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Research Article

Cosmology in the Universe with Distance Dependent Lorentz-Violating Background

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We consider a cosmological setup with inflaton in the presence of a redshift dependent Lorentz-violating time-like background to address the inflationary regime and other phases of the Universe. We also show that the regime of dark energy at large distances (low redshifts) is essentially dominated by the presence of the Lorentz-violating background.

1. Introduction

One of the first main issues in cosmology was explaining the problem of the horizon, flatness, and magnetic monopole, which was successfully explained with the advent of the inflationary phase in the early Universe [1–3]. This accelerated phase should end with the reheating of the Universe coinciding with the beginning of a radiation-dominated era and then a (dark) matter-dominated era of forming the first structures such as galaxies and clusters takes place. These are decelerated phases. However, in the year 1998, Riess et al. [4] and Perlmutter et al. [5] independently reported a present acceleration of the Universe. The source of this acceleration was denominated *dark energy*. Since then, several models have been considered in the literature in order to accomplish this new observation into the standard cosmology. Although its origin is not known yet some aspects are similar to inflationary phase; for example, its pressure should be negative in order to counteract the gravitational force and then provide accelerated expansion. The simplest candidate for the dark energy is the cosmological constant with the equation of state $\omega = -1$. However, it might well be possible that the origin of the dark energy is not due to a cosmological constant. If so it is necessary to look for alternative models to explain the present accelerated expansion of the Universe. Yet, as in the inflationary phase, one can model the source of the dark energy with scalar fields. The main difference is the too low scale compared with the inflation scale and that the scalar potentials should be flat enough to drive the present accelerated expansion of the Universe. The models with scalar field describing dark energy are basically those called quintessence [6–13] and k-essence [14–16]. In the former case one focuses on the use of scalar field with slowly varying potentials, while in the latter case the kinetic part drives the present accelerating phase of the Universe. One should mention also that these are not the only way to model the dark energy. To quote the main recent alternatives, we mention the models with modified gravity such as $f(R)$ gravity [17–19], $f(R,T)$ gravity [20, 21], scalar-tensor theories [22–26], and braneworld models [27, 28]. One might consider in this class of theories the ΛCDM model which is based on Einstein or $f(R)$ gravity with a cosmological constant. Nowadays all these models can be distinguished from each other through the use of observational data such as combined analysis of SN Ia, CMB, and BAO [4, 5, 29, 30]. These observations have constrained the equation of state of dark energy to the bound $-1.097 < \omega < -0.858$ at 95% of confidence [29, 30]. This favors the cosmological constant as a good candidate of dark energy, whereas it disfavors several other types of models. However,

further observations confirming or detecting deviations of the ΛCDM model are very important to shed new light on the origin of the dark energy.

The study of several Lorentz-violating cosmological scenarios in the inflationary phase has been addressed in the literature [31–38]. Between such works we can highlight, as motivations for our investigation, the approaches from Gasperini [31] and from Donnelly and Jacobson [37], for instance. In his work Gasperini proposes that the primordial phase of accelerated expansion of the Universe could be achieved if at some very early epoch the gravitational interactions were described by a nonlocally Lorentz invariant theory. It is also suggested that this additional mechanism for producing inflation could be used to solve some problems of the standard inflationary scenario. Besides, in [37] the authors consider a Lorentz-violating theory of inflation formed by an Einstein-aether theory coupled with a scalar field Lagrangian. There the authors determined cosmological parameters which are affected by the Lorentz violation but still allow for a natural end to inflation. On the other hand, application of such a scenario can also be found in dark energy, where it is shown that violation of the Lorentz invariance induces Lagrangians that are able to drive the current acceleration of the universe [39, 40]. As such, the former and the latter theories deal with Lorentz symmetry violation at small and large distances [41]. In our present study we consider these theories to guide ourselves to deal with a cosmological scenario where one can flow from small to large distances under a Lorentz-violating background. As we shall discuss below, this scenario captures the equations of state of both inflation and dark energy at asymptotic limits, which confirms the aforementioned efforts of producing accelerating Universe in the presence of Lorentz invariance violation.

In this paper we shall focus on a theoretical approach that deals with inflaton field in an early inflationary and late-accelerating Universe. The main point is that a time-like Lorentz-violating background permeating the space is responsible for dark energy at large distances (low redshifts). The reason to work with a time-like Lorentz operator is that a purely time-like background (or frame) is described by a small subset of Lorentz invariant violating operators which preserves rotational invariance—see, for instance, Passos et al. [42] for recent discussion. Besides, as discussed by Kostelecký and Mewes [43], the frame of the Cosmic Microwave Background stands as a natural choice for a preferred frame in this context.

