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Research Article

Nonlocal Black Hole Evaporation and Quantum Metric Fluctuations via Inhomogeneous Vacuum Density

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Inhomogeneity of the *actual* value of the vacuum energy density is considered in a black hole background. We examine the back-reaction of a Schwarzschild black hole to the highly inhomogeneous vacuum density and argue the fluctuations lead to deviations from general relativity in the near-horizon region. In particular, we found that vacuum fluctuations *onto* the horizon trigger adiabatic release of quantum information, while vacuum fluctuations in the vicinity of the horizon produce potentially observable metric fluctuations of order of the Schwarzschild radius. Consequently, we propose a form of strong nonviolent nonlocality in which we simultaneously get nonlocal release of quantum information and observable metric fluctuations.

1. Introduction

Recently, Unruh et al. argued (see [1]) the observed small nonnegative cosmological constant can be achieved without introducing new degrees of freedom, *e.g.,* negative-pressure scalar field. Instead, they address the cosmological constant problem by embracing the diverging value of the vacuum energy density as predicted by quantum field theory, without applying any renormalization procedures. Interestingly, by studying the gravitational effects of the constantly fluctuating vacuum, they found its local energy density to be highly inhomogeneous. As a result of this inhomogeneity, the spatial distance between a pair of neighboring points undergoes constant phase transitions in the form of rapid changes between expansion and contraction. Also, the singular expectation value of the local energy density of the vacuum was argued to be harmless at the energy levels of effective field theory since the fluctuations lead to huge cancellations on cosmological scales and ultimately to the observed accelerating expansion of the universe.

In a separate work [2], we studied the back-reaction of a Schwarzschild black hole geometry to quantum vacuum fluctuations. We examined how the black hole metric back-reacts to vacuum fluctuations in two regions, *onto* the horizon, and in the vicinity of the black hole. We considered vacuum fluctuations with local energy density *below* and *above* a certain threshold ζ. We found that *"strong"* quantum fluctuations (considered onto the horizon) lead to brief nonviolent departures from classicality, which allow for adiabatic leakage of low-temperature Hawking quanta at the necessary rate. In addition, we argued that quantum information can begin leaking out of the black hole as early as $r_S \log r_S$ after the initial collapse. The *"weak"* quantum fluctuations (considered in the vicinity of the black hole) were argued to be the microscopic source, which on scales of order of the Schwarzschild radius $\mathcal{O}(r_S)$ accumulates and produces metric fluctuations [2, 3].

The current model questions the classical black hole picture of local quantum field theory on a semiclassical geometry. In this work we apply the Unruh et al. model to Schwarzschild geometry; namely, we rewrite the equations of [1] in that background and study the gravitational effects of the inhomogeneous vacuum density. As a result, we propose a form of strong nonviolent nonlocality which yields significant modifications to the well-known general relativistic picture of black holes. The conjectured deviations from classicality lead simultaneously to nonlocal release of Hawking particles and quantum metric fluctuations. In fact, the present work may be thought of as the microscopic origin of the initially proposed by Giddings nonviolent nonlocality model [4–6].

The paper is organized as follows. In Section 2 we summarize the relevant work. In Section 3 we rewrite the equations of [1] in Schwarzschild black hole background and show that we can derive nontrivial modifications to general relativity in the near-horizon region.

2. Summary of Related Work

To make the paper self-contained, we begin by making a brief review of the main results of [1, 2]. Note the Unruh et al. model will serve as a basis for the scenario we will present later in Section 3. In Section 2.1 we demonstrate the physical interpretation of the conjectured in [1] inhomogeneous microscopically diverging vacuum energy density, and its effects on cosmological scales. Then, in Section 2.2, we review [2] to show how we initially derived the "soft" but nontrivial modifications to general relativity in the near-horizon region.

2.1. Accelerating Expansion via Inhomogeneous Quantum Vacuum Density. In the generic ΛCDM model of the universe the vacuum energy density is considered to be constant and homogeneous throughout space. As it was pointed out in [1], however, this basic assumption is only true for the expectation value of the vacuum energy density. Its *actual* value, *i.e.*, the one obtained by performing repeated measurements at a particular spatial point, constantly fluctuates. Consequently, we get a picture, where although the expectation value is effectively constant and homogeneous on cosmological scales, the actual value is rapidly changing in both time, as well as from point to point. Physically, this inhomogeneity of the vacuum density implies the spatial distance between any pair of nearby points constantly changes between phases of expansion and contraction. As we show in greater detail later, this leads to very important results in a black hole background.

Taking the constantly fluctuating inhomogeneous vacuum density as a starting point, we now briefly demonstrate its effects in a general spacetime [1]. Note that in Section 3 we apply those results to Schwarzschild black hole geometry.

Suppose the local energy density $\rho_{x,x'}$ between a pair of neighboring spatial points x and x' in some general metric $g_{\mu\nu}$ is [1]

$$\rho_{x,x'} = \frac{\mathrm{cov}(T_{00}(x), T_{00}(x'))}{\sigma_x \sigma_{x'}} \tag{1}$$

where

$$\sigma_x = \sqrt{\left\langle \left(T_{00}(x) - \langle T_{00}(x)\rangle\right)^2 \right\rangle} \tag{2}$$

Here, $T_{00}(x)$ and $T_{00}(x')$ are the local vacuum densities defined at the spatial points x and x', respectively.

