Research Article

Search for Vector Charmonium(-Like) States in the $e^+e^- \rightarrow \eta J/\psi$ Line Shape

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The cross section of $e^+e^- \rightarrow \eta J/\psi$ has been measured by BESIII and Belle experiments. Fit to the $e^+e^- \rightarrow \eta J/\psi$ line shape, the resonant structures are evident. The parameters for the three resonant structures are $M_1 = (3980 \pm 17 \pm 7)$ MeV/$c^2$, $\Gamma_1 = (104 \pm 32 \pm 13)$ MeV, $M_2 = (4219 \pm 5 \pm 4)$ MeV/$c^2$, $\Gamma_2 = (63 \pm 9 \pm 3)$ MeV, and $M_3 = (4401 \pm 12 \pm 4)$ MeV/$c^2$, $\Gamma_3 = (49 \pm 19 \pm 4)$ MeV, where the first uncertainties are statistical and the second are systematic. We attribute the three structures to $\psi(4040)$, $Y(4220)$, and $\psi(4115)$ states. The branching fractions $\mathcal{B}(\psi(4040) \rightarrow \eta J/\psi)$ and $\mathcal{B}(\psi(4115) \rightarrow \eta J/\psi)$ are given. If $Y(4220)$ is taken as $\psi(4S)$ state, the branching fraction $\mathcal{B}(\psi(4S) \rightarrow \eta J/\psi)$ is also given. Combining all $Y(4220)$ parameters obtained from different decays, we give average parameters for $Y(4220)$, which are $M_{Y(4220)} = (4220.8 \pm 2.4)$ MeV/$c^2$, $\Gamma_{Y(4220)} = (54.8 \pm 3.3)$ MeV.

The potential model works well in describing the heavy quarkonia states [1], especially for the charmonium states below the open-charm threshold. However, above this threshold, there are still many predicted states that have not been observed yet. In recent years, charmonium physics has gained renewed strong interest from both the theoretical and the experimental side, due to the observation of charmonium-like states, such as $Y(4260)$ [2], $Y(4360)$ [3], and $Y(4660)$ [4]. These $Y$-states are above the open-charm threshold and do not fit in the conventional charmonium spectroscopy, so they could be exotic states that lie outside the quark model [5–7]. The $1^-$ $Y$-states are all observed in $\pi^+ \pi^- J/\psi$ or $\pi^+ \pi^- \psi(3686)$, while recently, one state (called $Y(4220)$) was observed in $e^+e^- \rightarrow \omega \chi_{c0}$ [8] and two states (called $Y(4220)$ and $Y(4390)$) were observed in $e^+e^- \rightarrow \pi^+\pi^- h_c$ [9,10]. This indicates that the $Y$-states also can be searched by other charmonium transition decays. Among them, the cross section for $e^+e^- \rightarrow \eta J/\psi$ is relatively large, so we can search for $Y$-states in $\eta J/\psi$ line shape. The study of these $1^-$ $Y$-states is very helpful to clarify the missing predicted charmonium states in potential model. In all $Y$-states, perhaps some are conventional charmonium. So, it is important to confirm which $Y$-states are charmonium and which $Y$-states are exotic states.

Recently, the process $e^+e^- \rightarrow \eta J/\psi$ has been measured by BESIII [11, 12] and Belle [13] experiments. In [13], the authors claimed that $\eta J/\psi$ is from resonances $\psi(4040)$ and $\psi(4160)$. Figure 1 shows the cross sections from the two experiments for the center-of-mass energy from 3.80 to 4.65 GeV, and they are consistent with each other within error. The cross section of $e^+e^- \rightarrow \eta J/\psi$ is of the same order of magnitude as those of $e^+e^- \rightarrow \pi^+\pi^- J/\psi$ [14] or $e^+e^- \rightarrow \pi^+\pi^- \psi(3686)$ [15], but with a different line shape. This indicates that there is large coupling between $Y$-states and $\eta J/\psi$. So, we try to use BESIII and Belle measurements to extract the resonant structures parameters in $e^+e^- \rightarrow \eta J/\psi$. Currently, between 3.80 GeV and 4.65 GeV, all the observed vector charmonium(-like) states are $\psi(4040)$, $\psi(4160)$, $Y(4220)$, $Y(4360)$, $Y(4390)$, and $\psi(4415)$. In this paper, we try to search for these vector charmonium(-like) states in the $e^+e^- \rightarrow \eta J/\psi$ line shape.

