Research Article

The Texture One-Zero Neutrino Mass Matrix with Vanishing Trace

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In the light of latest neutrino oscillation data, we have investigated the one-zero Majorana neutrino mass matrix M_ν with zero sum condition of mass eigenvalues in the flavor basis, where charged lepton mass matrix is diagonal. Among the six possible one-zero cases, it is found that only five can survive the current experimental data, while case with (1, 1) vanishing element of M_ν is ruled out, if zero trace condition is imposed at 3σ confidence level (CL). Numerical and some approximate analytical results are presented.

1. Introduction

The Double Chooz, Daya Bay, and RENO Collaborations [1–3] have finally established the nonzero and relatively large value of the reactor mixing angle θ_{13} ; hence the number of known available neutrino oscillation parameters approaches five, namely, two mass-squared differences ($\delta m^2, \Delta m^2$) and three neutrino mixing angles ($\theta_{12}, \theta_{23}, \theta_{13}$). However, any general 3×3 neutrino mass matrix contains more parameters than can be measured in realistic experiments. In fact, assuming the Majorana-type nature of neutrinos, the neutrino mass matrix contains nine real free parameters: three neutrino masses (m_1, m_2, m_3), three flavor mixing angles ($\theta_{12}, \theta_{23}, \theta_{13}$), and three CP violating phases (δ, ρ, σ).

In order to reduce the number of free parameters, several phenomenological ideas, in particular texture zeros [4–26], have been widely adopted in the literature. The imposition of texture zeros in neutrino mass matrix leads to some important phenomenological relations between flavor mixing angles and fermion mass ratios [24–27]. In the flavor basis, where charged lepton mass matrix is diagonal, at most two zeros are allowed in neutrino mass matrix, which are consistent with neutrino oscillation data [25, 26]. The analysis of two texture zero neutrino mass matrices limits the number of experimentally viable cases to seven. The phenomenological implications of one texture zero neutrino mass matrix have

also been studied in the literature [20–23] and it has been observed that all the six cases are viable with experimental data. However, the imposition of single texture zero condition in neutrino mass matrix makes larger parametric space for viability with the data available compared with two-zero texture. In order to impart predictability to one-zero texture, additional constraints in the form of vanishing determinant [28] or trace can be incorporated. The phenomenological implication of determinantless condition on one-zero texture have been rigorously studied in [20, 28–32]. The implication of traceless condition was first put forward in [33] wherein the anomalies of solar and atmospheric neutrino oscillation experiments as well as the LSND experiment were simultaneously explained in the framework of three neutrinos. In [34], Zee has particularly investigated the case of CP conserving traceless neutrino mass matrix for explaining the solar and atmospheric neutrino deficits. Further motivation of traceless mass matrices can be provided by models wherein neutrino mass matrix can be constructed through a commutator of two matrices, as what happens in models of radiative mass generation [35]. In [36], Alhendi et al. have studied the case of two trackless submatrices of Majorana mass matrix in the flavor basis and carried out a detailed numerical analysis at 3σ confidence level. The phenomenological implications of traceless neutrino mass matrix on neutrino masses, CP violating phases, and effective neutrino mass term are also

studied in [37], for both normal and inverted mass ordering and in case of CP conservation and violation, respectively. In the present work we impose the traceless condition on texture one-zero Majorana mass matrix and investigate the outcomes of such condition on the parametric space of neutrino masses (m_1, m_2, m_3) and CP violating phases (δ, ρ, σ).

Assuming the Majorana nature of neutrinos, neutrino mass matrix is complex symmetric. In the flavor basis, if one of the elements is considered to be zero, the number of possible cases turns out to be six, which are given below:

$$\begin{aligned}
 T_1: & \begin{pmatrix} 0 & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix}, \\
 T_2: & \begin{pmatrix} \times & \times & \times \\ \times & 0 & \times \\ \times & \times & \times \end{pmatrix}, \\
 T_3: & \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & 0 \end{pmatrix}; \\
 T_4: & \begin{pmatrix} \times & 0 & \times \\ 0 & \times & \times \\ \times & \times & \times \end{pmatrix}, \\
 T_5: & \begin{pmatrix} \times & \times & 0 \\ \times & \times & \times \\ 0 & \times & \times \end{pmatrix}, \\
 T_6: & \begin{pmatrix} \times & \times & \times \\ \times & \times & 0 \\ \times & 0 & \times \end{pmatrix},
 \end{aligned} \tag{1}$$

where “ $\times$ ” stands for nonzero element and complex matrix element.

Among these possible cases, there exists a permutation symmetry between certain pair of cases, namely, (T_2, T_3) and (T_4, T_5), while cases T_1 and T_6 transform onto themselves independently. The origin of permutation symmetry is explained from the fact that these pairs are related by exchange of 2-3 rows and 2-3 columns of neutrino mass matrix. The corresponding permutation matrix is given by

$$P_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \tag{2}$$

which leads to the following relations among the neutrino oscillation parameters:

$$\begin{aligned}
 \theta_{12}^X &= \theta_{12}^Y, \\
 \theta_{23}^X &= 90^\circ - \theta_{23}^Y,
 \end{aligned}$$

$$\theta_{13}^X = \theta_{13}^Y,$$

$$\delta^X = \delta^Y \pm 180^\circ, \tag{3}$$

where X and Y superscripts denote the cases related by 2-3 permutation symmetry.

