Hindawi
Advances in High Energy Physics
Volume 2018, Article ID 2959354, 8 pages
https://doi.org/10.1155/2018/2959354

Research Article

The DKP Oscillator in Spinning Cosmic String Background

Mansoureh Hosseinpour and Hassan Hassanabadi

Faculty of Physics, Shahrood University of Technology, Shahrood, P.O. Box 3619995161-316, Iran

Correspondence should be addressed to Mansoureh Hosseinpour; hosseinpour.mansoureh@gmail.com

Received 11 June 2018; Revised 3 September 2018; Accepted 7 November 2018; Published 30 December 2018

Academic Editor: Diego Saez-Chillon Gomez

In this article, we investigate the behaviour of relativistic spin-zero bosons in the space-time generated by a spinning cosmic string. We obtain the generalized beta-matrices in terms of the flat space-time ones and rewrite the covariant form of Duffin-Kemmer-Petiau (DKP) equation in spinning cosmic string space-time. We find the solution of DKP oscillator and determine the energy levels. We also discuss the influence of the topology of the cosmic string on the energy levels and the DKP spinors.

1. Introduction

The Duffin-Kemmer-Petiau (DKP) equation has been used to describe relativistic spin-0 and spin-1 bosons [1–4]. The DKP equation has five- and ten-dimensional representation, respectively, for spin-0 and spin-1 bosons [5]. This equation is compared to the Dirac equation for fermions [6]. The DKP equation has been widely investigated in many areas of physics. The DKP equation has been investigated in the momentum space with the presence of minimal length [7, 8] and for spins 0 and 1 in a noncommutative space [9–12]. Also, the DKP oscillator has been studied in the presence of topological defects [13]. Recently, there has been growing interest in the so-called DKP oscillator [14–23] in particular in the background of a magnetic cosmic string [13]. The cosmic strings and other topological defects can form at a cosmological phase transition [24, 25]. The conical nature of the space-time around the string causes a number of interesting physical effects. Until now, some problems have been investigated in the gravitational fields of topological defects including the one-electron atom problem [26–28]. Spinning cosmic strings are similar usual cosmic string, characterized by an angular parameter α that depends on their linear mass density μ. The DKP oscillator is described by performing the nonminimal coupling with a linear potential. The name distinguishes it from the system called a DKP oscillator with Lorentz tensor couplings of [7–12, 14–16]. The DKP oscillator for spin-0 bosons has been investigated by Guo et al. in [10] in noncommutative phase space. The DKP oscillator with spin-0 has been studied by Yang et al. [11]. Exact solution of DKP oscillator in the momentum space with the presence of minimal length has been analysed in [8]. De Melo et al. construct the Galilean DKP equation for the harmonic oscillator in a noncommutative phase space [29]. Falek and Merad investigated the DKP oscillator of spins 0 and 1 bosons in noncommutative space [9]. Recently, there has been an increasing interest on the DKP oscillator [13–16, 22, 29–31]. The nonrelativistic limit of particle dynamics in curved space-time is considered in [32–36]. Also, the dynamics of relativistic bosons and fermions in curved space-time is considered in [17, 20, 31].

The influence of topological defect in the dynamics of bosons via DKP formalism has not been established for spinning cosmic strings. In this way, we consider the quantum dynamics of scalar bosons via DKP formalism embedded in the background of a spinning cosmic string. We solve DKP equation in presence of the spinning cosmic string space-time whose metric has off diagonal terms which involves time and space. The influence of this topological defect in the energy spectrum and DKP spinor presented graphically.

The structure of this paper is as follows: Section 2 describes the covariant form of DKP equation in a spinning cosmic string background. In Section 3, we introduce the DKP oscillator by performing the nonminimal coupling in this space-time, and we obtain the radial equations that are solved. We plotted the DKP spinor, density of probability, and

the energy spectrum for different conditions involving the deficit angle and the oscillator frequency. In the Section 4 we present our conclusions.

2. Covariant Form of the DKP Equation in the Spinning Cosmic String Background

We choose the cosmic string space-time background, where the line element is given by

$$ds^2 = -dT^2 + dX^2 + dY^2 + dZ^2 \tag{1}$$

The space-time generated by a spinning cosmic string without internal structure, which is termed ideal spinning cosmic string, can be obtain by coordinate transformation as

$$\begin{aligned} T &= t + a\alpha^{-1}\varphi \\ X &= r\cos(\varphi) \\ Y &= r\sin(\varphi) \\ \varphi &= \alpha\varphi' \end{aligned} \tag{2}$$

With this transformation, the line element ((1)) becomes [37–43]

$$\begin{aligned} ds^2 &= -(dt + ad\varphi)^2 + dr^2 + \alpha^2 r^2 d\varphi^2 + dz^2 \\ &= -dt^2 + dr^2 - 2adtd\varphi + \left(\alpha^2 r^2 - a^2\right) d\varphi^2 + dz^2 \end{aligned} \tag{3}$$

with $-\infty < z < \infty$, $\rho \geq 0$, and $0 \leq \varphi \leq 2\pi$. From this point on, we will take $c = 1$. The angular parameter α which runs in the interval $(0, 1]$ is related to the linear mass density μ of the string as $\alpha = 1 - 4\mu$ and corresponds to a deficit angle $\gamma = 2\pi(1-\alpha)$. We take $a = 4Gj$ where G is the universal gravitation constant and j is the angular momentum of the spinning string; thus a is a length that represents the rotation of the cosmic string. Note that, in this case, the source of the gravitational field relative to a spinning cosmic string possesses angular momentum and the metric (1) has an off diagonal term involving time and space.