We use a least $\chi^2$ method to fit the cross section. From Figure 1, we can see that there are three obvious structures around 4, 4.2, and 4.4 GeV in the line shape of $e^+e^- \rightarrow \eta J/\psi$. Assuming that $\eta J/\psi$ comes from three resonances, we fit
the cross section with coherent sum of three constant width relativistic Breit-Wigner (BW) functions (model 1, BW$_1$ + BW$_2$ + BW$_3$); that is,

$$
\sigma(\sqrt{s}) = \text{BW}_1(\sqrt{s}) \left[ \frac{\text{PS}(\sqrt{s})}{\text{PS}(M_1)} \right] + \text{BW}_2(\sqrt{s}) \left[ \frac{\text{PS}(\sqrt{s})}{\text{PS}(M_2)} e^{i\phi_1} \right]^2 + \text{BW}_3(\sqrt{s}) \left[ \frac{\text{PS}(\sqrt{s})}{\text{PS}(M_3)} e^{i\phi_2} \right]^2,
$$

where $\text{PS}(\sqrt{s}) = \rho/\sqrt{s}$ is the 2-body phase space factor, where $\rho$ is the $\eta$ or $J/\psi$ momentum in the $e^+e^-$ center-of-mass frame, $\phi_1$ and $\phi_2$ are relative phases, and $\text{BW}(\sqrt{s}) = \sqrt{2\pi} \Gamma_{ee} \mathcal{B}(\eta/\psi) \Gamma(s - M^2 + iM\Gamma)$ is the BW function for a vector state, with mass $M$, total width $\Gamma$, electron partial width $\Gamma_{ee}$, and the branching fraction to $\eta/\psi$, $\mathcal{B}(\eta/\psi)$. From the fit, $\Gamma_{ee}$ and $\mathcal{B}(\eta/\psi)$ cannot be obtained separately; we can only extract the product $\Gamma_{ee} \mathcal{B}(\eta/\psi)$.

$\chi^2$ is minimized to obtain the best estimation of the resonant parameters, and the statistical uncertainties are obtained when $\chi^2$ value changes to 1 compared with the minimum. Figure 2 shows the fit results. Two solutions are found with the same fit quality. The fits indicate the existence of three resonances (called $Y_1$, $Y_2$, and $Y_3$) with mass and width being $M_1 = (3980 \pm 17)$ MeV/$c^2$, $\Gamma_1 = (104 \pm 32)$ MeV; $M_2 = (4219 \pm 5)$ MeV/$c^2$, $\Gamma_2 = (63 \pm 9)$ MeV; and $M_3 = (4401 \pm 12)$ MeV/$c^2$, $\Gamma_3 = (49 \pm 19)$ MeV, and the goodness of the fit is $\chi^2/ndf = 51.8/50$, corresponding to a confidence level of 40%, where $ndf$ is the number of degrees of freedom. All fitted parameters of the cross section of $e^+e^- \rightarrow \eta/\psi$ are listed in Table 1.

There is only one $1^-$ charmonium state around 4.2 GeV, which is $\psi(4160)$. It is interesting to check whether the second structure in $e^+e^- \rightarrow \eta/\psi$ is from $\psi(4160)$, or whether there is contribution from $\psi(4160)$. If we fix $Y_2$ state's parameters to the mass and width of $\psi(4160)$ (model 2, BW$_1$ + $\psi(4160)$ + BW$_3$) [16], the goodness of fit is $\chi^2/ndf = 101.7/52$, corresponding to a confidence level of $5 \times 10^{-5}$. We also try to add $\psi(4160)$ resonance to fit the cross section (model 3, BW$_1$ + BW$_2$ + $\psi(4160)$ + BW$_3$), where the mass and width of $\psi(4160)$ are fixed at the world average values [16] in the fit. The goodness of fit is $\chi^2/ndf = 51.0/48$, corresponding to a confidence level of 36%.