The rest of the work is planned as follows: In Section 2, we discuss the methodology used to reconstruct the Majorana neutrino mass matrix and subsequently obtain some useful phenomenological relations of neutrino mass ratios and Majorana phases by incorporating texture one-zero and zero trace conditions simultaneously. In Section 3, we present the numerical analysis using some approximate analytical relations. In Section 4, we summarize our work.

2. Formalism

In the flavor basis, the Majorana neutrino mass matrix M_ν , depending on three neutrino masses (m_1, m_2, m_3) and the flavor mixing matrix can be expressed as

$$M_\nu = V \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} V^T. \tag{4}$$

The mixing matrix V can be written as $V = UP$, where U denotes the neutrino mixing matrix consisting of three flavor mixing angles and one Dirac-like CP violating phase, whereas the matrix P is a diagonal phase matrix; that is, $P = \text{diag}(e^{i\rho}, e^{i\sigma}, 1)$ with ρ and σ being the two Majorana CP violating phases. The neutrino mass matrix M_ν can then be rewritten as

$$M_\nu = \begin{pmatrix} M_{ee} & M_{e\mu} & M_{e\tau} \\ M_{e\mu} & M_{\mu\mu} & M_{\mu\tau} \\ M_{e\tau} & M_{\mu\tau} & M_{\tau\tau} \end{pmatrix} = U \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} U^T, \tag{5}$$

where $\lambda_1 = m_1 e^{2i\rho}$, $\lambda_2 = m_2 e^{2i\sigma}$, and $\lambda_3 = m_3$.

For the purpose of calculations, we have adopted the parameterization of the mixing matrix U considered by [5]; for example,

$$\begin{aligned}
 U &= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -c_{12}s_{23}s_{13} - s_{12}c_{23}e^{-i\delta} & -s_{12}s_{23}s_{13} + c_{12}c_{23}e^{-i\delta} & s_{23}c_{13} \\ -c_{12}c_{23}s_{13} + s_{12}s_{23}e^{-i\delta} & -s_{12}c_{23}s_{13} - c_{12}s_{23}e^{-i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{6}
 \end{aligned}$$

where $c_{ij} = \cos \theta_{ij}$, $s_{ij} = \sin \theta_{ij}$ for $i, j = 1, 2, 3$, and δ is the CP violating phase.

If one of the elements of M_ν is considered zero, that is, $M_{lm} = 0$, it leads to the following constraint equation:

$$U_{l1}U_{m1}\lambda_1 + U_{l2}U_{m2}\lambda_2 + U_{l3}U_{m3}\lambda_3 = 0, \tag{7}$$

where l, m run over e, μ , and τ .

TABLE 1: Current neutrino oscillation parameters from global fits at 1σ , 2σ , and 3σ confidence level [39]. No (IO) refers to normal (inverted) neutrino mass ordering.

Parameter	Best fit	1σ	2σ	3σ
δm^2 [10^{-5}eV^2]	7.60	7.42–7.79	7.26–7.99	7.11–8.18
$ \Delta m_{31}^2 $ [10^{-3}eV^2] (NO)	2.48	2.41–2.53	2.35–2.59	2.30–2.65
$ \Delta m_{31}^2 $ [10^{-3}eV^2] (IO)	2.38	2.32–2.43	2.26–2.48	2.20–2.54
θ_{12}	34.6°	33.6° – 35.6°	32.7° – 36.7°	31.8° – 37.8°
θ_{23} (NO)	48.9°	41.7° – 50.7°	40.0° – 52.1°	38.8° – 53.3°
θ_{23} (IO)	49.2°	46.9° – 50.7°	41.3° – 52.0°	39.4° – 53.1°
θ_{13} (NO)	8.6°	8.4° – 8.9°	8.2° – 9.1°	7.9° – 9.3°
θ_{13} (IO)	8.7°	8.5° – 8.9°	8.2° – 9.1°	8.0° – 9.4°
δ (NO)	254°	182° – 353°	0° – 360°	0° – 360°
δ (IO)	266°	210° – 322°	0° – $16^\circ \oplus 155^\circ$ – 360°	0° – 360°

The traceless condition implies that sum of the mass eigenvalues in neutrino mass matrix is zero; that is,

$$\lambda_1 + \lambda_2 + \lambda_3 = 0. \quad (8)$$

Using (7) and (8), we obtain

$$\begin{aligned} \frac{\lambda_1}{\lambda_3} &= \frac{U_{l2}U_{m2} - U_{l3}U_{m3}}{U_{l1}U_{m1} - U_{l2}U_{m2}}, \\ \frac{\lambda_2}{\lambda_3} &= \frac{U_{l3}U_{m3} - U_{l1}U_{m1}}{U_{l1}U_{m1} - U_{l2}U_{m2}}. \end{aligned} \quad (9)$$

The magnitudes of neutrino mass ratios are given by

$$\begin{aligned} \xi &\equiv \frac{m_1}{m_3} = \left| \frac{U_{l2}U_{m2} - U_{l3}U_{m3}}{U_{l1}U_{m1} - U_{l2}U_{m2}} \right|, \\ \zeta &\equiv \frac{m_2}{m_3} = \left| \frac{U_{l3}U_{m3} - U_{l1}U_{m1}}{U_{l1}U_{m1} - U_{l2}U_{m2}} \right|. \end{aligned} \quad (10)$$

