Hindawi
Advances in High Energy Physics
Volume 2018, Article ID 6045932, 10 pages
https://doi.org/10.1155/2018/6045932

Research Article

Analysis of $D_s^* D^* K^*$ and $D_{s1} D_1 K^*$ Vertices in Three-Point Sum Rules

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Received 3 February 2018; Revised 4 July 2018; Accepted 2 August 2018; Published 26 September 2018

Academic Editor: Ricardo G. Felipe

In this study, the coupling constants of $D_s^* D^* K^*$ and $D_{s1} D_1 K^*$ vertices were determined within the three-point Quantum chromodynamics sum rules method with and without consideration of the $SU_f(3)$ symmetry. The coupling constants were calculated for off-shell charm and K* cases. Considering the nonperturbative effect of the correlation function, as the most important contribution, the quark-quark, quark-gluon, and gluon-gluon condensate corrections were estimated and were compared with other predictive methods.

1. Introduction

Considerable attention has been focused on the strong form factors and coupling constants of meson vertices in the context of quantum chromodynamics (QCD) since the last decade. In high energy physics, understanding the functional form of the strong form factors plays very important role in explaining of the meson interactions. Therefore, accurate determination of the strong form factors and coupling constants associated with the vertices involving mesons has attracted great interest in recent studies of the high energy physics.

Quantum chromodynamics sum rules (QCDSR) formalism has been used extensively to study about the "exotic" mesons made of quark- gluon hybrid ($q\bar{q}g$), tetraquark states ($q\bar{q}q\bar{q}$), molecular states of two ordinary mesons, glueballs [1], and vertices involving charmed mesons such as $D^* D^* \rho$ [2, 3], $D^* D\pi$ [2, 4], $DD\rho$ [5], $D^* D\rho$ [6], DDJ/ψ [7], $D^* DJ/\psi$ [8], $D^* D_s K$, $D_s^* DK$, $D_0^* D_s K$, $D_{s0}^* DK$ [9], $D^* D^* P$, $D^* DV$, DDV [10], $D^* D^* \pi$ [11], $D_s D^* K$, $D_s^* DK$ [12], $DD\omega$ [13], $D_{s0} DK$ and $D_0 D_s K$ [14], $D_{s1} D^* K$, $D_{s1} D^* K_0^*$ [15, 16], $D_s D_s V$, $D_s^* D_s^* V$ [17, 18], and $D_1 D^* \pi$, $D_1 D_0 \pi$, $D_1 D_1 \pi$ [19].

In this study, 3-point sum rules (3PSR) method is used to calculate the strong form factors and coupling constants of the $D_s^* D^* K^*$ and $D_{s1} D_1 K^*$ vertices. The 3PSR correlation function is investigated from the phenomenological and the theoretical points of view. Regarding the phenomenological (physical) approach, the representation can be expressed in terms of hadronic degrees of freedom which can be considered as responsible for the introduction of the form factors, decay constant, and masses. The theoretical (QCD) approach usually can be divided into two main contributions as perturbative and nonperturbative. In this approach, the quark-gluon language and Wilson operator product expansion (OPE) are usually used to evaluate the correlation function in terms of the QCD degrees of freedom such as quark condensate, gluon condensate, etc. Equating the two sides and applying the double Borel transformations with respect to the momentum of the initial and final states to suppress the contribution of the higher states, and continuum, the strong form factors can be estimated.

The effective Lagrangian of the interaction for the $D_s^* D^* K^*$ and $D_{s1} D_1 K^*$ vertices can be written as [20]

$$\begin{aligned}\mathscr{L}_{D_s^* D^* K^*} = i g_{D_s^* D^* K^*} \Big[& D_s^{*\mu} \left(\partial_\mu K^{*\nu} \overline{D}_\nu^* - K^{*\nu} \partial_\mu \overline{D}_\nu^* \right) \\ & + \left(\partial_\mu D_s^{*\nu} K_\nu^* - D_s^{*\nu} \partial_\mu K_\nu^* \right) \overline{D}^{*\mu} \\ & + K^{*\mu} \left(D_s^{*\nu} \partial_\mu \overline{D}_\nu^* - \partial_\mu D_s^{*\nu} \overline{D}_\nu^* \right) \Big],\end{aligned} \tag{1}$$

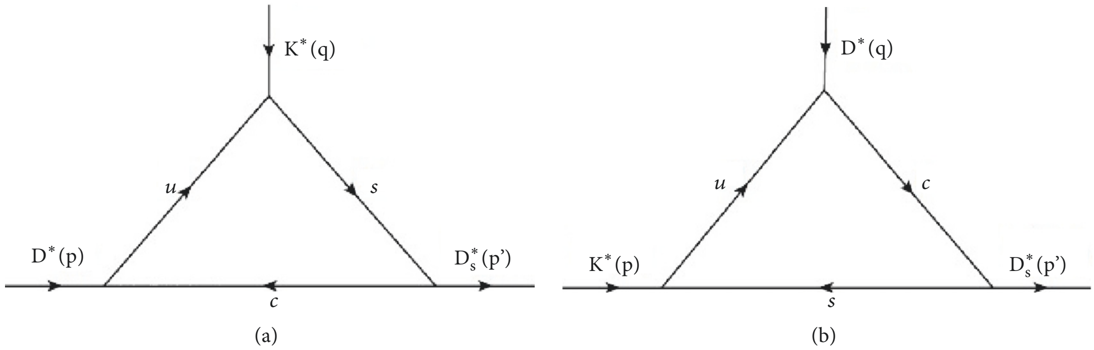


Figure 1: Perturbative diagrams for off-shell K^* (a) and off-shell D^* (b).

$$\begin{aligned}\mathscr{L}_{D_{s1}D_1K^*} = & \, ig_{D_{s1}D_1K^*}\left[D_{s1}{}^{\mu}\left(\partial_\mu K^{*\nu}\overline{D}_{1\nu} - K^{*\nu}\partial_\mu \overline{D}_{1\nu}\right)\right. \\ & + \left(\partial_\mu D_{s1}{}^{\nu}K^*_\nu - D_{s1}{}^{\nu}\partial_\mu K^*_\nu\right)\overline{D}_1^{\mu} \\ & \left. + K^{*\mu}\left(D_{s1}{}^{\nu}\partial_\mu \overline{D}_{1\nu} - \partial_\mu D_{s1}{}^{\nu}\overline{D}_{1\nu}\right)\right],\end{aligned} \tag{2}$$

where $g_{D_s^*D^*K^*}$ and $g_{D_{s1}D_1K^*}$ are the strong form factor. Using the introduced form of the Lagrangian, the elements related to the $D_s^*D^*K^*$ and $D_{s1}D_1K^*$ vertices can be derived in terms of the strong form factor as

$$\begin{aligned}&\left\langle D^*(p,\varepsilon) D_s^*(p',\varepsilon') \mid K^*(q,\varepsilon'')\right\rangle \\ &\quad = ig_{D_s^*D^*K^*}(q^2) \\ &\quad\quad \times \left[\left(q^\alpha + p'^\alpha\right)g^{\mu\nu} - \left(q^\mu + p^\mu\right)g^{\nu\alpha} + q^\nu g^{\alpha\mu}\right] \\ &\quad\quad \times \varepsilon_\alpha(p)\varepsilon'_\mu(p')\varepsilon''_\nu(q),\end{aligned} \tag{3}$$

$$\begin{aligned}&\left\langle D_1(p,\varepsilon) D_{s1}(p',\varepsilon') \mid K^*(q,\varepsilon'')\right\rangle \\ &\quad = ig_{D_{s1}D_1K^*}(q^2) \\ &\quad\quad \times \left[\left(q^\alpha + p'^\alpha\right)g^{\mu\nu} - \left(q^\mu + p^\mu\right)g^{\nu\alpha} + q^\nu g^{\alpha\mu}\right] \\ &\quad\quad \times \varepsilon_\alpha(p)\varepsilon'_\mu(p')\varepsilon''_\nu(q),\end{aligned} \tag{4}$$

where $q = p - p'$.