The systematic uncertainties will also influence the goodness of fit. The systematic uncertainties include the uncertainty of the center-of-mass energy determination, parametrization of the BW function, the cross section measurement, and the uncertainty of $\psi(4160)$'s mass and width. Since the uncertainty of the measured beam energy is about 0.8 MeV at BESIII, the beam energy $\sqrt{s}$ is varied within 0.8 MeV in the fit. To estimate the uncertainty from parametrization of BW function, the width $\Gamma$ of BW function is set to be the mass dependent width $\Gamma = \Gamma_0(\text{PS}(\sqrt{s})/\text{PS}(M))$ in the fit, where $\Gamma_0$ is the width of the resonance. The uncertainty of the cross section measurements will affect the goodness of fit; we vary the cross section values within the systematic uncertainty in the fit. To estimate the uncertainty from the uncertainty of $\psi(4160)$'s mass and width, we vary $\psi(4160)$'s mass and width within the uncertainty to refit. The results for different situations are also listed in Table 2.

From Table 2, we notice that model 2's confidence level is very bad, while those of model 1 and model 3 are all good. To get the significance of BW$_2$, we choose model 2 as zero hypothesis and choose model 3 as alternative hypothesis. When model 2 is at the minimum (maximum) goodness-of-fit value, model 3 is also at the minimum (maximum) goodness-of-fit value. So, the statistical significance of BW$_2$ is $(5.0\sigma, 8.4\sigma)$, comparing $\chi^2$'s change and taking into account the change of the number of degrees of freedom. To get the significance of $\psi(4160)$, we choose model 1 as zero hypothesis and choose model 3 as alternative hypothesis. The statistical significance of $\psi(4160)$ is $(0.2\sigma, 0.8\sigma)$, which means

![Figure 1: Cross section of $e^+e^- \rightarrow \eta/\psi$ from BESIII and Belle experiments.](image)

Table 1: The fitted parameters of the cross section of $e^+e^- \rightarrow \eta/\psi$. The first uncertainties are statistical, and the second are systematic.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Solution I</th>
<th>Solution II</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_1$ (MeV/$c^2$)</td>
<td>$3980 \pm 17 \pm 7$</td>
<td></td>
</tr>
<tr>
<td>$\Gamma_1$ (MeV)</td>
<td>$104 \pm 32 \pm 13$</td>
<td></td>
</tr>
<tr>
<td>$\Gamma_{ee}^{Y_1} \mathcal{B}(Y_1 \rightarrow \eta/\psi)$ (eV)</td>
<td>$3.6 \pm 0.9 \pm 0.3$</td>
<td>$4.1 \pm 1.2 \pm 0.4$</td>
</tr>
<tr>
<td>$M_2$ (MeV/$c^2$)</td>
<td>$4219 \pm 5 \pm 4$</td>
<td></td>
</tr>
<tr>
<td>$\Gamma_2$ (MeV)</td>
<td>$63 \pm 9 \pm 3$</td>
<td></td>
</tr>
<tr>
<td>$\Gamma_{ee}^{Y_2} \mathcal{B}(Y_2 \rightarrow \eta/\psi)$ (eV)</td>
<td>$3.6 \pm 1.1 \pm 0.3$</td>
<td>$6.7 \pm 1.3 \pm 0.4$</td>
</tr>
<tr>
<td>$M_3$ (MeV/$c^2$)</td>
<td>$4401 \pm 12 \pm 4$</td>
<td></td>
</tr>
<tr>
<td>$\Gamma_3$ (MeV)</td>
<td>$49 \pm 19 \pm 4$</td>
<td></td>
</tr>
<tr>
<td>$\Gamma_{ee}^{Y_3} \mathcal{B}(Y_3 \rightarrow \eta/\psi)$ (eV)</td>
<td>$0.7 \pm 0.3 \pm 0.2$</td>
<td>$1.4 \pm 0.7 \pm 0.2$</td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>$2.76 \pm 0.53 \pm 0.19$</td>
<td>$-2.64 \pm 0.43 \pm 0.18$</td>
</tr>
<tr>
<td>$\phi_2$</td>
<td>$-2.34 \pm 1.10 \pm 0.25$</td>
<td>$1.88 \pm 1.03 \pm 0.24$</td>
</tr>
</tbody>
</table>
that $\psi(4160)$ resonance is not significant. According to the above analysis, to describe the second structure in the fit, we only need one resonance BW$_1$; the resonance $\psi(4160)$ is not necessary.