Using (9), we find the following analytical relations for Majorana phases (ρ, σ):

$$\begin{aligned} \rho &= \frac{1}{2} \arg \left(\frac{U_{l2}U_{m2} - U_{l3}U_{m3}}{U_{l1}U_{m1} - U_{l2}U_{m2}} \right), \\ \sigma &= \frac{1}{2} \arg \left(\frac{U_{l3}U_{m3} - U_{l1}U_{m1}}{U_{l1}U_{m1} - U_{l2}U_{m2}} \right). \end{aligned} \quad (11)$$

Thus neutrino mass ratios (ξ, ζ) and two Majorana-type CP violating phases (ρ, σ) can fully be determined in terms of three mixing angles ($\theta_{12}, \theta_{23}, \theta_{13}$) and the Dirac-type CP violating phase (δ). The ratio of two neutrino mass-squared differences in terms of neutrino mass ratios ξ and ζ is given by

$$R_\nu = \frac{\delta m^2}{|\Delta m^2|} = \frac{2(\zeta^2 - \xi^2)}{|2 - (\zeta^2 + \xi^2)|}, \quad (12)$$

where $\delta m^2 = (m_2^2 - m_1^2)$ and $\Delta m^2 = |m_3^2 - (1/2)(m_1^2 + m_2^2)|$ [38] corresponds to solar and atmospheric neutrino squared differences, respectively. The sign of Δm^2 is still not known

experimentally; that is, $\Delta m^2 > 0$ or $\Delta m^2 < 0$ corresponds to the normal or inverted mass ordering of neutrinos.

The expressions for three neutrino masses (m_1, m_2, m_3) can be given as

$$\begin{aligned} m_3 &= \sqrt{\frac{\delta m^2}{(\zeta^2 - \xi^2)}}, \\ m_2 &= m_3 \zeta, \\ m_1 &= m_3 \xi. \end{aligned} \quad (13)$$

Thus the neutrino mass spectrum can be fully determined.

The expression for Jarlskog rephasing parameter J_{CP} , which is a measure of CP violation, is given by

$$J_{\text{CP}} = s_{12}c_{12}s_{23}c_{23}s_{13}^2 \sin \delta. \quad (14)$$

3. Numerical Results and Discussion

The experimental constraints on neutrino parameters at 1σ , 2σ , and 3σ confidence level (CL) are given in Table 1.

The effective Majorana mass term relevant for neutrino-less double beta ($0\nu\beta\beta$) decay is given by

$$|M|_{ee} = |m_1 c_{12}^2 c_{13}^2 e^{2i\rho} + m_2 s_{12}^2 c_{13}^2 e^{2i\sigma} + m_3 s_{13}^2|. \quad (15)$$

The future observation of $0\nu\beta\beta$ decay would imply lepton number violation and Majorana character of neutrinos. For recent reviews see [40–43]. There are a large number of projects such as CUORICINO [44], CUORE [45], GERDA [46], MAJORANA [47], SuperNEMO [48], EXO [49], and GENIUS [50] which target achieving a sensitivity up to 0.01 eV for $|M|_{ee}$. For the present analysis, we assume the upper limit on $|M|_{ee}$ to be less than 0.5 eV at 3σ CL [43]. The data collected from the Planck satellite [51] combined with other cosmological data put a limit on the sum of neutrino masses as

$$\Sigma = \sum_{i=1}^3 m_i < 0.23 \text{ eV} \quad \text{at } 95\% \text{ CL}. \quad (16)$$

Here, we take rather more conservative limit on sum of neutrino masses (Σ) (i.e., $\Sigma < 1 \text{ eV}$) at 3σ CL. We span

TABLE 2: The exact expression of neutrino mass ratios ξ and ζ of all the six one-zero textures with vanishing trace is shown. The symbols $c_{2(ij)} \equiv \cos 2\theta_{ij}$, $s_{2(ij)} \equiv \sin 2\theta_{ij}$ are defined.