The DKP equation in the cosmic string space-time (1) reads [13, 17, 31]

$$\left(\mathrm{i}\beta^{\mu}(x)\nabla_{\mu} - M\right)\Psi(x) = 0. \tag{4}$$

The covariant derivative in (4) is

$$\nabla_{\mu} = \partial_{\mu} + \Gamma_{\mu}(x) \tag{5}$$

where Γ_{μ} are the spinorial affine connections given by

$$\Gamma_{\mu} = \frac{1}{2}\omega_{\mu ab}\left[\beta^a, \beta^b\right]. \tag{6}$$

The matrices β^a are the standard Kemmer matrices in Minkowski space-time.

$$\beta^{\mu} = e^{\mu}_{a}\beta^{a} \tag{7}$$

The Kemmer matrices are an analogous to Dirac matrices in Dirac equation. There has been an increasing interest Dirac equation for spin half particles [44–47]. The matrices β^a satisfies the DKP algebra,

$$\beta^{\mu}\beta^{\nu}\beta^{\lambda} + \beta^{\lambda}\beta^{\nu}\beta^{\mu} = g^{\mu\nu}\beta^{\mu} + g^{\lambda\nu}\beta^{\mu}. \tag{8}$$

The conserved four-current is given by

$$J^{\mu} = \frac{1}{2}\overline{\Psi}\beta^{\mu}\Psi, \tag{9}$$

and the conservation law for J^{μ} takes the form

$$\nabla_{\mu}J^{\mu} + \frac{i}{2}\overline{\Psi}\left(U - \eta^0 U^{\dagger}\eta^0\right)\Psi = \frac{1}{2}\overline{\Psi}\left(\nabla_{\mu}\beta^{\mu}\right)\Psi \tag{10}$$

The adjoint spinor $\overline{\Psi}$ is defined as $\overline{\Psi} = \Psi^{\dagger}\eta^0$ with $\eta^0 = 2\beta^0\beta^0 - 1$, in such a way that $(\eta^0\beta^{\mu})^{\dagger} = \eta^0\beta^{\mu}$. The factor 1/2 which multiplies $\overline{\Psi}\beta^{\mu}\Psi$ is of no importance for the conservation law and ensures the charge density is compatible with the one used in the Klein-Gordon theory and its nonrelativistic limit. Thus, if U is Hermitian with respect to η^0 and the curved-space beta-matrices are covariantly constant, then the four-current will be conserved if [30]

$$\nabla_{\mu}\beta^{\mu} = 0. \tag{11}$$

The algebra expressed by these matrices generates a set of 126 independent matrices whose irreducible representations comprise a trivial representation, a five-dimensional representation describing the spin-zero particles, and a ten-dimensional representation associated with spin-one particles. We choose the 5×5 beta-matrices as follows [31]:

$$\begin{aligned} \beta^0 &= \begin{pmatrix} \theta & 0_{2\times3} \\ 0_{3\times2} & 0_{3\times3} \end{pmatrix}, \\ \vec{\beta} &= \begin{pmatrix} 0_{2\times2} & \vec{\tau} \\ -\vec{\tau}^{T} & 0_{3\times3} \end{pmatrix}, \\ \theta &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \\ \tau^1 &= \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\ \tau^2 &= \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\ \tau^3 &= \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix}. \end{aligned} \tag{12}$$

In (7), e^{μ}_{a} denote the tetrad basis that we can choose as

$$e^{\mu}_{a} = \begin{pmatrix} 1 & \dfrac{a\sin(\varphi)}{r\alpha} & \dfrac{-a\cos(\varphi)}{r\alpha} & 0 \\ 0 & \cos(\varphi) & \sin(\varphi) & 0 \\ 0 & \dfrac{-\sin(\varphi)}{r\alpha} & \dfrac{\cos(\varphi)}{r\alpha} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{13}$$

For the specific tetrad basis given by (13), we find from (7) that the curved-space beta-matrices read

$$\begin{aligned}
\beta^{(0)} &= e_a^t \beta^a = \beta^0 - \frac{a}{r\alpha}\beta^\varphi \\
\beta^{(1)} &= e_a^r \beta^a = \beta^r \\
\beta^r &= \cos\varphi\beta^1 + \sin\varphi\beta^2 \\
\beta^{(2)} &= e_a^2 \beta^a = \frac{\beta^\varphi}{r\alpha} \\
\beta^\varphi &= -\sin\varphi\beta^1 + \cos\varphi\beta^2 \\
\beta^{(3)} &= e_a^z \beta^a = \beta^3 = \beta^z
\end{aligned} \tag{14a}$$

and the spin connections are given by

$$\Gamma_\varphi = (1-\alpha)\left[\beta^1, \beta^2\right] \tag{15}$$

We consider only the radial component in the nonminimal substitution. Since the interaction is time-independent, one can write $\Psi(r,t) \propto e^{im\varphi}e^{ik_z z}e^{-iEt}\Phi(r)$, where E is the energy of the scalar boson, m is the magnetic quantum number, and k_z is the wave number. The five-component DKP spinor can be written as $\Phi^T = (\Phi_1, \Phi_2, \Phi_3, \Phi_4, \Phi_5)$, and the DKP equation (4) leads to the five

$$\begin{aligned}
&\big(r\alpha\left(-\mathrm{M}\Phi_1(r) + \mathrm{E}\Phi_2(r) + kz\Phi_5(r)\right) + \cos\varphi\big(a\mathrm{E} \\
&\quad + m)\,\Phi_4(r) - \mathrm{i}\left((-1+\alpha)\,\Phi_3(r) + r\alpha\Phi'_3(r)\right)\big) \\
&\quad - \sin\varphi\big((a\mathrm{E}+m)\,\Phi_3(r) \\
&\quad + \mathrm{i}\left((-1+\alpha)\,\Phi_4(r) + r\alpha\Phi'_4(r)\right)\big)\big) = 0 \\
&\left(\mathrm{E}\Phi_1(r) - \mathrm{M}\Phi_2(r)\right) = 0 \\
&\big((a\mathrm{E}+m)\sin\varphi\Phi_1(r) + r\alpha\big(-\mathrm{M}\Phi_3(r) + \mathrm{i} \\
&\quad \cdot \cos\phi\Phi'_1(r)\big)\big) = 0 \\
&\big(-(a\mathrm{E}+m)\cos\varphi\Phi_1(r) + r\alpha\big(-\mathrm{M}\Phi_4(r) + \mathrm{i} \\
&\quad \cdot \sin\varphi\Phi'_1(r)\big)\big) = 0 \\
&\left(kz + \mathrm{M}\psi\Phi_5(r)\right) = 0
\end{aligned} \tag{16}$$