2. The Model

Let us start with the Lagrangian of a 3 + 1-dimensional field theory in a curved background describing an inflaton field coupled to Lorentz-violating background:

$$e^{-1}\mathscr{L} = \frac{R}{2\kappa^2} - \frac{1}{2}\left(g_{\mu\nu} + \xi_1 k^1_{\mu\nu}\right)\partial^\mu\phi\partial^\nu\phi - V(\phi), \quad (1)$$

where $e = \sqrt{-g}$ and $\kappa^2 = 8\pi G$. $V(\phi)$ is the inflaton potential. The time-like tensors $k^i_{\mu\nu}$ $(i = 1, 2, \ldots)$ in principle could couple to several other fields in a more general Lagrangian which are such that only $k^i_{00} \neq 0$; that is,

$$k^i_{\mu\nu} = \begin{pmatrix} -\beta_i & 0 \\ 0 & 0 \end{pmatrix}, \quad (2)$$

where we have defined $k^1_{\mu\nu}$ as the tensor related to the inflaton. The tensor couples the inflaton field to the Lorentz-violating background through the coupling $\xi_1 > 0$. It is expected that at short distances $\beta_1 \to 0$ maintains all the inflationary dynamics depending only on the inflaton field, whereas for $\beta_1 \to -1$ the Lorentz-violating background becomes more effective at large distances and develops dark energy, as we shall see shortly.

We shall study cosmological issues in this model, so that our *background metric* is the Friedmann-Robertson-Walker (FRW) metric of a *flat* Universe given as

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a(t)^2\left(dx^2 + dy^2 + dz^2\right). \quad (3)$$

The *effective metric* due to presence of the background field is

$$\begin{aligned} d\tilde{s}^2 &= \tilde{g}_{\mu\nu}dx^\mu dx^\nu = \left(g_{\mu\nu} + \xi_1 k_{\mu\nu}\right)dx^\mu dx^\nu \\ &= -\left(1 + \xi_1\beta_1\right)c^2dt^2 + a(t)^2\left(dx^2 + dy^2 + dz^2\right), \end{aligned} \quad (4)$$

where c is the speed of light. The effective velocities for inflaton due to the background field are given by

$$v_1 = \sqrt{1 + \xi_1\beta_1}\,c. \quad (5)$$

The present choice of Lorenz-violating parameter just changes the Lorentz boosts because it affects only velocities.

Thus, in this sense we can define an analogous refractive index felt by the inflaton field as follows:

$$n \equiv \frac{c}{v_1} = \frac{1}{\sqrt{1 + \xi_1\beta_1}}. \quad (6)$$

This is because the refractive index is in general a wavelength dependent quantity given basically by

$$n^2 = 1 + \sum_{i=1}^{N} B_i \frac{\lambda_{0i}^2\lambda^2}{\lambda^2 - \lambda_{0i}^2} = 1 + NB_0\frac{\lambda_0^2\lambda^2}{\lambda^2 - \lambda_0^2}, \quad (7)$$

where in the last step we have assumed that all "molecules" resonate with same frequency, which means that $B_i = B_0$. This is an analog of the Sellmeier dispersion equation [44] which is well-known in optics. In the original formula, B_i are known as Sellmeier coefficients that are determined experimentally. From the cosmological perspective we make the identification $\lambda_0/\lambda \equiv a_0/a(t)$, for very small variations in time, where λ_0 and λ are the wavelengths observed now and then, respectively. Recalling the definition of redshift $z + 1 = \lambda_0/\lambda$, with $\lambda \leq \lambda_0$, we find

$$n^2 = 1 - \frac{C_1}{1 - (z+1)^2}, \quad (8)$$

where $C_1 = NB_0\lambda_0^2$. The extra minus before C_1 above is just to keep the analogy with Sellmeier equation normally applied, for example, to glass for $\lambda \geq \lambda_0$. Now comparing (8) with (6) we establish a relationship between the Lorentz-violating parameter and the cosmological redshift as follows:

$$\xi_1\beta_1(z) = -\frac{C_1}{z(z+2)+C_1}. \tag{9}$$

Notice that with (9) into (5) the inflaton field has velocity $v(z) \approx 0$ for very small redshifts but approaches velocity of light for very large redshifts. Thus, it is expected that in the reheating phase (at UV) the field is expected to develop a radiation-dominated phase, whereas in the current regime (at IR) it is expected to be responsible for the dark energy.