To check the statistical significance of the third structure, we choose BW$_1$ + BW$_2$ as zero hypothesis and then get the statistical significance of the third resonance as 5.6$\sigma$, comparing $\chi^2$'s change and taking into account the change of the number of degrees of freedom. So, to describe the $e^+e^-\rightarrow \eta/\psi$ line shape very well, three resonances are required; the fourth resonance is not necessary based on the current data. Model 1 (BW$_1$ + BW$_2$ + BW$_3$) can describe the line shape very well.

The systematic uncertainties on the resonant parameters are mainly from the uncertainty of the center-of-mass energy determination, parametrization of the BW function, and the cross section measurement. The details have been described above. By assuming that all these sources of systematic uncertainties are independent, we add them in quadrature.

From the above fit results using model 1, we notice that $Y_1$'s parameters are close to $\psi(4040)$ [16]; the differences are about 3$\sigma$ and less than 1$\sigma$ for mass and width. $Y_3$'s parameters are close to $\psi(4415)$ [16]; the differences are less than 2$\sigma$ and less than 1$\sigma$ for mass and width; so here we attribute the $Y_1$ and $Y_3$ states to $\psi(4040)$ and $\psi(4415)$ states. If we take $\Gamma(\psi(4040)\rightarrow e^+e^-)$ and $\Gamma(\psi(4415)\rightarrow e^+e^-)$ values from world averages [16], we can obtain the branching fractions $\mathcal{B}(\psi(4040)\rightarrow \eta/\psi) = (4.2\pm1.2)\times10^{-3}$ or $(4.8\pm1.5)\times10^{-3}$ and $\mathcal{B}(\psi(4415)\rightarrow \eta/\psi) = (1.2\pm0.6)\times10^{-3}$ or $(2.4\pm1.3)\times10^{-3}$. For $Y_2$ state, the mass and width are consistent with the state (called Y(4220)) found in $e^+e^-\rightarrow \omega\pi_0$ [8], $\pi^+\pi^-\eta_c$ [9], $\pi^+\pi^-J/\psi$ [14], $\pi^+D^0\bar{D}^{*-}$ [17], and $\pi^+\pi^-\eta$ [3686] [15], so it is reasonable that we take $Y_2$ state as Y(4220).

Table 3 lists the parameters for Y(4220) from different decay modes; the last line shows the fit results.
parameters from fit results, which are \( M_{Y(4220)} = (4220.8 \pm 2.4) \text{ MeV}/c^2 \), \( \Gamma_{Y(4220)} = (54.8 \pm 3.3) \text{ MeV} \).

\( Y_1 \) and \( Y_3 \) states are likely to be \( \psi(4040) \) and \( \psi(4415) \) states; it is reasonable that \( \psi(4040) \) and \( \psi(4415) \) can decay to \( \eta \bar{\psi}/\psi \) because they are considered as \( \psi(3S) \) and \( \psi(4S) \) states. For \( Y(4220) \) state, it also can decay to \( \eta \bar{\psi}/\psi \); so it is normal that \( Y(4220) \) is also considered as \( \psi(nS) \) state. In [18], the authors suggested that \( Y(4220) \) is a "missing \( \psi(4S) \)" state, which is predicted in [19, 20]. Now, from the fit results about \( Y(4220) \), the suggestion is reasonable. If we take \( Y(4220) \) as \( \psi(4S) \) state, then \( \psi(4415) \) will be \( \psi(5S) \) state. It is very clear for the \( \eta \bar{\psi}/\psi \) line shape that the three obvious structures are due to \( \psi(3S) \), \( \psi(4S) \), and \( \psi(5S) \) states. If we take the theoretical range \( \Gamma(\psi(4S) \rightarrow e^+e^-) = 0.63\sim0.66 \text{ keV} \) [18] and take fit results for \( Y_2 \) from Table 1, we can obtain the branching fraction \( \mathcal{B}(\psi(4S) \rightarrow \eta \bar{\psi}/\psi) = (5.5 \pm 1.7) \times 10^{-3} \) [18]. While [18] predicts that the upper limit of branching fraction of \( \psi(4S) \rightarrow \eta \bar{\psi}/\psi \) to be 1.9 \times 10^{-3}, the fit results have some deviations from theory prediction.