Then we obtain the following equation of motion for the first component Φ_1 of the DKP spinor:

$$\begin{aligned}
&\left(-\frac{(a\mathrm{E}+m)^2}{\alpha^2 r^2} + \mathrm{E}^2 - \mathrm{kz}^2 - \mathrm{M}^2\right)\Phi_1(r) \\
&\quad + \frac{(\alpha-1)\,\Phi'_1(r)}{\alpha r} + \Phi''_1(r) = 0
\end{aligned} \tag{17}$$

$$\Phi_1 = r^{1/2\alpha}R_{n,\ell}(r)\,. \tag{18}$$

Then (17) changes to

$$\begin{aligned}
&R''_{n,m}(r) + \frac{R'_{n,m}(r)}{r} \\
&\quad + \left(E^2 - k_z{}^2 - M^2 - \frac{1+4\,(aE+m)^2}{4r^2\alpha^2}\right)R(r) \\
&\quad = 0
\end{aligned} \tag{19}$$

By the change of variable $r = x\eta$, we can write (19) in the form

$$R''_{n,m}(x) + \frac{R'_{n,m}(x)}{x} + \left(1 - \frac{\lambda^2}{x^2}\right)R(x) = 0 \tag{20}$$

where $\lambda = ((1+4(aE+m)^2)/4\alpha^2)^{1/2}$ and $\eta = (E^2 - k_z{}^2 - M^2)^{-1/2}$. The physical solution of (20) is *Bessel J* function. Therefore the general solution to (20) is given by

$$R(r) = A_{\lambda\eta}J_\lambda\left(\frac{r}{\eta}\right) + B_{\lambda\eta}Y_\lambda\left(\frac{r}{\eta}\right) \tag{21}$$

where $Y_\lambda(r/\eta)$ is the Bessel function of the second kind. Sometimes this family of functions is also called Neumann functions or Weber functions. $J_\lambda(r/\eta)$ is the Bessel function of the first kind, given by

$$J_\nu(x) = \sum_{k=0}^{\infty}\frac{(-1)^k}{k!\Gamma(k+\nu+1)}\left(\frac{x}{2}\right)^{\nu+2k} \tag{22}$$

and $Y_\lambda(r/\eta)$ is the Bessel function of the second kind, given by

$$Y_\nu(x) = \frac{J_\nu(x)\cos(\nu\pi) - J_{-\nu}(x)}{\sin(\nu\pi)} \tag{23}$$

By considering the boundary condition for (21) such that $B_{\lambda\eta} = 0$, we find

$$R(r) = A_{\lambda\eta}J_\lambda\left(\frac{r}{\eta}\right) \tag{24}$$

3. The DKP Oscillator in Spinning Cosmic String Background

The DKP oscillator is introduced via the nonminimal substitution [17, 30, 31]

$$\frac{1}{i}\vec{\nabla}_\alpha \longrightarrow \frac{1}{i}\vec{\nabla}_\alpha - iM\omega\eta_0\vec{r} \tag{25}$$

where ω is the oscillator frequency, M is the mass of the boson already found in (4), and $\vec{\nabla}$ is defined in (5). We consider

only the radial component in the nonminimal substitution. The DKP equation (4) leads to the five equations:

$$\begin{aligned}
&\left(r\alpha\left(-M\Phi_1(r)+E\Phi_2(r)+kz\Phi_5(r)\right)+\left(\cos\varphi\,(aE\right.\right.\\
&\quad\left.+m)\,\Phi_4(r)\right)+\left(\mathrm{i}\cos\varphi\left(\Phi_3(r)\right.\right.\\
&\quad\left.\left.+\alpha\left(-1+r^2\mathrm{M}\omega\right)\Phi_3(r)-r\alpha\Phi'_3(r)\right)\right)\\
&\quad-\sin\varphi\left((aE+m)\,\Phi_3(r)\right.\\
&\quad\left.-\mathrm{i}\left(1-\alpha+r^2\alpha\mathrm{M}\omega\right)\Phi_4(r)+\mathrm{i}r\alpha\Phi'_4(r)\right)\Big)=0\\
&\left((aE+m)\sin\varphi\Phi_1(r)-Mr\alpha\Phi_3(r)+\mathrm{i}r\alpha\right.\\
&\quad\left.\cdot\cos\varphi\left(r\mathrm{M}\omega\Phi_1(r)+\Phi'_1(r)\right)\right)=0\\
&\left(-(aE+m)\cos\varphi\Phi_1(r)-Mr\alpha\Phi_4(r)+\mathrm{i}r\alpha\right.\\
&\quad\left.\cdot\sin\varphi\left(r\mathrm{M}\omega\Phi_1(r)+\Phi'_1(r)\right)\right)=0\\
&\left(E\Phi_1(r)-M\Phi_2(r)\right)=0\\
&\left(kz\Phi_1(r)+\mathrm{M}\Phi_4(r)\right)=0
\end{aligned}\tag{26}$$

By solving the above system of (26) in favour of Φ_1 we get

$$\begin{aligned}
\Phi_2(r)&=\frac{E}{M}\Phi_1(r)\\
\Phi_5(r)&=-\frac{k_z}{M}\Phi_1(r)\\
\Phi_4(r)&=\frac{-aE\cos\varphi\Phi_1(r)-m\sin\varphi\Phi_1(r)+\mathrm{i}\left(r^2\alpha\mathrm{M}\omega\sin\varphi\Phi_1(r)+r\alpha\sin\varphi\Phi'_1(r)\right)}{Mr\alpha}\\
\Phi_3(r)&=\frac{aE\sin\varphi\Phi_1(r)+m\sin\varphi\Phi_1(r)+\mathrm{i}\left(r^2\alpha\mathrm{M}\omega\cos\varphi\Phi_1(r)+r\alpha\cos\varphi\Phi'_1(r)\right)}{Mr\alpha}
\end{aligned}\tag{27}$$