The organization of the paper is as follows: In Section 2, the quark-quark, quark-gluon, and gluon-gluon condensate contributions, considering the nonperturbative effects of the Borel transform scheme, are discussed in order to calculate the strong form factors of the $D_s^*D^*K^*$ and $D_{s1}D_1K^*$ vertices in the framework of the 3PSR. The numerical analysis of the strong form factors estimation as well as the coupling constants, with and without consideration of the $SU_f(3)$ symmetry, is described in Section 3 and the conclusion is made in Section 4.

2. The Strong Form Factor of $D_s^*D^*K^*$ and $D_{s1}D_1K^*$ Vertices

To compute the strong form factor of the $D_s^*D^*K^*$ and $D_{s1}D_1K^*$ vertices via the 3PSR, we start with the correlation function. When the K^* meson is off-shell, the correlation function can be written in the following form:

$$\begin{aligned}\Pi^{K^*}_{\mu\nu\alpha}(p,p') = & \, i^2\int d^4x d^4y e^{i(p'x-py)} \\ & \cdot \left\langle 0 \mid \mathscr{T}\left\{j_\mu^{D_s^*}(x)\, j_\nu^{K^*\dagger}(0)\, j_\alpha^{D^*\dagger}(y)\right\} \mid 0\right\rangle,\end{aligned} \tag{5}$$

$$\begin{aligned}\Pi^{K^*}_{\mu\nu\alpha}(p,p') = & \, i^2\int d^4x d^4y e^{i(p'x-py)} \\ & \cdot \left\langle 0 \mid \mathscr{T}\left\{j_\mu^{D_{s1}}(x)\, j_\nu^{K^*\dagger}(0)\, j_\alpha^{D_1\dagger}(y)\right\} \mid 0\right\rangle.\end{aligned} \tag{6}$$

For off-shell charm meson, the correlation function can be written as

$$\begin{aligned}\Pi^{D^*}_{\mu\alpha\nu}(p,p') = & \, i^2\int d^4x d^4y e^{i(p'x-py)} \\ & \cdot \left\langle 0 \mid \mathscr{T}\left\{j_\mu^{D_s^*}(x)\, j_\alpha^{D^*\dagger}(0)\, j_\nu^{K^*\dagger}(y)\right\} \mid 0\right\rangle,\end{aligned} \tag{7}$$

$$\begin{aligned}\Pi^{D_1}_{\mu\alpha\nu}(p,p') = & \, i^2\int d^4x d^4y e^{i(p'x-py)} \\ & \cdot \left\langle 0 \mid \mathscr{T}\left\{j_\mu^{D_{s1}}(x)\, j_\alpha^{D_1\dagger}(0)\, j_\nu^{K^*\dagger}(y)\right\} \mid 0\right\rangle,\end{aligned} \tag{8}$$

where $j_\mu^{D_s^*} = \overline{c}\gamma_\mu s$, $j_\alpha^{D^*} = \overline{c}\gamma_\alpha u$, $j_\mu^{D_{s1}} = \overline{c}\gamma_\mu\gamma_5 s$, $j_\alpha^{D_1} = \overline{c}\gamma_\alpha\gamma_5 u$, and $j_\nu^{K^*} = \overline{u}\gamma_\nu s$ are interpolating currents with the same quantum numbers of D_s^*, D^*, D_{s1}, D_1, and K^* mesons. As described in Figure 1, $\mathscr{T}$, p, and p' are time ordering product and four momentum instances of the initial and final mesons, respectively.

Considering the OPE scheme in the phenomenological approach, the correlation functions (see (5) to (8)) can be written in terms of several tensor structures and their coefficients are found using the sum rules. It is clear from (3) and (4) that the form factor $g_{D_s^*D^*K^*}$ is used for the fourth Lorentz structure which can be extracted from the sum rules. We choose the Lorentz structure because of its fewer ambiguities in the 3PSR approach, i.e., less influence of higher dimension of the condensates and better stability as function of the Borel mass parameter [2]. For these reasons, the $g_{\mu\alpha}q_\nu$

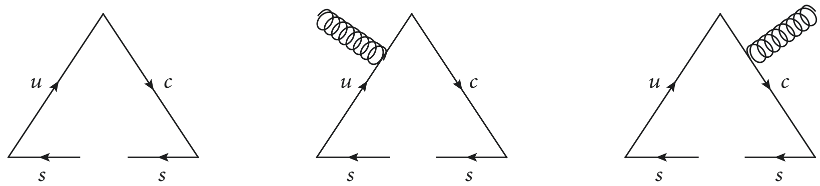


FIGURE 2: Nonperturbative diagrams for the off-shell D^* meson.

structure is chosen which is assumed to better formulate the problem.

In order to calculate the phenomenological part of the correlation functions in (5) to (8), three complete sets of intermediate states with the same quantum number as the currents $j_\mu^{D_s^*}$, $j_\alpha^{D^*}$, $j_\mu^{D_{s1}}$, $j_\alpha^{D_1}$, and $j_\nu^{K^*}$ are selected. The matrix elements $\langle 0 \mid j_\mu^{D_s^*} \mid D_s^*(p,\varepsilon)\rangle$, $\langle 0 \mid j_\alpha^{D^*} \mid D^*(p)\rangle$, $\langle 0 \mid j_\mu^{D_{s1}} \mid D_{s1}(p,\varepsilon)\rangle$, $\langle 0 \mid j_\alpha^{D_1} \mid D_1(p)\rangle$, and $\langle 0 \mid j_\nu^{K^*} \mid K^*(q,\varepsilon)\rangle$ are defined as

$$\left\langle 0 \mid j_\mu^V \mid V(p,\varepsilon)\right\rangle = m_V f_V \varepsilon_\mu(p), \tag{9}$$

where m_V and f_V are the masses and decay constants of mesons $V(D_s^*, D^*, D_{s1}, D_1, K^*)$ and ε_μ is introduced as the polarization vector of the vector meson $V(D_s^*, D^*, D_{s1}, D_1, K^*)$.

The phenomenological part of the $g_{\mu\alpha}q_\nu$ structure associated with the $D_s^* D^* K^*$ vertex for off-shell D^* and K^* mesons can be expressed as

$$\begin{aligned}
\Pi_{\mu\nu\alpha}^{D^*} &= -g_{D_s^* D^* K^*}^{D^*}\left(q^2\right) \\
&\cdot \frac{m_{K^*} f_{K^*} f_{D^*} f_{D_s^*}\left(3m_{D^*}^2 + m_{K^*}^2 - q^2\right)}{2m_{D_s^*}\left(q^2 - m_{D^*}^2\right)\left(p^2 - m_{K^*}^2\right)\left(p'^2 - m_{D_s^*}^2\right)} + \cdots, \\
\Pi_{\mu\nu\alpha}^{K^*} &= g_{D_s^* D^* K^*}^{K^*}\left(q^2\right) \\
&\cdot \frac{m_{K^*} f_{K^*} f_{D^*} f_{D_s^*}\left(3m_{D_s^*}^2 + m_{D^*}^2 - q^2\right)}{2m_{D_s^*}\left(q^2 - m_{K^*}^2\right)\left(p^2 - m_{D^*}^2\right)\left(p'^2 - m_{D_s^*}^2\right)} + \cdots,
\end{aligned} \tag{10}$$

The phenomenological part of the $g_{\mu\alpha}q_\nu$ structure associated with the $D_{s1}D_1K^*$ vertex for off-shell D_1 and K^* mesons can be expressed as

$$\begin{aligned}
\Pi_{\mu\nu\alpha}^{D_1} &= -g_{D_{s1} D_1 K^*}^{D_1}\left(q^2\right) \\
&\cdot \frac{m_{K^*} f_{K^*} f_{D_1} f_{D_{s1}}\left(3m_{D_1}^2 + m_{K^*}^2 - q^2\right)}{2m_{D_{s1}}\left(q^2 - m_{D_1}^2\right)\left(p^2 - m_{K^*}^2\right)\left(p'^2 - m_{D_{s1}}^2\right)} + \cdots, \\
\Pi_{\mu\nu\alpha}^{K^*} &= g_{D_{s1} D_1 K^*}^{K^*}\left(q^2\right) \\
&\cdot \frac{m_{K^*} f_{K^*} f_{D_1} f_{D_{s1}}\left(3m_{D_{s1}}^2 + m_{D_1}^2 - q^2\right)}{2m_{D_{s1}}\left(q^2 - m_{K^*}^2\right)\left(p^2 - m_{D^*}^2\right)\left(p'^2 - m_{D_{s1}}^2\right)} + \cdots.
\end{aligned} \tag{11}$$