Cases	Analytical expressions for ξ and ζ
T_1	$\xi = +\sec 2\theta_{12} (s_{12}^2 - t_{13}^2)$ $\zeta = -\sec 2\theta_{12} (c_{12}^2 - t_{13}^2)$
T_2	$\xi = \frac{(s_{12}^2 s_{13}^2 - c_{13}^2) s_{23}^2 + c_{12} c_{23} (c_{12} c_{23} e^{-i\delta} - 2s_{12} s_{23} s_{13}) e^{-i\delta}}{(s_{23}^2 s_{13}^2 - c_{23}^2 e^{-2i\delta}) c_{2(12)} + s_{2(12)} s_{2(23)} s_{13} e^{-i\delta}}$ $\zeta = \frac{(-c_{12}^2 s_{13}^2 + c_{13}^2) s_{23}^2 - s_{12} c_{23} (s_{12} c_{23} e^{-i\delta} + 2c_{12} s_{23} s_{13}) e^{-i\delta}}{(s_{23}^2 s_{13}^2 - c_{23}^2 e^{-2i\delta}) c_{2(12)} + s_{2(12)} s_{2(23)} s_{13} e^{-i\delta}}$
T_3	$\xi = \frac{(s_{12}^2 s_{13}^2 - c_{13}^2) c_{23}^2 + c_{12} s_{23} (c_{12} s_{23} e^{-i\delta} + 2s_{12} c_{23} s_{13}) e^{-i\delta}}{(c_{23}^2 s_{13}^2 - s_{23}^2 e^{-2i\delta}) c_{2(12)} - s_{2(12)} s_{2(23)} s_{13} e^{-i\delta}}$ $\zeta = \frac{(-c_{12}^2 s_{13}^2 + c_{13}^2) c_{23}^2 - s_{12} s_{23} (s_{12} s_{23} e^{-i\delta} - 2c_{12} c_{23} s_{13}) e^{-i\delta}}{(c_{23}^2 s_{13}^2 - s_{23}^2 e^{-2i\delta}) c_{2(12)} - s_{2(12)} s_{2(23)} s_{13} e^{-i\delta}}$
T_4	$\xi = \frac{-s_{12} c_{12} c_{23} e^{-i\delta} + s_{23} s_{13} (1 + s_{12}^2)}{s_{23} s_{13} (c_{12}^2 - s_{12}^2) + 2s_{12} c_{12} c_{23} e^{-i\delta}}$ $\zeta = \frac{-s_{12} c_{12} c_{23} e^{-i\delta} - s_{23} s_{13} (1 + c_{12}^2)}{s_{23} s_{13} (c_{12}^2 - s_{12}^2) + 2s_{12} c_{12} c_{23} e^{-i\delta}}$
T_5	$\xi = \frac{s_{12} c_{12} s_{23} e^{-i\delta} + c_{23} s_{13} (1 + s_{12}^2)}{c_{23} s_{13} (c_{12}^2 - s_{12}^2) - 2s_{12} c_{12} s_{23} e^{-i\delta}}$ $\zeta = \frac{s_{12} c_{12} s_{23} e^{-i\delta} - c_{23} s_{13} (1 + c_{12}^2)}{c_{23} s_{13} (c_{12}^2 - s_{12}^2) - 2s_{12} c_{12} s_{23} e^{-i\delta}}$
T_6	$\xi = \frac{s_{23} c_{23} (s_{12}^2 s_{13}^2 - c_{13}^2 - c_{12}^2 e^{-2i\delta}) - c_{12} s_{12} s_{13} (c_{23}^2 - s_{23}^2) e^{-i\delta}}{s_{23} c_{23} (s_{13}^2 + e^{-2i\delta}) c_{2(12)} e^{-i\delta} + 2s_{12} c_{12} s_{13} c_{2(23)} e^{-i\delta}}$ $\zeta = \frac{s_{23} c_{23} (-c_{12}^2 s_{13}^2 + c_{13}^2 + s_{12}^2 e^{-2i\delta}) - c_{12} s_{12} s_{13} (c_{23}^2 - s_{23}^2) e^{-i\delta}}{s_{23} c_{23} (s_{13}^2 + e^{-2i\delta}) c_{2(12)} e^{-i\delta} + 2s_{12} c_{12} s_{13} c_{2(23)} e^{-i\delta}}$

the parameter space of input neutrino oscillation parameters $(\theta_{12}, \theta_{23}, \theta_{13}, \delta m^2, \Delta m^2)$ by choosing the randomly generated points of the order of 10^{6-7} . Using Eq. (12), the parameter space of CP violating phases (δ, ρ, σ) , effective mass term $|M|_{ee}$, neutrino masses (m_1, m_2, m_3) can be subsequently constrained. In order to interpret the phenomenological results, some approximate analytical relations (up to certain leading order term of s_{13}) have been used in the following discussion. The exact analytical relations of neutrino mass ratios (ξ, ζ) have been provided in Table 2.

3.1. Case T_1 . Using (6) and (9), in the leading order term of θ_{13} , we obtain the following analytical relations:

$$\frac{\lambda_1}{\lambda_3} \approx \sec 2\theta_{12} s_{12}^2, \quad (17)$$

$$\frac{\lambda_2}{\lambda_3} \approx -\sec 2\theta_{12} c_{12}^2.$$

For NO, using (12), we obtain $R_\nu \approx \zeta^2 - \xi^2 \approx \sec 2\theta_{12}$. Using 3σ experimental range of oscillation parameters, we find $2.23 \leq R_\nu \leq 4.02$, which excludes the experimental range of R_ν and for IO we have

$$R_\nu \approx \frac{\sec 2\theta_{12}}{\sec^2 2\theta_{12} c_{12}^4 - 1}, \quad (18)$$

which is again inconsistent with current experimental data as $R_\nu > 0.75$. Therefore, Case T_1 is ruled out with the latest neutrino oscillation data at 3σ CL.

3.2. Case T_2 . Using (6) and (9), we obtain the following analytical relations in the leading order approximation of θ_{13} :

$$\frac{\lambda_1}{\lambda_3} \approx -\sec 2\theta_{12} (c_{12}^2 - t_{23}^2 e^{2i\delta}), \quad (19)$$

$$\frac{\lambda_2}{\lambda_3} \approx \sec 2\theta_{12} (s_{12}^2 - t_{23}^2 e^{2i\delta}).$$

From (19), one can obtain the neutrino mass ratios

$$\xi \approx \sec 2\theta_{12} \sqrt{c_{12}^4 + t_{23}^4 - 2c_{12}^2 t_{23}^2 \cos 2\delta}, \quad (20)$$

$$\zeta \approx \sec 2\theta_{12} \sqrt{s_{12}^4 + t_{23}^4 - 2s_{12}^2 t_{23}^2 \cos 2\delta}$$

and the Majorana CP violating phases

$$\rho \approx \frac{1}{2} \tan^{-1} \left(-\frac{t_{23}^2 \sin 2\delta}{c_{12}^2 - t_{23}^2 \cos 2\delta} \right) + O(s_{13}), \quad (21)$$

$$\sigma \approx \frac{1}{2} \tan^{-1} \left(-\frac{t_{23}^2 \sin 2\delta}{s_{12}^2 - t_{23}^2 \cos 2\delta} \right) + O(s_{13}).$$