Combining these results we obtain (23) of motion for the first component of the DKP spinor:

$$\begin{aligned}
&\Phi''_1(r)+\frac{(-1+\alpha)\,\Phi'_1(r)}{r\alpha}+\left(E^2-kz^2-M^2\right.\\
&\quad\left.+2M\omega-\frac{M\omega}{\alpha}-\frac{(aE+m)^2}{r^2\alpha^2}-r^2\mathrm{M}^2\omega^2\right)\Phi_1(r)\\
&\quad=0
\end{aligned}\tag{28}$$

Let us take Φ_1 as

$$\Phi_1=r^{1/2\alpha}R_{n,\ell}(r)\,.\tag{29}$$

Then (28) changes to

$$\begin{aligned}
&R''_{n,m}(r)+\frac{R'_{n,m}(r)}{r}+\left(E^2-k_Z{}^2-M^2+2M\omega\right.\\
&\quad\left.-\frac{M\omega}{\alpha}-\frac{1+4\,(aE+m)^2}{4r^2\alpha^2}-r^2\mathrm{M}^2\omega^2\right)R_{n,m}(r)=0
\end{aligned}\tag{30}$$

In order to solve the above equation, we employ the change of variable: $s=r^2$; thus we rewrite the radial equation (34) in the form

$$\begin{aligned}
&R_{n,m}{}''(s)+\frac{1}{s}R_{n,m}{}'(s)+\frac{1}{s^2}\left(-\xi_1s^2+\xi_2s-\xi_3\right)R_{n,m}(r)\\
&\quad=0
\end{aligned}\tag{31}$$

If we compare with this second-order differential equation with the Nikiforov-Uvarov (NU) form, given in (A.1) of Appendix, we see that

$$\begin{aligned}
\xi_1&=\frac{M^2\omega^2}{4}\\
\xi_2&=\frac{1}{4}\left(E^2-kz^2-M^2+2M\omega-\frac{M\omega}{\alpha}\right),\\
\xi_3&=\frac{1-\alpha^2+4\,(aE+m)^2}{16\alpha^2}
\end{aligned}\tag{32}$$

which gives the energy levels of the relativistic DKP equation from

$$(2n+1)\sqrt{\xi_1}-\xi_2+2\sqrt{\xi_3\xi_1}=0,\tag{33}$$

where

$$\begin{aligned}
\alpha_1&=1,\\
\alpha_2&=\alpha_3=\alpha_4=\alpha_5=0,\\
\alpha_6&=\xi_1,\\
\alpha_7&=-\xi_2,\\
\alpha_8&=\xi_3,\\
\alpha_9&=\xi_1,\\
\alpha_{10}&=1+2\sqrt{\xi_3},
\end{aligned}$$

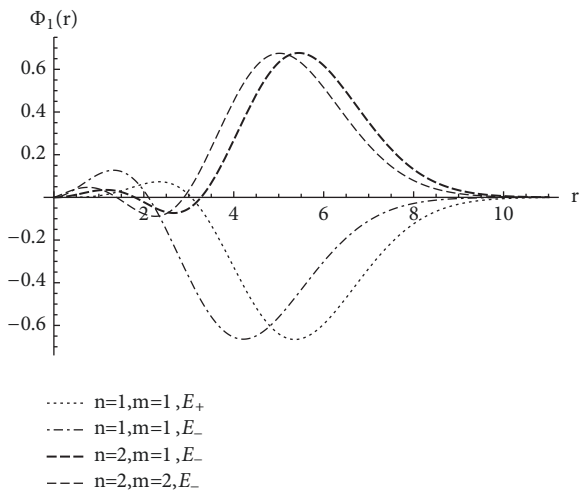

Figure 1: The wave function Φ_1 for $n = 1, 2$ and $0.0 \leq r \leq 10.0$ GeV $^{-1}$, with the parameters $M = 1$ GeV, $\alpha = 0.9$, $\omega = 0.25$, and $m = k_z = a = 1$.

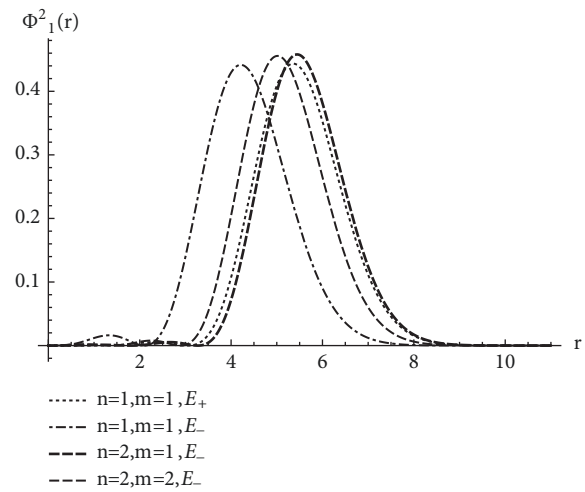

Figure 2: Density of probability $|\Phi_1|^2$ for $n = 1, 2$ and $0.0 \leq r \leq 10.0$ GeV $^{-1}$, with the parameters $M = 1$ GeV, $\alpha = 0.9$, $\omega = 0.25$, and $m = k_z = a = 1$.

$$\alpha_{11} = 2\sqrt{\xi_1}$$
$$\alpha_{12} = \sqrt{\xi_3},$$
$$\alpha_{13} = -\sqrt{\xi_1}. \tag{34}$$