Using the operator product expansion in Euclidean region and assuming $p^2, p'^2 \longrightarrow -\infty$, one can calculate the QCD side of the correlation function (see (5) to (8)) which contains perturbative and nonperturbative terms. Using the double dispersion relation for the coefficient of the Lorentz structure $g_{\mu\alpha}q_\nu$ appearing in the correlation function (see (3) and (4)), we get

$$\begin{aligned}
&\Pi_{per}^M\left(p^2, p'^2, q^2\right) \\
&\quad = -\frac{1}{4\pi^2}\int ds \int ds' \frac{\rho^M\left(s, s', q^2\right)}{\left(s - p^2\right)\left(s' - p'^2\right)} \\
&\qquad + \text{subtraction terms},
\end{aligned} \tag{12}$$

where $\rho^M(s, s', q^2)$ is spectral density and M stands for off-shell charm and K^* mesons. The Cutkoskys rule allows us to obtain the spectral densities of the correlation function for the Lorentz structure appearing in the correlation function. As shown in Figure 1, the leading contribution comes from the perturbative term. As a result, the spectral densities are obtained in the case of the double discontinuity in (12) for the vertices; see Appendix A.

In order to consider the nonperturbative part of the correlation functions for the case of spectator light quark (for off-shell charm meson), we proceed to calculate the nonperturbative contributions in the QCD approach which contain the quark-quark and quark-gluon condensates [23]. Figure 2 describes the important quark-quark and quark-gluon condensates from the nonperturbative contribution of the off-shell charm mesons [23].

In the 3PSR frame work, when the heavy quark is a spectator (for off-shell K^* meson), the gluon-gluon contribution can be considered. Figure 3 shows related diagrams of the gluon-gluon condensate. More details about the nonperturbative contributions $C_{D_s^* D^* K^*}^{D^*}$ and $C_{D_{s1} D_1 K^*}^{D_1}$ (sum contributions of quark-quark and quark-gluon condensates) and $C_{D_s^* D^* K^*}^{K^*}$ and $C_{D_{s1} D_1 K^*}^{K^*}$ (for gluon-gluon condensates) corresponding to Figures 2 and 3 are given in Appendix B, respectively.

Considering the perturbative and nonperturbative parts of the correlation function in order to suppress the contributions of the higher states, the strong form factors can be calculated in the phenomenological side by equating the two representations of the correlation function and applying the Borel transformations with respect to $p^2 (p^2 \longrightarrow M_1^2)$ and

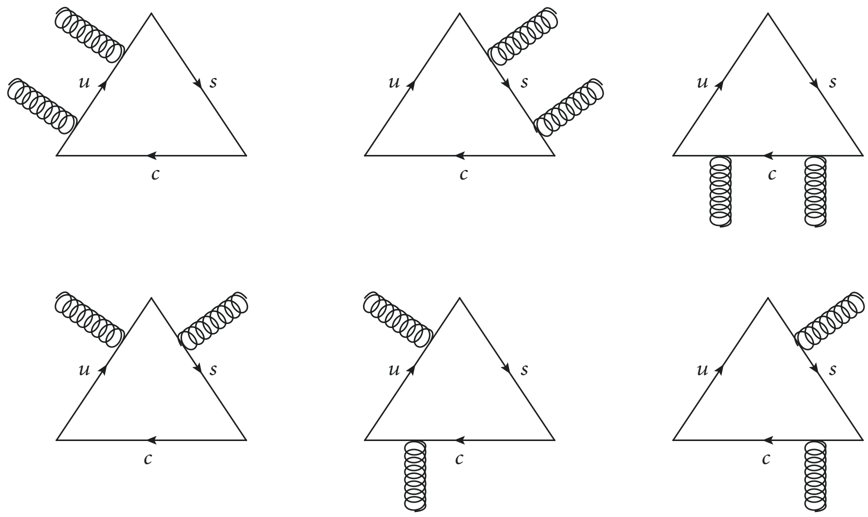


Figure 3: Nonperturbative diagrams for the off-shell K^* meson.

$p'^2 (p'^2 \longrightarrow M_2^2)$. The equations for the strong form factors $g^{K^*}_{D_s^* D^* K^*}$ and $g^{D^*}_{D_s^* D^* K^*}$ are obtained as

$$\begin{aligned} g^{K^*}_{D_s^* D^* K^*}\left(q^2\right) &= \frac{2 m_{D_s^*}\left(q^2 - m_{K^*}^2\right)}{m_{K^*} f_{K^*} f_{D^*} f_{D_s^*}\left(3 m_{D_s^*}^2 + m_{D^*}^2 - q^2\right)} \\ &\cdot e^{m_{D^*}^2/M_1^2} e^{m_{D_s^*}^2/M_2^2} \left\{ -\frac{1}{4\pi^2} \int_{(m_c+m_s)^2}^{s_0^{D_s^*}} ds' \right. \\ &\cdot \int_{(m_u+m_c)^2}^{s_0^{D^*}} ds \rho^{K^*}_{D_s^* D^* K^*}\left(s, s', q^2\right) e^{-s/M_1^2} e^{-s'/M_2^2} \\ &\left. - i M_1^2 M_2^2 \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \times C^{K^*}_{D_s^* D^* K^*} \right\}, \\ g^{D^*}_{D_s^* D^* K^*}\left(q^2\right) &= \frac{2 m_{D_s^*}\left(q^2 - m_{D^*}^2\right)}{m_{K^*} f_{K^*} f_{D^*} f_{D_s^*}\left(3 m_{D^*}^2 + m_{K^*}^2 - q^2\right)} \\ &\cdot e^{m_{K^*}^2/M_1^2} e^{m_{D_s^*}^2/M_2^2} \left\{ -\frac{1}{4\pi^2} \int_{(m_c+m_s)^2}^{s_0^{D_s^*}} ds' \right. \\ &\cdot \int_{(m_u+m_s)^2}^{s_0^{K^*}} ds \rho^{D^*}_{D_s^* D^* K^*}\left(s, s', q^2\right) e^{-s/M_1^2} e^{-s'/M_2^2} \\ &\left. + M_1^2 M_2^2 \langle s\bar{s} \rangle \times C^{D^*}_{D_s^* D^* K^*} \right\}. \end{aligned} \tag{13}$$

The equations describing the strong form factors $g^{K^*}_{D_{s1} D_1 K^*}$ and $g^{D_1}_{D_{s1} D_1 K^*}$ can be written as

$$\begin{aligned} g^{K^*}_{D_{s1} D_1 K^*}\left(q^2\right) &= \frac{2 m_{D_{s1}}\left(q^2 - m_{K^*}^2\right)}{m_{K^*} f_{K^*} f_{D_1} f_{D_{s1}}\left(3 m_{D_{s1}}^2 + m_{D_1}^2 - q^2\right)} \\ &\cdot e^{m_{D_1}^2/M_1^2} e^{m_{D_{s1}}^2/M_2^2} \left\{ -\frac{1}{4\pi^2} \int_{(m_c+m_s)^2}^{s_0^{D_{s1}}} ds' \right. \\ &\cdot \int_{(m_u+m_c)^2}^{s_0^{D_1}} ds \rho^{K^*}_{D_{s1} D_1 K^*}\left(s, s', q^2\right) e^{-s/M_1^2} e^{-s'/M_2^2} \\ &\left. - i M_1^2 M_2^2 \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \times C^{K^*}_{D_{s1} D_1 K^*} \right\}, \\ g^{D_1}_{D_{s1} D_1 K^*}\left(q^2\right) &= \frac{2 m_{D_{s1}}\left(q^2 - m_{D_1}^2\right)}{m_{K^*} f_{K^*} f_{D_1} f_{D_{s1}}\left(3 m_{D_1}^2 + m_{K^*}^2 - q^2\right)} \\ &\cdot e^{m_{K^*}^2/M_1^2} e^{m_{D_{s1}}^2/M_2^2} \left\{ -\frac{1}{4\pi^2} \int_{(m_c+m_s)^2}^{s_0^{D_{s1}}} ds' \right. \\ &\cdot \int_{(m_u+m_s)^2}^{s_0^{K^*}} ds \rho^{D_1}_{D_{s1} D_1 K^*}\left(s, s', q^2\right) e^{-s/M_1^2} e^{-s'/M_2^2} \\ &\left. + M_1^2 M_2^2 \langle s\bar{s} \rangle \times C^{D_1}_{D_{s1} D_1 K^*} \right\}. \end{aligned} \tag{14}$$

