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Research Article

Generalization of the Randall-Sundrum Model Using Gravitational Model $F(T, \Theta)$

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In this letter, we explore a generalized model based on two scenarios including the Randall-Sundrum model and gravity model $F(T, \Theta)$. We first study the standard Randall-Sundrum gravitational model and then add a function containing two parameters as torsion and trace energy-momentum tensor to the main action of the model. Next, we derive the equations of the generalized model and obtain a new critical value for the energy density of the brane. The results showed that inflation and the dark energy-dominated stage can be realized in this model. We pointed out one significant category of dark energy models that had greatly developed the knowledge about dark energy. To be specific, dark energy could either be quintessence-like, phantom-like, or the so-called "quintom"-like. The models of quintom type suggest that the equation of state parameter of dark energy can cross the cosmological constant boundary $\omega = -1$. Interestingly, this quintom scenario exactly appeared in this paper.

1. Introduction

According to the standard model of cosmology, the Planck Era refers to a time starting from the creation of the universe (about 14 billion years ago) until 10^{-43} seconds. In this time lapse, quantum effects are important parameters that can be formulated using the quantum gravity theory [1]. One of the ideas for describing quantum gravity is the string theory [2]. Some features of this theory include the replacement of one-dimensional quantum operators with pseudopoint operators in Hilbert space and the need for the presence of supersymmetry instead of symmetry [3]. The idea of the cosmology of the string emerged in the early 1990s by extending the string idea into the cosmological model [4]. According to this theory, from the mathematical point of view, we need a space with higher dimensions. The outcome of this idea is that the fundamental constants of the physics have a variable form with respect to time. The image of these constants in a (3 + 1)-dimensional world of our space-time universe is constant with respect to time [5].

Kaluza (1921) and Klein (1926) proposed the unified field theory of gravitation and electromagnetism, which needs an extra spatial dimension that is a function of the Planck length [6, 7].

Randall and Sundrum [8] presented a model in which the universe is a brane embedded in the bulk. In other words, the universe is located in a less spatial dimension than the main dimension of the bulk with five-dimensional space-time [9]. The main feature of the bulk is the gravity which can freely propagate in it. Because only a graviton particle can exist inside the bulk in this model, the other particles of the standard model are in the braneworld. Based on the number of branes available in this model, two Randall-Sundrum (RS) models (I) and (II) have been presented [8, 9]. The five-dimensional space-time determines the type of model. In type (I) Randall model, the distance L is limited while in type (II) Randall model L tends to infinity. The adjustable parameters of this model include cosmic bulk constant and energy density (tension) of the brane [8–10].

In this model, the torsional metric in the five-dimensional space-time is defined according to the following:

$$ds^2 = e^{-2A|y|}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2, \tag{1}$$

where $0 \leq y \leq \pi r_c$ is the fifth dimension of space and r_c is essentially a compactification "radius" [8].

Given the abovementioned explanations, we first study the Randall-Sundrum standard model. Then, we obtain the equations of the model by generalizing this model by adding a function $F(T, \Theta)$ to the main action of the model. Finally, we determine some cosmological parameters by our generalized model equations.

2. Teleparallel Gravity Model

The equations of the gravitational field in general relativity are based on the interaction between the curvature of space-time and the energy density content of the universe. Other models have been proposed for gravity [1–5] by generalizing a gravity model based on scalar and Ricci tensor (curvature). One of these models is based on the replacement of the torsion tensor rather than the curvature of space-time in the gravity model [11]. This modified model proves the inflation at the beginning of the universe.

In the following, a brief overview of the abovementioned model is presented. We assume Planck constant, Boltzmann constant, and the speed of light as $\hbar = K_B = c = 1$ and define the gravitational constant $8\pi G$ by the relation $K^2 = 8\pi/M_{pl}^2$ (where M_{pl} is Planck mass). In this model, the torsion tensor is denoted by the relation $T^{\rho}{}_{\mu\nu} = r^{\rho}{}_{\nu\mu} - r^{\rho}{}_{\mu\nu}$, where $r^{\rho}{}_{\nu\mu} = e^{\rho}_{A}\partial_{\mu}e^{A}_{\nu}$ is the connection without the Weitzenbock curvature [9]. The contortion tensor is also defined by the relation $K^{\mu\nu}{}_{\rho} = (1/2)(T^{\mu\nu}{}_{\rho} - T^{\nu\mu}{}_{\rho} - T_{\rho}{}^{\mu\nu})$. In addition, the torsion scalar is presented as $T = S_{\rho}{}^{\mu\nu}K_{\mu\nu}{}^{\rho}$ using the super potential $S_{\rho}{}^{\mu\nu}$ and the contortion tensor $K^{\rho}{}_{\mu\nu}$. The action of this modified model with the matter in terms of $F(T)$ is shown by the following [11]:

$$S = \int d^4x\,|e|\left(\frac{F(T)}{2K^2} + L_M\right), \tag{2}$$

where $|e| = \det(e^A_\mu) = \sqrt{-g}$ and L_M is the Lagrangian of matter. Regardless of material Lagrangian, gravity action in five dimensions is defined by the following relations:

$$\left({}^{(5)}S\right) = \int d^5x\left|{}^{(5)}e\right|\left(\frac{F\left({}^{(5)}T\right)}{2K_5^2}\right) \tag{3}$$

$$\left({}^{(5)}T\right) = \frac{1}{4}T^{abc}T_{abc} + \frac{1}{2}T^{abc}T_{cba} - T_{ab}{}^{a}T^{cb}{}_{c}, \tag{4}$$

where ${}^{(5)}e = \sqrt{{}^{(5)}g}$ and the gravitational constant and Planck mass in five dimensions are related to each other by the relation $K_5^2 = 8\pi G_5 = 8\pi/M_{pl}^2$ [11].

2.1. Compression of Kaluza-Klein. In this subsection, we need to explain the compression mechanism of the Kaluza-Klein [6, 7]. In these five dimensions, the metric is represented by the following diagonal matrix:

$${}^{(5)}g_{ab} = \begin{bmatrix} g_{\mu\nu} & 0 \\ 0 & -\varphi^2 \end{bmatrix}, \tag{5}$$

where the scalar field is uniform and dependent on time according to the relation $\varphi^2 = \mathfrak{R}^2\theta^2$. Here, φ is related to $\mathfrak{R}$ (compressed space radius) and the quadratic orthogonal components are represented in a compact space dimension with a dimensionless characteristic θ [11]. According to the relationship $g_{\mu\nu} = \eta_{AB}e^{A}{}_{\nu}e^{B}{}_{\nu}$, the second-order metric tensors in five dimensions and tangent spaces in five dimensions are defined according to the relations $\eta_{AB} = \mathrm{diag}(1, -1, -1, -1, -1)$ and $e^{A}{}_{\nu} = \mathrm{diag}(1, 1, 1, 1, \varphi)$, respectively. Therefore, using these relations in (3) and (4), the effective action is given by the compression mechanism of the Kaluza-Klein as follows:

$$S = \int d^4x\frac{1}{2K^2}\varphi\left|{}^{(4)}e\right|F\left(T + \varphi^{-2}\partial_\mu\varphi\partial^\mu\varphi\right), \tag{6}$$

where the shape of the gravitation function and $|{}^{(5)}e| = |e|\varphi$ does not change in comparison with relation (3). This function contains a torsion scalar component and a scalar field.

In the simplest case, assume a gravitational function in the form of $F(T) = T - 2\Lambda_4$, where Λ_4 is a positive cosmological constant. If we define a scalar field σ in the form of $\varphi = \sigma^2\xi$, where $\xi = 1/4$, by rewriting (6), the effective action is obtained as follows:

$$S_{KK}^{eff} = \int d^4x\frac{1}{K^2}\left|{}^{(4)}e\right|\left[\frac{1}{8}\sigma^2T + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \Lambda_4\right], \tag{7}$$

where the metric in the flat universe of FLRW is defined as $ds^2 = dt^2 - a^2(t)\Sigma_{i=1,2,3}(dx^i)^2$, a is the scale factor, and $H = \dot{a}/a$ is the Hubble parameter. Moreover, the vector of the space tangent and metric are defined with the relations $e^A_\mu = \mathrm{diag}(1, a, a, a)$ and $g_{\mu\nu} = \mathrm{diag}(1, -a^2, -a^2, -a^2)$, respectively. If $T = -6H^2$, the equations of the gravity field are calculated by the following [11]:

$$\frac{1}{2}\dot{\sigma}^2 - \frac{3}{4}H^2\sigma^2 + \Lambda_4 = 0 \tag{8}$$

$$\dot{\sigma} + H\sigma\dot{\sigma} + \frac{1}{2}\dot{H}\sigma^2 = 0, \tag{9}$$

And the equation of motion for the scalar field is obtained by

$$\ddot{\sigma} + 3H\dot{\sigma} + \frac{3}{2}H^2\sigma = 0. \tag{10}$$

By combining the abovementioned gravity field equations, the following is obtained:

$$\frac{3}{2}H^2\sigma^2 - 2\Lambda_4 + H\sigma\dot{\sigma} + \frac{1}{2}\dot{H}\sigma^2 = 0. \tag{11}$$

With solving this equation in terms of H, the obtained value (H) is achieved corresponding to the Hubble parameter in the inflation time [11].