As the final step, it should be mentioned that the corresponding wave function is

$$R_{n,m}(r) = N r^{2\alpha_{12}} e^{\alpha_{13} r^2} L_n^{\alpha_{10}-1}\left(\alpha_{11} r^2\right). \tag{35}$$

where N is the normalization constant. In limit a$\longrightarrow$ 0 we have the usual metric in cylindrical coordinates where described by the line element

$$ds^2 = -dt^2 + dr^2 + \alpha^2 r^2 d\varphi^2 + dz^2 \tag{36}$$

as pointed out by authors in [17] dynamic of DKP oscillator in the presence of this metric describe by

$$\varphi''_1(r) + \frac{\alpha - 1}{\alpha r}\varphi'_1(r) + \left(E^2 - M^2 - kz^2 + \frac{(2\alpha - 1) M\omega}{\alpha} - \frac{m^2}{\alpha^2 r^2} - M^2\omega^2 r^2\right)\varphi_1(r) = 0 \tag{37}$$

and the corresponding wave function is

$$\varphi(r) = N r^{2A} e^{Br^2} L_n^{C-1}\left(Dr^2\right) \tag{38}$$

where A,B,C, and D are constant and L_n^{C-1} denotes the generalized Laguerre polynomial. In Figure 1, $\Phi_1(r)$ is plotted versus r for different quantum number with the parameters listed under it. The density of probability $|\Phi_1|^2$ is shown in

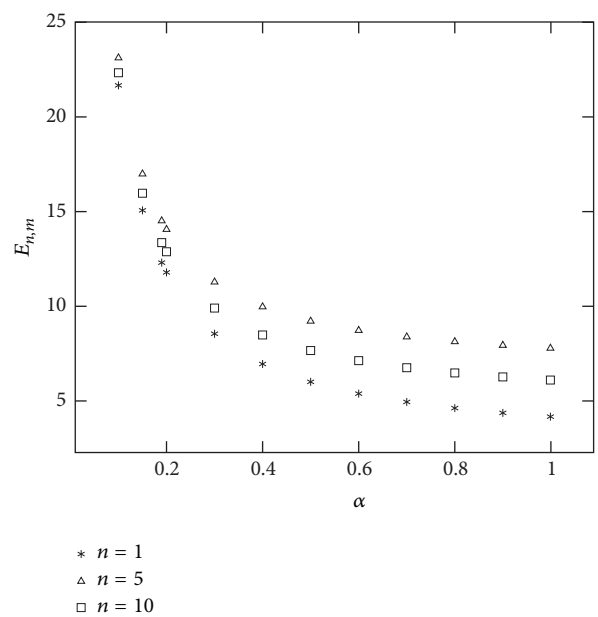

Figure 3: The energy as a function of α with $M = m = a = \omega = k_z = 1$.

Figure 2. The negative and positive solution of energy versus α is shown in Figures 3 and 4 for $n = 1, 5$ and 10. As in Figures 3 and 4, we observe that the absolute value of energy decreases with α. Also in Figure 5 energy is plotted versus ω for quantum numbers. We see that absolute value of energy increases with ω. The negative and positive solution of energy versus n is shown in Figure 6 for different parameter α. We obtained the energy levels of the DKP oscillator in that background and observed that the energy increases with the level number. In Figure 7, energy is plotted versus a for different quantum numbers. We see energy increases with parameter a. Also we observed that the energy levels of the

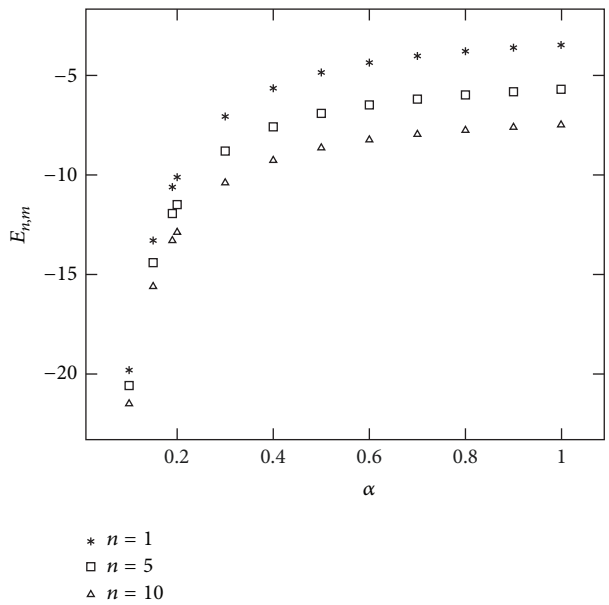

Figure 4: The energy as a function of α with $M = m = a = \omega = k_z = 1$.

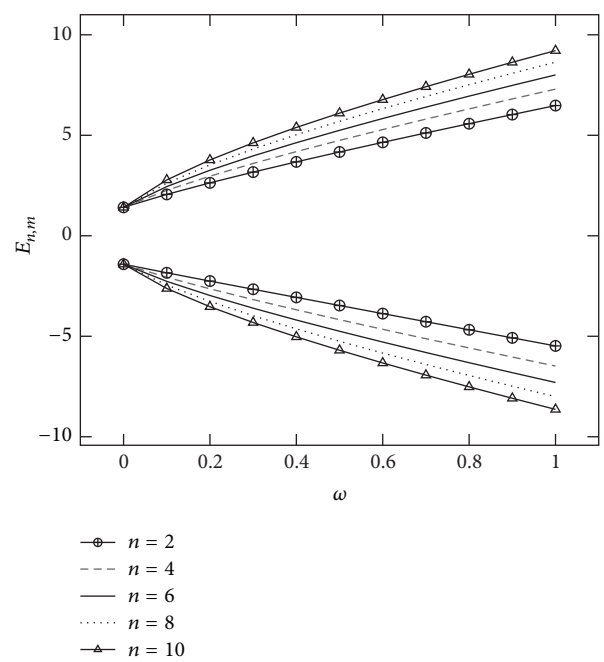

Figure 5: The energy as a function of ω with $M = 1$, $k_z = 1$, $m = 1$, $a = 1$, and $\alpha = 0.5$.