Evidently from (1), the value of $\rho_{x,x'}$ is determined by the covariant vacuum densities, defined at the neighboring spatial coordinates, namely, $T_{00}(x)$ and $T_{00}(x')$. Effectively, one can think of $\rho_{x,x'}$ as a 2-point correlation function. That is, in order for $\rho_{x,x'}$ to have a nontrivial value, it must always be evaluated between close spacetime points. Otherwise, $\rho_{x,x'} \longrightarrow 0$ as the separation between x and x' becomes large, in which case $T_{00}(x)$ and $T_{00}(x')$ are no longer correlated and thus evolve independently. The requirement that x and x' are close comes from the limited domain of dependence of individual vacuum fluctuations, that is, their high momentum/short wavelength.

Assuming $\rho_{x,x'}$ is nonvanishing, then in order for it to be positive/negative, both $T_{00}(x)$ and $T_{00}(x')$ need to be, respectively, above/below the zero threshold of $\langle T_{00}\rangle$ [1]

$$\rho_{x,x'} = \begin{cases} > 0 & if\ T_{00}(x), T_{00}(x') > 0 \\ < 0 & if\ T_{00}(x), T_{00}(x') < 0 \end{cases} \tag{3}$$

Therefore, the coefficient $\rho_{x,x'}$ shows the correlation between vacuum densities defined between a pair of nearby points x and x'. One should note that since in this model we do not apply any renormalization procedures, we cannot use the generic stress-energy tensor as a source in the Einstein field equations. Instead, we must slightly modify the stress tensor in order to account for the diverging expectation value of the vacuum fluctuations.

Studying the gravitational effects of the inhomogeneous vacuum density requires inhomogeneity of the underlying metric as well. So in this scenario, the scale factor has to have an extra stochastic component which would allow it to account for that inhomogeneity. Therefore, following (1), the generic scale factor of the standard Friedmann-Robertson-Walker metric is modified as

$$ds^2 = -dt^2 + a^2(t,x)\left(dx^2 + dy^2 + dz^2\right) \tag{4}$$

As a result of the vacuum inhomogeneity, the scale factor $a(t,x)$ now has additional degrees of freedom in the form of a space-dependent coupling term. That is, when the local scale factor is evaluated at a given spacetime point, its dynamics is dictated (sourced) by the stochastically varying vacuum fluctuations at that point. This richer structure of the scale factor allows for a pair of nearby points to be expanding or contracting, depending on the sign of $\rho_{x,x'}$, (3). In particular, for $\rho_{x,x'} > 0$ the spatial separation between the pair of points increases, while for $\rho_{x,x'} < 0$, the spatial separation decreases.

It was then shown in [1] that one could consider a local Hubble rate term and evaluate it between the neighboring x and x'

$$\nabla H = -4\pi G J \tag{5}$$

where $H \equiv \dot{a}/a$ is the Hubble parameter and J denotes the energy flux of the vacuum, which accumulates over a given region of space. Hence J can be thought of as a functional of the local energy density in that neighborhood $\langle J(x,x')\rangle \sim \int_x^{x'} \rho_{x,x'}$.

Interestingly enough, following the extra degrees of freedom of $a(t,x)$, ∇H was found to be *constantly* fluctuating with energy, sourced by the accumulation of the vacuum density.

The general solution of (5) reads

$$H(t,x) = H(t,x_0) - 4\pi G \int_x^{x'} J(t,x')\left(dx', dy', dz'\right) \tag{6}$$

As (6) shows, the local Hubble rate depends on the spatial accumulation of J in the region between x and x'. To be more precise, we expect there may be some dissipation of the accumulated energy to nearby coordinates. For simplicity, however, throughout the paper we ignore all such effects and focus solely on individual 2-point functions.

Using this more complex spacetime dynamics induced by the inhomogeneous vacuum density, it was proposed that the equation of motion of ∇H goes as

$$\ddot{a} + \Omega^2 (t, x) a = 0 \tag{7}$$

where

$$\Omega^2 (t, x) = \frac{4\pi G}{3} \left(\rho + \sum_{i=1}^{3} P_i \right), \quad \rho = T_{00}, \; P_i = \frac{T_{ii}}{a^2} \tag{8}$$

In this case, (7) simply states that ∇H has the behavior of a harmonic oscillator. That is, it constantly fluctuates around its equilibrium point. Of course, every crossing of the equilibrium point is associated with a change of sign.

Let us briefly summarize the spacetime back-reaction to the conjectured in [1] constantly fluctuating inhomogeneous vacuum density. Physically, (5) and (7) describe a picture where the separation between a pair of neighboring points, $\Delta x \equiv |x - x'|$, is constantly fluctuating between phases of expansion and contraction. In fact, when Δx (herein defined as the proper distance) is expanding in some region, a neighboring region must be contracting, and vice versa. When a vacuum fluctuation with local energy density *above* its equilibrium point is considered between x and x', Δx (i.e., the proper distance) grows. On the other hand, if the local energy density is *below* the equilibrium point, the proper distance Δx decreases. This enhanced spacetime dynamics is characterized by constant local phase transitions between expansion and contraction which happen as the local energy density of the vacuum goes through its equilibrium point and changes sign as it does so. Those constant phase changes accumulate and lead to massive cancellations on cosmological scales. However, assuming a slight *positive* excess, we get the observed cosmological expansion. Although the microscopic values of ρ may be huge, their infrared effects are small, *i.e.,* the wild fluctuations do not lead to $\mathcal{O}(1)$ corrections in weak gravitational regimes. Besides, (8) tell us that the scale factor $a(t, x)$, in a given neighborhood, depends on the time-dependent frequency $\Omega(t, x)$ which exhibits quasiperiodic dynamics.

