majorana fermion or singlet scalar as candidates of dark matter has been pointed out [169]. Realisation of doubly coexisting dark matter candidates in the context of extended seesaw framework has been pointed out [170].

7.1.2. Leptogenesis in SUSY SO(10).

In non-SUSY minimal LR models where $M_D$ is similar to charged lepton mass matrix successful leptogenesis emerges with intermediate scale hierarchical RH neutrino masses [259]. In SUSY SO(10) the underlying quark-lepton unification forces the Dirac neutrino mass to be similar to the up quark mass matrix. This pushes the type I seesaw scale closer to the GUT scale $M_X \gtrsim 10^{14}$ GeV and rules out the possibility of low scale $W_R$ bosons accessible to accelerator searches in foreseeable future unless the canonical seesaw ansatz is given up, for example, in favour of inverse seesaw with TeV scale pseudo Dirac neutrinos and $W_R$ bosons [164, 165, 199]. With heavy right-handed Majorana neutrinos and GUT scale LR breaking scale, successful leptogenesis has been implemented in realistic SUSY SO(10) [260]. With the help of an effective dim.5 operator ansatz which originates from renormalisable interactions at GUT-Planck scale in SUSY SO(10) (without using 126$_L$) both thermal and nonthermal leptogenesis [261–264] have been discussed with heavy hierarchical RH neutrino of masses [265, 266]. Possible solutions to the allowed parameter space to evade gravitino constraint have been also discussed in this work.

Apart from the models with resonant leptogenesis, possibility of leptogenesis under gravitino constraint in SUSY SO(10) has been realised with hierarchical RH neutrinos assisted by sterile neutrinos. As already noted above, in these cases the extended seesaw formula controls the neutrino mass as a result of cancellation of type I seesaw contribution. Gauge coupling unification in these SO(10) models requires the $G_{213}$ symmetry to occur at the intermediate scale in the renormalisable model [173]. A common feature of both these models [172, 173] is the generation of lepton asymmetry through the decay of hierarchical sterile neutrinos through their respective mixings with heavier RH neutrinos which are also hierarchical.

An important and specific advantage of heavy gauge singlet neutrino decay to achieve leptonic CP asymmetry is the following: The singlet neutrino of mass $\sim 10^9$ GeV which decays to $h\phi$ though its mixing with RH neutrino of mass $\sim 10^{10}$ GeV has a small mixing angle $\sim 10^{-5}$. This small mixing ensures out-of-equilibrium condition by making the decay rate smaller than the Hubble expansion rate in arriving at CP asymmetry at lower temperatures $\sim 300$ GeV.

8. Singlet Fermion Assisted Leptogenesis in Non-SUSY SO(10)

An extensive review of thermal leptogenesis with reference to LFV is available in [267]. With the neutrino mass following a modified type I seesaw at a scale $\gtrsim 10^6$ GeV, thermal leptogenesis has been investigated in [267–273]. It is well known that TeV scale RH neutrinos can participate in resonant leptogenesis contributing to enhanced generation of leptonic CP asymmetry that is central to generation of baryon asymmetry of the universe via sphaleron interactions. Here we briefly discuss a recent work where quasi-degenerate sterile neutrinos at the TeV scale in non-SUSY SO(10) have been shown to achieve resonant leptogenesis through their decays. The Feynman diagrams at the tree level and with vertex and self-energy corrections are shown in Figure 9.

The fermion-Higgs coupling in all the diagrams is $V_{hi}$ instead of the standard Higgs-Yukawa coupling $h = M_D/V_{wK}$, where $|\gamma = M/M_{N_1}$, $M_D$ is given in (32), and $V_{wK} \approx 174$ GeV. Denoting the mass eigenvalue of a sterile neutrino by $m_{\tilde{\nu}_i}$ ($k = 1, 2, 3$), for computation of baryon asymmetry $Y_B$ of the Universe with a washout factor $K_L$, we utilise the ansatz [124]
leading to $K$ leading to $K$

Case 2

The depleted washout factor is [274]

$$K_{ij} = \frac{\delta^2 K_i}{K_j},$$

$H(\hat{m}_S)$ being the Hubble parameter at temperature $\hat{m}_S$. Defining

$$\delta_i = \frac{|\hat{m}_i - \hat{m}_j|}{\Gamma_i}, \quad (i \neq j),$$

the depleted washout factor is [274]

$$K_{ij}^\text{eff} = \delta^2 K_i.$$

Here we discuss two cases for the sterile neutrino contribution towards leptogenesis and baryon asymmetry: (1) $\hat{m}_S$ is light, $\hat{m}_{S_1}$ and $\hat{m}_{S_3}$ are quasi-degenerate; (2) $\hat{m}_S$ is light, $\hat{m}_{S_2}$ and $\hat{m}_{S_3}$ are quasi-degenerate.