2.2. Effective Gravity in Randall-Sundrum Model. Considering the solution of vacuum equations in a five-dimensional space-time and (RS) type (II) model, which has only one

mass within a five-dimensional bulk with positive energy density, it is concluded that the five-dimensional space-time is an anti-desitter (AdS) space. In the following, according to Randall model type II [12, 13] in the teleparallel gravity theory, Friedmann equations for the brane in the FLRW background metric for effective gravity $F(T)$ are determined as follows:

$$H^2 \frac{dF(T)}{dT} = -\frac{1}{12}\left[F(T) - 4\Lambda - 2K^2\rho_M - \left(\frac{K_5^2}{2}\right)Q\rho_m^2\right] \tag{12}$$

where $Q = (11 - 60\omega_M + 93\omega_M^2)/4$. In this relation, $\omega_M = P_M/\rho_M$ corresponds to the perfect fluid equation of state for the pressure and density of the confined matter in the brane. $\Lambda = \Lambda_5 + (K_5^2/2)^2\lambda^2$ with $0 < \lambda$ brane tension, and $G = [1/(3\pi)](K_5^2)^2\lambda$. With substituting $F(T) = T - 2\Lambda_5$ in (11), the approximate solution of desitter for brane is obtained as $H = H_{DE} = \sqrt{(\Lambda_5 + K_5^4\lambda^2/6)}$, where $a(t) = a_{DE}e^{(H_{DE}t)}$ with $a_{DE} > 0$, and $T = -6H^2$ is assumed. Therefore, this relation can describe the accelerating expansion of the universe [11].

3. Teleparallel Gravity Model with a Trace of Momentum-Energy Tensor

Another example of the generalized model based on the $F(T)$ model that we discuss here is the $F(T,\Theta)$ model. There are two main reasons for implementing the $F(T,\Theta)$ model. First, it is a novel gravitational theory. Here, the only restriction imposed on F is that it must be an analytic function, meaning that $F(T,\Theta)$ is a real function locally given by a convergent power series and is infinitely differentiable. Second, due to the extra freedom in the Lagrangian imposed, the $F(T,\Theta)$ cosmology allows for a vast class of scenarios and behaviors. For example, the scalar perturbations at the linear level reveal that $F(T,\Theta)$ cosmology can be free of ghosts and instabilities for a wide class of ansatzes and model parameters [13, 14].

An important feature of this model is the effect of geometry and the universe content through space-time torsion and trace of momentum-energy tensors, where T is torsion scalar and Θ is energy-momentum tensor trace. Therefore, in order to study this model, we consider the proposed gravity function $F(T,\Theta)$ in a five-dimensional space-time compressed by the Kaluza-Klein theory [11, 14]:

$$F(T,\Theta) = \alpha(-T)^n(\Theta)^m \tanh\left(\frac{T_0}{T}\right), \tag{13}$$

where $\alpha = 1, m = 1, n = 2, T_0 = -6H_0^2$. The physical motivation, in this case, is a possible expression that leads to an accelerated expansion phase. It is particularly interesting to look at models that are able to give an effective equation of state with crossing the phantom divide. Also, interpreting gravitational interactions in terms of the torsion rather than the scalar curvature results in the equivalent teleparallel formulation of the general relativity. Now, let us use the abovementioned gravity function in a type II Randall-Sundrum model.

First, we transfer the five-dimensional space-time in which the gravity model is related to the scalar field and coupled with the additional dimension of space, into a four-dimensional space-time in a brane with a flat metric FLRW using the compression mechanism of the Kaluza-Klein. We have already investigated that in the gravity model $F(T)$ in the four-dimensional space we get the Friedmann equations according to (12) [11]. Therefore, due to the characteristics of the gravity model $F(T,\Theta)$ mentioned above, we rewrite Friedmann equations for this gravitational model. In this model, the torsion is based on the relation $T = -6H^2$ and Θ is the momentum-energy tensor trace. The total momentum-energy tensor on brane is written according to (14) [13]:

$$T_B^A = S_B^A\delta(y), \tag{14}$$

where $S_B^A = \mathrm{diag}(-\rho_b, P_b, P_b, P_b, 0)$ and P_b, ρ_b are pressure and density of the total brane energy, respectively. By the conjugation condition in $y = 0$ in the five-dimensional space and simplifying the relations, we have

$$H^2 = \left(\varepsilon\frac{\rho_b^2}{36}\chi^4\right) + \frac{\Lambda}{6} - \frac{k}{a^2} + \frac{C}{a^4}, \tag{15}$$

where C is constant of integration. It is of note that this relation is established in the brane and the energy of conservation law will be conserved according to the following relation:

$$\dot{\rho}_b + 3H(\rho_b + P_b) = 0. \tag{16}$$

Assuming that $\rho_b = \rho + \lambda$, where λ is a brane tension and $\Lambda = \varepsilon\lambda^2(\chi^4/6)(\varepsilon = 1$, if the extra dimension is space-like, while $\varepsilon = -1$, if it is time-like), we get the following relation in a flat metric ($k = 0$), where $C = 0$ is the constant of integration and called dark radiation:

$$H^2 = \left(\frac{\rho}{3}\right)\left(1 + \frac{\rho}{2\lambda}\right). \tag{17}$$

Now, by applying the gravity function $F(T,\Theta)$, (13), in the Friedman equation obtained from the compression of space KK, (12), we have

$$\tanh\left(\frac{T_0}{T}\right)\left[A(-T)^n + BT^{n-1} + C(-T)^{n+1}\right] = \frac{8\pi G}{12(3\omega - 1)} + \frac{\pi G}{16\lambda(3\omega - 1)}\left(11 - 60\omega + 93\omega^2\right)\rho, \tag{18}$$

where $A = -n/6, B = T_0/6, C = 1/72$. Assuming $n = 2$ and due to the relation of momentum-energy tensor (14), we get $\Theta = 3P - \rho$, which can be rewritten as $\Theta = (-1 + 3\omega)$ using the equation of state. Using the Maclaurin expansion for the hyperbolic tangent and using the first-order approximation, we have

$$T = \frac{T_0}{72}\left[\frac{8\pi G}{12(3\omega - 1)} + \frac{\pi G}{16\lambda(3\omega - 1)}\left(11 - 60\omega + 93\omega^2\right)\rho\right] - 12. \tag{19}$$

Here, assuming that $\rho_b = \lambda\sqrt{\rho}$ and substituting it in (14), we get (18). Moreover, the Hubble parameter is obtained according to (20) using the relation $T = -6H^2$. Therefore, we have

$$H = \pm\left(-\frac{T_0}{432}\left[\frac{8\pi G}{12(3\omega-1)} + \frac{\pi G}{16\lambda(3\omega-1)}\left(11-60\omega+93\omega^2\right)\rho\right]+2\right)^{1/2}. \tag{20}$$

Using (20) and assuming that $\rho_b = \lambda\sqrt{\rho}$, the critical value for the energy density of the brane is determined according to the following equation:

$$\rho_b < \sqrt{\frac{863\Theta\lambda^3}{\pi G T_0}}. \tag{21}$$

Now, we can obtain the universe scale factor from the combination of Hubble parameter and relation (20), for the inflation period using the hyperbolic tangent series and the first-order approximation. Also, using an approximate of the deSitter solution on the brane (assuming $T_0 = -6H_0^2 = -24\times10^{-36}$), we obtain scale factor $a(t)$, shown in Figure 1. To calculate the equation of state $\omega(t)$, it is necessary to solve simultaneously three nonlinear differential equations according to the obtained relations for the gravity model $F(T,\Theta)$ in the five-dimensional space. For this purpose, first, by combining (16) and the equation of state $p = \omega\rho c^2$, we get the differential equation between the energy density of the brane and $\omega(t)$ as follows:

$$\dot{\rho}_b + 3H\rho_b(1+\omega(t)) = 0. \tag{22}$$

Now, according to the pressure and energy relation for the teleparallel gravity model [15] and combining it with the equation of state, we get the nonlinear differential equation for the scalar field and the Hubble parameter as follows:

$$\left(\frac{\dot{\varphi}}{2} - \frac{3}{4}\varphi^2H^2 + \Lambda_4\right)\omega = \frac{\dot{\varphi}^2}{2} + H\varphi\dot{\varphi} + \frac{3}{4}H^2\varphi^2 + \frac{1}{2}\varphi^2\dot{H} - \Lambda_4. \tag{23}$$

By solving (22) and (23) simultaneously and also the equation of motion for the scalar field (10), the equation of state $\omega(t)$ is obtained. In the following, we used the first-order approximation to solve differential equations. Moreover, by adjusting the brane tension parameter, the energy density, and the potential of the scalar field with the cosmological constant on the brane, we get the equation of state in terms of time shown in Figure 2. As shown in this model, the phantom boundary crossing occurred [16, 17].