2.2. Unitary Black Hole Evolution and Horizon Fluctuations via Quantum Vacuum Fluctuations. In [2] we argued that treating gravity in the near-horizon region of Schwarzschild black hole as a field theory and considering its coupling to the matter fields in this region $\int \phi^{\mu\nu} T_{\mu\nu}$ produce fluctuations which modify the general relativistic description. We considered fluctuations with local energy density *below* and *above* ζ, where ζ is an arbitrary threshold. We then studied how the black hole geometry back-reacts to those fluctuations in two distinct regions, *i.e., onto* the horizon and just outside the black hole.

More precisely, we studied how the horizon geometry back-reacts to "strong" quantum fluctuations and how the near-horizon region back-reacts to "weak" fluctuations. The analysis was carried out under the assumptions that (i) black holes are fast scramblers [7], (ii) quantum information is found in the emitted Hawking particles, and (iii) the scrambled infallen information *need not* be embedded uniformly across the horizon.

Let us now precisely define what we mean by "strong" and "weak" quantum fluctuations.

(A) In a broader sense, we take a fluctuation to be *strong* if, when considered at asymptotic infinity, its local energy density leads to a localized particle production

$$a_i^{\dagger} |0\rangle = |x\rangle \tag{9}$$

Therefore, if we had a measuring apparatus counting the strong fluctuations in a given spacetime region Σ at asymptotic infinity, its results would be consistent with the expectation value of the number operator $\langle N \rangle$ in that region

$$\langle N \rangle = \sum_{i}^{\mathcal{N}} \int_{\Sigma} \varphi_i^{strong} \tag{10}$$

More specifically, in a black hole background, we argue a "strong" quantum fluctuation *onto* the horizon yields brief departure from local quantum field theory.

(B) A quantum fluctuation is taken as *weak* if its local energy density is below the threshold ζ. Because of the small local energy density, we assume the back-reaction of the background metric would be negligible if we considered weak fluctuations in a relatively small part of the near-horizon region. That is why we are interested in how the near-horizon metric back-reacts when the weak fluctuations are taken on scale $\mathcal{O}(r_S)$.

We argue that weak fluctuations on scale $\mathcal{O}(r_S)$ lead to nonperturbative effects which manifest in potentially observable metric fluctuations that can play an important role in observer complementarity and in gravitational wave astronomy in the form of detectable "echoes" and deviations from general relativity close to the horizon. We assume that away from a black hole the weak fluctuations do not lead to perturbations and leave the geodesic equation invariant. One should keep in mind that, although effectively negligible at infinity, when examined locally, the weak fluctuations still cause a pair of points to rapidly change phases between expansion and contraction.

To demonstrate how we define the weak fluctuations in the vicinity of the horizon we adopted the following analogy. Suppose we interpret individual fluctuations as harmonic oscillators, denoted by χ_i. Imagine we place the harmonic oscillators on a string in the vicinity of the horizon, Figure 1. Using an arbitrary normalized spacing ϵ, we can generally describe the string as

$$\sum_{i=1}^{N} \int_{S2} d\varphi \left(n_i \varphi_i \right) \tag{11}$$

where φ_i is the oscillation frequency of the different harmonic oscillators.

In this picture we get an ensemble of fluctuations which, as we will argue later, yield coherent Schwarzschild-scale metric fluctuations. We assume separate harmonic oscillators *need not* have the same frequency. In fact, due to their limited domain of dependence (of order of the wavelength of the fluctuation), even neighboring harmonic oscillators have different frequencies. Thus an evolution equation for a particular spacetime region can be given in terms of a linear combination of the harmonic oscillators in that region. Like we mentioned earlier, despite the arbitrarily high (diverging) oscillating frequency a single harmonic oscillator may have microscopically, its effect in an infrared cut-off is negligible, and we do not expect $\mathcal{O}(1)$ corrections to the background metric.

Our analysis in [2] lead us to the following two main results.

First, we found that "strong" fluctuations, considered onto the horizon, lead to nonlocal release of quantum information via substantial deviations from classicality (for similar results [4–6]). We also demonstrated the model achieves Page-like evaporation spectrum [8, 9] and predicts black holes begin emitting Hawking particles in time, logarithmic in the entropy, *i.e.*, $t_* \sim \mathcal{O}(r_S \log r_S)$, where $t_* \ll M^3$.

Second, we found that "weak" quantum fluctuations, considered in the vicinity of the horizon, produce metric fluctuations which, although locally negligible, were shown to accumulate and on scale of order of the Schwarzschild radius lead to significant modifications to general relativity. Physically, the horizon periodically shifts (with frequency ω) radially outward with an amplitude δ, where $l_P \ll \delta < M$. In Schwarzschild coordinates the metric fluctuations translate to shift of the horizon from $r = 2M$ to $r = 2M+\delta$. One can think of the weak fluctuations as exerting a drag-like force on the black hole which produces observable macroscopic quantum gravity effects. Schematically

$$\omega \sim M_{BH}^{-1} \tag{12}$$

where ω is the frequency of the metric fluctuations.

