

Research Article

Observation of Gaussian Pseudorapidity Distributions for Produced Particles in Proton-Nucleus Collisions at Tevatron Energies

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The statistical event-by-event analysis of inelastic interactions of protons in emulsion at 800 GeV reveals the existence of group of events with Gaussian pseudorapidity distributions for produced particles, as suggested by hydrodynamic-tube model. Events belong to very central collisions of protons with heavy emulsion nuclei with probability of realization of less than 1% and with multiplicity of shower particles exceeding (2–3 times) the average multiplicity in proton-nucleus collisions in emulsion. Bjorken's energy density for these events reaches 2.0 GeV per fm³. The data are interpreted as a result of the QCD phase transition in proton-tube collisions at Tevatron energies.

Measurements at RHIC and CERN have discovered that the central collisions of heavy ions at these energies result initially in production of hadronic matter in the form of very hot compressed and a nearly frictionless liquid, quark-gluon plasma (QGP), whose evolution and decay produce the final state particles (for reviews see, e.g., [1, 2]). The data were analyzed in the framework of various theoretical approaches, including different, sometimes very sophisticated, versions of the hydrodynamic model [3–5]. Recent data from the LHC [6–9] reveal unexpected indications on formation of QGP in pp and p²⁰⁸Pb interactions and provide support to the idea of hydrodynamic model, interaction of an incident proton with a tube of nuclear matter.

The hydrodynamic model of multiparticle production was introduced for the head-on nucleon-nucleon collisions at very high (> 1 TeV) energies of a projectile [10] and then was generalized to the case of nucleon-nucleus collisions [11]. In the latter case the projectile nucleon can cut out in the nucleus a tube whose cross section is equal to the cross section of the nucleon and interacts only with this part of the target nucleus. The length of a tube may vary in dependence on the geometry of an interaction. The projectile and the tube undergo a

strong Lorentz contraction in the reference frame where their velocities are equal to each other. In contrast to the case of a nucleon-nucleon collision, an intricate mechanism of compression of nuclear matter treated as a continuous medium comes into play at the first stage of collision with a tube. The hadronic matter within the tube has very high density and high temperature $T \gg \mu c^2$, where μ is the pion mass, so that, following modern concepts it consists of point-like quarks and gluons, rather than usual hadrons. It is the quark-gluon plasma and it expands according to the laws of relativistic hydrodynamics of ideal fluid. While expanding, it becomes cooler. When the temperature of hadronic matter reaches $T \approx \mu c^2$, the plasma decays producing the final state particles, mostly pions.

The original Landau hydrodynamic model is, to our best knowledge, the only model suggesting some certain shape for pseudorapidity distributions of produced particles in both nucleon-nucleon and nucleon-nucleus collisions at very high energies, Gaussian distribution. Of course, it is necessary to note that only in the case of a very high multiplicity does the pseudorapidity distribution of produced particles in an individual event become a meaningful concept.

Simplicity of the model probably is one of the reasons why it is considered to be a "wildly extremal proposal" [1]. Of course, the scope of the original model was rather narrow. It was introduced to describe only few general characteristics of multiparticle production, pseudorapidity distribution of charged particles representing one of the simplest characteristics of the production process. More advanced versions of the hydrodynamic model are needed to explain more complex characteristics and features of the process and from this point of view they are more plausible but maybe less certain in predictions.

Pseudorapidity distributions of particles produced in interactions with nuclei at very high energies were discussed in many papers but in most cases the data were presented for inclusive and semi-inclusive reactions. At the same time when discussing the shape of pseudorapidity distributions of produced particles in the framework of the hydrodynamic model it is better to analyze the experimental data on the event-by-event basis in order, at least, to avoid problems related to the geometry of a hadron-nucleus interaction.

In the present paper we are analyzing on the event-by-event basis the shape of pseudorapidity distributions of relativistic singly charged (shower in emulsion terminology) particles (mostly pions) produced in inelastic incoherent interactions of 800 GeV protons with emulsion nuclei. We are looking at the possibility that these distributions for the individual central collisions are Gaussian distributions as suggested by the original hydrodynamic model.

The experimental data of the present study were collected in the framework of the Baton-Rouge-Krakow-Moscow-Tashkent Collaboration. We use for the analysis 1800 inelastic incoherent events. In each event found, the multiplicity of different types of charged particles was determined and spatial (θ) and azimuthal (φ) emission angles were measured.