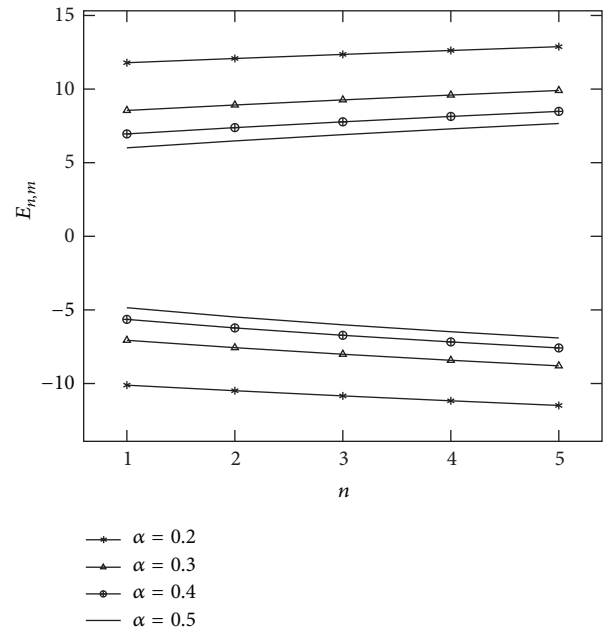

Figure 6: The energy as a function of n with $M = k_z = m = 1 = \omega = 1$, and $\alpha = 0.5$.

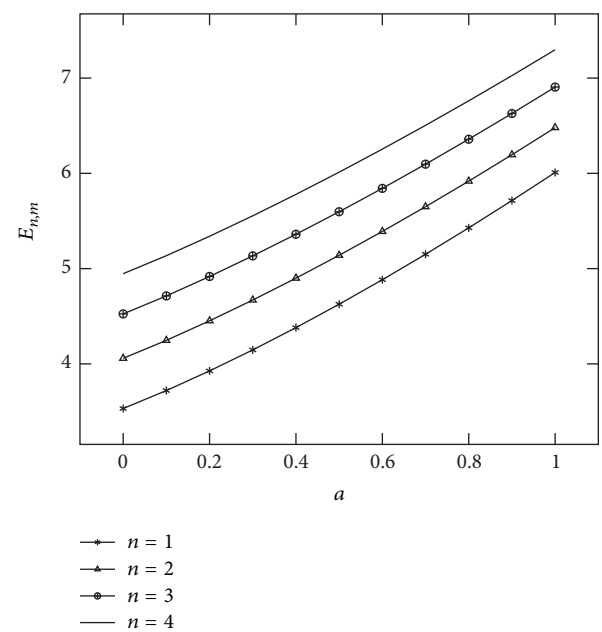

Figure 7: The energy as a function of a with $M = m = \omega = k_z = 1$, and $\alpha = 0.5$.

DKP oscillator in that background increases with the level number.

4. Conclusion

The overall objective of this paper is the study of the relativistic quantum dynamics of a DKP oscillator field for spin-0 particle in the spinning cosmic string space-time. The line element in this background is obtained by coordinate transformation of Cartesian coordinate. The metric has off diagonal terms which involves time and space. We considered the covariant form of DKP equation in the spinning cosmic string background and obtained the solutions of DKP equation for spin-0 bosons. Second we introduced DKP oscillator via the nonminimal substitution and considered DKP oscillator in that background. From the corresponding DKP equation, we obtained a system of five equations. By

combining the results of this system we obtained a second-order differential equation for first component of DKP spinor that the solutions are Laguerre polynomials. We see that the results are dependent on the linear mass density of the cosmic string. In the limit case of $a = 0$ and $\alpha = 1$, i.e., in the absence of a topological defect, recover the general solution for flat space- time. We plotted $\Phi_1(r)$, for $n = 1, 2$. We examined the behaviour of the density of probability $|\Phi_1|^2$. We observed that $|\Phi_1|^2$ for any parameter by increasing r have a very small peak at beginning and then have a taller peak and then by increasing r it tends to zero. We obtained the behaviour of energy spectrum as a function of α. We see that the absolute value of energy decreases as α increasing.

Appendix

Nikiforov-Uvarov (NU) Method

The Nikiforov-Uvarov method is helpful in order to find eigenvalues and eigenfunctions of the Schrödinger equation, as well as other second-order differential equations of physical interest. More details can be found in [48, 49]. According to this method, the eigenfunctions and eigenvalues of a second- order differential equation with potential are

$$\Phi''(s) + \frac{\alpha_1 - \alpha_2 s}{s(1-\alpha_3 s)}\Phi'(s) + \frac{1}{(s(1-\alpha_3 s))^2}\left(-\xi_1 s^2 + \xi_2 s - \xi_3\right)\Phi(s) = 0 \tag{A.1}$$

According to the NU method, the eigenfunctions and eigenenergies, respectively, are

$$\Phi(s) = s^{\alpha_{12}}(1-\alpha_3 s)^{-\alpha_{12}-(\alpha_{13}/\alpha_3)} \cdot P_n^{(\alpha_{10}-1,(\alpha_{11}/\alpha_3)-\alpha_{10}-1)}(1-2\alpha_3 s) \tag{A.2}$$

and

$$\alpha_2 n - (2n+1)\alpha_5 + (2n+1)\left(\sqrt{\alpha_9} + \alpha_3\sqrt{\alpha_8}\right) + n(n-1)\alpha_3 + \alpha_7 + 2\alpha_3\alpha_8 + 2\sqrt{\alpha_8\alpha_9} = 0 \tag{A.3}$$

where

$$\begin{aligned}
\alpha_4 &= \frac{1}{2}(1-\alpha_1),\\
\alpha_5 &= \frac{1}{2}(\alpha_2 - 2\alpha_3),\\
\alpha_6 &= \alpha_5^2 + \xi_1,\\
\alpha_7 &= 2\alpha_4\alpha_5 - \xi_2,\\
\alpha_8 &= \alpha_4^2 + \xi_3,\\
\alpha_9 &= \alpha_3\alpha_7 + \alpha_3^2\alpha_8 + \alpha_6,\\
\alpha_{10} &= \alpha_1 + 2\alpha_4 + 2\sqrt{\alpha_8},\\
\alpha_{11} &= \alpha_2 - 2\alpha_5 + 2\left(\sqrt{\alpha_9} + \alpha_3\sqrt{\alpha_8}\right),\\
\alpha_{12} &= \alpha_4 + \sqrt{\alpha_8},\\
\alpha_{13} &= \alpha_5 - \left(\sqrt{\alpha_9} + \alpha_3\sqrt{\alpha_8}\right)
\end{aligned} \tag{A.4}$$