**Case 1 ($\hat{m}_S$ light, $\hat{m}_{S_1}$ and $\hat{m}_{S_3}$ heavy and quasi-degenerate).**

Using an allowed interesting region of the parameter space $M = \text{diag}(146, 3500, 3500)$ GeV, $V_R = 10^{4}$ GeV, and $M_N = f V_R$, we get

$$\hat{m}_S = \text{diag}(1.0, 595.864\ldots, 595.864\ldots) \text{ GeV},$$

leading to $K_2 = 2.7 \times 10^7$. Using $(\hat{m}_{S_2} - \hat{m}_{S_3}) = 2 \times 10^{-7}$ GeV, we obtain

$$\epsilon_{S_1} = 0.824,$$

$$Y_B = 1.5 \times 10^{-10}.\quad (41)$$

**Case 2 ($\hat{m}_S$ light, $\hat{m}_{S_2}$ and $\hat{m}_{S_3}$ heavy and quasi-degenerate).**

Choosing another allowed region of the parameter space $M = \text{diag}(3200, 146, 3200)$ GeV, similarly we get

$$\hat{m}_S = \text{diag}(500.567\ldots, 1.0, 500.567\ldots) \text{ GeV},\quad (42)$$

leading to $K_1 = 4 \times 10^6$. Using $(\hat{m}_{S_2} - \hat{m}_{S_3}) = 7 \times 10^{-5}$ GeV, we obtain

$$\epsilon_{S_2} = 0.682,$$

$$Y_B = 4 \times 10^{-10}.\quad (43)$$

In Case 1 with $\hat{m}_{S_1} \sim \mathcal{O}(1)$ GeV, the lightest sterile neutrino acts as the most dominant source of $0\nu\beta\beta$ decay whereas the heavy quasi-degenerate pair of sterile neutrinos $S_2$ and $S_3$ mediate resonant leptogenesis. Similarly in the alternative scenario of Case 2 with $\hat{m}_{S_2} \sim \mathcal{O}(1)$ GeV, the second generation light sterile neutrino acts as the mediator of dominant double beta decay while the heavy quasi-degenerate pair of the first and the third-generation sterile neutrinos mediate resonant leptogenesis. Because of the resonant leptogenesis constraint, we note that either Case 1 or Case 2 is permitted, but not both.

Our predictions for the double beta decay half-life and the baryon asymmetry in Cases 1 and 2 are presented in Table 4. It is clear that for smaller mass eigenvalues of sterile neutrinos in Case 1 or Case 2 it is possible to saturate current experimental limit on the double beta decay half-life while explaining the right order of magnitude of the baryon asymmetry. Thus, in addition to the Case 1 found in [112], we have shown another possible alternative scenario as Case 2.

Before concluding this section certain interesting results on thermal leptogenesis derived earlier are noted. Thermal leptogenesis with a hybrid seesaw and RH neutrino dark matter have been proposed by introducing additional $U(1)$ gauge symmetry [136]. Thermal leptogenesis in extended seesaw models has been investigated earlier [268–272] which are different from our cases reported here and earlier [112, 113]. Possibilities of falsifying high scale leptogenesis on the basis of certain LHC results and also on the basis of LFV and $0\nu\beta\beta$ decay results have been suggested [273, 275]. Prospects of dark matter in the minimal inverse seesaw model have been also investigated in ref. [276, 277].

### 9. Summary and Discussion

Reviewing the contributions already made, we have discussed how the added presence of singlet fermions which manifest as singlet neutrinos of Majorana type effectively cancels out the would-be dominant type I seesaw contribution to light...
neutrino masses. The neutrino masses consistent with the oscillation data are now governed by the classic inverse seesaw formula (renamed as extended seesaw formula) or type II seesaw, or even linear or double seesaw under their respective limiting conditions. But the dominant part of the singlet neutrino mass is given by the type I seesaw mechanism where the role of the Dirac neutrino mass matrix $M_D$ is replaced by the $N$-$S$ mixing matrix $M$. The cancellation mechanism of type I seesaw term for the active light neutrino masses is universal in the sense that appropriate extensions of SM, left-right gauge theories, Pati-Salam Model, SO(10), SUSY, or non-SUSY can accommodate it leading to the dominance of seesaw mechanism of another type. In the cases of extended seesaw, linear seesaw, or type II seesaw dominance, the double beta decay rates in the $W_L$-$W_R$ channel are dominated by the exchange of light sterile neutrinos with masses in the range of $O(1-10)$ GeV. With type II dominance, the second- and third-generation sterile neutrinos could be heavy and quasi-degenerate and explain baryon asymmetry of the universe through resonant leptogenesis. The models also predict a rich variety of results for LFV decays and lepton nonunitarity effects. In SUSY GUTs in the absence of added fermion singlets baryogenesis via leptogenesis through decays of heavy RH neutrinos is usually affected by gravitino problem. A possible solution to this is the well known TeV scale resonant leptogenesis which requires extremely small mass difference between the heavy quasi-degenerate pair leading to their unity mass ratio up to very high degree of accuracy. The presence of sterile neutrinos considerably alleviates this problem by changing this fine-tuned mass ratio to a value as large as ~10. In SUSY SO(10) leptogenesis under gravitino constraint is achieved even for large hierarchical masses of RH neutrinos. When the model is extended by fermion singlets, the singlet neutrinos which mix with RH neutrinos also acquire hierarchical masses below $10^6$ GeV. The hierarchical singlet neutrinos decay through their mixings with RH neutrinos to generate the desired lepton CP asymmetry. In these models also the light neutrino mass formula is due to the extended seesaw (or inverse seesaw). One simple reason for the success of the sterile neutrino assisted leptogenesis in SUSY SO(10) is the smallness of mixing angle $\xi$ between the lighter sterile neutrino and the heavy RH neutrino with $\sin \xi \sim M/M_N \sim 10^{-5} \rightarrow 10^{-6}$. This reduces the decay rate of the sterile neutrino considerably to satisfy the out-of-equilibrium condition which forms an important ingredient for the generation of CP asymmetry.