where the quantities $s_0^{D_s^*}$, $s_0^{D^*}$ $s_0^{D_{s1}}$, $s_0^{D_1}$, and $s_0^{K^*}$ are introduced as the continuum thresholds in D_s^*, D^* D_{s1}, D_1, and K^* mesons, respectively, and $\rho^{K^*}_{D_s^* D^* K^*}$, $\rho^{D^*}_{D_s^* D^* K^*}$, $C^{K^*}_{D_s^* D^* K^*}$, $C^{D^*}_{D_s^* D^* K^*}$, $\rho^{K^*}_{D_{s1} D_1 K^*}$, $\rho^{D_1}_{D_{s1} D_1 K^*}$, $C^{K^*}_{D_{s1} D_1 K^*}$, and $C^{D_1}_{D_{s1} D_1 K^*}$ are defined in Appendices A and B.

3. Numerical Analysis

In order to numerically estimate the strong form factors and coupling constants of the vertices $D_s^* D^* K^*$ and $D_{s1} D_1 K^*$, the values of the quark and meson masses are chosen as $m_s = 0.14 \pm 0.01$ GeV, $m_{K^*} = 0.89$ GeV, $m_{D_s^*} = 2.11$ GeV, $m_{D_{s1}} = 2.46$ GeV, and $m_{D_1} = 2.42$ GeV [24]. Moreover, the leptonic decay constants of the vertices are $f_{K^*} = 220 \pm 5$ [24], $f_{D_s^*} = 314 \pm 19$ [25], $f_{D^*} = 242 \pm 12$ [25], $f_{D_{s1}} = 225 \pm 20$ [26], and $f_{D_1} = 219 \pm 11$ [27]

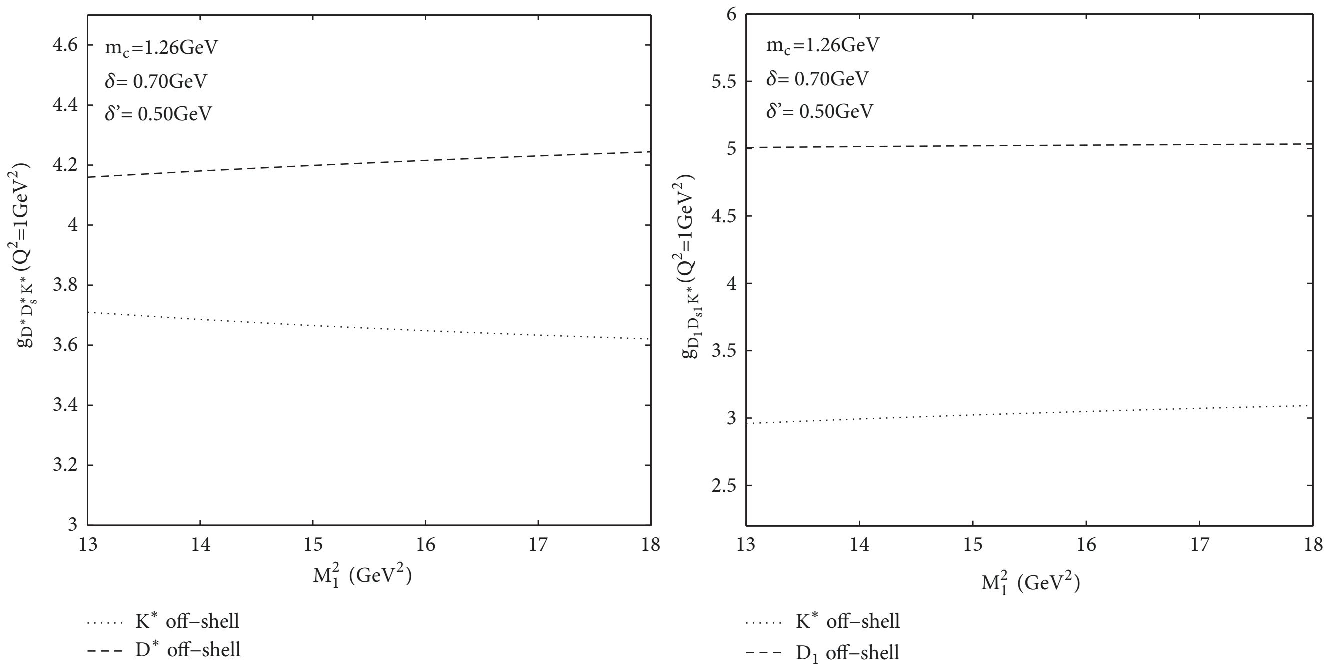


FIGURE 4: The strong form factors $g_{D_s^*D^*K^*}$ (left) and $g_{D_{s1}D_1K^*}$ (right) as functions of the Borel mass parameter M_1^2 for off-shell charm and K^* mesons.

TABLE 1: Parameters appearing in the fit functions for the $D_s^*D^*K^*$ and $D_{s1}D_1K^*$ vertices for various m_c and (δ,δ'), where $(\delta_1,\delta_1') = (0.50, 0.30)$, $(\delta_2,\delta_2') = (0.70, 0.50)$, and $(\delta_3,\delta_3') = (0.90, 0.70)$ GeV.

	Set I						Set II	
Form factor	$A(\delta_1,\delta_1')$	$B(\delta_1,\delta_1')$	$A(\delta_2,\delta_2')$	$B(\delta_2,\delta_2')$	$A(\delta_3,\delta_3')$	$B(\delta_3,\delta_3')$	$A(\delta_2,\delta_2')$	$B(\delta_2,\delta_2')$
$g^{K^*}_{D_s^*D^*K^*}(Q^2)$	4.04	183.10	4.95	197.49	5.93	215.62	4.43	266.52
$g^{D^*}_{D_s^*D^*K^*}(Q^2)$	3.67	61.46	4.42	52.33	5.56	47.96	4.18	57.95
$g^{K^*}_{D_{s1}D_1K^*}(Q^2)$	4.19	15.24	4.39	21.18	4.68	29.21	4.22	7.69
$g^{D_1}_{D_{s1}D_1K^*}(Q^2)$	2.62	13.27	3.13	19.47	4.02	25.32	3.03	14.87

There are four auxiliary parameters containing the Borel mass parameters M_1 and M_2 and continuum thresholds $s_0^{K^*}$, $s_0^{D^*(D_1)}$, and $s_0^{D_s^*(D_{s1})}$ in ((13) and (14)). The strong form factors and coupling constants are physical quantities which are independent of the mass parameters and continuum thresholds. However, the continuum thresholds are not completely arbitrary and can be related to the energy of the first exited state. The values of the continuum thresholds are taken to be $s_0^{K^*} = (m_{K^*} + \delta)^2$, $s_0^{D^*(D_1)} = (m_{D^*(D_1)} + \delta')^2$, and $s_0^{D_s^*(D_{s1})} = (m_{D_s^*(D_{s1})} + \delta')^2$ where 0.50 GeV $\le \delta \le$ 0.90 GeV and 0.30 GeV $\le \delta' \le$ 0.70 GeV [2–4].