According to the terminology adopted in emulsion experiments, depending on the ionization produced, the charged particles emitted during the interaction were divided into the following groups:

(1) "shower" or s -particles: singly charged particles with a speed of $\beta \geq 0.7$. These are mainly particles produced by the interaction of particles (mainly π^- and K^- mesons) and singly charged projectile fragments. Ionization on the tracks of these particles is $I < 1.4I_0$, where I_0 is the minimal ionization on the tracks of singly charged particles.

(2) "gray" or g -particles: particles moving at a speed of $\beta < 0.7$ and leaving the tracks with the length of > 3 mm and ionization $I > 1.4I_0$ in the emulsion. They mainly consist of protons knocked out of the target nucleus in the process of interaction and having a momentum of $0.2 \leq p \leq 1$ GeV/c, with a small admixture of π -mesons with a momentum of $60 \leq p \leq 170$ GeV/c.

(3) "black" or b -particles: most of them are protons with a momentum of $p \leq 0.2$ GeV/c and heavier fragments of the target nucleus, leaving the tracks with the length of < 3 mm and ionization $I > 1.4I_0$ in the emulsion.

Details of the experiment together with the main experimental results on multiplicities and pseudorapidity distributions were published in [12].

For the analysis of experimental data on the shape of pseudorapidity distributions of relativistic shower particles in individual events we have utilized the statistical approach described in detail in [13]. We use the coefficient of skewness g_1 , as a measure of asymmetry, and the coefficient of excess g_2 , as a measure of flattening, which represent parametrically invariant quantities defined as

$$\begin{aligned} g_1 &= m_3 m_2^{-3/2}, \\ g_2 &= m_4 m_2^{-2} - 3, \\ m_k &= \frac{1}{n} \sum_{i=1}^n (\eta_i - \bar{\eta})^k, \quad \bar{\eta} = \frac{1}{n} \sum_{i=1}^n \eta_i \end{aligned} \quad (1)$$

where m_k are the central moments of η -distributions and $n = n_s$ stands here for the multiplicity of s -particles in an event.

It follows from the mathematical statistics that if quantities $\eta_1, \eta_2, \dots, \eta_n$ are independent of one another in events of a subensemble and obey Gaussian distributions, the distribution of these parametrically invariant quantities does not depend on the parameters of the Gaussian distributions, and the number n of particles in the subensemble event uniquely determines the distribution of parametrically invariant quantities. In this case the mathematical expectation values and variances of g_1 and g_2 are as follows:

$$\begin{aligned} \nu_{g_1}(n) &= 0, \\ \sigma_{g_1}^2(n) &= 6(n-2)(n+1)^{-1}(n+3)^{-1}, \\ \nu_{g_2}(n) &= -6(n+1)^{-1}, \\ \sigma_{g_2}^2(n) &= 24n(n-2)(n-3)(n+1)^{-2}(n+3)^{-1}(n+5)^{-1}. \end{aligned} \quad (2)$$

We refer to the model described above, where the pseudorapidities obey a Gaussian distribution, as the G model.

From the mathematical point of view, our goal is to test the hypothesis that pseudorapidities in the events with different and sufficiently large multiplicity n are finite representative random samples with the volume n from the single infinite parent population (see Sect.13.3 in [13]), in which pseudorapidities are distributed according to the Gaussian law. To test this hypothesis, we use the central limit theorem (see Sections 17.1-17.4 in [13]), which asserts that the sum of a large number of independent and equally distributed so-called normalized random variables (see Sect.15.6 in [13]) has a normal distribution in the limit. In mathematical statistics, these normalized quantities are constructed from the random variable and the mathematical expectation and variance obtained from these random variables (see Sect.15.6 in [13]). However, our goal is to test the hypothesis of the normality of pseudorapidity distribution in individual experimental events (that is, in the individual finite samples from an infinite parent population). Therefore, we construct a normalized random variable in a different way; namely, when constructing it for each individual event with a multiplicity of n_s , we calculate the quantities g_1 and g_2 (see (1)), using

the experimental values of the event pseudorapidities, and the variances and mathematical expectations are determined by theoretical formulas (2) (see Eq. (29.3.7) in [13]) for a normally distributed quantity.