The inverse proportionality between the frequency of the metric fluctuations and the mass of the black hole was argued to lead to thermodynamic instability at late times. Imagine a freely evaporating Schwarzschild black hole with no perturbations being introduced to it. Given $M_{BH} = 1/T$, we expect T to monotonically increase as the black hole evaporates. As we can see, (12) dictates that, as the black hole loses mass, ω steadily increases. Tracing that evolution to the final stages of the evaporation we assume the black hole becomes thermodynamically unstable as $M_{BH} \longrightarrow m_P$, where m_P is the Planck mass. At that point, the black hole was conjectured to explode [10]. Here, late-time black hole evolution is in agreement with the thermodynamic instability first proposed in [10].

Recently, the authors of [11] numerically solved the Einstein equations modified with metric fluctuations. Similar to the current work, they showed that the proposed metric fluctuations lead to major deviations from the traditional general relativistic black hole description. In agreement with our results (in particular (12)), their analysis shows there is an inverse proportionality between the frequency of the fluctuations, and black hole's mass, where for a black hole binary merger, the overlap between the classical and the modified pictures of the waveform of the emitted gravitational waves decreases as the mass of the binary gets smaller. That is, as the frequency of the metric fluctuations increases, the near-horizon region deviates from general relativity even more significantly.

3. The Quantum Vacuum Origin of Metric Fluctuations and Nonlocal Evaporation

In the current section we adopt the Unruh et al. model and study its effects in Schwarzschild background. Consequently, we propose a dynamical mechanism for a form of strong nonviolent nonlocality which significantly modifies the traditional field theory picture in the near-horizon region. Specifically, we demonstrate how the conjectured in [1] inhomogeneous vacuum density leads simultaneously to quantum metric fluctuations [2, 4–6, 11–13] and nonlocal release of quantum information. One should note we do not attempt to quantize gravity in the current paper. Although the considered vacuum fluctuations may not be normalized, which modifies the stress-energy tensor and makes it nongeneric, they still contribute to its expectation value.

We aim to make the transition from cosmological scales to a black hole case more consistent. That is why we would like to first expand more on the implications of (1) for black hole backgrounds.

So let us now focus in more detail on how a pair of neighboring spacetime points x and x' is affected by the suggested extremely inhomogeneous vacuum density. Consider the following [1]:

$$\Delta\rho^2 (\Delta x) = \frac{\left\langle \left\{ T_{00}(t,x) - T_{00}(t,x') \right)^2 \right\} \right\rangle}{(4/3)\left\langle T_{00}(t,x) \right\rangle^2} \tag{13}$$

where $T_{00}(x)$ is the vacuum density at x and $\Delta x \equiv |x - x'|$ denotes the separation between x and x'.

Recall our earlier discussion about the evolution of the local energy density. Specifically, $\rho \longrightarrow 0$ as Δx gets large, in which case $T_{00}(x)$ and $T_{00}(x')$ evolve independently.

Keeping that in mind, suppose a classical Schwarzschild solution

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega^2 \tag{14}$$

where $d\Omega^2 = (d\theta^2 + \sin^2\theta d\phi^2)$.

Particularly, we are interested in rewriting (13) in Schwarzschild geometry. Working in a black hole background requires us to take into account the large gradient with respect to the horizon

$$\rho = \partial_r (\partial_r \phi)^2 \tag{15}$$

where ρ is the local energy density and ϕ is the gravitational potential. Thus near a matter source (15) is very sensitive to changes in the radial coordinate.

Therefore, by taking into account (15), we can rewrite (13) in Schwarzschild background as

$$\int_R \int_x^{x'} J^2 = \int_R \int_x^{x'} \frac{\left\langle \left\{ \partial_r J(t,x) - \partial_r J(t,x') \right)^2 \right\} \right\rangle}{(4/3) \left\langle \partial_r J(t,x) \right\rangle^2} \tag{16}$$

where R denotes the near-horizon region and J is the vacuum accumulation term.

Notice that since we no longer work on cosmological scales, (15) becomes relevant, and so we substituted the vacuum energy density terms, defined on particular spacetime points, with J. Both terms $J(x)$ and $J(x')$ are calculated with respect to the horizon. Here, J is very sensitive to radial changes because of (15). In case x and x' both lie on the same $r = const$ surface, the expectation value of J between them varies stochastically.

Because J is of particular importance in the near-horizon neighborhood, let us now focus on its general properties.