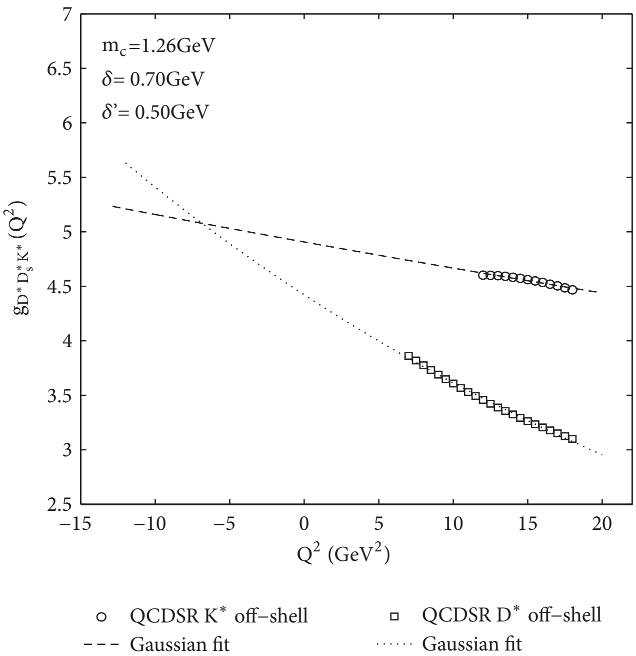
Our results should be almost insensitive to the intervals of the Borel parameters. In this work, the Borel masses are related as $M_1^2/M_2^2 = (m_{K^*}^2 + m_c^2)/m_{D_s^*(D_{s1})}^2$ and $M_1^2 = M_2^2$ for off-shell charm mesons and K^*, respectively [5, 6]. The form factors for the $D_s^*D^*K^*$ and $D_{s1}D_1K^*$ vertices with respect to the Borel parameters M_1^2 are shown in Figure 4. It is found from the figure that the stability of the form factors, as function of Borel parameters, is good in the region of 13 GeV^2 $< M_1^2 <$ 18 GeV^2 for off-shell K^* and charm mesons. We get $M_1^2 = 15$ GeV^2 and calculate the strong form factors $g_{D_s^*D^*K^*}$ in some points of Q^2 via the 3PSR formalism.

To extract the coupling constants from the form factors, it is needed to extend the Q^2 dependency of the strong form factors to the ranges that the sum rule results are not valid. Therefore, we fitted two sets of points (boxes and circles) imposing the condition that the two resulting parameterizations lead to the same result for $Q^2 = -m_m^2$, where m_m is the mass of the off-shell mesons. This procedure is sufficient to reduce the uncertainties. It is found that the sum rule predictions of the form factors in ((13) and (14)) are well fitted to the function

$$g\left(Q^2\right) = Ae^{-Q^2/B}. \quad (15)$$

The values of the parameters A and B are given in Table 1.

Variations of the strong form factors $g^{K^*}_{D_s^*D^*K^*}$ and $g^{D^*}_{D_s^*D^*K^*}$ for $D_s^*D^*K^*$ vertex and $g^{K^*}_{D_{s1}D_1K^*}$ and $g^{D_1}_{s1D_1K^*}$ for $D_{s1}D_1K^*$ vertex with respect to the Q^2 parameter are shown in Figure 5. The boxes and circles show the results of the



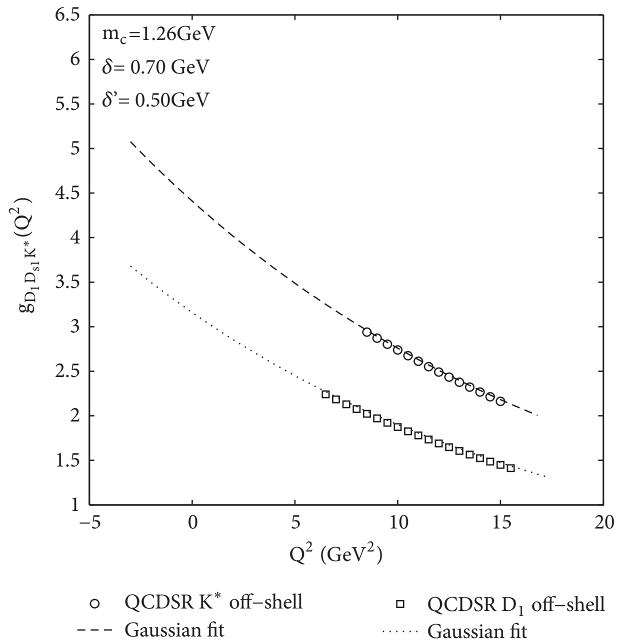


Figure 5: The strong form factors $g_{D_s^* D^* K^*}$ (left) and $g_{D_{s1} D_1 K^*}$ (right) as functions of Q^2 for off-shell charm and K* mesons.

Table 2: The coupling constant of the vertices $D_s^* D^* K^*$ and $D_{s1} D_1 K^*$.

g	Set I		Set II	
	off-shell charmed	off-shell K^*	off-shell charmed	off-shell K^*
$g_{D_s^* D^* K^*}$	4.77 ± 0.63	4.96 ± 0.64	4.48 ± 0.58	4.45 ± 0.58
$g_{D_{s1} D_1 K^*}$	4.22 ± 0.55	4.56 ± 0.59	4.48 ± 0.58	4.67 ± 0.62

Table 3: Parameters appearing in the fit functions for the $g_{D_s^* D^* K^*}$ and $g_{D_{s1} D_1 K^*}$ form factors in $SU_f(3)$ symmetry with $m_c = 1.26$ GeV and $(\delta, \delta') = (0.70, 0.50)$ GeV.

Form factor	A	B	Form factor	A	B
$g^{K^*}_{D_s^* D^* K^*}(Q^2)$	5.01	218.91	$g^{K^*}_{D_{s1} D_1 K^*}(Q^2)$	4.56	20.63
$g^{D^*}_{D_s^* D^* K^*}(Q^2)$	4.54	52.08	$g^{D_1}_{D_{s1} D_1 K^*}(Q^2)$	3.66	20.89

numerical evaluation of the form factors via the 3PSR. It is clear from the figure that the form factors are in good agreement with the fitted function.

So-called harder is used. In the present analysis we find that the form factor is harder when the lighter meson is off-shell. This is in line with the results of our previous work [17], whereas this is in contrast with other previous calculations quoted by the authors [28, 29].

The value of the strong form factors at $Q^2 = -m_m^2$ is defined as coupling constant. Calculation results of the coupling constant of the vertices $D_s^* D^* K^*$ and $D_{s1} D_1 K^*$ are summarized in Table 2. It should be noted that the coupling constants $g_{D_s^* D^* K^*}$ and $g_{D_{s1} D_1 K^*}$ are in the unit of GeV^{-1}.

In order to estimate the error of the calculated parameters, variations of the Borel parameter, continuum thresholds, and leptonic decay constants, as the most significant reasons of the uncertainties, are considered.

To investigate the value of the strong coupling constant via the $SU_f(3)$ symmetry, the mass of the s quark is ignored in all equations. Calculated parameters A and B for the $g_{D_s^* D^* K^*}$ and $g_{D_{s1} D_1 K^*}$ vertices, considering $(\delta, \delta') = (0.70, 0.50)$ GeV, are given in Table 3.

Estimated coupling constants of the vertices $D_s^* D^* K^*$, and $D_{s1} D_1 K^*$, considering the $SU_f(3)$ symmetry, are summarized in Table 4. The comparisons of the coupling constants $g_{D_s^* D^* K^*}$ with $g_{D^* D^* \rho}$, considering other methods described in [2, 3], are given in Table 5. It is found that the results of the calculated parameters are in reasonable agreement with that of [2, 3] and a factor of two orders of magnitude larger in comparison with [21, 22].

4. Conclusion

Strong form factors and coupling constants of $D_s^* D^* K^*$ and $D_{s1} D_1 K^*$ vertices were calculated in the frame work of

TABLE 4: The coupling constant of the vertices $D_s^* D^* K^*$ and $D_{s1} D_1 K^*$, in $SU_f(3)$ symmetry.

g	off-shell charmed	off-shell K^*	g	off-shell charmed	off-shell K^*
$g_{D_s^* D^* K^*}$	4.88 ± 0.64	5.03 ± 0.65	$g_{D_{s1} D_1 K^*}$	4.85 ± 0.63	4.75 ± 0.62

TABLE 5: Values of the strong coupling constant reporting different reference of the coupling constant $g_{D^* D^* \rho}$ [2, 3, 21, 22].

g	Ours	Reference [2, 3]	Reference [21, 22]
$g_{D_s^* D^* K^*}$	4.95 ± 0.64	6.60 ± 0.30	2.52

3-point sum rules of quantum chromodynamics with and without consideration of the $SU_f(3)$ symmetry. Considering nonperturbative contributions of the correlation functions, the quark-quark, quark-gluon, and gluon-gluon condensate corrections were estimated as the most effective terms. It was found from the numerical results that the obtained coupling constants are in good agreement with the other prediction methods described in [2, 3].