The correlation plots for case T_2 have been compiled in Figures 1(a), 1(b), 1(c), and 1(d) and Figures 2(a), 2(b), 2(c),

FIGURE 1: Case T_2 (NO): scattering plots of Majorana phases, Dirac CP violating phase (δ), effective neutrino mass $|M|_{ee}$, and Jarlskog rephrasing invariant (J_{CP}) have been shown. All the phase angles (δ, ρ, σ) are measured in degrees and $|M|_{ee}$ is in eV unit.

and 2(d), respectively. It is found from the analysis that case T_2 favors both normal (NO) and inverted mass ordering (IO) at 3σ CL. The parameter space of CP violating phases δ, ρ, σ is found to be constrained to very small ranges for NO (Figures 1(a) and 1(b)). However, for IO, comparatively significant allowed parameter space is available for δ, ρ, σ (Figures 2(a) and 2(b)).

In the leading order approximation of s_{13} , the effective mass term in $0\nu\beta\beta$ decay turns out to be

$$|M|_{ee} \approx m_3 t_{23}^2 \approx 2.32 \times 10^{-2} \text{ eV}, \quad (22)$$

which lies well within the sensitivity limit of neutrinoless double beta decay experiments. Figures 1(c) and 2(c) show the correlation plot between $|M|_{ee}$ and δ for NO and IO, respectively. The Jarlskog rephrasing parameter J_{CP} is found to be nonvanishing for NO (Figure 1(d)); however, $J_{CP} = 0$ cannot be excluded for IO (Figure 2(d)).

3.3. Case T_3 . With the help of (6) and (9), we deduce the following analytical expressions in the leading order of s_{13} term.

$$\begin{aligned} \frac{\lambda_1}{\lambda_3} &\approx \sec 2\theta_{12} \left(c_{12}^2 - \frac{1}{t_{23}^2} e^{2i\delta} \right), \\ \frac{\lambda_2}{\lambda_3} &\approx \sec 2\theta_{12} \left(s_{12}^2 - \frac{1}{t_{23}^2} e^{2i\delta} \right). \end{aligned} \quad (23)$$

From (23), one can obtain the neutrino mass ratios

$$\begin{aligned} \xi &\approx \sec 2\theta_{12} \sqrt{c_{12}^4 + \frac{1}{t_{23}^4} - 2c_{12}^2 \frac{1}{t_{23}^2} \cos 2\delta}, \\ \zeta &\approx \sec 2\theta_{12} \sqrt{s_{12}^4 + \frac{1}{t_{23}^4} - 2s_{12}^2 \frac{1}{t_{23}^2} \cos 2\delta} \end{aligned} \quad (24)$$

FIGURE 2: Case T_2 (IO): scattering plots of Majorana phases, Dirac CP violating phase (δ), effective neutrino mass $|M|_{ee}$, and Jarlskog rephrasing invariant (J_{CP}) have been shown. All the phase angles (δ, ρ, σ) are measured in degrees and $|M|_{ee}$ is in eV unit.

and the Majorana CP violating phases

$$\begin{aligned}\rho &\approx \frac{1}{2} \tan^{-1} \left(-\frac{\sin 2\delta}{c_{12}^2 t_{23}^2 - \cos 2\delta} \right) + O(s_{13}), \\ \sigma &\approx \frac{1}{2} \tan^{-1} \left(-\frac{\sin 2\delta}{s_{12}^2 t_{23}^2 - t_{23}^2 \cos 2\delta} \right) + O(s_{13}).\end{aligned}\quad (25)$$

Cases T_2 and T_3 are related via permutation symmetry; therefore the phenomenological results for case T_3 can be obtained from case T_2 by using (3). The correlation plots for case T_3 have been compiled in Figures 3(a), 3(b), 3(c), and 3(d) (NO) and Figures 4(a), 4(b), 4(c), and 4(d) (IO), indicating the parameter space of $\rho, \sigma, \delta, |M|_{ee}, J_{CP}$.

3.4. Case T_4 . Using (6) and (9), we deduce the following analytical expressions in the leading order of s_{13} term:

$$\begin{aligned}\frac{\lambda_1}{\lambda_3} &\approx \frac{\lambda_2}{\lambda_3} \approx -0.5, \\ \xi &\approx \zeta \approx 0.5.\end{aligned}\quad (26)$$

Since $R_\nu = 0$ in the leading order approximation of s_{13} , we have to work next to leading order, and we get

$$\begin{aligned}\frac{\lambda_1}{\lambda_3} &\approx -\frac{1}{2} \left(1 - \frac{3}{2} \frac{s_{23}s_{13}}{c_{12}s_{12}c_{23}} e^{i\delta} \right) + O(s_{13}^2), \\ \frac{\lambda_2}{\lambda_3} &\approx -\frac{1}{2} \left(1 + \frac{3}{2} \frac{s_{23}s_{13}}{c_{12}s_{12}c_{23}} e^{i\delta} \right) + O(s_{13}^2).\end{aligned}\quad (27)$$