In the presence of sterile neutrinos, type II seesaw dominance is achieved with $U(1)_{B-L}$ breaking scale much lower than the mass of the Higgs triplet $\Delta_{L}(3, 1, -2, 1)$. This mechanism makes it possible to have type II seesaw formula even with TeV scale of $W_R$ or $Z_R$ boson whereas in the conventional attempts in SUSY or non-SUSY SO(10), or split-SUSY models, the type II seesaw dominance required these $W_R$ and $Z_R$ boson masses to be near the GUT-Planck scale. In the standard lore in the proposed models of type II seesaw dominance with very large $W_R$ and RH neutrino masses, the nonstandard contribution to double beta decay in the $W_L$-$W_R$ channel, damped by the heavy $\Delta_{L}(3, 1, -2, 1)$ boson propagator, is negligible. But this concept is overthrown when type II seesaw dominance is assisted by sterile neutrinos. In the new scheme even though the heavy $\Delta_{L}(3, 1, -2, 1)$ boson exchange contribution is negligible, the double beta decay mediated by the exchange of light sterile neutrinos in the $W_L$-$W_R$ channel predicts dominant decay rates saturating the current experimental limits. This new model of type II seesaw dominance predicts resonant leptogenesis in the non-SUSY SO(10) originating from the near TeV scale masses of quasi-degenerate sterile neutrinos of the other two generations.

While GUTs like SUSY SO(10) have a rich structure of dark matter candidates, even in the presence of sterile neutrinos, some aspects of embedding dark matter and their detection possibilities in the SM extensions have been discussed in [169–171]. The SUSY and non-SUSY SO(10) GUTs considered under these seesaw mechanisms satisfy coupling unification and proton life time constraints, the latter being accessible to ongoing search experiments [109, 110, 112, 173, 176, 177].

In this review we have considered the class of models where heavy RH Majorana mass terms are present satisfying the conditions $M_N > M \gg M_{DH}$ under which the generalised form of the neutral fermion matrix gives different seesaw formulas. In these models the type I seesaw, if not cancelled by using the decoupling criteria and two-step block diagonalisation process, would have given dominant contributions. A common feature of all these models is dominant double beta decay in the $W_L$-$W_R$ channel mediated by light sterile neutrinos as well as leptogenesis generated by heavier sterile neutrinos of the other two generations.

The singlet neutrinos needed for these models are found to have mass ranges between few GeV to ~1 TeV. They are the mixed states of added fermion singlets $S_i$ and heavy RH neutrinos $N_i$ where the latter are in the spinorial representation of SO(10). The mass terms of fermion singlets violate the global lepton number symmetry of the SM. As such these masses are required to be as light as possible according to ’t Hooft’s naturalness criteria [163]. In other words the global lepton number symmetry protects these masses naturally and prevents them from becoming superheavy. This is a special advantage in favour of TeV scale seesaw mechanisms as well as the new type II intermediate scale seesaw mechanism [112] due to the cancellation of the type I seesaw. Further we have brought down the $U(1)_R \times U(1)$ breaking scale in non-SUSY SO(10) to be accessible to LHC and future accelerator searches by a number of new physical processes including the $Z'$ boson. The predicted proton lifetime has been noted to be accessible to ongoing searches [278–280].

There are a number of interesting models assisted by TeV scale pseudo Dirac neutrinos [109, 164–168, 199] where such heavy RH Majorana neutrino masses are either absent or, if present, they do not satisfy the decoupling criteria. Details of phenomenology and predictions of such models are beyond the scope of the present review. Likewise the interesting possibilities of detection of gauge singlet neutrinos through their displaced vertices [113, 117–119] and the renormalisation group impacts [281] in the presence of inverse seesaw have been excluded from present discussions.
Competing Interests
While submitting this manuscript for publication, the authors declare that they have no conflict of interests with any individual or organisation, private or governmental, whatsoever.

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