Appendix

A. Explicit Expressions of Spectral Densities

In this appendix, the explicit expressions of spectral densities are given as

$$\begin{aligned}
\rho^{D^*}_{D_s^* D^* K^*} &= 3I_0 \left[3m_s^2 - 2m_c m_s - s - \Delta + 4A \right. \\
&\quad \left. + (C_1 - C_2)(2u + 2m_c m_s - 2s) - 8(E_1 - E_2)\right] \\
\rho^{K^*}_{D_s^* D^* K^*} &= 3I_0 \left[3m_c^2 - 2m_c m_s - s - \Delta' + 4A' \right. \\
&\quad \left. + 2\left(C_1' - C_2'\right)(u + 2m_c m_s - 2s) - 8\left(E_1' - E_2'\right)\right] \\
\rho^{D_1}_{D_{s1} D_1 K^*} &= 3I_0 \left[3m_s^2 + 2m_c m_s - s - \Delta + 4A \right. \\
&\quad \left. + (C_1 - C_2)(2u - 2m_c m_s - 2s) - 8(E_1 - E_2)\right] \\
\rho^{K^*}_{D_{s1} D_1 K^*} &= 3I_0 \left[3m_c^2 + 2m_c m_s - s - \Delta' + 4A' \right. \\
&\quad \left. + 2\left(C_1' - C_2'\right)(u - 2m_c m_s - 2s) - 8\left(E_1' - E_2'\right)\right]
\end{aligned} \tag{A.1}$$

where coefficients in the spectral densities are given as

$$\begin{aligned}
I_0\left(s, s', q^2\right) &= \frac{1}{4\lambda^{1/2}(s, s', q^2)}, \\
\lambda(a, b, c) &= a^2 + b^2 + c^2 - 2ac - 2bc - 2ac, \\
\Delta &= s' + m_s^2 - m_c^2, \\
\Delta' &= s' + m_c^2 - m_s^2, \\
\Delta'' &= s + m_s^2, \\
u &= s + s' - q^2, \\
C_1 &= \frac{1}{\lambda(s, s', q^2)}\left[2s'\Delta'' - u\Delta\right], \\
C_2 &= \frac{1}{\lambda(s, s', q^2)}\left[2s\Delta - u\Delta''\right], \\
A &= -\frac{1}{2\lambda(s, s', q^2)}\left[4ss'm_s^2 - s\Delta^2 - s'\Delta''^2 - m_s^2 u^2 \right. \\
&\quad \left. + u\Delta\Delta''\right], \\
E_1 &= \frac{1}{2\lambda^2(s, s', q^2)}\left[8ss'^2 m_s^2\Delta'' - 2s'm_s^2 u^2\Delta'' \right. \\
&\quad - 4ss'm_s^2 u\Delta + m_s^2 u^3\Delta - 2s'^2\Delta''^3 + 3s'u\Delta\Delta''^2 \\
&\quad \left. - 2ss'\Delta^2\Delta'' - u^2\Delta^2\Delta'' + su\Delta^3\right], \\
E_2 &= \frac{1}{2\lambda^2(s, s', q^2)}\left[8s^2 s'm_s^2\Delta - 2sm_s^2 u^2\Delta'' \right. \\
&\quad - 4ss'm_s^2 u\Delta'' + m_s^2 u^3\Delta'' - 2s^2\Delta^3 + 3su\Delta^2\Delta'' \\
&\quad \left. - 2ss'\Delta\Delta''^2 - u^2\Delta\Delta''^2 + s'u\Delta''^3\right],
\end{aligned} \tag{A.2}$$

and also $A' = A_{|m_c \longleftrightarrow m_s}$, $C_1' = C_{1|m_c \longleftrightarrow m_s}$, $C_2' = C_{2|m_c \longleftrightarrow m_s}$, $E_1' = E_{1|m_c \longleftrightarrow m_s}$, and $E_2' = E_{2|m_c \longleftrightarrow m_s}$.

B. Explicit Expressions of the Coefficients

In this appendix, the explicit expressions of the coefficients of the quark and gluon condensate contributions of the strong form factors in the Borel transform scheme for all the vertices are presented.

$$\begin{aligned}
C^{D^*}_{D_s^* D^* K^*} &= \left(-6\frac{m_s {m_c}^2}{{M_2}^2} - 2\frac{{m_0}^2 m_c}{{M_2}^2} + 6\frac{m_c {m_s}^2}{{M_2}^2} \right. \\
&\quad + 3\frac{m_s q^2}{{M_2}^2} + 3\frac{m_c q^2 {m_s}^2}{{M_1}^2 {M_2}^2} - \frac{{m_0}^2 m_c q^2}{{M_1}^2 {M_2}^2} - 3\frac{{m_c}^3 {m_s}^2}{{M_1}^2 {M_2}^2} \\
&\quad \left. + \frac{{m_0}^2 {m_c}^3}{{M_1}^2 {M_2}^2} - 3\frac{{m_c}^3 {m_s}^2}{{M_2}^4} + \frac{3}{2}\frac{{m_0}^2 {m_c}^3}{{M_2}^4} + 3m_s\right) \\
&\quad \times e^{-m_c^2/M_2^2}, \\
C^{D_1}_{D_{s1} D_1 K^*} &= \left(2\frac{{m_0}^2 m_c}{{M_2}^2} - 6\frac{m_s {m_c}^2}{{M_2}^2} + 3\frac{m_s q^2}{{M_2}^2}\right.
\end{aligned}$$

$$-6\frac{m_c m_s{}^2}{M_2{}^2}-\frac{m_0{}^2 m_c{}^3}{M_1{}^2 M_2{}^2}-3\frac{m_c q^2 m_s{}^2}{M_1{}^2 M_2{}^2}+\frac{m_0{}^2 m_c q^2}{M_1{}^2 M_2{}^2}+3\frac{m_c{}^3 m_s{}^2}{M_1{}^2 M_2{}^2}+3\frac{m_c{}^3 m_s{}^2}{M_2{}^4}-\frac{3}{2}\frac{m_0{}^2 m_c{}^3}{M_2{}^4}+3m_s\Bigg)\times e^{-m_c^2/M_2^2}, \tag{B.1}$$