In the rather more special case of $\alpha = 0$,

$$\begin{aligned}
\lim_{\alpha_3\longrightarrow 0} P_n^{(\alpha_{10}-1,(\alpha_{11}/\alpha_3)-\alpha_{10}-1)}(1-2\alpha_3 s) &= L_n^{\alpha_{10}-1}(\alpha_{11}s)\\
\lim_{\alpha_3\longrightarrow 0}(1-\alpha_3 s)^{-\alpha_{12}-(\alpha_{13}/\alpha_3)} &= e^{\alpha_{13}s}
\end{aligned} \tag{A.5}$$

and, from (14a), we find for the wave function

$$\Phi(s) = s^{\alpha_{12}} e^{\alpha_{13}s} L_n^{\alpha_{10}-1}(\alpha_{11}s) \tag{A.6}$$

where $L_n^{\alpha_{10}-1}$ denotes the generalized Laguerre polynomial.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

References

[1] N. Kemmer, "Quantum theory of einstein-bose particles and nuclear interaction," *Proceedings of the Royal Society A Mathematical, Physical and Engineering Sciences*, vol. 166, no. 924, pp. 127–153, 1938.

[2] R. J. Duffin, "On the characteristic matrices of covariant systems," *Physical Review A: Atomic, Molecular and Optical Physics*, vol. 54, no. 12, article no. 1114, 1938.

[3] N. Kemmer, "The particle aspect of meson theory," *Proceedings of the Royal Society A Mathematical, Physical and Engineering Sciences*, vol. 173, no. 952, pp. 91–116, 1939.

[4] G. Petiau, University of Paris, Thesis (1936). Published in Acad. R. Belg. Cl. Sci. Mem. Collect. 8 16 (2) (1936).

[5] E. M. Corson, *Introduction to Tensors, Spinors Relativistic Wave Equations*, Chelsea Publishing, 1953.

[6] W. Greiner, *Relativistic Quantum Mechanics*, Springer, Berlin, Germany, 2000.

[7] M. Falek and M. Merad, "Bosonic oscillator in the presence of minimal length," *Journal of Mathematical Physics*, vol. 50, no. 2, Article ID 023508, 2009.

[8] M. Falek and M. Merad, "A generalized bosonic oscillator in the presence of a minimal length," *Journal of Mathematical Physics*, vol. 51, no. 3, Article ID 033516, 2010.

[9] M. Falek and M. Merad, "DKP oscillator in a noncommutative space," *Communications in Theoretical Physics*, vol. 50, no. 3, pp. 587–592, 2008.

[10] G. Guo, C. Long, Z. Yang, and S. Qin, "DKP oscillator in noncommutative phase space," *Canadian Journal of Physics*, vol. 87, no. 9, pp. 989–993, 2009.

[11] Z.-H. Yang, C.-Y. Long, S.-J. Qin, and Z.-W. Long, "DKP oscillator with spin-0 in three-dimensional noncommutative phase space," *International Journal of Theoretical Physics*, vol. 49, no. 3, pp. 644–651, 2010.

[12] H. Hassanabadi, Z. Molaee, and S. Zarrinkamar, "DKP oscillator in the presence of magnetic field in (1+2)-dimensions for spin-zero and spin-one particles in noncommutative phase space," *The European Physical Journal C*, vol. 72, no. 11, article no. 2217, 2012.

[13] L. B. Castro, "Quantum dynamics of scalar bosons in a cosmic string background," *The European Physical Journal C*, vol. 75, no. 6, article no. 287, 2015.

[14] N. Debergh, J. Ndimubandi, and D. Strivay, "On relativistic scalar and vector mesons with harmonic oscillator - like interactions," *Zeitschrift für Physik C Particles and Fields*, vol. 56, pp. 421–425, 1992.

[15] Y. Nedjadi and R. C. Barrett, "The Duffin-Kemmer-Petiau oscillator," *Journal of Physics A: Mathematical and General*, vol. 27, no. 12, pp. 4301–4315, 1994.

[16] Y. Nedjadi, S. Ait-Tahar, and R. C. Barrett, "An extended relativistic quantum oscillator for $S = 1$ particles," *Journal of Physics A: Mathematical and General*, vol. 31, no. 16, pp. 3867–3874, 1998.

[17] M. Hosseinpour, H. Hassanabadi, and F. M. Andrade, "The DKP oscillator with a linear interaction in the cosmic string spacetime," *The European Physical Journal C*, vol. 78, no. 2, p. 93, 2018.

[18] A. Boumali and L. Chetouani, "Exact solutions of the Kemmer equation for a Dirac oscillator," *Physics Letters A*, vol. 346, no. 4, pp. 261–268, 2005.

[19] I. Boztosun, M. Karakoc, F. Yasuk, and A. Durmus, "Asymptotic iteration method solutions to the relativistic Duffin-Kemmer-Petiau equation," *Journal of Mathematical Physics*, vol. 47, no. 6, Article ID 062301, 2006.

[20] M. de Montigny, M. Hosseinpour, and H. Hassanabadi, "The spin-zero Duffin-Kemmer-Petiau equation in a cosmic-string space-time with the Cornell interaction," *International Journal of Modern Physics A*, vol. 31, no. 36, Article ID 1650191, 2016.