Since the local energy density in a spacetime region which includes a black hole exhibits Gaussian-like distribution (15), we assume that as $r \longrightarrow \infty$, $\langle J \rangle \longrightarrow 0$. Although this may be true for the expectation value of J, its *actual* value must still constantly fluctuate due to the extremely inhomogeneous vacuum density. Clearly, we can see that, in the near-horizon region, J has a nonzero expectation value $\int_R \langle J \rangle \gg 0$. Therefore, the strongly radial-dependent behavior of J in the near-horizon region is trivially given as

$$\int_R \frac{\partial \langle J \rangle}{\partial r} \tag{17}$$

Where depending on r with respect to the horizon, the general solutions to (17) are

$$\int_R \frac{\partial \langle J \rangle}{\partial r} = \begin{cases} 0 & for\ r > r_S \\ > 0 & for\ 2M < r < r_S \end{cases} \tag{18}$$

Lastly, we can easily extend (17) to include an arbitrary number of gauge fields as

$$\int_R \left\langle T_{\mu\nu} \right\rangle + \frac{\partial \lambda}{\partial r} \tag{19}$$

where λ is the vacuum and $T_{\mu\nu}$ is the stress-energy tensor.

Note that, at constant r from the horizon, the energy density of the vacuum fluctuations depends on the internal degrees of freedom of the black hole and varies stochastically.

3.1. Metric Fluctuations: Weak Quantum Fluctuations = Local Phase Transitions. In this subsection we demonstrate how the proposed inhomogeneous vacuum density can modify the classical near-horizon physics and thus yield metric fluctuations. Specifically, by embracing the harmonic-oscillator-like constant phase transitions of ∇H, we rewrite (5) in Schwarzschild background and study how the metric back-reacts. Hence, we show that in this scenario one can obtain the conjectured quantum corrections to general relativity in the region just outside the black hole. Like we saw earlier, the microscopically extremely inhomogeneous vacuum density gives freedom to the scale factor to be locally expanding or contracting. Recall that in (5) we defined a local Hubble rate term ∇H and argued that, due to the inhomogeneous vacuum density, it has the dynamics of a harmonic oscillator (7). Namely, it constantly changes between phases of expansion and contraction.

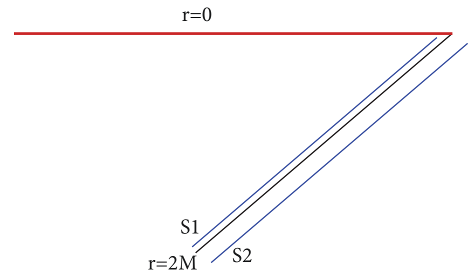

Figure 1: The bold red line is the singularity, and the bold black line, $r = 2M$, is the horizon. Imagine that $S1$ coincides with the horizon, and thus modes located on $S1$ are spacelike separated from an observer outside the black hole. Further, imagine that $S2$ is placed in the vicinity of the horizon.

For the sake of completeness we will examine coarse-grained and fine-grained version of the argument.

3.1.1. Coarse-Grained. In the particular case we neglect contributions coming from individual degrees of freedom and instead only focus on the effective (macroscopic) back-reaction on scale of order of the Schwarzschild radius.

Keeping in mind the large field-strength radial dependence in this region (15), we can rewrite (5) as

$$\int_{S1}^{S2} \nabla H = -4\pi G \int_{S1}^{S2} \partial_t \langle J \rangle \tag{20}$$

Unlike earlier, where we considered ∇H between the neighboring points x and x', we now evaluate it in the region between *S2* and *S1* (the horizon), Figure 1.

To better demonstrate the back-reaction of the near-horizon geometry (20), consider the following *gedanken* experiment. Imagine *S2* is a timelike hypersurface the size of the Schwarzschild radius just outside the black hole, coupled to the horizon degrees of freedom. Suppose we wish to evaluate the constant ∇H phase transitions in the region between the hypersurface and the horizon. We assume this region constantly undergoes uniform phase transitions with characteristic oscillation cycle time T, where by "uniform" we mean that, once every T, a phase change of ∇H in that region takes place. Unlike the cosmological case [1], here we assume that on every T the phase transitions average out to a small *negative* value. Here, T is given as

$$T = \frac{2\pi}{\Omega} \tag{21}$$

where Ω is time-dependent frequency which depends on $\langle T_{00} \rangle$.

That constant change between phases of expansion and contraction just outside the black hole (assuming a slight

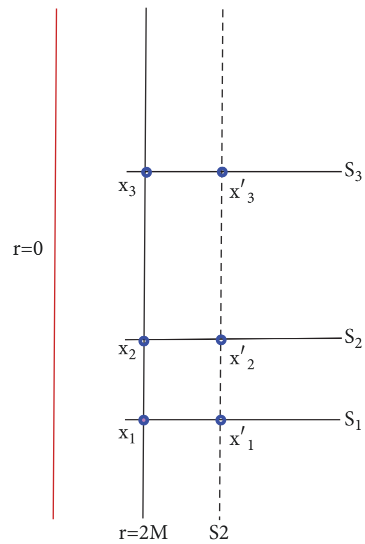

Figure 2: Similar to Figure 1, imagine $S1$ coincides with the horizon. The dotted line, $S2$, is an artificial (nonphysical) timelike hypersurface in the vicinity of the horizon. Note that the blue dots x_i and x'_i are pairs of corresponding points, where $x_i \in S1$, $x'_i \in S2$, and $x_i, x'_i \in S_i$. Also, $\sum_i x_i \equiv S1$ and $\sum_i x'_i \equiv S2$.

negative excess every T) yields the dragging-like effect on the horizon. For simplicity, one can imagine the conjectured dynamics outside the black hole as the back-reaction of the horizon to a vector field coupled to matter fields in Rindler space, where we consider only contributions from modes very close to the horizon.