$$\begin{aligned}
C^{K^*}_{D_s^* D^* K^*} ={}& \widehat{I}_0(3,2,2)m_c^6-\widehat{I}_0(3,2,2)m_c^5 m_s \\
&+\widehat{I}_0(3,2,2)m_c^3m_s^3+\widehat{I}_1^{[1,0]}(3,2,2)m_c^4 \\
&+3\widehat{I}_0(4,1,1)m_c^4-\widehat{I}_2^{[1,0]}(3,2,2)m_c^4+2\widehat{I}_6(3,2,2) \\
&\cdot m_c^4+2\widehat{I}_2(2,1,3)m_c^3m_s+2\widehat{I}_0(2,1,3)m_c^3m_s \\
&-2\widehat{I}_1(2,2,2)m_c^3m_s-2\widehat{I}_1(2,1,3)m_c^3m_s \\
&-\widehat{I}_1(3,2,1)m_c^3m_s+\widehat{I}_2(3,2,1)m_c^3m_s \\
&-3\widehat{I}_0(4,1,1)m_c^3m_s+2\widehat{I}_2(2,2,2)m_c^3m_s \\
&+\widehat{I}_0(2,2,2)m_c^2m_s^2-2\widehat{I}_6(3,2,2)m_c^2m_s^2 \\
&+\widehat{I}_0^{[0,1]}(3,2,2)m_c^2m_s^2+\widehat{I}_2^{[1,0]}(3,2,2)m_c^2m_s^2 \\
&-\widehat{I}_1^{[1,0]}(3,2,2)m_c^2m_s^2+2\widehat{I}_2(3,1,2)m_cm_s^3 \\
&-\widehat{I}_2^{[1,0]}(3,2,2)m_cm_s^3-2\widehat{I}_1(3,1,2)m_cm_s^3 \\
&+6\widehat{I}_0(1,1,4)m_cm_s^3+\widehat{I}_1^{[1,0]}(3,2,2)m_cm_s^3 \\
&+\widehat{I}_0(3,1,2)m_s^4+2\widehat{I}_6^{[1,0]}(3,2,2)m_c^2+\widehat{I}_0(2,1,2) \\
&\cdot m_c^2+8\widehat{I}_8(3,2,1)m_c^2+4\widehat{I}_8^{[0,1]}(3,2,2)m_c^2 \\
&+2\widehat{I}_6^{[0,1]}(3,2,2)m_c^2-4\widehat{I}_6(3,2,1)m_c^2 \\
&-4\widehat{I}_7^{[0,1]}(3,2,2)m_c^2+3\widehat{I}_1^{[1,0]}(4,1,1)m_c^2 \\
&+6\widehat{I}_6(4,1,1)m_c^2+6\widehat{I}_6(3,1,2)m_c^2-8\widehat{I}_7(3,2,1) \\
&\cdot m_c^2-12\widehat{I}_7(4,1,1)m_c^2+2\widehat{I}_0(2,2,1)m_c^2 \\
&+12\widehat{I}_8(4,1,1)m_c^2-3\widehat{I}_2^{[1,0]}(4,1,1)m_c^2 \\
&+8\widehat{I}_6(2,1,3)m_cm_s+4\widehat{I}_6(2,2,2)m_cm_s \\
&-\widehat{I}_0^{[1,1]}(3,2,2)m_cm_s-3\widehat{I}_1(1,3,1)m_cm_s \\
&-2\widehat{I}_1^{[0,1]}(3,1,2)m_cm_s+3\widehat{I}_1^{[1,0]}(3,2,1)m_cm_s \\
&-2\widehat{I}_6(3,2,1)m_cm_s+3\widehat{I}_2(1,3,1)m_cm_s \\
&+2\widehat{I}_2^{[0,1]}(3,1,2)m_cm_s-\widehat{I}_0(2,1,2)m_cm_s \\
&-2\widehat{I}_6(3,1,2)m_cm_s-3\widehat{I}_2^{[1,0]}(3,2,1)m_cm_s \\
&-2\widehat{I}_6^{[1,0]}(3,2,2)m_s^2-12\widehat{I}_6(1,1,4)m_s^2 \\
&-\widehat{I}_0(2,1,2)m_s^2-3\widehat{I}_0(1,1,3)m_s^2-2\widehat{I}_0^{[0,1]}(3,1,2) \\
&\cdot m_s^2+3\widehat{I}_0^{[1,0]}(1,1,4)m_s^2+\widehat{I}_0^{[1,1]}(3,2,2)m_s^2 \\
&+6\widehat{I}_2^{[1,0]}(1,1,4)m_s^2+\widehat{I}_0(3,1,1)m_s^2 \\
&+24\widehat{I}_7(1,1,4)m_s^2-24\widehat{I}_8(1,1,4)m_s^2+4\widehat{I}_6(2,2,2) \\
&\cdot m_s^2-6\widehat{I}_1^{[1,0]}(1,1,4)m_s^2-2\widehat{I}_0^{[1,0]}(2,2,2)m_s^2 \\
&+3\widehat{I}_0^{[0,1]}(1,1,4)m_s^2+2\widehat{I}_6(3,1,2)m_s^2 \\
&+3\widehat{I}_1^{[0,1]}(2,1,2)-2\widehat{I}_2(2,1,1)+3\widehat{I}_6(1,3,1) \\
&-3\widehat{I}_2^{[0,1]}(2,1,2)-4S_1,1(3,2,2)+4\widehat{I}_7^{[1,0]}(3,2,1) \\
&-8\widehat{I}_8(2,2,1)-2\widehat{I}_6^{[0,1]}(2,2,2)-2\widehat{I}_6^{[0,1]}(3,1,2) \\
&+8\widehat{I}_7(2,2,1)-4\widehat{I}_8^{[1,0]}(3,2,1)-4\widehat{I}_6^{[1,0]}(3,2,1) \\
&+2\widehat{I}_6^{[1,1]}(3,2,2)-12\widehat{I}_8(3,1,1)+3\widehat{I}_1^{[1,0]}(3,1,2) \\
&+2\widehat{I}_1(2,1,1)-2\widehat{I}_2^{[1,0]}(1,2,2)+2\widehat{I}_1^{[1,0]}(1,2,2) \\
&+4\widehat{I}_7^{[0,1]}(2,2,2)-4\widehat{I}_8^{[0,1]}(2,2,2)+4\widehat{I}_6(2,2,1) \\
&-\widehat{I}_0^{[0,1]}(3,1,1)-3\widehat{I}_2^{[1,1]}(3,1,2)+\widehat{I}_2^{[2,0]}(2,2,2) \\
&-\widehat{I}_1^{[2,0]}(2,2,2)+12\widehat{I}_7(3,1,1)-8\widehat{I}_6(2,1,2) \\
&+4\widehat{I}_8^{[1,1]}(3,2,2)+\widehat{I}_0^{[2,0]}(3,2,1)-2\widehat{I}_0^{[1,0]}(1,2,2) \\
&-\widehat{I}_0^{[0,1]}(2,1,2)-3\widehat{I}_0(2,1,1)-4\widehat{I}_6(3,1,1) \\
&-2\widehat{I}_6^{[1,0]}(2,2,2)+4\widehat{I}_7^{[1,0]}(2,2,2)-4\widehat{I}_8^{[1,0]}(2,2,2) \\
&+2\widehat{I}_0(1,1,2)-\widehat{I}_0^{[1,0]}(3,1,1)-6\widehat{I}_7(1,3,1) \\
&+6\widehat{I}_8(1,3,1),
\end{aligned}$$

$$\begin{aligned}
C^{K^*}_{D_{s1} D_1 K^*} ={}& \widehat{I}_0(3,2,2)m_c^6+\widehat{I}_2(3,2,2)m_c^3m_s^3 \\
&-\widehat{I}_1(3,2,2)m_c^3m_s^3-\widehat{I}_0(3,2,2)m_c^3m_s^3 \\
&-4\widehat{I}_7(3,2,2)m_c^4+3\widehat{I}_0(2,2,2)m_c^4+2\widehat{I}_6(3,2,2) \\
&\cdot m_c^4+4\widehat{I}_8(3,2,2)m_c^4+3\widehat{I}_0(4,1,1)m_c^4 \\
&-2\widehat{I}_2(2,2,2)m_c^3m_s+2\widehat{I}_1(2,2,2)m_c^3m_s \\
&-2\widehat{I}_2(2,1,3)m_c^3m_s+2\widehat{I}_1(2,1,3)m_c^3m_s \\
&+\widehat{I}_1(3,2,1)m_c^3m_s+\widehat{I}_0^{[0,1]}(3,2,2)m_c^3m_s \\
&-\widehat{I}_2(3,2,1)m_c^3m_s-\widehat{I}_0(3,1,2)m_c^3m_s \\
&-2\widehat{I}_6(3,2,2)m_c^2m_s^2-\widehat{I}_1^{[1,0]}(3,2,2)m_c^2m_s^2 \\
&-6\widehat{I}_0(1,1,4)m_c^2m_s^2+\widehat{I}_2^{[1,0]}(3,2,2)m_c^2m_s^2 \\
&+6\widehat{I}_2(1,1,4)m_cm_s^3-6\widehat{I}_1(1,1,4)m_cm_s^3
\end{aligned}$$