Using (27), the neutrino mass ratios can be given as

$$\begin{aligned}\xi &\approx \frac{1}{2} \sqrt{1 + \frac{9}{4} \frac{s_{23}^2 s_{13}^2}{c_{12}^2 s_{12}^2 c_{23}^2} - \frac{3s_{23}s_{13}}{c_{12}s_{12}c_{23}} \cos \delta}, \\ \zeta &\approx \frac{1}{2} \sqrt{1 + \frac{9}{4} \frac{s_{23}^2 s_{13}^2}{c_{12}^2 s_{12}^2 c_{23}^2} + \frac{3s_{23}s_{13}}{c_{12}s_{12}c_{23}} \cos \delta},\end{aligned}\quad (28)$$

FIGURE 3: Case T_3 (NO): scattering plots of Majorana phases, Dirac CP violating phase (δ), effective neutrino mass $|M|_{ee}$, and Jarlskog rephrasing invariant (J_{CP}) have been shown. All the phase angles (δ, ρ, σ) are measured in degrees and $|M|_{ee}$ is in eV unit.

and the Majorana CP violating phases are as

$$\begin{aligned}\rho &\approx \frac{1}{2} \tan^{-1} \left(\frac{3s_{23}s_{13} \sin \delta}{3s_{23}s_{13} \cos \delta - 2c_{12}s_{12}c_{23}} \right) + O(s_{13}^2), \\ \sigma &\approx \frac{1}{2} \tan^{-1} \left(\frac{3s_{23}s_{13} \sin \delta}{3s_{23}s_{13} \cos \delta + 2c_{12}s_{12}c_{23}} \right) + O(s_{13}^2).\end{aligned}\quad (29)$$

Using the best fit values from latest global fits on neutrino oscillation data (Table 1), the neutrino mass spectrum can be given as follows:

$$\begin{aligned}m_3 &= \sqrt{\frac{\delta m^2}{(\zeta^2 - \xi^2)}} \approx 4.87 \times 10^{-2} \text{ eV}, \\ m_2 &= m_3 \zeta \approx 2.85 \times 10^{-2} \text{ eV}, \\ m_1 &= m_3 \xi \approx 2.71 \times 10^{-2} \text{ eV},\end{aligned}\quad (30)$$

implying that only NO is allowed. Figures 5(a) and 5(b) show the correlation plot between Majorana phases (ρ, σ) and Dirac CP violating phase (δ). The parameter space for δ is found to be restricted near 90° and 270° . The prediction is significant considering the latest hint on δ near 270° in the recent global fits on neutrino oscillation data (Table 1). The Majorana phases (ρ, σ) are found to be constrained near -90° and 90° . In Figure 5(d), it is explicitly shown that J_{CP} is nonzero implying that case T_5 is necessarily CP violating.

In the leading order of s_{13} , the effective mass term in $0\nu\beta\beta$ decay can be approximated as

$$|M|_{ee} \approx \frac{m_3}{2} \approx 2.43 \times 10^{-2} \text{ eV}, \quad (31)$$

which is well within the accessible limit of next generation neutrinoless double decay experiments. The correlation plot between $|M|_{ee}$ and δ has been provided for case T_4 in Figure 5(c).

FIGURE 4: Case T_3 (IO): scattering plots of Majorana phases, Dirac CP violating phase (δ), effective neutrino mass $|M|_{ee}$, and Jarlskog rephrasing invariant (J_{CP}) have been shown. All the phase angles (δ, ρ, σ) are measured in degrees and $|M|_{ee}$ is in eV unit.

3.5. Case T_5 . With the help of (6) and (9), we obtain the following analytical expressions in the leading order of s_{13} term:

$$\begin{aligned} \frac{\lambda_1}{\lambda_3} &\approx \frac{\lambda_2}{\lambda_3} \approx 0.5, \\ \xi &\approx \zeta \approx 0.5. \end{aligned} \quad (32)$$

Since $R_\nu = 0$ in the leading order approximation of s_{13} , we have to work next to leading order, and we obtain

$$\begin{aligned} \frac{\lambda_1}{\lambda_3} &\approx -\frac{1}{2} \left(1 + \frac{3}{2} \frac{c_{23}s_{13}}{c_{12}s_{12}s_{23}} e^{i\delta} \right) + O(s_{13}^2), \\ \frac{\lambda_2}{\lambda_3} &\approx -\frac{1}{2} \left(1 - \frac{3}{2} \frac{c_{23}s_{13}}{c_{12}s_{12}s_{23}} e^{i\delta} \right) + O(s_{13}^2). \end{aligned} \quad (33)$$

From (33), the neutrino mass ratios can be given as follows:

$$\begin{aligned} \xi &\approx \frac{1}{2} \sqrt{1 + \frac{9}{4} \frac{c_{23}^2 s_{13}^2}{c_{12}^2 s_{12}^2 s_{23}^2} - \frac{3c_{23}s_{13}}{c_{12}s_{12}s_{23}} \cos \delta}, \\ \zeta &\approx \frac{1}{2} \sqrt{1 + \frac{9}{4} \frac{c_{23}^2 s_{13}^2}{c_{12}^2 s_{12}^2 s_{23}^2} + \frac{3c_{23}s_{13}}{c_{12}s_{12}s_{23}} \cos \delta}, \end{aligned} \quad (34)$$

and the Majorana CP violating phases are given as

$$\begin{aligned} \rho &\approx \frac{1}{2} \tan^{-1} \left(\frac{3c_{23}s_{13} \sin \delta}{3c_{23}s_{13} \cos \delta + 2c_{12}s_{12}s_{23}} \right) + O(s_{13}^2), \\ \sigma &\approx \frac{1}{2} \tan^{-1} \left(\frac{3c_{23}s_{13} \sin \delta}{3c_{23}s_{13} \cos \delta - 2c_{12}s_{12}s_{23}} \right) + O(s_{13}^2). \end{aligned} \quad (35)$$