Recently, the measurement of \( e^+e^- \rightarrow \pi^+\pi^- \bar{\psi}/\psi \) [14] from BESIII indicates that the previous \( Y(4260) \) structure may be the combination of two resonances; the lower resonance is \( Y(4220) \). BESIII also has observed process \( e^+e^- \rightarrow \gamma X(3872) \) [21] around 4.23 and 4.26 GeV. In [22–24], the authors suggested that \( X(3872) \) can be identified as a \( 2^3P_1 \bar{c}\bar{c} \) state with effect from \( DD^* \) threshold. If we assume that \( X(3872) \) structure is due to \( \chi_{c2}(2P) \) state and take \( Y(4220) \) as \( \psi(4S) \) state, the process \( e^+e^- \rightarrow \gamma X(3872) \) [21] around 4.23 and 4.26 GeV observed by BESIII will be a simple transition \( \psi(4S) \rightarrow \gamma\chi_{c2}(2P) \). We think this is a reasonable explanation for \( X(3872) \) and \( Y(4220) \). This indicates that perhaps some exotic structures are due to the conventional charmonium.

In summary, we fit to the \( e^+e^- \rightarrow \eta \bar{\psi}/\psi \) line shape three evident resonant structures. The parameters for the three resonant structures are \( M_1 = (3980 \pm 17 \pm 7) \text{ MeV}/c^2 \), \( \Gamma_1 = (104 \pm 32 \pm 13) \text{ MeV} \), \( M_2 = (4219 \pm 5 \pm 4) \text{ MeV}/c^2 \), \( \Gamma_2 = (63 \pm 9 \pm 3) \text{ MeV} \), and \( M_3 = (4401 \pm 12 \pm 4) \text{ MeV}/c^2 \), \( \Gamma_3 = (49 \pm 19 \pm 4) \text{ MeV} \), where the first uncertainties are statistical and the second are systematic. We attribute the three structures to \( \psi(4040) \), \( Y(4220) \), and \( \psi(4415) \) states. The branching fractions are \( \mathcal{B}(\psi(4040) \rightarrow \eta \bar{\psi}/\psi) = (4.2 \pm 1.2) \times 10^{-3} \) or \( (4.8 \pm 1.5) \times 10^{-3} \) and \( \mathcal{B}(\psi(4415) \rightarrow \eta \bar{\psi}/\psi) = (1.2 \pm 0.6) \times 10^{-3} \) or \( (2.4 \pm 1.3) \times 10^{-3} \). We emphasize that the second structure in \( e^+e^- \rightarrow \eta \bar{\psi}/\psi \) is not from \( \psi(4160) \): it is more consistent with \( Y(4220) \). Combining all \( Y(4220) \) parameters obtained from different decays, we give average parameters for \( Y(4220) \), which are \( M_{Y(4220)} = (4220.8 \pm 2.4) \text{ MeV}/c^2 \), \( \Gamma_{Y(4220)} = (54.8 \pm 3.3) \text{ MeV} \). If we take \( Y(4220) \) as \( \psi(4S) \) state, then \( \psi(4415) \) will be \( \psi(5S) \) state. It is very clear for the \( \eta \bar{\psi}/\psi \) line shape that the three obvious structures are due to \( \psi(3S) \), \( \psi(4S) \), and \( \psi(5S) \) states. If we take the theoretical range \( \Gamma(\psi(4S) \rightarrow e^+e^-) = 0.63\sim0.66 \text{ keV} \) [18], we can obtain the branching fraction \( \mathcal{B}(\psi(4S) \rightarrow \eta \bar{\psi}/\psi) = (5.5 \pm 1.7) \times 10^{-3} \) [18], which has some deviations from theory prediction [18]. At present, the number of high precision data points in \( e^+e^- \rightarrow \eta \bar{\psi}/\psi \) line shape is relatively small, so more measurements around this energy region are desired; this can be achieved in BESIII and BelleII experiments in the future.

**Conflicts of Interest**

The authors declare that there are no conflicts of interest regarding the publication of this paper.

**Acknowledgments**

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References