Thus, if our hypothesis of normality is true (if the G-model is realized), then by our construction, the normalized quantities d_1 and d_2 (see Sect.15.6 in [13])

$$\begin{aligned} d_1 &= [g_1 - \nu_{g_1}(n)] \sigma_{g_1}^{-1}(n), \\ d_2 &= [g_2 - \nu_{g_2}(n)] \sigma_{g_2}^{-1}(n) \end{aligned} \quad (3)$$

have dispersions equal to 1 and mathematical expectations equal to 0 both in the subensemble of events (with the fixed number of particles n) and, consequently, in the ensemble of the events (where n can take any possible values).

Moreover, if the hypothesis of the normality of the pseudorapidity distribution is true, then, according to the central limit theorem of mathematical statistics, for a sufficiently large number N of independent random samples (that is, the number of interaction events) the sums of these independent and identically distributed normalized quantities

$$\begin{aligned} \overline{d}_1 \sqrt{N} &= \frac{1}{\sqrt{N}} \sum_{i=1}^N d_{1,i}, \\ \overline{d}_2 \sqrt{N} &= \frac{1}{\sqrt{N}} \sum_{i=1}^N d_{2,i} \end{aligned} \quad (4)$$

should be less than 2 with the probability of 95% (see Sections 17.1-17.4 in [13]).

If the hypothesis of normality is true, then the G-model is quite realistic and for a small N we can use the asymptotic normality of g_1 and g_2 in the subensemble of the events described by the G-model. Then the normalized quantities d_1 and d_2 are equally distributed with parameters 0 and 1 in both the subensemble and in the ensemble of events with the large enough n^{min} , which is selected to make the notion of distribution meaningful. In this case (for the G-model) the sums (4) have the same restrictions.

In this paper, the sums (4) were calculated for events with the multiplicity of shower particles n_s within the limits from n_s^{min} to n_s^{max} . Calculations were repeated for different intervals (n_s^{min}, n_s^{max}) with fixed n_s^{max} , whereas the value of n_s^{min} was changing from some minimum value of n_s to the maximum value of $n_s = n_s^{max}$, which was defined from the experiment.

The procedure described was applied to the experimental data. In Figure 1 we show dependence of the parameters $\overline{d}_1 \sqrt{N}$ and $\overline{d}_2 \sqrt{N}$ on the multiplicity n^{min} for proton-nucleus interactions considered. It is seen that both parameters $\overline{d}_1 \sqrt{N}$ and $\overline{d}_2 \sqrt{N}$ decrease in their absolute magnitude with increasing n^{min} . The data reveal the existence in p -nucleus interactions of a small group of events with the values of parameters $\overline{d}_1 \sqrt{N}$ and $\overline{d}_2 \sqrt{N}$ which are simultaneously less than 2 in their absolute magnitudes. It follows from our consideration that pseudorapidity distributions of shower particles in these events are representative samplings from the

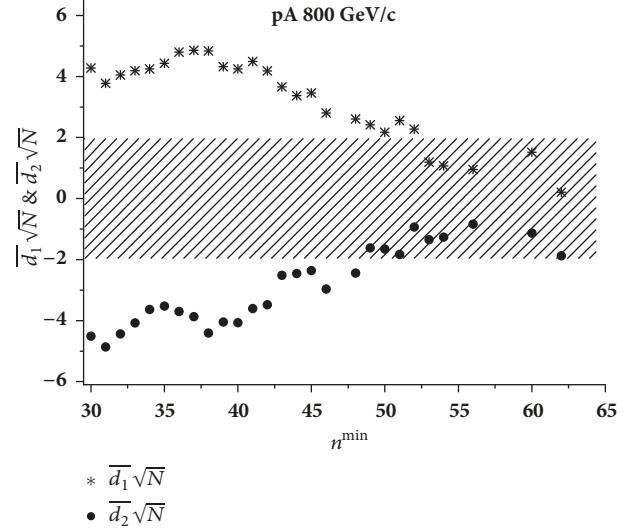


FIGURE 1: Dependence of parameters $\overline{d}_1 \sqrt{N}$ and $\overline{d}_2 \sqrt{N}$ on n^{min} for proton-nucleus interactions in emulsion at 800 GeV. The shaded area is the area where $|\overline{d}_1 \sqrt{N}| & |\overline{d}_2 \sqrt{N}| < 2$.