3. From Inflationary to Dark Energy Regime

Let us now study the regime where the inflaton is the dominant species. The Einstein field equations lead to the Friedmann equation $H^2 = (8\pi G/3)\rho_\phi$ whose inflaton density ρ_ϕ is governed by the inflaton field with energy-momentum tensor:

$$\begin{aligned} T_{\mu\nu} &= \partial_\mu\phi\partial_\nu\phi + g_{\mu\nu}\mathscr{L}_\phi, \\ T^\mu_\nu &= \operatorname{diag}\left(-\rho_\phi, p_\phi, p_\phi, p_\phi\right), \end{aligned} \tag{10}$$

where $\mathscr{L}_\phi$ is

$$\mathscr{L}_\phi = -\frac{1}{2}\left(g_{\mu\nu} + \xi_1 k^1_{\mu\nu}\right)\partial^\mu\phi\partial^\nu\phi - V(\phi), \tag{11}$$

and p_ϕ is the pressure component of the energy-momentum tensor due to the scalar field. From now on, we are going to work with $c = 1$. Since we will be interested in homogeneous inflaton configurations $\phi \equiv \phi(t)$, we can write down the energy density ρ_ϕ and pressures $p_i = p_\phi\,(i = 1,2,3)$ as follows:

$$\rho_\phi = \frac{1}{2}(1-\xi_1\beta_1)\dot\phi^2 + V(\phi), \tag{12a}$$

$$p_\phi = \frac{1}{2}(1+\xi_1\beta_1)\dot\phi^2 - V(\phi), \tag{12b}$$

where the dot stands for derivative with respect to the temporal coordinate.

The equation of state for the inflaton field can be readily found and is given by

$$\omega \equiv \frac{p}{\rho} = \frac{(1/2)(1+\xi_1\beta_1)\dot\phi^2 - V(\phi)}{(1/2)(1-\xi_1\beta_1)\dot\phi^2 + V(\phi)}. \tag{13}$$

Note that when the potential part dominates one finds the usual $\omega \simeq -1$ and an inflationary regime takes place, where we may constrain $\xi_1\beta_1$ via calculation of the number of e-folds—see the following. On the other hand, when the kinetic part *at the end of inflation* dominates over the scalar potential part we find the interesting equation of state

$$\omega \simeq \frac{1+\xi_1\beta_1}{1-\xi_1\beta_1}, \tag{14}$$

that agrees with radiation-dominated regime $\omega \to 1/3$ as $\xi_1\beta_1 \to -1/2$ and also matter-dominated regime $\omega = 0$ at $\xi_1\beta_1 \to -1$. These values are completely in accord with (9) since $\xi_1\beta_1 \in (-1,0)$. Furthermore, it is interesting to notice that in general the equation of state (13) has the following behaviors by power expanding it around $z = 0$ (IR) and $z = \infty$ (UV):

$$\begin{aligned} \omega_0(z) &= -\frac{V}{\dot\phi^2 + V} + \mathcal{O}(z), \\ \omega_\infty(z) &= \frac{(1/2)\dot\phi^2 - V}{(1/2)\dot\phi^2 + V} + \mathcal{O}\left(\frac{1}{z^2}\right) \end{aligned} \tag{15}$$

with $\omega_\infty(z) \in (-1,1)$ and $\omega_0(z) \in (-1,0)$ which means that, in the *slow-roll* regime, where $\dot\phi^2 \ll V(\phi)$, the equation of state at inflationary phase $\omega_\infty(z) \simeq -1$ is somehow redshifted to the equation of state of dark energy $\omega_0(z) \simeq -1$.

Let us now focus on such aforementioned regime at inflationary phase. The modified equation of motion for the inflaton field is

$$\ddot\phi + 3H\dot\phi + \left(\frac{1}{1+\xi_1\beta_1}\right)V' = 0; \quad V' = \frac{\partial V}{\partial\phi}, \tag{16}$$

whereas the modified Friedmann equation reads

$$H^2 = \frac{8\pi G}{3}\left[\frac{1}{2}(1-\xi_1\beta_1)\dot\phi^2 + V(\phi)\right]. \tag{17}$$

Note the presence of the Lorentz-violating background in both equations which will drive new effects as we shall see below. In (16) we may find a slow-roll regime, that is, when the friction term $3H\dot\phi$ dominates over the acceleration term $\ddot\phi$. This is also accompanied by the condition $\dot\phi^2 \ll V(\phi)$ into (17). Thus we find now the following equations:

$$\begin{aligned} 3H\dot\phi + \left(\frac{1}{1+\xi_1\beta_1}\right)V' &\simeq 0, \\ H \equiv \left(\frac{d\ln a}{dt}\right) &\simeq \sqrt{\frac{8\pi G}{3}V(\phi)}. \end{aligned} \tag{18}$$