It is shown that inflation or the dark energy-dominated stage can be realized only by the effect of the torsion and trace energy-momentum tensor without the curvature. As a result, it can be interpreted that these models may be equivalent to the Kaluza-Klein and RS models without gravitational effects of the curvature but just due to those of the torsion and trace energy-momentum tensor in teleparallelism. Indeed, this is the new work on the concrete cosmological solutions to describe the cosmic accelerated expansion of the KK and RS models in $F(T,\Theta)$ gravity. Based on these results, it can be stated that phenomenological $F(T,\Theta)$ gravity models in the four-dimensional space-time can be derived from more fundamental theories. In this regard, the observational constraints on the derivative of $F(T)$ and similar function as $F(T,\Theta)$ until the fifth order were presented in [18] with cosmographic parameters acquired from the observational data of Supernovae Ia and the baryon acoustic oscillations. The results of the model presented in this work, as a concrete example of $F(T,\Theta)$ gravity models, are consistent with those obtained in [18].

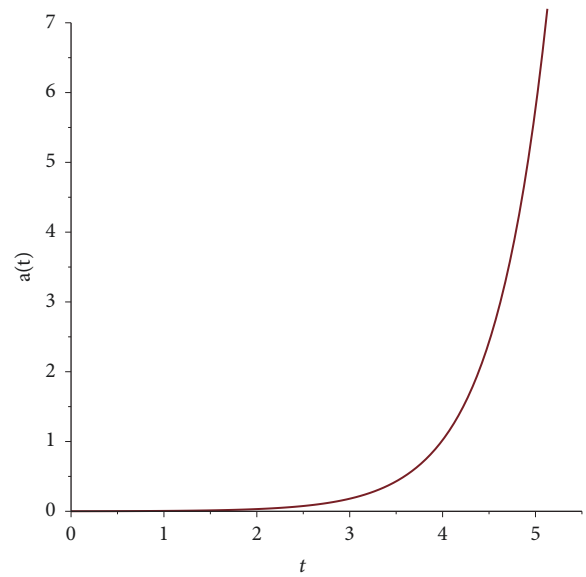

Figure 1: The universe scale factor for the inflation period (where $\alpha = 1, m = 1, n = 2$.)

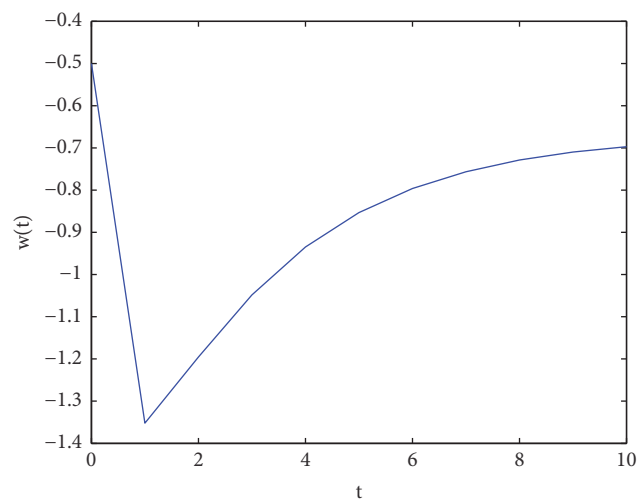

Figure 2: The equation of state $\omega(t)$ in terms of time for generalized RS model with $F(T,\Theta)$ for $\alpha = 1, m = 1, n = 2$.

4. Conclusion

In this paper, a generalized gravity model was proposed based on the time-space torsion and interaction with the universe content. The study of this gravity model in a five-dimensional space according to the Randall-Sundrum approach included a four-dimensional brane in a five-dimensional bulk. In this regard, the Kaluza-Klein theory was used to compress the fifth dimension of space in this gravity model. Then, in accordance with the inflation period of the standard cosmological model, the new critical value for the energy density of the brane, the Hubble parameter, and the scale factor were obtained. Finally, it has been illustrated that, in $F(T, \Theta)$ gravity, inflation in the early universe and the late-time cosmic acceleration can be realized.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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