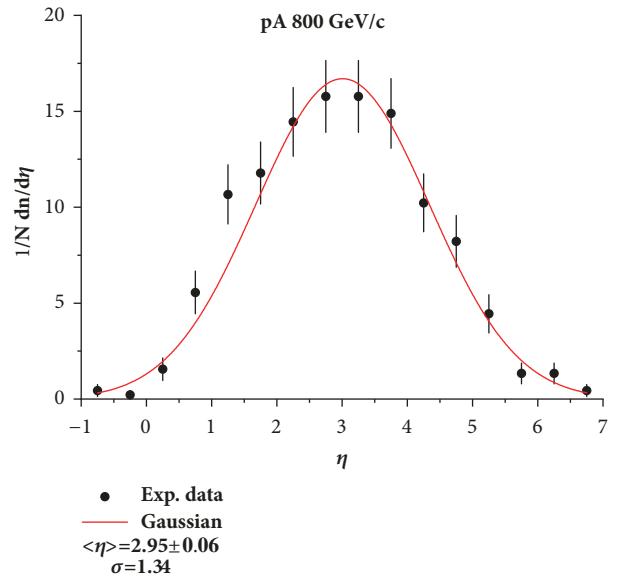


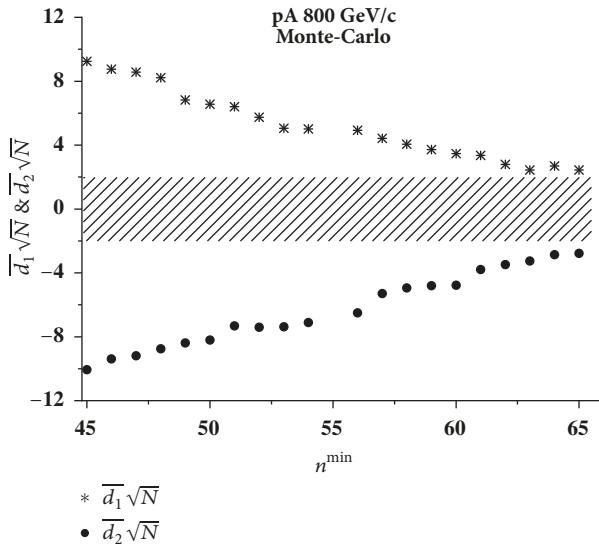
FIGURE 2: Pseudorapidity distributions of shower particles in selected proton-nucleus collisions. Curve is a Gaussian distribution.

parent Gaussian distribution. Pseudorapidity distributions in these individual events obey the Gaussian law. The number of these events in proton-nucleus collisions considered is equal to 9. Characteristics of these selected events are presented in Table 1 and in Figure 2 we show the summary pseudorapidity distribution for these events.

In order to verify our experimental results we have utilized this statistical approach to the samples of Monte Carlo events generated following the simple Independent Emission Model (IEM) [14, 15]. In the framework of this model we assume that (i) multiplicity (n_s) distributions of simulated events reproduce the experimental distributions

TABLE 1: Characteristics of 9 events selected by the statistical approach from proton-nucleus collisions.

No	n_s	N_h	pA 800 GeV/c			
			$\langle \eta \rangle$	$\sigma(\eta)$	$d_1 \sqrt{N}$	$d_2 \sqrt{N}$
1	53	37	2.74 ± 0.35	2.58	-0.87	-0.042
2	53	14	3.29 ± 0.16	1.15	1.62	-0.27
3	55	13	2.83 ± 0.17	1.23	-0.11	-1.03
4	55	9	2.92 ± 0.19	1.41	0.82	-0.43
5	59	19	3.29 ± 0.19	1.44	-0.91	0.39
6	61	11	2.46 ± 0.14	1.10	1.66	-0.21
7	61	19	3.08 ± 0.18	1.42	1.08	0.59
8	65	27	2.82 ± 0.17	1.40	0.53	-1.09
9	65	18	3.11 ± 0.14	1.12	0.24	-1.55

FIGURE 3: Dependence of parameters $\overline{d}_1 \sqrt{N}$ and $\overline{d}_2 \sqrt{N}$ on n^{min} for 5000 Monte Carlo generated events. The shaded area is the area where $|\overline{d}_1 \sqrt{N}| & |\overline{d}_2 \sqrt{N}| < 2$.

for the interactions considered; (ii) one-particle pseudorapidity distributions of s -particles in each one of simulated subensembles of events (within, for instance, the fixed range of n_s) reproduce the experimental distribution for the same range of n_s ; (iii) emission angles of s -particles in each one of simulated events are statistically independent.