Physically, the back-reaction of the near-horizon region manifests as a slight outward shift of the horizon from $r = 2M$ to $r = 2M + \delta$ in Schwarzschild coordinates.

Imagine a far-away observer who cannot probe the region near the horizon. As far as she is concerned, she may effectively interpret those metric fluctuations as a Planck scale structure just outside the horizon (similar to the stretched horizon in observer complementarity or gravitational wave “echoes”). Different proposals have recently been made regarding the possibility of observing such quantum gravity effects (see [11–13]).

In conclusion, we can see that by rewriting (5) in a Schwarzschild black hole background we can effectively derive the proposed in [2–6, 11–13] metric fluctuations.

3.1.2. Fine-Grained. In this case we focus on contributions coming from individual degrees of freedom, *i.e.,* phase transitions of ∇H between a pair of corresponding points (say, x and x'), where both lie on the same spacelike hypersurface S_i in the near-horizon region. Namely, $x \in S1$, $x' \in S2$, and $x, x' \in S_i$, Figure 2.

Suppose we foliate the black hole spacetime into $t = const$ spacelike hypersurfaces. The particular foliation $\mathscr{F}$ forms a family of extrinsically flat slicing. The slicing is generic and compatible with the Killing symmetries of the spherically symmetric spacetime. Thus $\mathscr{F}$ preserves the spherical symmetry of the horizon geometry. The foliation can be intuitively identified with a family of observers that only have time and radial components. We have chosen this more trivial geometry-preserving foliation as we aim to make the study of the back-reaction of the metric to individual 2-point functions easier.

Considering the particular foliation, we can straightforwardly rewrite (20) as

$$\sum_{i=1}^{\mathcal{N}} \nabla H_i = -4\pi G \sum_{i=1}^{\mathcal{N}} \left\langle J\left(x_i, x'_i\right)\right\rangle \tag{22}$$

where x and x' lie on the same spacelike hypersurface S_i and also $\sum_i x_i \equiv S1$ and $\sum_i x'_i \equiv S2$. We think of $S2$ as an artificial (nonphysical) $r = const$ timelike hypersurface, and we assume it respects the same foliation as the horizon, Figure 2.

Evidently from (22), and in agreement with what we have argued earlier (5), the phase transitions of ∇H between a pair of neighboring points depend on the vacuum accumulation term J between them. Note that, due to the stochastic nature of the inhomogeneous vacuum density, the r.h.s. of (22) fluctuates constantly with field strength of order of the expectation value of the vacuum density in the region $\langle J(x, x')\rangle \approx \int_x^{x'} \langle T_{00}\rangle$.

However, (22) does not suffice for a complete description of the near-horizon dynamics. In addition, we also need to examine the evolution of the local scale factor. The equation of motion of $a(t, x)$ away from matter source is given as [1]

$$\begin{aligned} \tan \Theta_x = {} & \frac{\Omega(0, x)}{\Omega(0, x')} \tan \Theta_x \\ & + \frac{4\pi G}{\Omega(0, x')} \int_x^{x'} J\left(0, x'\right) dl' \end{aligned} \tag{23}$$

where Θ_x is the initial phase of $a(t, x)$ at some arbitrary x.

Similar to what we did with (22), we now wish to express the equation of motion of $a(t, x)$ in the vicinity of the horizon. We do that in terms of the vacuum accumulation between a pair of corresponding points both of which lie on the same spacelike hypersurface. Neglecting the initial phase of $a(t, x)$ because of its constant fluctuations, we get

$$\tan \Theta_{x,x'} = \int_x^{x'} \frac{\Omega(x)}{\Omega(x')} \tan \Theta_{x,x'} + \frac{4\pi G}{\Omega(x')} \left\langle J\left(x, x'\right)\right\rangle \tag{24}$$

where $\langle J(x, x')\rangle \approx \int_x^{x'} \langle T_{00}\rangle$. One should recall we focus on the small *negative* excess of the fluctuations every T.

The back-reaction of the near black hole geometry to the fluctuations between a pair of points on a single slice (22) is negligible, regardless of their microscopically singular expectation value. However, the spatial separation between the pair, Δx, is constantly fluctuating in a harmonic-oscillator-like manner between phases of expansion and contraction due to the inhomogeneous vacuum density. Consequently, by considering the constant fluctuations of Δx on all hypersurfaces of order of the Schwarzschild radius and assuming a subtle *negative* excess on each slice every $T = 2\pi/\Omega$, the back-reaction accumulates and leads to deviations from general relativity which manifest in metric fluctuations.

In case $M_{BH} = const$, then $\langle J(x, x')\rangle$ depends on the dimensionality of the internal Hilbert space of the black hole and thus fluctuates stochastically

$$\left\langle J\left(x, x'\right)\right\rangle \sim \int_{x}^{x'} \langle T_{00}\rangle \sim \dim\left(\mathscr{H}_{int}\right) \tag{25}$$

Therefore, following (15) and considering the microscopic back-reaction of the metric to the constantly fluctuating inhomogeneous vacuum density, the action for the near-horizon region reads

$$I_{near} = \int_{R} \int dx \int dt \left[\partial_r \lambda + T_{00}\left(x, x'\right) \partial_r \left(x, x'\right) \partial_t \lambda\right] \tag{26}$$

where the radial-dependent term of the vacuum $\partial_r \lambda$ is given by (15). The time-dependent vacuum term $\partial_t \lambda$ is taken on a $r = const$ surface with respect to the horizon and varies with the change in the mass of the black hole. The action is given in terms of couplings between the inhomogeneous constantly fluctuating and radially dependent vacuum term λ and the local energy density.