FIGURE 5: Case T_4 (NO): scattering plots of Majorana phases, Dirac CP violating phase (δ), effective neutrino mass $|M|_{ee}$, and Jarlskog rephrasing invariant (J_{CP}) have been shown. All the phase angles (δ, ρ, σ) are measured in degrees and $|M|_{ee}$ is in eV unit.

Using the best fits from latest global neutrino oscillation data, the neutrino mass spectrum can be given as follows:

$$\begin{aligned}
 m_3 &= \sqrt{\frac{\delta m^2}{(\zeta^2 - \xi^2)}} \approx 4.87 \times 10^{-2} \text{ eV}, \\
 m_2 &= m_3 \zeta \approx 2.85 \times 10^{-2} \text{ eV}, \\
 m_1 &= m_3 \xi \approx 2.71 \times 10^{-2} \text{ eV},
 \end{aligned} \tag{36}$$

indicating that only NO is allowed. Since T_4 and T_5 are related due to permutation symmetry, therefore their phenomenological implications are similar. The phenomenological results for case T_5 can be derived from case T_4 using (3). The correlation plots for $\rho, \sigma, \delta, |M|_{ee}, J_{CP}$ have been compiled in Figure 6.

In the leading order of s_{13} term, the effective mass term in $0\nu\beta\beta$ decay can be approximated as

$$|M|_{ee} \approx \frac{m_3}{2} \approx 2.43 \times 10^{-2} \text{ eV}, \tag{37}$$

which lies within the sensitivity limits of future $0\nu\beta\beta$ decay experiments.

3.6. Case T_6 . With the help of (6) and (9), we deduce the following analytical expressions in the leading order of s_{13} term

$$\begin{aligned}
 \frac{\lambda_1}{\lambda_3} &\approx -\sec 2\theta_{12} (c_{12}^2 + e^{2i\delta}) e^{i\delta}, \\
 \frac{\lambda_2}{\lambda_3} &\approx \sec 2\theta_{12} (s_{12}^2 + e^{2i\delta}) e^{i\delta}.
 \end{aligned} \tag{38}$$

FIGURE 6: Case T_5 (NO): scattering plots of Majorana phases, Dirac CP violating phase (δ), effective neutrino mass $|M|_{ee}$, and Jarlskog rephrasing invariant (J_{CP}) have been shown. All the phase angles (δ, ρ, σ) are measured in degrees and $|M|_{ee}$ is in eV unit.

Using (38), we obtain the approximate relations for neutrino mass ratios

$$\begin{aligned}\xi &\approx \sec 2\theta_{12} \sqrt{c_{12}^4 + 2c_{12}^2 \cos 2\delta + 1}, \\ \zeta &\approx \sec 2\theta_{12} \sqrt{s_{12}^4 + 2s_{12}^2 \cos 2\delta + 1},\end{aligned}\quad (39)$$

and the Majorana CP violating phases

$$\begin{aligned}\rho &\approx \frac{1}{2} \tan^{-1} \left(\frac{\sin \delta (\cos 2\delta + c_{12}^2) + \sin 2\delta \cos \delta}{\cos \delta (\cos 2\delta + c_{12}^2) - \sin 2\delta \sin \delta} \right) \\ &\quad + O(s_{13}^2),\end{aligned}$$

$$\begin{aligned}\sigma &\approx \frac{1}{2} \tan^{-1} \left(\frac{\sin \delta (\cos 2\delta + s_{12}^2) + \sin 2\delta \cos \delta}{\cos \delta (\cos 2\delta + s_{12}^2) - \sin 2\delta \sin \delta} \right) \\ &\quad + O(s_{13}^2).\end{aligned}\quad (40)$$

The correlation plots for $\rho, \sigma, \delta, |M|_{ee}, J_{CP}$ have been compiled in Figures 7(a), 7(b), 7(c), and 7(d). In Figures 7(a) and 7(b) the parameter space of δ is found to be confined to very small regions. Also the Majorana phases ρ and σ also get restricted to -54.7° – $-44.7^\circ \oplus 45.3^\circ$ – 54.5° . However, $J_{CP} = 0$ as evident in Figure 7(d), which implies case T_6 points out the CP conservation.

From the analysis, out of six cases, only T_1 is found to be inconsistent with the experimental data for both normal and inverted mass ordering. For remaining cases the parameter space of CP violating phases (δ, σ, ρ), effective mass term $|M|_{ee}$, and neutrino masses (m_1, m_2, m_3) is found to be

TABLE 3: The allowed ranges of Dirac-like CP violating phase δ , the Majorana phases ρ, σ , and three neutrino masses m_1, m_2, m_3 for the experimentally viable cases at 3σ CL. Masses are in eV.