Let us now consider the simplest inflaton potential and the quadratic potential

$$V(\phi) = \frac{1}{2}m^2\phi^2. \tag{19}$$

For the slow-roll condition (17) simplifies to

$$H = \sqrt{\frac{4\pi G}{3}}m\phi, \tag{20}$$

and (18) leads to

$$\dot\phi = -\frac{m}{\sqrt{12\pi G}(1+\xi_1\beta_1)}; \tag{21}$$

that is

$$\phi(t) = \phi_0 - \frac{m}{\sqrt{12\pi G}(1+\xi_1\beta_1)}t. \tag{22}$$

Finally using (20) we obtain the scale factor

$$a(t) = a_0 \exp\left[\sqrt{\frac{4\pi G}{3}} m\phi_0 t - \frac{m^2}{6(1+\xi_1\beta_1)} t^2\right]. \quad (23)$$

Now to understand the effect of the Lorentz-violating background on the inflationary phase of the Universe we make use of the e-folds number defined as

$$N_e = \int_{t_i}^{t_f} H\, dt, \quad (24)$$

and making use of (18) we find

$$N_e = \int_{t_i}^{t_f} H\, dt = \int_{t_i}^{t_f} H \frac{d\phi}{d\phi} dt = \int_{\phi(t_i)}^{\phi(t_f)} \frac{H}{\dot{\phi}} d\phi \quad (25)$$

so

$$\begin{aligned} N_e &= -\frac{3}{1+\xi_1\beta_1} \int_{\phi(t_i)}^{\phi(t_f)} \frac{H^2}{V'} d\phi \\ &= -\frac{8\pi G}{1+\xi_1\beta_1} \int_{\phi(t_i)}^{\phi(t_f)} \frac{V}{V'} d\phi. \end{aligned} \quad (26)$$

For the inflaton potential defined in (19) we find the modified e-fold number:

$$\begin{aligned} N_e &= -\frac{4\pi G}{1+\xi_1\beta_1} \int_{\phi(t_i)}^{\phi(t_f)} \phi\, d\phi \\ &= \frac{2\pi G}{1+\xi_1\beta_1} \left(\phi(t_i)^2 - \phi(t_f)^2\right). \end{aligned} \quad (27)$$

Now assuming $\phi(t_f) \approx 0$ we obtain

$$N_e = \frac{2\pi G}{1+\xi_1\beta_1} \phi(t_i)^2, \quad (28)$$

which implies

$$\xi_1\beta_1 = \frac{2\pi G \phi(t_i)^2}{N_e} - 1. \quad (29)$$

For $N_e \simeq 60$, $\phi(t_i)^2 \simeq 4m_{pl}^2$, and $G \simeq 1/m_{pl}^2$ we find for (29)

$$\beta_1 \simeq \left(\frac{8\pi}{60} - 1\right)\xi_1^{-1} \simeq -\frac{0.5811}{\xi_1}, \quad \xi_1 > 0. \quad (30)$$

Notice this agrees with the end of inflation (and also with the beginning of the radiation) regime described by (14) where Lorentz-violating background field assumes the value $\xi_1\beta_1 = -1/2$.

Besides the approaches above, there are several interesting cosmological properties which may be derived from the slow-roll parameters, as well as from the power spectrum perturbations. Let us first observe the consequences of this inflationary model for the two first slow-roll parameters, whose explicit forms in the slow-roll regime are [45]

$$\begin{aligned} \epsilon &= \frac{1}{16\pi G}\left(\frac{V'}{V}\right)^2; \\ \eta &= \frac{1}{8\pi G}\frac{V''}{V}. \end{aligned} \quad (31)$$

The previous equations yield

$$\epsilon = \eta = \frac{1}{4\pi G\phi^2}, \quad (32)$$

which is consistent with a chaotic inflation driven by a quadratic potential. As in the standard inflationary scenario, the strength of the tensor perturbations is directly related to the magnitude of the energy density. It is well-known that the power spectrum for scalar perturbation has the form [45]

$$P_\zeta = \frac{H^4}{4\pi^2\dot{\phi}^2}, \quad (33)$$

when we deal with one scalar field Lagrangian. It is important to point out that all quantities here are determined at the horizon crossing [45, 46]. Such a parameter enables us to compute the so-called scalar spectral index, which is given by [45]

$$n_s - 1 \equiv \frac{1}{HP_\zeta}\frac{dP_\zeta}{dt}, \quad (34)$$

and this parameter is remarkably important as a test for cosmological models, since it is directly measured in the CMB. An equivalent way to represent n_s is through the expression