In summary, we demonstrated that when we consider the accumulated back-reaction of the background metric to the weak quantum vacuum fluctuations (*i.e.,* constant phase transitions of ∇H on different hypersurfaces) across the whole horizon area, $\mathcal{O}(r_S)$, we effectively get the proposed quantum metric fluctuations. The current derivation of these "soft" quantum modifications to the effective black hole geometry may be thought of as the microscopic origin of the metric fluctuations proposed by Giddings [4–6].

3.2. Nonlocal Black Hole Evaporation via Strong Quantum Vacuum Fluctuations. In this subsection we continue the study of the back-reaction of a Schwarzschild black hole to the constantly fluctuating inhomogeneous vacuum density. More precisely, we examine what effects the conjectured in [2] strong quantum vacuum fluctuations have, when they are considered *onto* the horizon. Specifically, we focus on how the locality constraint of local quantum field theory is modified due to the back-reaction of the metric to those fluctuations. As a result, we argue the horizon geometry *need not* always respect locality.

Given the fundamental degrees of freedom are not continuously distributed, we assume the semiclassical geometry to be just an effective field theory. Thus we expect the local quantum field theory evolution to only be approximately correct. As a result, deviations from it should be present in high energy regimes. For instance, black holes are one such place where we expect nonlocal corrections to manifest. That is the case since due to the large field-strength radial dependence in the near-horizon region, $\langle T_{00}(x)\rangle \gg \langle T_{00}(y)\rangle$, where $2M \leq x \leq R$ and $y > R$.

The particular strong vacuum fluctuations are assumed to have high enough local energy density as to yield "soft" (*i.e.,* brief and highly localized) nonlocal corrections to local quantum field theory. Similar to [1], we assume these soft corrections manifest in the form of local singularity points.

Because in [1] the local scale factor $a(t, x)$, defined at each spacetime point, was argued to have a harmonic-oscillator-like behavior, as it changes phases, $a(t, x)$ must inevitably go through zero, *i.e.,* yield a local singularity point. However, this was shown to not be physically problematic. Since we interpret the local scale factor as a harmonic oscillator, $a(t, x) = 0$ is just a generic part of the oscillation cycle which takes place every T. Namely, a harmonic oscillator cannot change sign (phase) without passing through zero. Therefore, in a generic oscillation cycle a singularity point disappears almost immediately.

Similarly here, because we interpret individual strong vacuum fluctuations *onto* the horizon as harmonic oscillators, the microscopically singular expectation value (as predicted by quantum field theory) of a single strong fluctuation at a given point on the horizon should not be problematic. Likewise, we assume the local singularity point disappears almost immediately.

Physically, a strong vacuum fluctuation on the horizon briefly violates the generic locality constraint of local quantum field theory in that small region. As a result, this deviation from classicality allows quantum information to escape to asymptotic infinity at the necessary rate. Due to the small domain of dependence and brief lifespan of the local singularity points, their effects on a freely falling observer are negligible. These nonlocal corrections to the semiclassical geometry, although "soft," have significant effect on the black hole over periods compared to its lifetime. That being said, we assume that, since the inhomogeneous vacuum density is constantly fluctuating, it will carry out quantum information at a rate of $\mathcal{O}(1)$ per light-crossing time as to restore unitary quantum mechanics. Such local quantum field theory modifications allow evaporation of information-carrying Hawking particles to begin as early as the scrambling time.

Let us clarify what we mean when we characterize the nonlocal corrections as "soft." The local quantum field theory deviations (*i.e.,* local singularity points) are "brief" in the sense that they have very short lifespan of order of the lifetime of the fluctuation. Furthermore, the deviations are considered to be "highly localized" (*i.e.,* short wavelength/high momentum) since they manifest on scales of order of the wavelength of the fluctuation. Thus a strong vacuum fluctuation has a limited domain of dependence, and cannot lead to $\mathcal{O}(1)$ corrections to the background metric of a solar mass black hole.

Let us now point out an important distinction between the local singularity points on cosmological scales (away from a black hole) [1] and onto the horizon. We claim there is an intrinsic difference in the stage of the oscillation cycle during which a singularity point is produced. More precisely, a strong vacuum fluctuation (onto the horizon) produces a local singularity when, during an oscillation cycle T, it reaches the maximum of its local energy density, which also happens to be above a certain threshold ζ. On the other hand, the local scale factor $a(t, x)$ (away from a black hole) produces a singularity point when it reaches zero during its oscillation cycle.

Moreover, the local singularity points, although similar, should not be mistaken with spacetime defects (see [14, 15]). For instance, (i) at a local singularity point/spacetime defect the curvature is divergent, and (ii) particle passing through (near) a spacetime defect/local singularity will be scattered off, *i.e.,* experience a local Lorentz boost.