Cases	Normal mass ordering (NO)		Inverted mass ordering (IO)	
T_1	$\times$	$\times$	$\times$	$\times$
	$\times$	$\times$	$\times$	$\times$
	$\times$	$\times$	$\times$	$\times$
T_2	$\delta = 153.7^\circ - 158.4^\circ \oplus 204.1^\circ - 206.3^\circ$	$m_1 = 0.0680 - 0.311$	$\delta = 0^\circ - 21.06^\circ \oplus 126^\circ - 156^\circ$	$m_1 = 0.0425 - 0.357$
	$\rho = -64.9^\circ - -58.5^\circ \oplus 58.6^\circ - 64.3^\circ$	$m_2 = 0.0712 - 0.314$	$\oplus$	$m_2 = 0.0435 - 0.359$
	$\sigma = -64.5^\circ - -58.89^\circ \oplus 57.8^\circ - 64.3^\circ$	$m_3 = 0.0842 - 0.343$	$204^\circ - 233^\circ$	$m_3 = 0.00098 - 0.359$
T_3			$\oplus$	
			$340^\circ - 360^\circ$	
			$\rho = -60^\circ - 60^\circ$	
T_4			$\sigma = -90^\circ - -48.1^\circ \oplus 48.2^\circ - 90^\circ$	
	$\delta = 12.49^\circ - 25.9^\circ \oplus 334.9^\circ - 348.8^\circ$	$m_1 = 0.0291 - 0.690$	$\delta = 23.7^\circ - 50.48^\circ$	$m_1 = 0.0422 - 0.377$
	$\rho = -71.49^\circ - -60.3^\circ \oplus 59.36^\circ - 69.7^\circ$	$m_2 = 0.0307 - 0.690$	$\oplus$	$m_2 = 0.0435 - 0.377$
T_5	$\sigma = -73.34^\circ - -59.62^\circ \oplus 57.87^\circ - 72.7^\circ$	$m_3 = 0.0480 - 0.690$	$159.6^\circ - 201.2^\circ$	$m_3 = 0.00095 - 0.377$
			$\oplus$	
			$309.2^\circ - 338.1^\circ$	
T_6			$\rho = -60^\circ - 60^\circ$	
			$\sigma = -90^\circ - -48^\circ \oplus 48^\circ - 90^\circ$	
	$\delta = 86.68^\circ - 94.95^\circ \oplus 264.8^\circ - 273.3^\circ$	$m_1 = 0.0269 - 0.0406$	$\times$	$\times$
T_7	$\rho = -82.87^\circ - -1.29^\circ \oplus 69.57^\circ - 82.08^\circ$	$m_2 = 0.0280 - 0.0417$	$\times$	$\times$
	$\sigma = -82.87^\circ - -71.7^\circ \oplus 70.5^\circ - 82.08^\circ$	$m_3 = 0.0516 - 0.0682$	$\times$	$\times$
T_8	$\delta = 85.99^\circ - 94.48^\circ \oplus 266.45^\circ - 274.49^\circ$	$m_1 = 0.0269 - 0.0406$	$\times$	$\times$
	$\rho = -82.87^\circ - -72.22^\circ \oplus 71.88^\circ - 83.01^\circ$	$m_2 = 0.0280 - 0.0417$	$\times$	$\times$
	$\sigma = -82.87^\circ - -72.22^\circ \oplus 71.88^\circ - 83.01^\circ$	$m_3 = 0.0516 - 0.0682$	$\times$	$\times$
T_9	$\times$	$\times$	$\delta = 53.25^\circ - 68.5^\circ$	$m_1 = 0.0449 - 0.0664$
	$\times$	$\times$	$\oplus$	$m_2 = 0.0456 - 0.0672$
	$\times$	$\times$	$110.2^\circ - 125^\circ$	$m_3 = 0.0099 - 0.0400$
T_{10}	$\times$	$\times$	$\oplus$	
	$\times$	$\times$	$235.7^\circ - 250^\circ$	
	$\times$	$\times$	$\oplus$	
T_{11}	$\times$	$\times$	$291.3^\circ - 309^\circ$	
	$\times$	$\times$	$\rho = -54.73^\circ - -44.7^\circ \oplus 45.36^\circ - 54.5^\circ$	
	$\times$	$\times$	$\sigma = -54.73^\circ - -44.7^\circ \oplus 45.36^\circ - 54.5^\circ$	

constrained to an appreciable extent at 3σ CL. The allowed ranges of all the five viable cases for Dirac CP violating phase (δ), Majorana phases (ρ, σ), and neutrino masses (m_1, m_2, m_3) have been summarized in Table 3.

4. Summary and Conclusion

In the present work, we have systematically analyzed the texture one-zero Majorana mass matrix along with zero trace condition. In our analysis, we find that case T_1 with vanishing (1, 1) element of M_ν is ruled out with current experimental

data. Therefore, out of six possible cases of one-zero texture with zero trace, only five, namely, T_2, T_3, T_4, T_5 , and T_6 , can survive the current experimental tests at 3σ CL. The ongoing and future neutrino based experiments including neutrinoless double beta decay and cosmological experiments would test the validity of present texture zero analysis.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

FIGURE 7: Case T_6 (IO): scattering plots of Majorana phases, Dirac CP violating phase (δ), effective neutrino mass $|M|_{ee}$, and Jarlskog rephrasing invariant (J_{CP}) have been shown. All the phase angles (δ, ρ, σ) are measured in degrees and $|M|_{ee}$ is in eV unit.

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