In Figure 3 we show the values of parameters $\overline{d}_1 \sqrt{N}$ and $\overline{d}_2 \sqrt{N}$ in dependence on the multiplicity n^{min} for Monte Carlo events generated in the framework of IEM following the experimental multiplicity and pseudorapidity distributions of shower particles in proton-nucleus interactions in emulsion at 800 GeV. We see that absolute values of both $\overline{d}_1 \sqrt{N}$ and $\overline{d}_2 \sqrt{N}$ decrease with increasing n^{min} , but we found no events with Gaussian pseudorapidity distributions. We conclude from these results that the probability of accidental formation of Gaussian pseudorapidity distributions in individual events, not recognizable by the statistical approach utilized, is negligibly small for our experimental conditions.

It is necessary to note that the experimental events found by the statistical analysis are very rare with probability of realization less than 1%. They belong to central interactions of hadrons with heavy emulsion nuclei. For instance, the average multiplicity of shower particles in these 9 events $n_s = (58.6 \pm 1.6)$ exceeds almost three times the average multiplicity in proton-nucleus interactions in emulsion at 800 GeV, which is equal to (20.0 ± 0.3) [12]. Average multiplicities of black and grey particles, representing, following emulsion terminology, fragments of the target nucleus equal, respectively, for these events $n_b = (10.0 \pm 1.7)$, $n_g = (8.6 + 1.9)$, indicating that central interactions of protons indeed took place with heavy (Br,Ag) nuclei in emulsion. Note that $N_h = n_g + n_b$ in Table 1.

It follows from the data on average pseudorapidities and dispersions of pseudorapidity distributions in these selected events (see Table 1) that they do fluctuate considerably. Therefore the sum of pseudorapidity distributions in selected individual events shown in Figure 2 do not demonstrate very good agreement with the Gaussian shape. From Figure 2 for the density $(1/N)(dN/d\eta)$ in the central region for selected proton-nucleus collisions we have (15.9 ± 1.8) .

The experimental observations of the present paper encourage us to interpret the existence of events with the Gaussian pseudorapidity distributions of produced particles in central relativistic proton-nucleus interactions as a result of a proton-tube collisions and subsequent formation in the course of an interaction of a droplet of hadronic matter, the quark-gluon plasma, i.e., the primordial high density state, whose expansion and cooling lead to its decay with production of final state particles. This interpretation may be supported by following considerations.

Calculations in the framework of lattice QCD show [16, 17] that, at the energy densities exceeding a critical value of about 1 to 1.5 GeV per fm³, achievable at incident energies of about $\sqrt{s_{NN}} \gtrsim 5$ GeV, the hadronic phase of matter disappears giving rise to the primordial high density state (QGP) whose evolution is governed by the elementary interactions of quarks and gluons. From the radius of a tube equal 1 fm and the experimental value of the density $(1/N)(dN/d\eta)$ for selected proton-nucleus events we have for Bjorken's energy density [18] approximately 2.0 GeV per fm³ what is more than the critical value of the density.

It is known from simple kinematics that the rapidity of the center of mass frame in a proton-tube collision, where a tube consists of k nucleons, is shifted from that of a proton-proton collision on the value $\Delta y = (1/2) \ln(k)$. So, from the value of this shift it is possible to estimate the average number of nucleons in the tube. From the experimental data of Figure 2 for selected proton-nucleus collisions we have an estimate $k = 4.7$, which leads to the corresponding estimate of the energy of proton-tube interactions $\sqrt{s} \sim 80$ GeV. At the same time the Glauber model gives for the average number of intranuclear collisions 2.75 and 3.20 for p -Em and p -BrAg interactions, respectively [19, 20].

Of course, realization of the phase transition cannot be easily expected in proton-nucleus collisions at these energies, even central ones. If it nevertheless does happen it must be a rare and random phenomenon with fluctuations and instabilities playing significant role in outcome of an interaction, so that the produced intermediate QCD objects may vary in some important initial characteristics, in the volume, for example. Similar situation was considered for heavy-ion collisions at SPS energies [21]. It was shown that big droplets of quark-matter may be formed at this energy densities due to fluctuations, but not in average events. The percolation model was used to reflect the complexity of the process. Therefore it was recommended to search for these objects on the event-by-event basis. Evolution of these objects follows obviously general principles taken into account by the original hydrodynamic model and may lead to the Gaussian distributions of final state particles in pseudorapidities. Therefore we believe that it is important to study and to confirm this possibility in other experiments as well.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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