4. Casimir Stress Interpretation

In [1] the Casimir effect was used as a tool for illustrating the effects of vacuum fluctuations. In the current Section we present a toy model in which we restate the conjectured quantum metric fluctuations in terms of the Casimir effect in the near-horizon region. Specifically, we present the metric fluctuations in the language of the Casimir effect just outside the black hole (between *S1* and *S2*, Figure 1).

Likewise, we will examine two distinct cases: coarse-grained and fine-grained.

In its general form, the Casimir stress equation is given as [1]

$$S(t,x,y) = T_{zz}^{inside} - T_{zz}^{outside} \tag{27}$$

In this picture, imagine the role of the pair of conducting plates is played by *S1* and *S2*, Figure 1.

4.1. Coarse-Grained. Similar to Section 3, we are only interested in the effective (macroscopic) dynamics and thus neglect contributions from individual degrees of freedom.

We can straightforwardly expand (27) in the near-horizon region as

$$\int_{S1}^{S2} S(t,x,y) = \int_{S1}^{S2} \langle \partial_r \rho_{in} \rangle - \int_{S2}^{\infty} \langle \partial_r \rho_{out} \rangle \tag{28}$$

where, as we showed earlier, the expectation value of the radially dependent local energy density $\langle \partial_r \rho_{in} \rangle$ is of order of the vacuum accumulation term in that region

$$\int_{S1}^{S2} \langle \partial_r \rho_{in} \rangle \sim \int_{S1}^{S2} \partial_r \langle J \rangle \tag{29}$$

Following (18), we expect $\int_R \langle J \rangle \gg 0$ and $\int_\infty \langle J \rangle = 0$. That is, the second term on the r.h.s. of (28) vanishes at the asymptotic limit; namely $\langle \rho_{out} \rangle \longrightarrow 0$ as $r \longrightarrow \infty$.

Therefore, we generally get

$$\left\langle T_{zz}^{inside} \right\rangle \gg \left\langle T_{zz}^{outside} \right\rangle \tag{30}$$

One should keep in mind that, regardless of whether $\langle J \rangle$ vanishes or not, it constantly fluctuates due to the inhomogeneous vacuum density.

Notice that (30) is in agreement with the general construction of the Casimir effect (27).

Considering the microscopic back-reaction of the spherically symmetric background metric that we discussed earlier and the greater energy density in the vicinity of the horizon (30), one can easily see how the proposed quantum metric fluctuations of order of the Schwarzschild radius emerge in this setting.

4.2. Fine-Grained. Similar to our approach in Section 3.1.2, we begin by foliating the horizon region into individual spacelike hypersurfaces. As a result, we can now express the l.h.s. of (28) as

$$\int_R S(t,x,y) = \sum_{i=1}^{\mathcal{N}} \left\langle J\left(x_i, x_i'\right) \right\rangle \tag{31}$$

Evidently, the nonvanishing l.h.s. of (28) can straightforwardly be rewritten as a linear combination of vacuum accumulation terms, defined on individual slices.

Due to the inhomogeneous vacuum density, Δx on any given slice constantly fluctuates between phases of expansion and contraction. Moreover, because of the strong field gradient (15) and considering (30), we assume that in the large $\mathcal{N}$ limit (*i.e.,* of order of the Schwarzschild radius) the accumulated back-reaction is positive. Thus, an observer at asymptotic infinity sees oscillations of the horizon as a back-reaction of the underlying black hole metric.

In summary, we see that with minimal assumptions one can restate our earlier argument about quantum metric fluctuations in terms of the Casimir effect in a near black hole region.

5. Conclusions

In the current work we studied how a Schwarzschild black hole back-reacts to the constantly fluctuating inhomogeneous vacuum density proposed in [1]. More precisely, embracing the microscopically singular expectation value of the local energy density of the inhomogeneous vacuum fluctuations (predicted by quantum field theory), we examined how a black hole metric back-reacts in two distinct regions: the vicinity of the black hole and onto the horizon. As a result, we demonstrated that vacuum fluctuations above a given threshold, considered onto the horizon, cause deviations from local quantum field theory. Meanwhile, fluctuations below that threshold, considered in the vicinity of the horizon, lead to potentially observable metric fluctuations of order of the Schwarzschild radius.

Physically, the conjectured modifications of local quantum field theory, induced by the strong vacuum fluctuations onto the horizon, were argued to lead to nonlocal release of information-carrying Hawking particles. In fact, we argued that in this scenario a black hole can begin radiating quantum information to infinity as early as the scrambling time [2].

On the other hand, we argued that weak fluctuations in the near-horizon region yield observable macroscopic quantum gravity effects in the form of metric fluctuations of order of the Schwarzschild radius, that is, constant oscillations of the horizon between $r = 2M$ and $r = 2M + \delta$. As far as a distant observer is concerned, we assume she may interpret the conjectured metric fluctuations as a physical membrane just outside the horizon. Thus the proposed metric fluctuations may serve as the microscopic origin of the stretched horizon in observer complementarity [16]. Also, we assume the proposed metric fluctuations play a significant role in binary black hole mergers. In particular, they may produce observable postmerger gravitational wave "echoes" similar to [17].

The recent advances in gravitational wave astronomy have opened new possibilities for experimentally testing models, similar to this one, which predict deviations from general relativity in the near-horizon region. In addition, the current scenario could also be approached from an accretion disk perspective as we believe the metric fluctuations may have measurable effects on accretion disk flows around a black hole.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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