$$
\begin{aligned}
&+ \widehat{I}_0 (3,1,2) m_s^4 + \widehat{I}_1 (2,1,2) m_c^2 - \widehat{I}_1 (3,1,1) m_c^2 \\
&+ 2\widehat{I}_0 (2,2,1) m_c^2 + \widehat{I}_1^{[2,0]} (3,2,2) m_c^2 + 8\widehat{I}_8 (3,2,1) \\
&\cdot m_c^2 + 4\widehat{I}_8^{[0,1]} (3,2,2) m_c^2 + 2\widehat{I}_6^{[0,1]} (3,2,2) m_c^2 \\
&- 12\widehat{I}_7 (4,1,1) m_c^2 - 4\widehat{I}_7^{[0,1]} (3,2,2) m_c^2 \\
&+ 4\widehat{I}_8^{[1,0]} (3,2,2) m_c^2 + \widehat{I}_2 (3,1,1) m_c^2 \\
&+ 2\widehat{I}_8^{[1,0]} (3,2,2) m_c^2 - \widehat{I}_2^{[2,0]} (3,2,2) m_c^2 \\
&- 4\widehat{I}_6 (3,2,1) m_c^2 - 8\widehat{I}_7 (3,2,1) m_c^2 - \widehat{I}_0^{[0,1]} (2,2,2) \\
&\cdot m_c^2 + \widehat{I}_0 (2,1,2) m_c^2 + 6\widehat{I}_6 (4,1,1) m_c^2 \\
&- \widehat{I}_2 (2,1,2) m_c^2 - 4\widehat{I}_7^{[1,0]} (3,2,2) m_c^2 + 6\widehat{I}_6 (3,1,2) \\
&\cdot m_c^2 + 12\widehat{I}_8 (4,1,1) m_c^2 - 4\widehat{I}_1 (1,1,3) m_c m_s \\
&+ 4\widehat{I}_2 (1,1,3) m_c m_s - 4\widehat{I}_6 (2,2,2) m_c m_s \\
&- 3\widehat{I}_2 (1,3,1) m_c m_s + 2\widehat{I}_0 (1,2,2) m_c m_s \\
&+ 2\widehat{I}_6 (3,1,2) m_c m_s + \widehat{I}_0 (2,1,2) m_c m_s \\
&- 2\widehat{I}_2^{[0,1]} (3,1,2) m_c m_s - 8\widehat{I}_6 (2,1,3) m_c m_s \\
&+ 2\widehat{I}_0^{[1,0]} (2,1,3) m_c m_s + 2\widehat{I}_6 (3,2,1) m_c m_s \\
&+ \widehat{I}_2 (2,1,2) m_c m_s + 2\widehat{I}_1^{[0,1]} (3,1,2) m_c m_s \\
&- \widehat{I}_1 (2,1,2) m_c m_s + 3\widehat{I}_1 (1,3,1) m_c m_s \\
&+ 3\widehat{I}_0 (2,2,1) m_c m_s - 3\widehat{I}_0 (1,1,3) m_s^2 + 4\widehat{I}_6 (2,2,2) \\
&\cdot m_s^2 - 6\widehat{I}_1^{[1,0]} (1,1,4) m_s^2 + 3\widehat{I}_0^{[0,1]} (1,1,4) m_s^2 \\
&- 12\widehat{I}_6 (1,1,4) m_s^2 - 2\widehat{I}_0^{[0,1]} (3,1,2) m_s^2 \\
&- \widehat{I}_0 (2,1,2) m_s^2 + 6\widehat{I}_2^{[1,0]} (1,1,4) m_s^2 + 2\widehat{I}_6 (3,1,2) \\
&\cdot m_s^2 - 2\widehat{I}_6^{[1,0]} (3,2,2) m_s^2 - 2\widehat{I}_2^{[1,0]} (1,2,2) \\
&+ \widehat{I}_2^{[1,0]} (3,1,1) - 4\widehat{I}_6^{[1,0]} (3,2,1) - \widehat{I}_0^{[1,0]} (3,1,1) \\
&- 8\widehat{I}_8 (2,2,1) - \widehat{I}_0^{[0,1]} (2,1,2) - 12\widehat{I}_8 (3,1,1) \\
&- 2\widehat{I}_6^{[1,0]} (2,2,2) + 12\widehat{I}_7 (3,1,1) - 8\widehat{I}_8 (2,1,2) \\
&- 8\widehat{I}_6 (2,1,2) - \widehat{I}_1^{[1,0]} (3,1,1) + 8\widehat{I}_7 (2,1,2) \\
&+ 4\widehat{I}_7^{[1,0]} (3,2,1) - 4\widehat{I}_8^{[1,0]} (3,2,1) + 4\widehat{I}_7^{[0,1]} (3,1,2) \\
&- 4\widehat{I}_8^{[0,1]} (3,1,2) + 2\widehat{I}_1^{[0,1]} (1,2,2) + 2\widehat{I}_1 (2,1,1) \\
&- 4\widehat{I}_7^{[1,1]} (3,2,2) + 4\widehat{I}_8^{[1,1]} (3,2,2) + 3\widehat{I}_6 (1,3,1) \\
&- 2\widehat{I}_6^{[0,1]} (3,1,2) - 2\widehat{I}_6^{[0,1]} (2,2,2) - 4\widehat{I}_6 (3,1,1) \\
&+ \widehat{I}_0^{[0,2]} (3,1,2) - \widehat{I}_0^{[0,1]} (3,1,1) - 3\widehat{I}_0 (2,1,1) \\
&+ 4\widehat{I}_6 (2,2,1) + 2\widehat{I}_6^{[1,1]} (3,2,2) - 2\widehat{I}_2 (2,1,1) \\
&+ 8\widehat{I}_7 (2,2,1) - 2\widehat{I}_0^{[0,1]} (2,2,1),
\end{aligned} \tag{B.2}
$$

where

$$
\begin{aligned}
\widehat{I}_\mu^{[\alpha,\beta]} (a,b,c) &= \left[M_1^2\right]^\alpha \\
&\cdot \left[M_2^2\right]^\beta \frac{d^\alpha}{d\left(M_1^2\right)^\alpha} \frac{d^\beta}{d\left(M_2^2\right)^\beta} \left[M_1^2\right]^\alpha \left[M_2^2\right]^\beta \widehat{I}_\mu (a,b,c), \\
\widehat{I}_k (a,b,c) &= i \frac{(-1)^{a+b+c+1}}{16\pi^2 \Gamma(a)\Gamma(b)\Gamma(c)} \left(M_1^2\right)^{1-a-b+k} \\
&\cdot \left(M_2^2\right)^{4-a-c-k} U_0 (a+b+c-5, 1-c-b), \\
\widehat{I}_m (a,b,c) &= i \frac{(-1)^{a+b+c+1}}{16\pi^2 \Gamma(a)\Gamma(b)\Gamma(c)} \left(M_1^2\right)^{-a-b-1+m} \\
&\cdot \left(M_2^2\right)^{7-a-c-m} U_0 (a+b+c-5, 1-c-b), \\
\widehat{I}_6 (a,b,c) &= i \frac{(-1)^{a+b+c+1}}{32\pi^2 \Gamma(a)\Gamma(b)\Gamma(c)} \left(M_1^2\right)^{3-a-b} \left(M_2^2\right)^{3-a-c} \\
&\cdot U_0 (a+b+c-6, 2-c-b), \\
\widehat{I}_n (a,b,c) &= i \frac{(-1)^{a+b+c}}{32\pi^2 \Gamma(a)\Gamma(b)\Gamma(c)} \left(M_1^2\right)^{-4-a-b+n} \\
&\cdot \left(M_2^2\right)^{11-a-c-n} U_0 (a+b+c-7, 2-c-b),
\end{aligned} \tag{B.3}
$$

where $k = 1, 2$, $m = 3, 4, 5$ and $n = 7, 8$. We can define the function $U_0(a,b)$ as

$$
\begin{aligned}
U_0 (a,b) &= \int_0^\infty dy \left(y + M_1^2 + M_2^2\right)^a \\
&\cdot y^b \exp\left[-\frac{B_{-1}}{y} - B_0 - B_1 y\right],
\end{aligned} \tag{B.4}
$$

where

$$
\begin{aligned}
B_{-1} &= \frac{1}{M_2^2 M_1^2} \left(m_s^2 \left(M_1^2 + M_2^2\right)^2 - M_2^2 M_1^2 Q^2\right), \\
B_0 &= \frac{1}{M_1^2 M_2^2} \left(m_s^2 + m_c^2\right)\left(M_1^2 + M_2^2\right), \\
B_1 &= \frac{m_c^2}{M_1^2 M_2^2}.
\end{aligned} \tag{B.5}
$$

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors of the manuscript declare that there are no conflicts of interest regarding publication of this article.

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