$$n_s = -6\epsilon + 2\eta = -4\epsilon, \quad (35)$$

where we use the fact that $\epsilon = \eta$ due to the slow-roll approximation. The previous equation together with (28) and (32) allows us to write n_s as

$$n_s - 1 = -\frac{2}{1+\xi_1\beta_1 N_e} \approx -\frac{2}{N_e} + \delta_{n_s}; \quad \delta_{n_s} = \frac{2}{N_e}\xi_1\beta_1, \quad (36)$$

where we consider $\phi(t) = \phi(t_i)$ in (32) at the horizon crossing; moreover we also expanded the right-hand side around small values of $\xi_1\beta_1$.

Recently, Planck collaboration [30] strongly constrained that the scalar spectral index has the value $n_s = 0.9655 \pm 0.0062$; therefore, we are able to use (36) to establish a bound for the product $\xi_1\beta_1$ related to the CMB data. This bound is stronger than the one derived from (29), once we do not need to impose any initial value for $\phi(t_i)$. Following the procedure adopted in [47], we assume the equalities

$$\begin{aligned} n_s &= 1 - \frac{2}{N_e} = 0.9655; \\ \delta_{n_s} &= \frac{2}{N_e}\xi_1\beta_1 \sim -10^{-3}, \end{aligned} \quad (37)$$

where we associated the Lorentz-breaking parameter with the order of the experimental error for n_s. Therefore, we find that $N_e \sim 57.97$ and $\xi_1\beta_1 \sim -10^{-2}$. From (20), (22), and (34), we derive n_s as a function of time, yielding

$$n_s = 1 - \frac{36(\beta_1\xi_1 + 1)}{\left(\sqrt{3}mt - 6\sqrt{\pi G}\phi_0(\beta_1\xi_1 + 1)\right)^2}. \quad (38)$$

This last result is going to be useful to determine a relation between the scalar spectral index with the denominated tensor-scalar ratio, whose definition is

$$r = \frac{P_T}{P_\zeta}; \quad P_T = 64\pi G\left(\frac{H}{2\pi}\right)^2. \tag{39}$$

Thus, taking (20) and (22), the explicit time dependence of r is

$$r = \frac{144}{\left(\sqrt{3}mt - 6\sqrt{\pi G}\phi_0\left(\beta_1\xi_1 + 1\right)\right)^2}, \tag{40}$$

and, therefore, by combining n_s with r we are able to find the relation

$$r = \frac{4\left(1 - n_s\right)}{\beta_1\xi_1 + 1}, \tag{41}$$

unveiling that the Lorentz-breaking parameter changes the standard dependence between n_s and r (see (2.7) at [45] for comparison). Furthermore, from (36) we determine that

$$r = \frac{1}{N_e}\frac{8}{\left(1 + \xi_1\beta_1\right)^3} \approx \frac{8}{N_e}\left(1 - 3\beta_1\xi_1\right), \tag{42}$$

and, then, using the previous constraints to N_e and to $\xi_1\beta_1$, we yield $r \sim 10^{-1}$, which is compatible with the Planck collaboration measurements for cosmological parameters [30].

4. Conclusions

We present a new model to address the dark energy problem by changing the kinetic part of the inflaton field with a Lorentz-violating time-like background that plays the role of a medium that affects the inflaton velocity, as one can see in (5). Such a behavior is responsible for changing how long the inflationary process can last. By identifying an analogous refractive index (see (6)), we were able to relate this background with the cosmological redshift. Such a relation leads us to conclude that a time-like Lorentz-violating background can be responsible for dark energy at low energy (low redshifts). Furthermore, this is in accord with the aforementioned fact that it has been shown that violation of the Lorentz invariance induces Lagrangians that can drive the present acceleration of the universe. More interestingly, the combined slow-roll and dominated-Lorentz-violating background regimes develop an equation of state for the dark energy which approaches the equation of state of the cosmological constant, as pointed in (15). This feature is remarkably consistent with the expansion phase that our Universe has been passing through. Another interesting result of our investigation was the determination of the scalar spectral index (n_s) and of the tensor-scalar ratio (r), for this cosmological scenario. Note that the Lorentz-breaking parameter shifted the relation between both cosmological parameters. Moreover, the bound for the product $\xi_1\beta_1$ derived from n_s resulted in a value for tensor-scalar ratio which agrees with Planck's data for cosmological parameters. Such a test strengthens the potential of our work, and further studies should be addressed elsewhere.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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