[21] F. Yasuk, M. Karakoc, and I. Boztosun, "The relativistic Duffin–Kemmer–Petiau sextic oscillator," *Physica Scripta*, vol. 78, no. 4, Article ID 045010, 2008.

[22] A. Boumali, "On the eigensolutions of the one-dimensional Duffin-Kemmer-Petiau oscillator," *Journal of Mathematical Physics*, vol. 49, no. 2, Article ID 022302, 2008.

[23] Y. Kasri and L. Chetouani, "Energy spectrum of the relativistic Duffin-Kemmer-Petiau equation," *International Journal of Theoretical Physics*, vol. 47, no. 9, pp. 2249–2258, 2008.

[24] A. Vilenkin and E. P. Shellard, *Cosmic Strings and Other Topological Defects*, Cambridge University Press, Cambridge, UK, 1994.

[25] A. Vilenkin, "Cosmic strings and domain walls," *Physics Reports*, vol. 121, no. 5, pp. 263–315, 1985.

[26] N. G. Marchuknmarchuk, "Dirac equation in Riemannian space without tetrads," *Il Nuovo Cimento B*, vol. 115, no. 11, 2000.

[27] L. D. Landau and E. M. Lifshitz, *Quantum Mechanics, Non-Relativistic Theory*, Pergamon, New York, USA, 1977.

[28] G. de A. Marques and V. B. Bezerra, "Hydrogen atom in the gravitational fields of topological defects," *Physical Review D: Covering Particles, Fields, Gravitation and Cosmology*, vol. 66, no. 10, Article ID 105011, 2002.

[29] G. R. de Melo, M. de Montigny, and E. S. Santos, "Spinless Duffin-Kemmer-Petiau oscillator in a Galilean non-commutative phase space," *Journal of Physics: Conference Series*, vol. 343, Article ID 012028, 2012.

[30] L. B. Castro, "Noninertial effects on the quantum dynamics of scalar bosons," *The European Physical Journal C*, vol. 76, no. 2, article no. 61, 2016.

[31] H. Hassanabadi, M. Hosseinpour, and M. de Montigny, "Duffin-Kemmer-Petiau equation in curved space-time with scalar linear interaction," *The European Physical Journal Plus*, vol. 132, no. 12, p. 541, 2017.

[32] R. Bausch, R. Schmitz, and L. A. Turski, "Single-Particle Quantum States in a Crystal with Topological Defects," *Physical Review Letters*, vol. 80, no. 11, pp. 2257–2260, 1998.

[33] E. Aurell, "Torsion and electron motion in quantum dots with crystal lattice dislocations," *Journal of Physics A: Mathematical and General*, vol. 32, no. 4, article no. 571, 1999.

[34] C. R. Muniz, V. B. Bezerra, and M. S. Cunha, "Landau quantization in the spinning cosmic string spacetime," *Annals of Physics*, vol. 350, pp. 105–111, 2014.

[35] V. B. Bezerra, "Global effects due to a chiral cone," *Journal of Mathematical Physics*, vol. 38, no. 5, pp. 2553–2564, 1997.

[36] C. Furtado, V. B. Bezerra, and F. Moraes, "Quantum scattering by a magnetic flux screw dislocation," *Physics Letters A*, vol. 289, no. 3, pp. 160–166, 2001.

[37] M. S. Cunha, C. R. Muniz, H. R. Christiansen, and V. B. Bezerra, "Relativistic Landau levels in the rotating cosmic string spacetime," *The European Physical Journal C*, vol. 76, no. 9, p. 512, 2016.

[38] G. Clément, "Rotating string sources in three-dimensional gravity," *Annals of Physics*, vol. 201, no. 2, pp. 241–257, 1990.

[39] C. Furtado, F. Moraes, and V. B. Bezerra, "Global effects due to cosmic defects in Kaluza-Klein theory," *Physical Review D: Particles, Fields, Gravitation and Cosmology*, vol. 59, no. 10, Article ID 107504, 1999.

[40] R. A. Puntigam and H. H. Soleng, "Volterra distortions, spinning strings, and cosmic defects," *Classical and Quantum Gravity*, vol. 14, no. 5, article no. 1129, 1997.

[41] P. S. Letelier, "Spinning strings as torsion line spacetime defects," *Classical and Quantum Gravity*, vol. 12, no. 2, 1995.

[42] P. O. Mazur, "Spinning cosmic strings and quantization of energy," *Physical Review Letters*, vol. 57, no. 8, pp. 929–932, 1986.

[43] J. R. Gott and M. Alpert, "General relativity in a (2+1)-dimensional space-time," *General Relativity and Gravitation*, vol. 16, no. 3, pp. 243–247, 1984.

[44] K. Bakke and C. Furtado, "On the interaction of the Dirac oscillator with the Aharonov–Casher system in topological defect backgrounds," *Annals of Physics*, vol. 336, pp. 489–504, 2013.

[45] G. Q. Garcia, J. R. de S. Oliveira, K. Bakke, and C. Furtado, "Fermions in Gödel-type background space-times with torsion and the Landau quantization," *The European Physical Journal Plus*, vol. 132, no. 3, article no. 123, 2017.

[46] J. Carvalho, C. Furtado, and F. Moraes, "Dirac oscillator interacting with a topological defect," *Physical Review A: Atomic, Molecular, and Optical Physics and Quantum Information*, vol. 84, no. 3, Article ID 032109, 2011.

[47] E. R. F. Medeiros and E. R. B. de Mello, "Relativistic quantum dynamics of a charged particle in cosmic string spacetime in the presence of magnetic field and scalar potential," *The European Physical Journal C*, vol. 72, no. 6, article no. 2051, 2012.

[48] A. F. Nikiforov and V. B. Uvarov, *Special Functions of Mathematical Physics*, Birkhäauser, Basel, Switzerland, 1988.

[49] C. Tezcan and R. Sever, "A general approach for the exact solution of the Schrödinger equation," *International Journal of Theoretical Physics*, vol. 48, no. 2, pp. 337–350, 2009.